2024-DA

2024-DA

Subject	**Data Science & AI-2024-DA-FN**
No.of Sections	**2**
Total Marks	**100**

Duration: **180 mins**
Start Time: **31-10-2024 00:00:00**
End Time: **16-02-2025 23:59:00**

General Aptitude

Section

Total Questions : **10** Max Marks : **15**

1. If '→' denotes increasing order of intensity, then the meaning of the words [sick → infirm → moribund] is analogous to [silly → _____ → daft]. Which one of the given options is appropriate to fill the blank?

A) frown

B) fawn

C) vein

D) vain

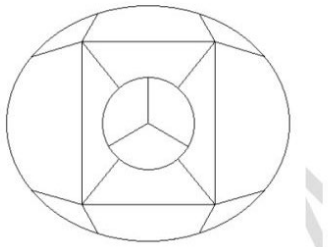
You have 327 wrongly answered questions, Revisit them.

Correct Answer: **D** Your Answer: **B** Marks : 1 / 0.33 (-Ve)

Hide Solution

Vain means marked by fulfility or ineffectualness.
sick → infirm → moribund is analogous to silly → vain → daft.

2. The 15 parts of the given figure are to be painted such that no two adjacent parts with shared boundaries (excluding corners) have the same color. The minimum number of colors required is



A)

4

B)
3

C)
5

D)
6

Correct Answer: **A** Your Answer: **B** Marks : 1 / 0.33 (-Ve)

Hide Solution

3. How many 4-digit positive integers divisible by 3 can be formed using only the digits {1, 3, 4, 6, 7}, such that no digit appears more than once in a number?

A)
24

B)
48

C)
72

D)
12

Correct Answer: **B** Your Answer: **B** Marks : 1 / 0.33 (-Ve)

Hide Solution

{1,3,4,6,7}
1 + 3 + 4 + 6 + 7 = 21
Sum of digits are exactly divisible by '3' then total number is divisible by '3'.
(1 3 4 6 7)
Which 'no' we have to remove and again
It is divisible by '3'.
Like, 3 (or) 6 remove.
We need 4 digit number
"1 4 6 7" $\rightarrow$ 4!
"1 3 7 4" $\rightarrow$ 4!
48

4. The sum of the following infinite series is

$$2+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{8}+\frac{1}{9}+\frac{1}{16}+\frac{1}{27}+\ldots$$

A)
11/3

B)
7/2

C)
13/4

D)
9/2

Correct Answer: **B** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

$$2+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{8}+\frac{1}{9}+\frac{1}{16}+\frac{1}{27}+\ldots\ldots$$

$$2+\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots.\right]+\left[\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots.\right]$$

$$2+\left[\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots.\right]+\left[\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots.\right]$$

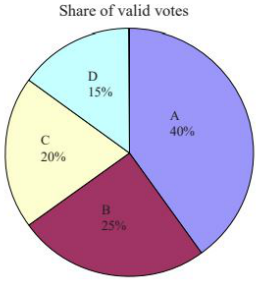
$$2+\left[\frac{1}{2}\left(1+\frac{1}{2}+\ldots.\right)\right]+\left[\frac{1}{3}\left(1+\frac{1}{3}+\frac{1}{3^2}\ldots.\right)\right]$$

$$2+\left[\frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right)\right]+\left[\frac{1}{3}\left(\frac{1}{1-\frac{1}{3}}\right)\right]$$

$$2+\left[\frac{1}{2}(2)\right]+\left[\frac{1}{3}\left(\frac{3}{2}\right)\right]$$

$$2+\left[1+\frac{1}{2}\right]=\frac{7}{2}$$

5. In an election, the share of valid votes received by the four candidates A, B, C, and D is represented by the pie chart shown. The total number of votes cast in the election were 1,15,000, out of which 5,000 were invalid.



Based on the data provided, the total number of valid votes received by the candidates B and C is

A)
45,000

B)
49,500

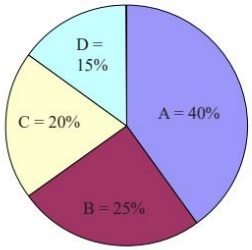
C)
51,750

D)
54,000

Correct Answer: **B** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

Share of valid votes,



Total votes = 1,15,000 + 5000 = 1,10,000
From pie chart 100% = 1,10,000
B & C together (45%) = ?

$$= \frac{110000 \times 45}{100}$$
$$= 49,500$$

6. Thousands of years ago, some people began dairy farming. This coincided with a number of mutations in a particular gene that resulted in these people developing the ability to digest dairy milk. Based on the given passage, which of the following can be inferred?

A)
All human beings can digest dairy milk.

B)
No human being can digest dairy milk.

C)
Digestion of dairy milk is essential for human beings.

D)
In human beings, digestion of dairy milk resulted from a mutated gene.

Correct Answer: **D** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

It is very clear from the paragraph that we can infer a particular gene that resulted in these people developing the ability to digest dairy milk.

7. The probability of a boy or a girl being born is 1/2. For a family having only three children, what is the probability of having two girls and one boy?

A)
3/8

B)
1/8

C)
1/4

D)
1/2

Correct Answer: **A** Not Attempted : ☐ Marks : 2 / 0.66 (-Ve)

☐ Hide Solution

$P(\text{Boy}) = P(\text{Girl}) = \frac{1}{2}$

Let x represent number of girls in a family. Clearly

x is a Binomial random variable with $n = 3,\ P = \frac{1}{2}$

$P(X=2) = {}^{3}C_{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{1} = \frac{3}{8}$

8. Person 1 and Person 2 invest in three mutual funds A, B, and C. The amounts they invest in each of these mutual funds are given in the table.

	Mutual fund A	Mutual fund B	Mutual fund C
Person 1	₹10,000	₹20,000	₹20,000
Person 2	₹20,000	₹15,000	₹15,000

At the end of one year, the total amount that Person 1 gets is ₹500 more than Person 2. The annual rate of return for the mutual funds B and C is 15% each. What is the annual rate of return for the mutual fund A?

A)
7.5%

B)
10%

C)
15%

D)
20%

Correct Answer: **B** Not Attempted : ☐ Marks : 2 / 0.66 (-Ve)

☐ Hide Solution

Person - 1 gets 500 /- more than that person - 2

B & C $\rightarrow$ 15% each

Person - 1:

$(10000 \times x\%) + (20000 + 20000) \times 15\% - [(20000 \times x\%) + (15000 + 15000) \times 15]\% = 500$

$-(10000)x\% + (10000) \times \frac{5}{100} = 500$

$1500 - 1500 = 10000 \times \frac{x}{100}$

$1000 = 100\ x$

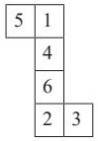
$\Rightarrow x = 10\%$

9.

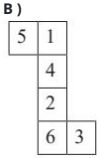
Three different views of a dice are shown in the figure below.

The piece of paper that can be folded to make this dice is

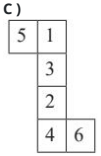
A)



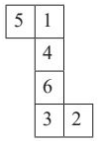
B)



C)



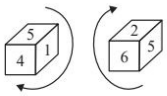
D)



Correct Answer: **A** Not Attempted : □ Marks : 2 / 0.66 (-Ve)

□ Hide Solution

Given Dice is general dice.



we had one common number and different surfaces, then from the common number we have to write the remaining numbers as clockwise/anticlockwise, then corresponding numbers are opposite surfaces.

than, $5 \rightarrow 1, 4$

$5 \rightarrow 6, 2$

$\begin{Bmatrix} 6-1 \\ 4-2 \\ 5-3 \end{Bmatrix}$

10.

Visualize two identical right circular cones such that one is inverted over the other and they share a common circular base. If a cutting plane passes through the vertices of the assembled cones, what shape does the outer boundary of the resulting cross-section make?

A)
A rhombus

B)
A triangle

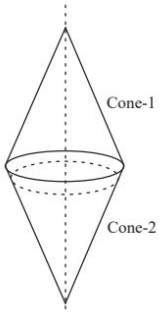
C)
An ellipse

D)
A hexagon

Correct Answer: **A** Not Attempted : □ Marks : 2 / 0.66 (-Ve)

□ Hide Solution

Rhombus



Data Science & AI

Section

Total Questions : **55** Max Marks : **85**

11. Consider the following statements:

(i) The mean and variance of a Poisson random variable are equal.

(ii) For a standard normal random variable, the mean is zero and the variance is one.

Which ONE of the following options is correct?

A)
Both (i) and (ii) are true

B)
(i) is true and (ii) is false

C)
(ii) is true and (i) is false

D)
Both (i) and (ii) are false

Correct Answer: **A** Not Attempted : □ Marks : 1 / 0.33 (-Ve)

□ Hide Solution

Both (i) and (ii) are true statements.

12. Three fair coins are tossed independently. T is the event that two or more tosses result in heads. S is the event that two or more tosses result in tails. What is the probability of the event T∩S ?

A)
0

B)
0.5

C)
0.25

D)
1

Correct Answer: **A** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

Sample space {TTT, TTH, THT, THH, HTT, HTH, HHT, HHH}
T = {THH, HTH, HHT, HHH}
S = {TTT, TTH, THT, HTT}
$T \cap S = \phi$
$P(T \cap S) = 0$

13. Consider the matrix $M = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$.

Which ONE of the following statements is TRUE?

A)
The eigenvalues of M are non-negative and real.

B)
The eigenvalues of M are complex conjugate pairs.

C)
One eigenvalue of M is positive and real, and another eigenvalue of M is zero.

D)
One eigenvalue of M is non-negative and real, and another eigenvalue of M is negative and real.

Correct Answer: **B** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

$$M-\lambda I = \begin{bmatrix} 2-\lambda & -1 \\ 3 & 1-\lambda \end{bmatrix}$$

$|M-\lambda I| = 0$

$(2-\lambda)(1-\lambda) + 3 = 0$

$2-2\lambda-\lambda+\lambda^2+3 = 0$

$\lambda^2-3\lambda+5 = 0$

$$\lambda = \frac{3 \pm \sqrt{9-4(1)(5)}}{2}$$

$$= \frac{3 \pm \sqrt{11}\,i}{2}$$

The eigen value of M are complex conjugate pairs.

14. Consider performing depth-first search (DFS) on an undirected and unweighted graph G starting at vertex s. For any vertex u in G, d[u] is the length of the shortest path from s to u. Let (u, v) be an edge in G such that d[u] < d[v]. If the edge (u, v) is explored first in the direction from u to v during the above DFS, then (u, v)becomes a _____ edge.

A)
tree

B)
cross

C)
back

D)
gray

Correct Answer: **A** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

If the edge <u, v> explored first in the direction from u to v during the DFS, then <u, v> is called tree edge.

15. For any twice differentiable function f: R → R, if at some $x^* \in R$, $f^1(x^*) = 0$ and $f^{11}(x^*) > 0$, then the function f necessarily has a _____ at $x = x^*$.
Note: R denotes the set of real numbers.

A)
local minimum

B)
global minimum

C)
local maximum

D)
global maximum

Correct Answer: **A** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

If $f^1(x^*) = 0$ and $f^{11}(x^*) > 0$ then f(x) has local minimum at $x = x^*$

16. Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) First In First Out	(i) Stacks
(q) Lookup Operation	(ii) Queues
(r) Last In First Out	(iii) Hash Tables

A)
(p) – (ii), (q) – (iii), (r) – (i)

B)
(p) – (ii), (q) – (i), (r) – (iii)

C)
(p) – (i), (q) – (ii), (r) – (iii)

D)
(p) – (i), (q) – (iii), (r) – (ii)

Correct Answer: **A** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

Queue: First in First out
Stack: Last in First out
Hash table: Look up operation

17. Consider the dataset with six data points: $\{(x_1,y_1), (x_2,y_2),\ldots., (x_6,y_6)\}$, where $x_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $x_2 = \begin{bmatrix}0\\1\end{bmatrix}$, $x_3 = \begin{bmatrix}0\\-1\end{bmatrix}$, $x_4 = \begin{bmatrix}-1\\0\end{bmatrix}$, $x_5 = \begin{bmatrix}2\\2\end{bmatrix}$, $x_6 = \begin{bmatrix}-2\\-2\end{bmatrix}$, and the labels are given by $y_1 = y_2 = y_5 = 1$, and $y_3 = y_4 = y_6 = -1$. A hard margin linear support vector machine is trained on the above dataset.
Which ONE of the following sets is a possible set of support vectors?

A)
$\{x_1, x_2, x_5\}$

B)
$\{x_3, x_4, x_5\}$

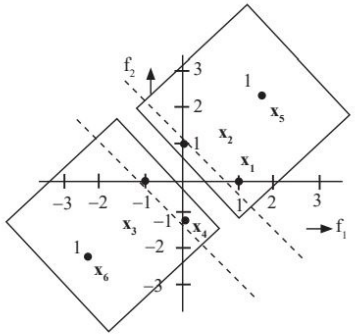
C)
$\{x_4, x_5\}$

D)
$\{x_1, x_2, x_3, x_4\}$

Correct Answer: **D** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

It is given that $x_1=\begin{bmatrix}1\\0\end{bmatrix}$, $x_2=\begin{bmatrix}0\\1\end{bmatrix}$, $x_3=\begin{bmatrix}0\\-1\end{bmatrix}$, $x_4=\begin{bmatrix}-1\\0\end{bmatrix}$, $x_5=\begin{bmatrix}2\\2\end{bmatrix}$, $x_6=\begin{bmatrix}-2\\-2\end{bmatrix}$, and the labels are given by $y_1=y_2=y_5=1$, and $y_3=y_4=y_6=-1$.



Here we have six data points and each data point is having two features namely f_1 and f_2. For example, if we consider the data point x_1 its corresponding $f_1=1$ and $f_2=0$.

Each data point is marked with a circle and its corresponding label value is shown in the above figure.

Now we can separate these data points based on their label. x_1, x_2 and x_5 with label 1can be separated from the data points x_3, x_4 and x_6 with label -1. As hard margin linear support vector machine is trained possible set of support vectors will be $\{x_1, x_2, x_3, x_4\}$

18. Match the items in Column 1 with the items in Column 2 in the following table:

Column 1	Column 2
(p) Principal Component Analysis	(i) Discriminative Model
(q) Naive Bayes Classification	(ii) Dimensionality Reduction
(r) Logistic Regression	(iii) Generative Model

A)
(p) – (iii), (q) – (i), (r) – (ii)

B)
(p) – (ii), (q) – (i), (r) – (iii)

C)
(p) – (ii), (q) – (iii), (r) – (i)

D)
(p) – (iii), (q) – (ii), (r) – (i)

Correct Answer: **C** Not Attempted : □ Marks : 1 / 0.33 (-Ve)

□ Hide Solution

Principal component analysis is dimensionality reduction technique.
Naive Bayes is a generative model.
Logistic regression is discriminative model.

19. Euclidean distance based k-means clustering algorithm was run on a dataset of 100 points with k = 3. If the points $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$ are both part of cluster 3, then which ONE of the following points is necessarily also part of cluster 3?

A)
$\begin{bmatrix}0\\0\end{bmatrix}$

B)
$\begin{bmatrix}0\\2\end{bmatrix}$

C)
$\begin{bmatrix}0\\2\end{bmatrix}$

D)
$\begin{bmatrix}0\\1\end{bmatrix}$

Correct Answer: **D** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

Given that the points (1, 1) & (–1, 1) are clustered together, it implies centroid of this cluster lies at the mid point of these two points.

Mid point = $\frac{1+(-1)}{2}, \frac{(1+1)}{2} = (0, 1)$

So the centroid of the cluster is (0, 1) which itself lies in the same cluster with least possible euclidean distance.
Thus the correct answer is $\begin{bmatrix}0\\1\end{bmatrix}$.

20. Given a dataset with k binary-valued attributes (where k > 2) for a two-class classification task, the number of parameters to be estimated for learning a naive Bayes classifier is

A)
$2^k + 1$

B)
$2k + 1$

C)
$2^{k+1} + 1$

D)
k^2+1

Correct Answer: **B** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

From Bayes' theorem, we can write

$$P(y/x_1, x_2, x_3....x_k) = \frac{P(x_1, x_2, x_3.....x_k/y)\,P(y)}{P(x_1, x_2, x_3.....x_k)}$$

from conditional independence,

$P(x_1, x_2, x_3....x_k/y)\ P(y) = P(x_1/y)\ P(x_2/y)\ P(x_3/y)....$ $P(x_k/y)\ P(y)$

As we need to perform two class classification, in every probability term we have two probabilities such that we need 2k parameters and from P(y) one parameter, so total (2k+1) parameters.

21. Consider performing uniform hashing on an open address hash table with load factor $\alpha = \frac{n}{m} < 1$, where n elements are stored in the table with m slots. The expected number of probes in an unsuccessful search is at most $\frac{1}{1-\alpha}$. Inserting an element in this hash table requires at most _____ probes, on average.

A)

$\ln\left(\frac{1}{1-\alpha}\right)$

B)

$\frac{1}{1-\alpha}$

C)

$1+\frac{\alpha}{2}$

D)

$\frac{1}{1+\alpha}$

Correct Answer: **B** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

Given load factor $\alpha = \frac{n}{m}$

Expected no. of probes in an unsuccessful search is atmost $\frac{1}{1-\alpha}$. The expected no. of probes gives us an average estimate of the number of probes required for insertion. So we need $\frac{1}{1-\alpha}$ probes on average.

22.

For any binary classification dataset, let $S_B \in R^{d\times d}$ and $S_w \in R^{d\times d}$ be the between-class and within-class scatter (covariance) matrices, respectively. The Fisher linear discriminant is defined by $u^* \in R^d$, that maximizes

$$J(u) = \frac{u^T S_B u}{u^T S_w u}$$

If $\lambda=J(u^*)$, S_w is non-singular and $S_B \neq 0$, then (u^*, λ) must satisfy which ONE of the following equations?

Note: R denotes the set of real numbers.

A)

$S_w^{-1} S_B u^* = \lambda u^*$

B)

$S_w u^* = \lambda S_B u^*$

C)

$S_B S_w u^* = \lambda u^*$

D)

$u^{*T} u^* = \bar{\lambda^2}$

Correct Answer: **A** Not Attempted : ☐ Marks : 1 / 0.33 (-Ve)

☐ Hide Solution

In the problem statement, it is given that S_B be the between-class and S_W be the within-class covariance matrices respectively.
The fisher linear discriminant is given by u* and u* maximizes J(u) i.e.,

$$J(u) = \frac{u^T S_B u}{u^T S_w u}$$

Here, u* that solves $\max_u J(u)$ must be an eigen vector of $S_w^{-1} S_B$ corresponding to the largest eigen value of $S_w^{-1} S_B$.
Thus, $S_w^{-1} S_B u^* = \lambda u^*$
Thus option (a) is correct.

23. Let h_1 and h_2 be two admissible heuristics used in A* search.
Which ONE of the following expressions is always an admissible heuristic?

A)

$h_1 + h_2$

B)

$h_1 \times h_2$

C)

$h_1/h_2, (h2 \neq 0)$

D)

$|h_1 - h_2|$

Correct Answer: **D** Not Attempted : ☐ Marks : 1 / 0.33 (-Ve)

☐ Hide Solution

Option (a): If h_1, h_2 are admissible,then h_1+h_2 may or may not be admissible,because:
For a node "n",
Consider h*(n) = 6 (where, h* function value is lesser than or equal to all actual true path costs)
Let $h_1(n) = 3 \rightarrow h_1(n) \leq h^*(n)$, Hence h_1 is admissible.
Let $h_2(n) = 4 \rightarrow h_2(n) \leq h^*(n)$, Hence h_2 is admissible.
But $h_1+h_2 = 7$ is not less than h*(n), Hence $h_1 + h_2$ is not admissible.

Option (b): If h_1, h_2 are admissible,then $h_1 \times h_2$ may or may not be admissible,because:
For a node "n",
Consider h*(n) = 6 (where, h* function value is lesser than or equal to all actual true path costs)
Let $h_1(n) = 3 \rightarrow h_1(n) \leq h^*(n)$, Hence h_1 is admissible.
Let $h_2(n) = 4 \rightarrow h_2(n) \leq h^*(n)$, Hence h_2 is admissible.
But $h_1 \times h_2 = 12$ is not less than h*(n), Hence $h_1 \times h_2$ is not admissible.

Option (c):
If h_1, h_2 are admissible,then h_1/h_2 may or may not be admissible,because:
For a node "n",
Consider h*(n) =0.6 (where, h* function value is lesser than or equal to all actual true path costs)
Let $h_1(n) = 0.4 \rightarrow h1(n) \leq h^*(n)$, Hence h_1 is admissible.
Let $h_2(n) = 0.2 \rightarrow h2(n) \leq h^*(n)$, Hence h_2 is admissible.
But $h_1/h_2 = 2$ is not less than h*(n),Hence h_1/h_2 is not admissible.

Option (d):
If h_1, h_2 are admissible,then $|h_1-h_2|$ is always admissible.

24. Consider five random variables U, V, W, X, and Y whose joint distribution satisfies:
$P(U,V,W,X,Y)=P(U)P(V)P(W|U,V)P(X|W)P(Y|W)$
Which ONE of the following statements is FALSE?

A)
Y is conditionally independent of V given W

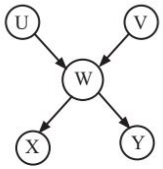
B)
X is conditionally independent of U given W

C)
U and V are conditionally independent given W

D)
Y and X are conditionally independent given W

Correct Answer: **C** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution



Option (c) is False because
U and V are conditionally independent not given W.

25. Consider the following statement:
In adversarial search, α–β pruning can be applied to game trees of any depth where α is the (m) value choice we have formed so far at any choice point along the path for the MAX player and β is the (n) value choice we have formed so far at any choice point along the path for the MIN player.
Which ONE of the following choices of (m) and (n) makes the above statement valid?

A)
(m) = highest, (n) = highest

B)
(m) = lowest, (n) = highest

C)
(m) = highest, (n) = lowest

D)
(m) = lowest, (n) = lowest

Correct Answer: **C** Not Attempted : Marks : 1 / 0.33 (-Ve)

Hide Solution

In adversarial search, α-β pruning can be applied to game trees of any depth where α is the highest value choice we have formed so far at any choice point along the path for the MAX player and β is the lowest value choice we have formed so far at any choice point along the path for the MIN player.

26.

Consider a database that includes the following relations:
Defender(name, rating, side, goals)
Forward(name, rating, assists, goals)
Team(name, club, price)
Which ONE of the following relational algebra expressions checks that every name occurring in Team appears in either Defender or Forward, where ϕ denotes the empty set?

A)
Π_{name} (Team)\(Π_{name} (Defender)∩ Π_{name} (Forward)) = ϕ

B)
Π_{name} (Defender)∩(Π_{name} (Forward))\ Π_{name} (Team)) = ϕ

C)
Π_{name}(Team)\(Π_{name} (Defender)∪ Π_{name} (Forward)) = ϕ

D)
(Π_{name} (Defender)∪ Π_{name} (Forward)) \Π_{name} (Team) = ϕ

Correct Answer: **C** Not Attempted : □ Marks : 1 / 0.33 (-Ve)

□ Hide Solution

(Π_{name} Team)/(Π_{name} Defender ∪ Π_{name} Forward)=ϕ
Checks that every name occuring in Team appears in either defender or forward.

27. Let the minimum, maximum, mean and standard deviation values for the attribute income of data scientists be ₹46000, ₹170000, ₹96000, and ₹21000, respectively.
The z-score normalized income value of ₹106000 is closest to which ONE of the following options?

A)
0.217

B)
0.476

C)
0.623

D)
2.304

Correct Answer: **B** Not Attempted : □ Marks : 1 / 0.33 (-Ve)

□ Hide Solution

The z-score normalized value of x is $\frac{x-\mu}{\sigma}$.
The z-score normalized value of 106000 is $\frac{106000-96000}{21000} = 0.476$

28.

Consider the following tree traversals on a full binary tree:
(i) Preorder (ii) Inorder (iii) Postorder
Which of the following traversal options is/are sufficient to uniquely reconstruct the full binary tree?

A)
(i) and (ii)

B)
(ii) and (iii)

C)
(i) and (iii)

D)
(ii) only

Correct Answer: **A,B,C** Not Attempted : Marks : 1 / 0 (-Ve)

Hide Solution

Since it is given full binary tree so we can construct it by using any of two combination of inorder, preorder, postorder.

29. Let x and y be two propositions. Which of the following statements is a tautology /are tautologies?

A)
$(\neg x \wedge y) \Rightarrow (y \Rightarrow x)$

B)
$(x \wedge \neg y) \Rightarrow (\neg x \Rightarrow y)$

C)
$(\neg x \wedge y) \Rightarrow (\neg x \Rightarrow y)$

D)
$(x \wedge \neg y) \Rightarrow (y \Rightarrow x)$

Correct Answer: **B,C,D** Not Attempted : Marks : 1 / 0 (-Ve)

Hide Solution

When x is False, y is true
Option (a) is False, Hence It is not a tautology.

30. Consider sorting the following array of integers in ascending order using an in-place
Quick sort algorithm that uses the last element as the pivot.

60	70	80	90	100

The minimum number of swaps performed during this Quick sort is _____.

Correct Answer: **0** Not Attempted : Marks : 1 / 0 (-Ve)

Hide Solution

1	2	3	4	5
60	70	80	90	100

Let p = 1
r = 5
x = 100 (pivot)
i = 0
j = 1 Compare A[j] ≤ x, TRUE so i = 1
j = 2 Compare A[j] ≤ x, TRUE so i = 2
j = 3 Compare A[j] ≤ x, TRUE so i = 3
j = 4, Compare A[j] ≤ x, TRUE so i = 4

```
Partition (A, P, r)
{
   x= A[r];
   i = P–1
for (j = P to r–1)
{
  if (A[j] ≤ x)
  {
     i = i+1;
   exchange A[i] with A[j]
   }
}
exchange A[i+1] with A[r];
return i+1;
}
Quick sort (A, p, r)
{ if (p<r)
  { q = partition (A, p, r);
     Quick sort (A, p, q–1);
     Quick sort (A, q+1, r);
   }
}
```

By using this algorithm, every element swapped by itself, if we ignore self swaps then min no swap "0".

31.

Consider the following two tables named Raider and Team in a relational database maintained by a Kabaddi league. The attribute ID in table Team references the primary key of the Raider table, ID.

Raider			
ID	Name	Raids	RaidPoints
1	Arjun	200	250
2	Ankush	190	219
3	Sunil	150	200
4	Reza	150	190
5	Pratham	175	220
6	Gopal	193	215

Team		
City	ID	BidPoints
Jaipur	2	200
Patna	3	195
Hyderabad	5	175
Jaipur	1	250
Patna	4	200
Jaipur	6	200

The SQL query described below is executed on this database:

```
SELECT *
FROM Raider, Team
WHERE Raider.ID=Team.ID AND City="Jaipur"
AND
RaidPoints > 200;
```

The number of rows returned by this query is _____.

Correct Answer: **3** Answer Range: **(3 , 3)** Not Attempted : □ Marks : 1 / 0 (-Ve)

□ Hide Solution

Sol: Output of the query is

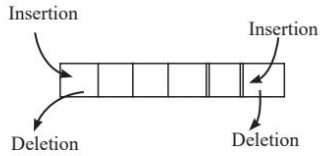
id	name	Raids	Raid points	city	Team id	Bid pionts
2	Ankush	192	219	Jaipur	2	200
1	Arjun	200	250	Jaipur	1	250
6	Gopal	193	215	Jaipur	6	215

32.

The fundamental operations in a double-ended queue D are:
insertFirst(e) – Insert a new element e at the beginning of D.
insertLast(e) – Insert a new element e at the end of D.
removeFirst() – Remove and return the first element of D.
removeLast() – Remove and return the last element of D.
In an empty double-ended queue, the following operations are performed:
insertFirst(10)
insertLast(32)
a ←removeFirst()
insertLast(28)
insertLast(17)
a ←removeFirst()
a ← removeLast()
The value of a is _____.

Correct Answer: **17** Answer Range: **(17 , 17)** Not Attempted : ☐ Marks : 1 / 0 (-Ve)

☐ Hide Solution



insertFirst(10) → D contain 10 at beginning
insertLast(32) → D contain 32 at end
a ←removeFirst()→ 'a' assigned with 10
insertLast(28) → 'D' contain 28 at end
insertLast(17) → D contain 17 at end
a ←removeFirst() →a assigned with 28
a ← removeLast() →a assigned with 17.
hence finally 'a' contain 17. The value of a is 17.

33.

Let f: R → R be the function $f(x) = \frac{1}{1+e^{-x}}$.
The value of the derivative of f at x where f(x) = 0.4 is _____
(rounded off to two decimal places).
Note: R denotes the set of real numbers.

Correct Answer: **0.24** Answer Range: **(0.24 , 0.24)** Not Attempted : ☐ Marks : 1 / 0 (-Ve)

☐ Hide Solution

$$f(x) = \frac{1}{1+e^{-x}} = 0.4$$

$$1 + e^{-x} = 2.5$$

$$e^{-x} = 1.5$$

$$f^1(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$f^1(x) = \frac{1.5}{(1+1.5)^2} = 0.24$$

34. The sample average of 50 data points is 40. The updated sample average after including a new data point taking the value of 142 is _____.

Correct Answer: **42** Answer Range: **(42 , 42)** Not Attempted : ☐ Marks : 1 / 0 (-Ve)

☐ Hide Solution

Given $\frac{x_1 + x_2 + \ldots x_{50}}{50} = 40$

$\Rightarrow x_1 + x_2 + \ldots + x_{50} = 2000$

updated sample average $= \frac{x_1 + x_2 + \ldots x_{50} + 142}{51}$

$= \frac{2000 + 142}{51}$

$= 42.$

35. Consider the 3×3 matrix $M = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \\ 4 & 3 & 6 \end{bmatrix}$.

The determinant of $(M^2 + 12M)$ is _____.

Correct Answer: **0** Not Attempted : ☐ Marks : 1 / 0 (-Ve)

☐ Hide Solution

$|M^2 + 12M| = |M(M+12I)| = |M|\,|M+12I|$

we can observe that $R_1 + R_2 = R_3$ in matrix M.

Hence $|M| = 0$

$\therefore |M|\,|M+12I| = 0.$

36. A fair six-sided die (with faces numbered 1, 2, 3, 4, 5, 6) is repeatedly thrown independently. What is the expected number of times the die is thrown until two consecutive throws of even numbers are seen?

A)
2

B)
4

C)
6

D)
8

Correct Answer: **C** Not Attempted : ☐ Marks : 2 / 0.66 (-Ve)

☐ Hide Solution

Let the expected no. of die throws be x.

1. If the first throw is odd no. then we have wasted one throw. The probability of this event is $\frac{1}{2}$ and the total no. of throws is x+1

2. If the first throw is even no. and second throw is odd then we have wasted two throws. The probability of this event is $\frac{1}{4}$ and total no. of throws required is x+2.

If the first throw is even and the second throw is also even then we are done. The probability of this event is $\frac{1}{4}$ and the total no. of throws required is 2

from the above cases, we have

$$x = \frac{1}{2}(x+1) + \frac{1}{4}(x+2) + \frac{1}{4}(2)$$

$$x = \frac{2x+2+x+2+2}{4}$$

$$4x = 3x + 6$$

$$\Rightarrow x = 6.$$

37. Let $f: R \to R$ be a function. Note: R denotes the set of real numbers.

$$f(x) = \begin{cases} -x, & \text{if } x < -2 \\ ax^2 + bx + c, & \text{if } x \in [-2, 2] \\ x, & \text{if } x > 2 \end{cases}$$

Which ONE of the following choices gives the values of a, b, c that make the function f continuous and differentiable?

A)

$a = \frac{1}{4},\ b = 0,\ c = 1$

B)

$a = \frac{1}{2},\ b = 0,\ c = 0$

C)

$a = 0,\ b = 0,\ c = 0$

D)

$a = 1,\ b = 1,\ c = -4$

Correct Answer: **A** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$

$\lim_{x\to -2^-} -x = \lim_{x\to -2^+} ax^2 + bx + c$

$-(-2) = 4a - 2b + c$

$\Rightarrow 4a - 2b + c = 2$ _____(1)

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\lim_{x\to 2^-} ax^2 + bx + c = \lim_{x\to 2^+} x$

$4a + 2b + c = 2$ _____(2)

adding (1) and (2)

$4a - 2b + c = 2$

$4a + 2b + c = 2$

$8a + 2c = 4$

$\Rightarrow 4a + c = 2$

Subtracting (1) and (2)

$4a - 2b + c = 2$

$4a + 2b + c = 2$

$-4b = 0$

$\Rightarrow b = 0$

$$f'(x) = \begin{cases} -1 & x < -2 \\ 2ax + b & x \in [-2, 2] \\ 1 & x > 2 \end{cases}$$

$f'(2^-) = f'(2^+)$

$2a(2) + b = 1$

$4a + 0 = 1$

$\Rightarrow a = \frac{1}{4}$

$4\left(\frac{1}{4}\right) + c = 2$

$\Rightarrow c = 1$

38. Consider the following Python code:

```
def count(child_dict, i):
    if i not in child_dict.keys():
        return 1
    ans = 1
    for j in child_dict[i]:
        ans += count(child_dict, j)
    return ans
child_dict = dict()
child_dict[0] = [1,2]
child_dict[1] = [3,4,5]
child_dict[2] = [6,7,8]
print(count(child_dict,0))
```

Which ONE of the following is the output of this code?

A)

6

B)

1

C)

8

D)

9

Correct Answer: **D** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

Child-dict is a dictionary where each key represents a parent node and the corresponding value is a list (or) set of child nodes
The function count() recursively count the no. of nodes in the tree starting from the given node 'i'
The base case is when the node 'i' is not in child-dict. keys(). Which means it is a leaf node in this it returns 1 for non leaf node it recursively counts the no. of nodes in its children and address 1 for itself.
Finally it returns the total count of nodes.

39. Consider the function computeS(X) whose pseudocode is given below:

```
computeS(X)
S[1] ← 1
for i ←2 to length (X)
  S[i] ← 1
  if X[i − 1] ≤ X[i]
    S[i] ← S[i] + S[i − 1]
  end if
end for
return S
```

Which ONE of the following values is returned by the function compute S(X) for X = [6, 3, 5, 4, 10]?

A)

[1, 1, 2, 3, 4]

B)

[1, 1, 2, 3, 3]

C)

[1, 1, 2, 1, 2]

D)

[1, 1, 2, 1, 5]

Correct Answer: **C** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

Step1: S[1] ← 1
loop i is running from 2 to 5 where
i = 2, S[2] = 1;
i = 3, S[3] = 2
i = 4, S[4] = 1
i = 5, S[5] = 2
hence finally we get S = [1 1 2 1 2]

40. Let F(n) denote the maximum number of comparisons made while searching for an entry in a sorted array of size n using binary search.
Which ONE of the following options is TRUE?

A)
$F(n) = F(\lfloor n/2 \rfloor) + 1$

B)
$F(n) = F(\lfloor n/2 \rfloor) + F(\lfloor n/2 \rfloor)$

C)
$F(n) = F(\lfloor n/2 \rfloor)$

D)
$F(n) = F(n-1) + 1$

Correct Answer: **A** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

By using divide and conquer strategy, the recurrence equation of binary search is

$$F(n) = F\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + 1$$

41. Consider the following Python function:

```
def fun(D, s1, s2):
if s1 < s2:
   D[s1], D[s2] = D[s2], D[s1]
  fun(D, s1+1, s2-1)
```

What does this Python function fun() do? Select the ONE appropriate option below.

A)
It finds the smallest element in D from index s1 to s2, both inclusive.

B)
It performs a merge sort in-place on this list D between indices s1 and s2, both inclusive.

C)
It reverses the list D between indices s1 and s2, both inclusive.

D)
It swaps the elements in D at indices s1 and s2, and leaves the remaining elements unchanged.

Correct Answer: **C** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

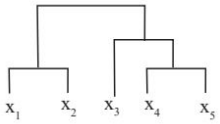
The provided python function fun(D, S_1, S_2) takes a dictionary D and two indices S_1 and S_2.
If the values associated with S_1 in the dictionary "D" is less than the value associated with "S_2," the function swaps the element between S_1 and S_2 effectively reverse the elements in that range.

42. Consider the table below, where the $(i,j)^{th}$ element of the table is the distance between points x_i and x_j. Single linkage clustering is performed on data points, x_1, x_2, x_3, x_4, x_5.

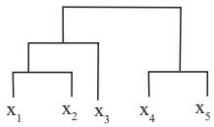
	x_1	x_2	x_3	x_4	x_5
x_1	0	1	4	3	6
x_2	1	0	3	5	3
x_3	4	3	0	2	5
x_4	3	5	2	0	1
x_5	6	3	5	1	0

Which ONE of the following is the correct representation of the clusters produced?

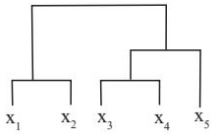
A)



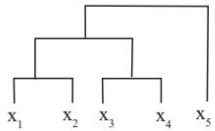
B)



C)



D)



Correct Answer: **A** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

	x_1	x_2	x_3	x_4	x_5
x_1	0	1	4	3	6
x_2	1	0	3	5	3
x_3	4	3	0	2	5
x_4	3	5	2	0	1
x_5	6	3	5	1	0

Single linkage clustering is performed on the above data points x_1, x_2, x_3, x_4, x_5.
The distance between one data point to every other data point is given in the table.
'D_1' is the lowest distance possible between any two data points. From the table it is observed that $D_1 = 1$ is between (x_1, x_2) and (x_4, x_5)
So we need to cluster elements (x_1, x_2) and (x_4, x_5) we then proceed to update the initial proximity values into new proximity values given by D_2.

$D_2((x_1, x_2), x_3) = \min(D_1(x_1, x_3), D_1(x_2, x_3),$
$= \min(4, 3) = 3$

Similarly,

$D_2((x_4, x_5), x_3) = \min(D_1(x_3, x_4), D_1(x_3, x_5),$
$= \min(2, 5) = 2$

If we consider,

$\min([(D_2(x_1, x_2), x_3), (D_2(x_4, x_5), x_3]) = 2$

$\therefore$ We need to cluster x_3 with (x_4, x_5)

Which is given in option (a).

43. Consider the two neural networks (NNs) shown in Figures 1 and 2, with ReLU activation (ReLU(z)) = max{0, z}, $\forall z \in R$). R denotes the set of real numbers. The connections and their corresponding weights are shown in the Figures. The biases at every neuron are set to 0. For what values of p, q, r in Figure 2 are the two NNs equivalent, when x_1, x_2, x_3 are positive?

Figure 1

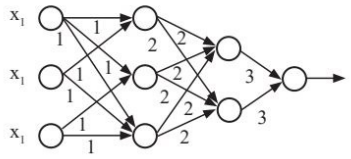
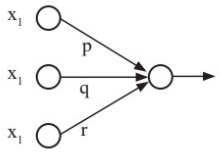


Figure 2



A)

$p = 36, q = 24, r = 24$

B)

p = 24, q = 24, r = 36

C)

p = 18, q = 36, r = 24

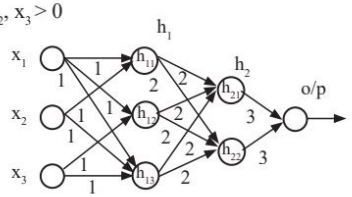
D)

p = 36, q = 36, r = 36

Correct Answer: **A** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

It is given that activation function is ReLu and x_1, x_2, $x_3 > 0$



As x_1, x_2, $x_3 > 0$ and all weights are positive ReLu activation function works as Linear activation function.

From the given network at the first hidden layer,

$h_{11} \Rightarrow x_1 + x_2$

$h_{12} \Rightarrow x_1 + x_3$

$h_{13} \Rightarrow x_1 + x_2 + x_3$

Next at second hidden layer h_2,

$h_{21} \Rightarrow 2(x_1 + x_2) + 2(x_1 + x_3) + 2(x_1 + x_2 + x_3)$

$\therefore h_{21} \Rightarrow 6x_1 + 4x_2 + 4x_3$

As the inputs to h_{21} and h_{22} are same,

$h_{22} \Rightarrow 6x_1 + 4x_2 + 4x_3$

Now at the output node,

output $\Rightarrow 3(6x_1 + 4x_2 + 4x_3) + 3(6x_1 + 4x_2 + 4x_3)$

$\therefore$ output $\Rightarrow 36x_1 + 24x_2 + 24x_3$

$\therefore p = 36, q = 24$ & $r = 24$.

44. Consider a state space where the start state is number 1. The successor function for the state numbered n returns two states numbered n+1 and n+2. Assume that the states in the unexpanded state list are expanded in the ascending order of numbers and the previously expanded states are not added to the unexpanded state list.

Which ONE of the following statements about breadth-first search (BFS) and depth-first search (DFS) is true, when reaching the goal state number 6?

A)

BFS expands more states than DFS.

B)

DFS expands more states than BFS.

C)

Both BFS and DFS expand equal number of states.

D)

Both BFS and DFS do not reach the goal state number 6.

Correct Answer: **C** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

Let the successor function be "S"

S(1) returns states 2&3. But state 2 should be expanded.

S(2) returns states 3&4. But state 3 should be expanded.

S(3) returns states 4&5. But state 4 should be expanded.

S(4) returns states 5&6. But state 5 should be expanded.

S(5) returns states 6&7. But state 6 should be expanded.

BFS: 1 -2-3-4-5-6

DFS:1-2-3-4-5-6

45. Consider the following sorting algorithms:

(i) Bubble sort

(ii) Insertion sort

(iii) Selection sort

Which ONE among the following choices of sorting algorithms sorts the numbers in the array [4, 3, 2, 1, 5] in increasing order after exactly two passes over the array?

A)

(i) only

B)

(iii) only

C)

(i) and (iii) only

D)

(ii) and (iii) only

Correct Answer: **B** Not Attempted : Marks : 2 / 0.66 (-Ve)

Hide Solution

In selection sort algorithm, in every pass the smallest element of unsorted part of array will be placed in its correct place. With this property, in first pass element "1" will be placed in its proper place, in second pass element "2" will be placed in its proper place, and remaining elements are automatically in sorted order.

46. Given the relational schema R = (U, V, W, X, Y, Z) and the set of functional dependencies:
{U→V, U→W, WX→Y, WX→Z, V→X}
Which of the following functional dependencies can be derived from the above set?

A)
$VW \rightarrow YZ$

B)
$WX \rightarrow YZ$

C)
$VW \rightarrow U$

D)
$VW \rightarrow Y$

Correct Answer: **A,B,D** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

option (a): $VW^+ = VWXZY$ determining $VW \rightarrow Y$.
option (b): $VW^+ = VWXZY$ determining $VW \rightarrow YZ$
option (c): $VW^+ = VWXZY$ not determining $VW \rightarrow U$
option (d): $WX^+ = WXYZ$ determining $WX \rightarrow YZ$.

47. Select all choices that are subspaces of R^3.
Note: R denotes the set of real numbers.

A)
$$\left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3 : \; x = \alpha \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \alpha, \beta \in R \right\}$$

B)
$$\left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3 : \; x = \alpha^2 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \beta^2 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \alpha, \beta \in R \right\}$$

C)
$$\left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3 : 5x_1 + 2x_3 = 0, 4x_1 - 2x_2 + 3x_3 = 0 \right\}$$

D)
$$\left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3 : 5x_1 + 2x_3 = 4 = 0 \right\}$$

Correct Answer: **A,C** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

(a) and (c) satisfy all the properties of subspace. For option (b) as α^2 and β^2 are positive $-x$ cannot be part of the given set. Hence not a subspace. For option (d), zero vector is not present in the given set hence not a subspace

48. Which of the following statements is/are TRUE?
Note: R denotes the set of real numbers.

A)
There exist $M \in R^{3\times3}$, $p \in R^3$, and $q \in R^3$ such that $Mx = p$ has a unique solution and $Mx = q$ has infinite solutions.

B)
There exist $M \in R^{3\times3}$, $p \in R^3$, and $q \in R^3$ such that $Mx = p$ has no solutions and $Mx = q$ has infinite solutions.

C)
There exist $M \in R^{2\times3}$, $p \in R^3$, and $q \in R^2$ such that $Mx=p$ has a unique solution and $Mx=q$ has infinite solutions.

D)
There exist $M \in R^{3\times2}$, $p \in R^3$, and $q \in R^3$ such that $Mx=p$ has a unique solution and $Mx=q$ has no solutions.

Correct Answer: **B,D** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

(a) If MX= P has a unique solutions then Rank (M) = Rank (M/P) = 3. If Rank (M) = 3 then MX= Q cannot have infinite solutions. Option (a) is false.

(b) If MX = P has no solution then this can be one of the possibility Rank (M) = 2 Rank (M/P)=3. Now with Rank (M) = 2 and Rank (M/Q)=2. MX=Q will have infinite solutions. Option (b) is true.

(c) MX=P can never have unique solution as Rank (M) $\neq$ Number of variables. Option C is false.

(d) If MX = P has unique solution then Rank (M) = Rank (M/P) = 2. Now with Rank (M) = 2 and Rank (M/Q) = 3, MX = Q has no solution. Option (d) is true.

49. Let R be the set of real numbers, U be a subspace of R^3 and $M \in R^{3\times3}$ be the matrix corresponding to the projection on to the subspace U.
Which of the following statements is/are TRUE?

A)

If U is a 1-dimensional subspace of R^3, then the null space of M is a 1-dimensional subspace.

B)

If U is a 2-dimensional subspace of R^3, then the null space of M is a 1-dimensional subspace.

C)

$M^2 = M$

D)

$M^3 = M$

Correct Answer: **B,C,D** Not Attempted : ☐ Marks : 2 / 0 (-Ve)

☐ Hide Solution

The matrix M corresponding to the projection on to the subspace U is given by $M = A(A^TA)^{-1}A^T$
Where the columns of A are basis for the subspace U.

$M^2 = M.M = A(A^TA)^{-1}A^T.A(A^TA)^{-1}A^T$

$= A(A^TA)^{-1}A^T$

$\Rightarrow M^2 = M$ option (c) is true

$M^3 = M^2.M = M.M = M^2 = M$

$\Rightarrow M^3 = M$ option (d) is true

According to rank plus nullity theorem, we have $\dim(R(u)) + \dim(N(u)) = 3$
where R(u) indicates range space of u and N(u) indicates null space of u.
option (a) does not satisfy the above condition where as
option (b) satisfies the above condition.

50. Consider the function f: R→R where R is the set of all real numbers.

$$f(x) = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{3x^2}{2} + 1$$

Which of the following statements is/are TRUE?

A)

x = 0 is a local maximum of f

B)

x = 3 is a local minimum of f

C)

x = −1 is a local maximum of f

D)

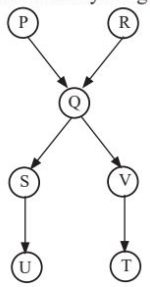
x = 0 is a local minimum of f

Correct Answer: **A,B** Not Attempted : ☐ Marks : 2 / 0 (-Ve)

☐ Hide Solution

$f^1(x) = x^3 - 2x^2 - 3x$
$x^3 - 2x^2 - 3x = 0$
$x(x^2 - 2x - 3) = 0$
$x^2 - 3x + x - 3 = 0$
$x(x-3)+1\ (x-3)= 0$
$(x-3)(x+1) = 0$
stationary points = $-1, 0, 3$
$f^{11}(x) = 3x^2 - 4x - 3$
$f^{11}(-1) = 3 + 4 - 3 = 4 > 0$
At $x = -1$ $f(x)$ has local min
$f^{11}(0) = -3 < 0$
At $x = 0$ $f(x)$ has local max
$f^{11}(3) = 27-12-3 = 12 > 0$
At $x = 3$ $f(x)$ has local min.

51. Consider the directed acyclic graph (DAG) below:



Which of the following is/are valid vertex orderings that can be obtained from a topological sort of the DAG?

A)
P Q R S T U V

B)
P R Q V S U T

C)
P Q R S V U T

D)
P R Q S V T U

Correct Answer: **B,D** Not Attempted : □ Marks : 2 / 0 (-Ve)

□ Hide Solution

i) Topological sort always starting from the vertex whose indegree is "0"
ii) By using DFS algorithm, by arranging finishing time in the decreasing order we get topological ordering (OR) start from the vertex whose indegree "0" and remove that vertex together with its edge and include this vertex into topological ordering continue this process for all vertexes.
P R Q V S U T, P R Q S V T U are valid.
Option (a) invalid as we cannot get "Q" after P.
Option (c) invalid as we cannot get "Q" after P.

52. Let H, I, L, and N represent height, number of internal nodes, number of leaf nodes, and the total number of nodes respectively in a rooted binary tree.

Which of the following statements is/are always TRUE?

A)

$L \leq I + 1$

B)

$H + 1 \leq N \leq 2^{H+1} - 1$

C)

$H \leq I \leq 2^{H} - 1$

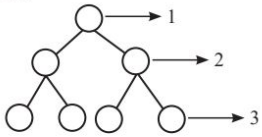
D)

$H \leq L \leq 2^{H-1}$

Correct Answer: **A,B,C** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

Level No.



(here I = 3, L = 4, H = 2, N = 7)

Maximum no. of nodes possible if it is full binary tree. No. of nodes in full binary tree of height "h" is

$= 2^0 + 2^1 + 2^2 + ... + 2^h$

$n = 2^{h+1} - 1$

Minimum no. of nodes possible in skewed binary tree so $n = h+1$

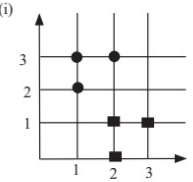
So $h+1 \leq n \leq 2^{h+1} - 1$

and from the above diagram we can conclude $L \leq I + 1$, $H \leq I \leq 2^{H} - 1$ but $H \leq L \leq 2^{H-1}$ is not True.

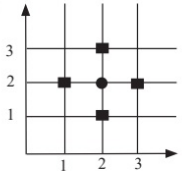
53.

Consider the following figures representing datasets consisting of two-dimensional features with two classes denoted by circles and squares.

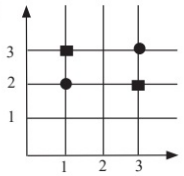
(i)



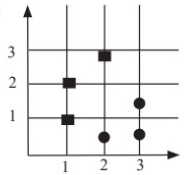
(ii)



(iii)



(iv)



Which of the following is/are TRUE?

A)

(i) is linearly separable

B)

(ii) is linearly separable

C)

(iii) is linearly separable

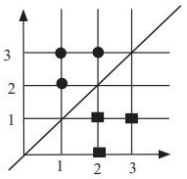
D)

(iv) is linearly separable

Correct Answer: **A,D** Not Attempted : □ Marks : 2 / 0 (-Ve)

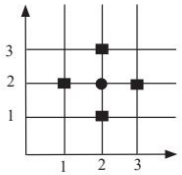
□ Hide Solution

(i)



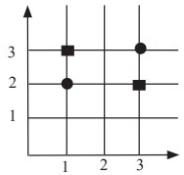
⇒Linearly separable

ii)



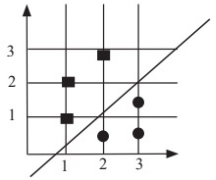
⇒Linearly non-separable

iii)



⇒Same type of data points on the diagonal, so linearly non separable.

iv)



⇒Linearly separable

54. Let game(ball, rugby) be true if the ball is used in rugby and false otherwise.
Let shape(ball, round) be true if the ball is round and false otherwise.
Consider the following logical sentences:
s1: ∀ ball ¬ game(ball, rugby) ⇒shape(ball, round)
s2: ∀ball ¬ shape(ball, round) ⇒game(ball, rugby)
s3: ∀ball game(ball, rugby) ⇒ ¬ shape(ball, round)
s4: ∀ball shape(ball, round) ⇒ ¬ game(ball, rugby)
Which of the following choices is/are logical representations of the assertion,
"All balls are round except balls used in rugby"?

A)

$s1 \wedge s3$

B)

$s1 \wedge s2$

C)

$s2 \wedge s3$

D)

$s3 \wedge s4$

Correct Answer: **A,C** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

s1: ∀ ball ¬ game(ball, rugby) ⇒shape (ball, round)
Any ball that is not used in rugby, that ball must be round in shape.
s2: ∀ball ¬ shape(ball, round) ⇒game (ball, rugby)
Any ball that is not round in shape, that ball must be used in Rugby.
s3: ∀ball game(ball, rugby) ⇒ ¬ shape (ball, round)
All the balls used in rugby must not be round in shape.
s4: ∀ball shape(ball, round) ⇒ ¬ game (ball, rugby)
All the balls that are round in shape, must not be used in rugby.
s1 ∧ s2, s2∧s3 are equivalent to "All balls are round except the balls used in rugby

55. An OTT company is maintaining a large disk-based relational database of different movies with the following schema:

Movie(ID, CustomerRating)
Genre(ID, Name)
Movie_Genre(MovieID, GenreID)

Consider the following SQL query on the relation database above:

```
SELECT *
FROM Movie, Genre, Movie_Genre
WHERE
  Movie.CustomerRating > 3.4 AND
  Genre.Name = "Comedy" AND
  Movie  Genre.MovieID = Movie.ID AND
  Movie_Genre.GenreID = Genre.ID;
```

This SQL query can be sped up using which of the following indexing options?

A)

B^+ tree on all the attributes.

B)

Hash index on Genre.Name and B^+ tree on the remaining attributes.

C)

Hash index on Movie.CustomerRating and B^+ tree on the remaining attributes.

D)

Hash index on all the attributes.

Correct Answer: **A,B** Not Attempted : □ Marks : 2 / 0 (-Ve)

□ Hide Solution

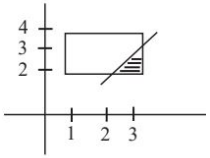
Hash index is used for direct search and B^+-Tree index is used for both direct and ordered access.

56. Let X be a random variable uniformly distributed in the interval [1, 3] and Y be a random variable uniformly distributed in the interval [2, 4]. If X and Y are independent of each other, the probability $P(X \geq Y)$ is _____ (rounded off to three decimal places)

Correct Answer: **0.125** Answer Range: **(0.125 , 0.125)** Not Attempted : □ Marks : 2 / 0 (-Ve)

□ Hide Solution

Let us consider rectangular grid where $1 \leq x \leq 3$ and $2 \leq y \leq 4$. $Y = x$ is a straight line and $X \geq Y$ is the region which is below the line as shown below.



$$P(X \geq Y) = \frac{\text{Area of shaded Region}}{\text{Area of total Region}}$$

$$= \frac{\frac{1}{2}(1)(1)}{2(2)} = \frac{1}{8}$$

$$= 0.125$$

57. Let X be a random variable exponentially distributed with parameter $\lambda > 0$. The probability density function of X is given by:

$$f_x(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

If $5E(X) = Var(X)$, where $E(X)$ and $Var(X)$ indicate the expectation and variance of X, respectively, the value of λ is _____ (rounded off to one decimal place).

Correct Answer: **0.2** Answer Range: **(0.2 , 0.2)** Not Attempted : □ Marks : 2 / 0 (-Ve)

□ Hide Solution

$$5E(X) = Var(X)$$
$$5\left(\frac{1}{\lambda}\right) = \frac{1}{\lambda^2}$$
$$5 = \frac{1}{\lambda}$$
$$\Rightarrow \lambda = \frac{1}{5} = 0.2$$

58. Consider two events T and S. Let $\overline{T}$ denote the complement of the event T. The probability associated with different events are given as follows:
$P(\overline{T}) = 0.6,\ \ P(S|T) = 0.3, P(S|\overline{T}) = 0.6$
Then, P(T|S) is _____ (rounded off to two decimal places).

Correct Answer: **0.25** Answer Range: **(0.25 , 0.25)** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

$$P(S|T) = 0.3$$
$$\frac{P(S \cap T)}{P(T)} = 0.3$$
$$\frac{P(S \cap T)}{0.4} = 0.3$$
$$P(S \cap T) = 0.12$$
$$P(S|\overline{T}) = 0.6$$
$$\frac{P(S \cap \overline{T})}{P(\overline{T})} = 0.6$$
$$\frac{P(S) - P(S \cap T)}{0.6} = 0.6$$
$$P(S) - 0.12 = 0.36$$
$$P(S) = 0.48$$
$$P(T \mid S) = \frac{P(S \cap T)}{P(S)}$$
$$= \frac{0.12}{0.48}$$
$$= 0.25$$

59. Consider a joint probability density function of two random variables X and Y

$$f_{XY}(x,y) = \begin{cases} 2xy, & 0 < x < 2, \quad 0 < y < x \\ 0, & \text{otherwise} \end{cases}$$

Then, E[Y|X = 1.5] is _____.

Correct Answer: **1** Answer Range: **(1 , 1)** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

$$f_{y/x}(y/x) = \frac{f(x,y)}{f_x(x)}$$

$$f_x(x) = \int_{y=0}^{x} f(x,y)\,dy$$

$$= \int_{y=0}^{x} 2xy\,dy$$

$$= 2x\left(\frac{y^2}{2}\right)_0^x$$

$$f_x(x) = x^3$$

$$f_{y/x}(y/x) = \frac{2xy}{x^3} = \frac{2y}{x^2}$$

$$E(Y/X = x) = \int_{y=0}^{x} y\, f_{y/x}(y/x)\,dy$$

$$\int_{y=0}^{x} y\left(\frac{2y}{x^2}\right)dy$$

$$= \frac{2}{x^2}\left(\frac{y^3}{3}\right)_0^x$$

$$\frac{2}{x^2}\left(\frac{x^3}{3}\right)$$

$$E[Y/X=x] = \frac{2x}{3}$$

$$E[Y/X = 1.5] = \frac{2(1.5)}{3}$$

$$= 1.$$

60. Evaluate the following limit:

$$\lim_{x\to 0} \frac{\ln((x^2+1)\cos x)}{x^2} = \_\_\_\_\_.$$

Correct Answer: **0.5** Answer Range: **(0.5 , 0.5)** Not Attempted : ☐ Marks : 2 / 0 (-Ve)

☐ Hide Solution

$$\lim_{x\to 0} \frac{\ln((x^2+1)\cos x)}{x^2} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\ln((x^2+1)\cos x)}{x^2} = \lim_{x\to 0} \frac{\ln(x^2+1) + \ln\cos x}{x^2}$$

$$\lim_{x\to 0} \frac{\frac{2x}{x^2+1} - \frac{\sin x}{\cos x}}{2x}$$

$$\lim_{x\to 0} \frac{1}{x^2+1} - \frac{1}{2}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\lim_{x\to 0}\frac{1}{\cos x}$$

$$= 1 - \frac{1}{2} = 0.5$$

61. Let $u = \begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}$, and let $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$, be the singular values of the matrix $M = uu^T$(where u^T is the transpose of u). The value of $\sum_{i=1}^{5} \sigma_i$ is _____.

Correct Answer: **55** Answer Range: **(55 , 55)** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

$\text{Rank}(m) = \text{Rank}(uu^T) = \text{Rank}(u) = 1$

$m = uu^T$

$m^T = (uu^T)^T = (u^T)^T u^T) = uu^T = m$

$\Rightarrow$ m is symmetric matrix.

Since m is symmetric matrix with rank 1 it has only one non-zero eigen value and remaining four eigen values are 0.

Let λ be the non-zero eigen value of m. We know that sum of eigen values of m is equal trace of m

$\lambda + 0 + 0 + 0 + 0 = \text{trace}(m)$

$\lambda = \text{trace}(uu^T)$

$\lambda = u^T u$

$$= (1\ 2\ 3\ 4\ 5)\begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}$$

$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$= 55 > 0$

Since m is symmetric with rank 1 it has only one non-zero singular value. For symmetric matrices, the eigenvalues and singular values are closely related. A non-negative eigen value $\lambda \geq 0$ is also a singular value $\sigma = \lambda$

$\Rightarrow \sigma_1 = 55, \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0$

$\sum_{i=1}^{5} \sigma_i = 55 + 0 + 0 + 0 + 0 = 55.$

62.

Details of ten international cricket games between two teams "Green" and "Blue" are given in Table C. This table consists of matches played on different pitches, across formats along with their winners. The attribute Pitch can take one of two values: spin-friendly (represented as S) or pace-friendly (represented as F). The attribute Format can take one of two values: one-day match (represented as O) or test match (represented as T).

A cricket organization would like to use the information given in Table C to develop a decision-tree model to predict outcomes of future games between these two teams.

To develop such a model, the computed Information Gain(C, Pitch) with respect to the Target is _____ (rounded off to two decimal places).

Table C

Match Number	Pitch	Format	Winner (Target)
1	S	T	Green
2	S	T	Blue
3	F	O	Blue
4	S	O	Blue
5	F	T	Green
6	F	O	Blue
7	S	O	Green
8	F	T	Blue
9	F	O	Blue
10	S	O	Green

Correct Answer: **0.12** Answer Range: **(0.11 , 0.13)** Not Attempted : □ Marks : 2 / 0 (-Ve)

□ Hide Solution

As a first step calculate the entropy for the target.
It is given by entropy (c).
In the target column winner count,
Green-4, Blue-6

$\therefore$ Entropy (c) = $\frac{-4}{10}\log 2^{\frac{4}{10}} - \frac{6}{10}\log 2^{\frac{6}{10}} = 0.971$

Next we need to calculate the information gain with respect to the attribute 'pitch'.
For slow pitch (P_s), Green-3, Blue-2
For fast pitch (P_F), Green-1, Blue-4

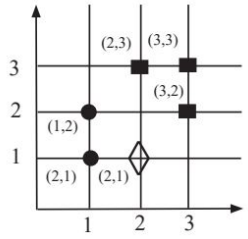
$\therefore$ Information gain (C, pitch) = Entropy (C) $-\frac{5}{10}$ Entropy (S) $-\frac{5}{10}$ Entropy (F)

$$\text{calculate, Entropy (S)} = -\frac{2}{5}\log 2^{(2/5)} - \frac{3}{5}\log 2^{(3/5)}$$
$$= 0.52 + 0.44 = 0.98$$

$$\text{Entropy(F)} = -\frac{1}{5}\log 2^{(1/5)} - \frac{4}{5}\log 2^{(4/5)}$$
$$= 0.46 + 0.26 = 0.72$$

$\therefore$ Information gain (C, pitch)
$= 0.971 - 0.5 \times 0.98 - 0.5 \times 0.72$
$= 0.971 - 0.49 - 0.36$
$= 0.12.$

63. Given the two-dimensional dataset consisting of 5 data points from two classes (circles and squares) and assume that the Euclidean distance is used to measure the distance between two points. The minimum odd value of k in k-nearest neighbor algorithm for which the diamond (◊) shaped data point is assigned the label square is _____.

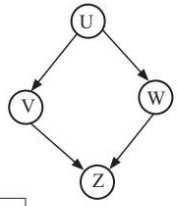


Correct Answer: **5** Answer Range: **(5 , 5)** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

Consider different possible odd values for 'k' such as, K=1⇒ Nearest Point = circle.
Diamond is classified as circle.
K=3⇒Nearest points = 2 circles, 1 square
∵Majority vote is towards circle,
Diamond is classified as circle.
K = 5 ⇒ Nearest points = 2 circles, 3 squares
∴Majority vote is towards square
Diamond is classified as square.
⇒correct answer is 5.

64. Given the following Bayesian Network consisting of four Bernoulli random variables and the associated conditional probability tables:



	P(·)
U = 0	0.5
U = 1	0.5

	P(V=0|·)	P(V=1|·)
U = 0	0.5	0.5
U = 1	0.5	0.5

	P(W=0|·)	P(W=1|·)
U = 0	1	0
U = 1	0	1

		P(Z=0|·)	P(Z=1|·)
V = 0	W = 0	0.5	0.5
V = 0	W=1	1	0
V = 1	W = 0	1	0
V = 1	W = 1	0.5	0.5

The value of P(U=1, V=1, W=1, Z=1) = _____ (rounded off to three decimal places).

Correct Answer: **0.125** Answer Range: **(0.125 , 0.125)** Not Attempted : Marks : 2 / 0 (-Ve)

Hide Solution

P(U=1,V=1,W=1,Z=1)
= P(U=1,V=1|U=1,W=1|U=1,Z=1|V,W=1)
= 0.5 x 0.5 x 1x 0.5
= 0.125

65.

Two fair coins are tossed independently. X is a random variable that takes a value of 1 if both tosses are heads and 0 otherwise. Y is a random variable that takes a value of 1 if at least one of the tosses is heads and 0 otherwise.
The value of the covariance of X and Y is _____ (rounded off to three decimal places)

Correct Answer: **0.0625** Answer Range: **(0.062 , 0.063)** Not Attempted : □ Marks : 2 / 0 (-Ve)

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HH $\Rightarrow$X=1 $\Rightarrow$P(X=1) = $\frac{1}{4}$

HT, TH, TT $\Rightarrow$ X = 0 $\Rightarrow$P(X=0) = $\frac{3}{4}$

X	0	1
P(X)	$\frac{3}{4}$	$\frac{1}{4}$

HH, HT, TH $\Rightarrow$Y =1 $\Rightarrow$P(Y=1) = $\frac{3}{4}$

TT $\Rightarrow$ Y = 0 $\Rightarrow$ P (Y=0) = $\frac{1}{4}$

Y	0	1
P(Y)	$\frac{1}{4}$	$\frac{3}{4}$

$E(X)= \ 0\left(\frac{3}{4}\right)+1\left(\frac{1}{4}\right)=\frac{1}{4}$

$E(Y)=0\left(\frac{1}{4}\right)+1\left(\frac{3}{4}\right)=\frac{3}{4}$

XY = 1 when X = 1 and Y = 1

XY = 0, when X = 0, Y = 1 or X = 1, Y = 0 or X = 0, Y = 0

XY	0	1
P(XY)	$\frac{3}{4}$	$\frac{1}{4}$

$E(XY)) = 0\left(\frac{3}{4}\right)+1\left(\frac{1}{4}\right)=\frac{1}{4}$

Cov(X, Y) = E(XY) –E(X) E(Y)

$=\frac{1}{4}-\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)$

$=\frac{1}{16}$

= 0.0625.