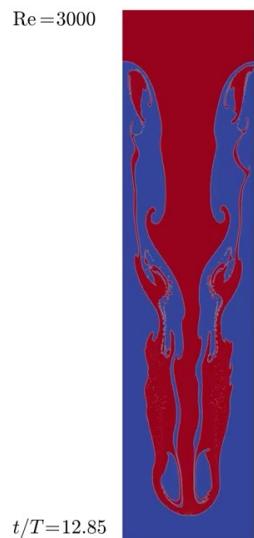


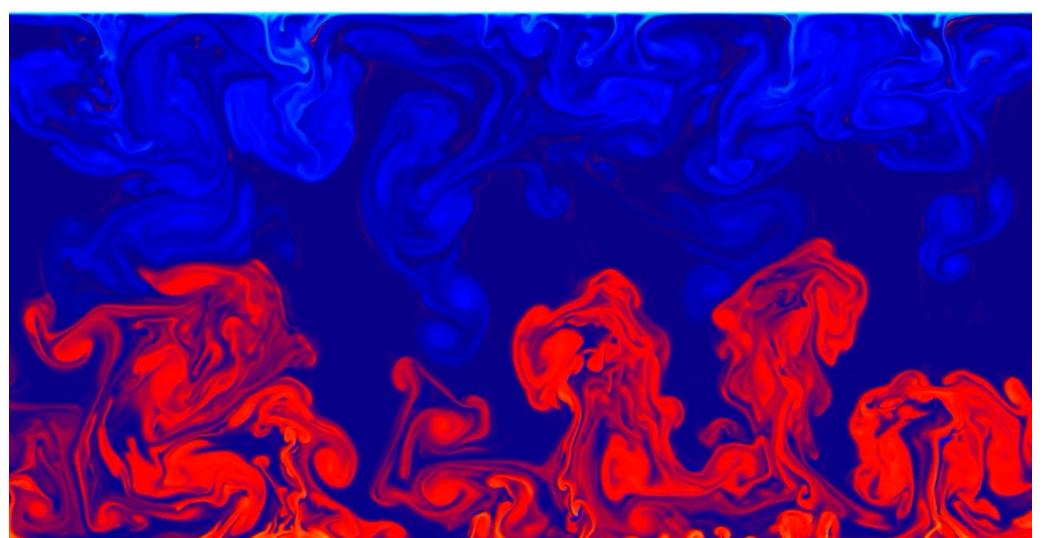
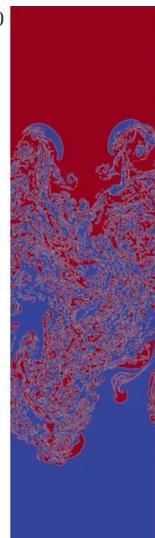
Re=3000



Re = 10000



Re = 30000



De Rosis & Enan
Phys Fluids 2021

Fluid dynamics and beyond: opportunities with the lattice Boltzmann method

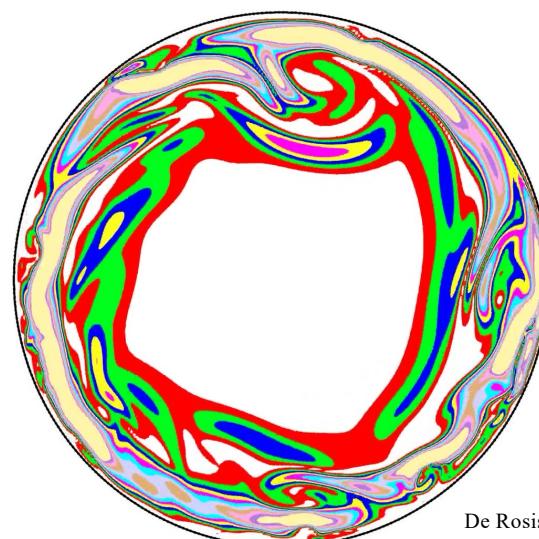
Alessandro De Rosis

Lecturer in Virtual Engineering

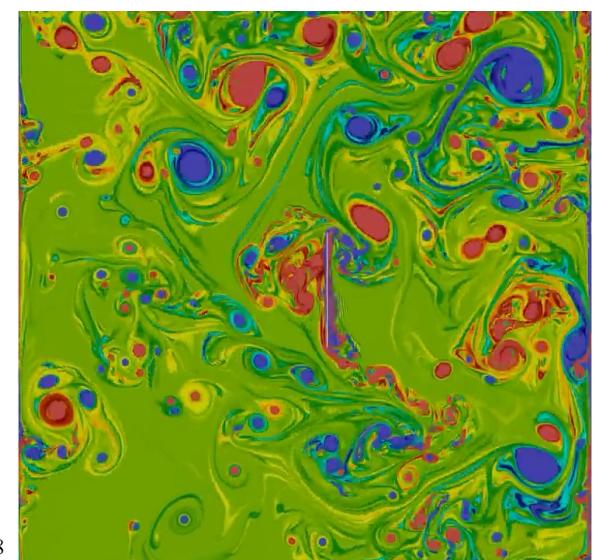
Department of Fluids and Environment

The University of Manchester

alessandro.derosis@manchester.ac.uk

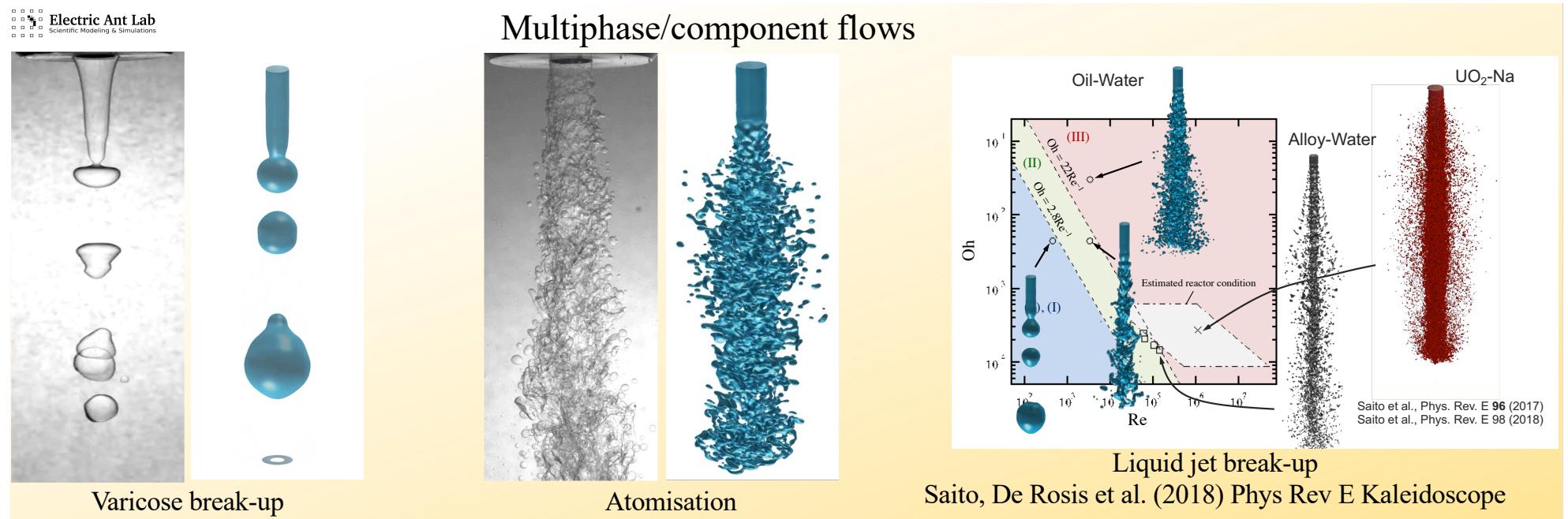
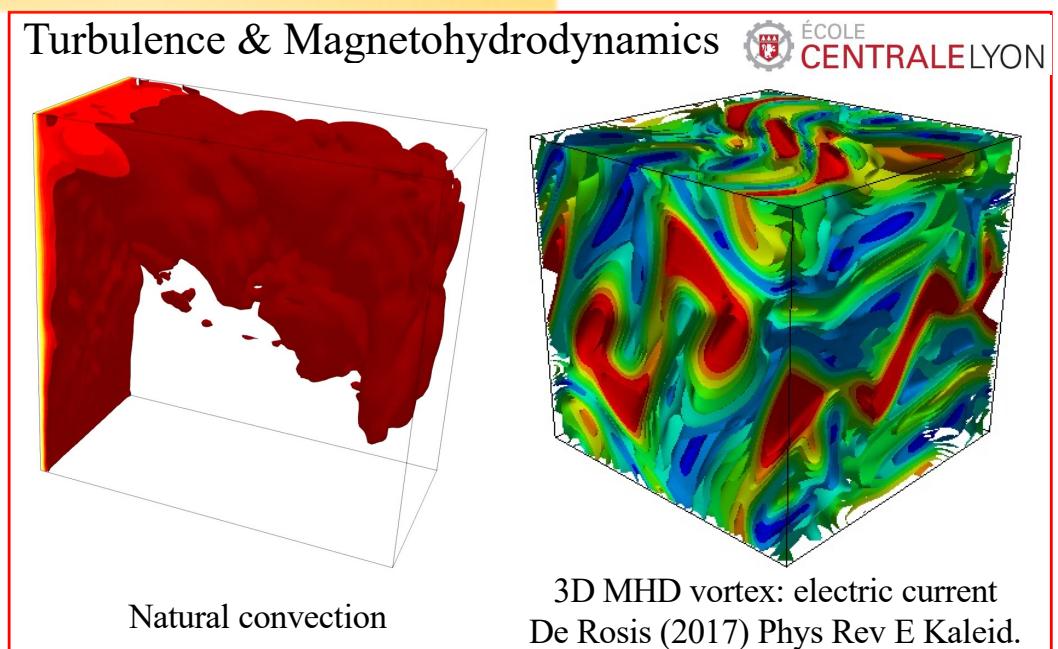
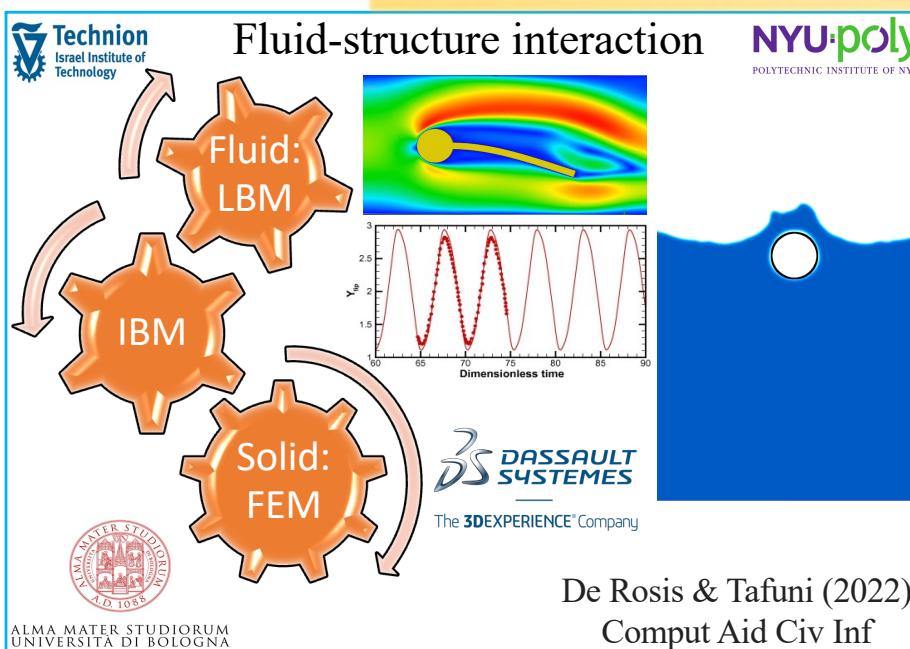


De Rosis
Europhys Lett 2018



Multiphysics modelling by LBM

THEORY OF LBM & NUMERICAL SIMULATIONS



Magnetohydrodynamics modelling

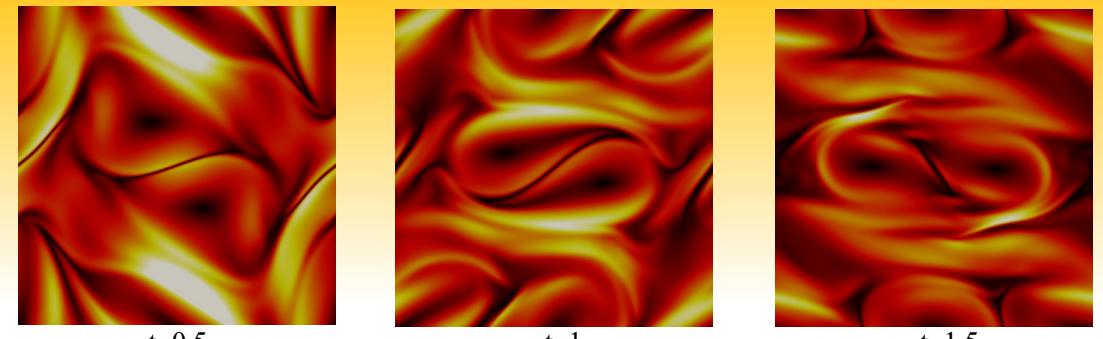
Macroscopic

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla p}{\rho} + \nu \Delta \mathbf{u} + \frac{\mathbf{j} \times \mathbf{B}}{\rho}$$

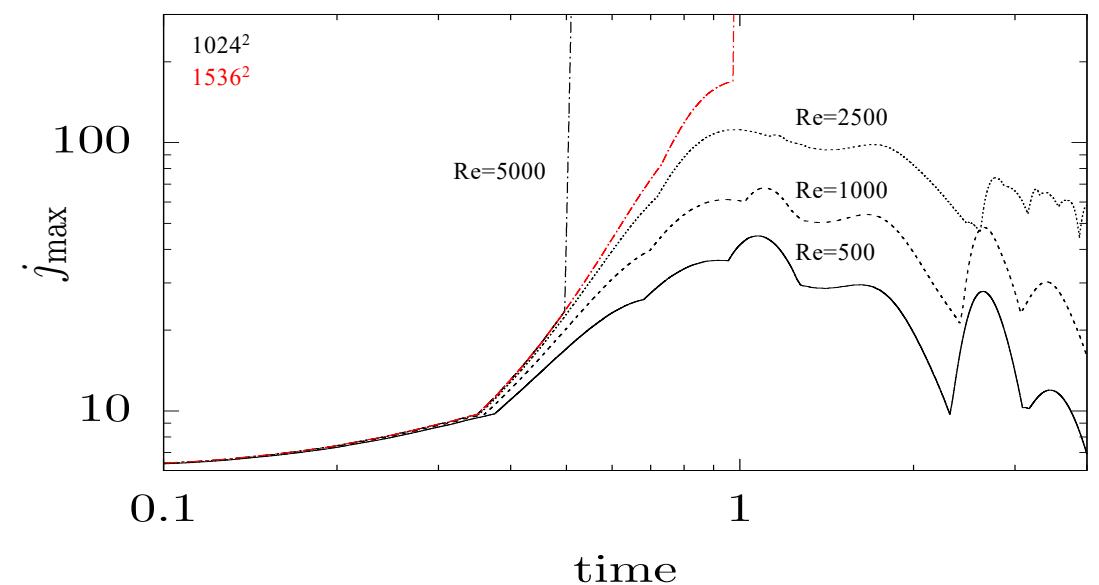
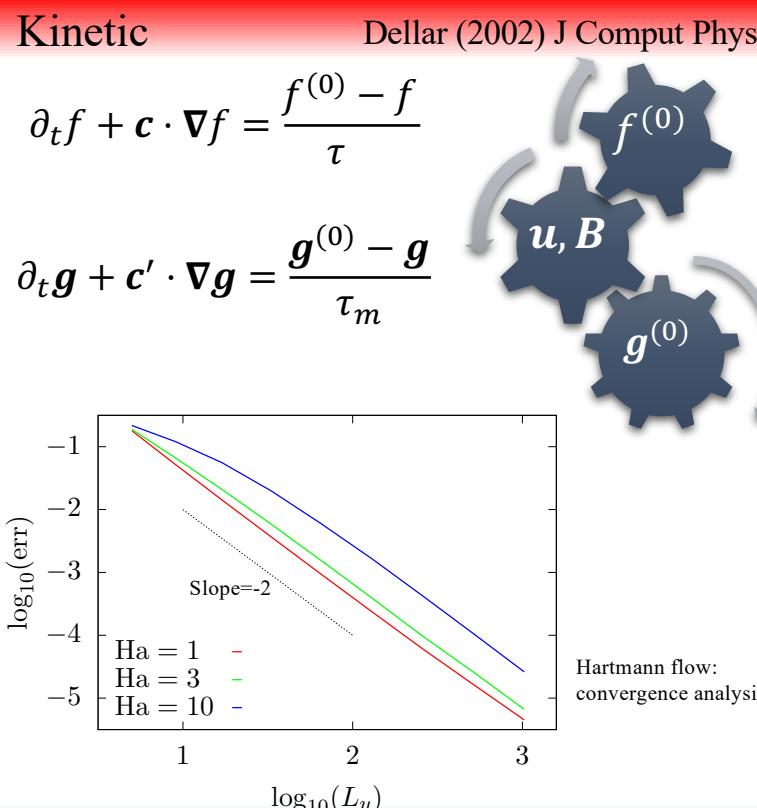
$$\partial_t \mathbf{B} + (\mathbf{u} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{u} + \lambda \Delta \mathbf{B}$$

$$\nabla \cdot \mathbf{u} = 0, \nabla \cdot \mathbf{B} = 0$$

A tough problem: 2D Orszag-Tang vortex

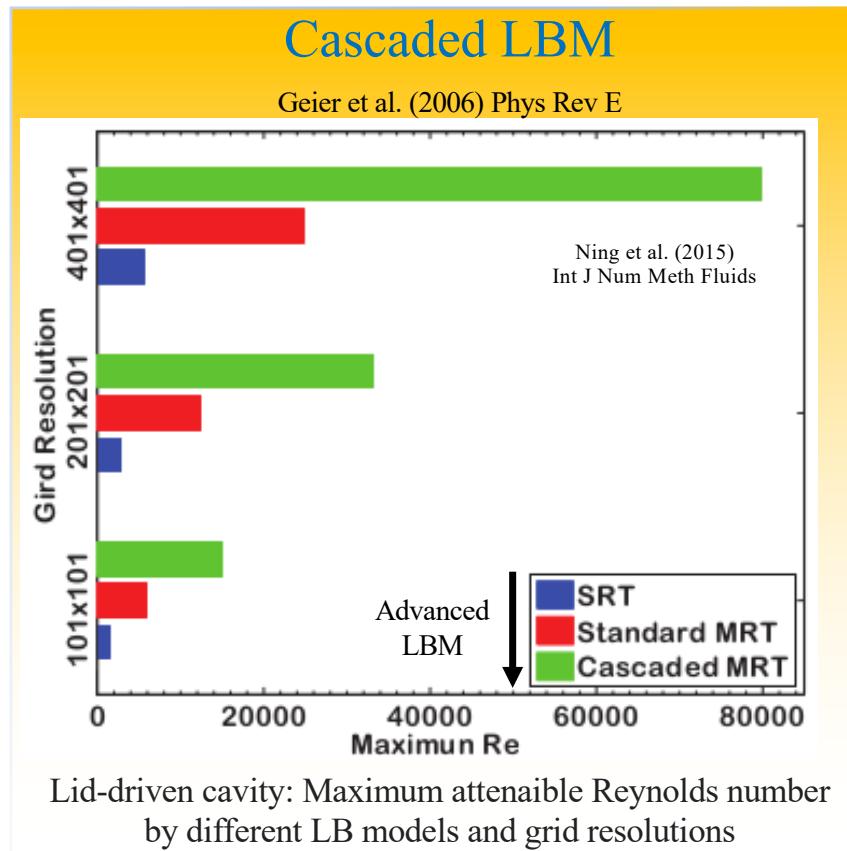


Evolution of the magnetic field at Re=2500

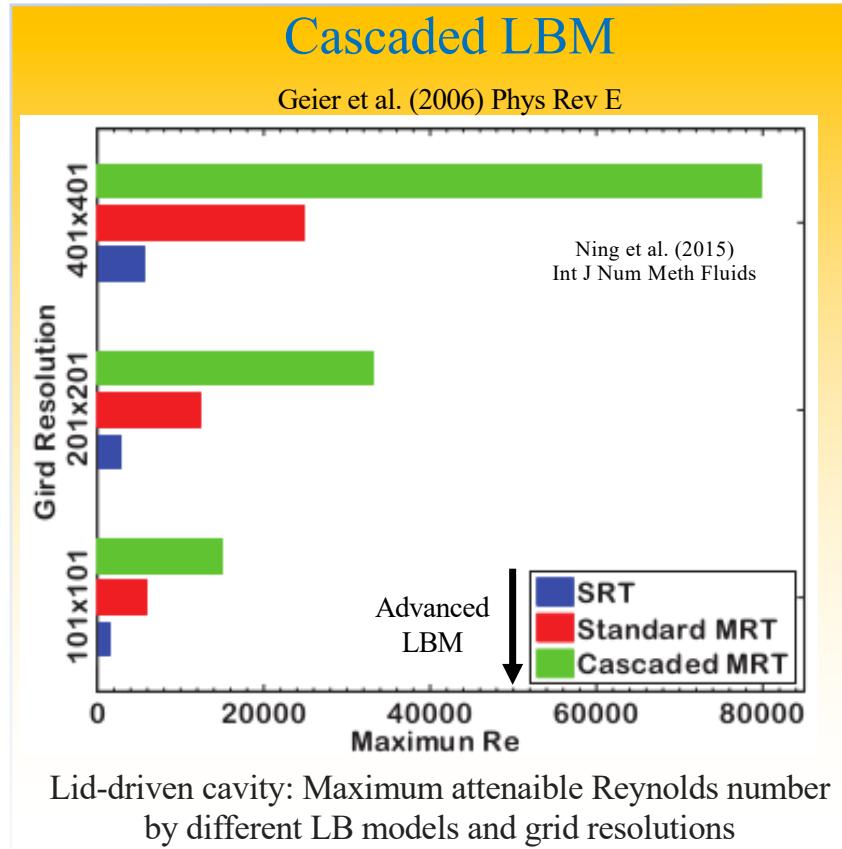


Maxima of the electric current at different Reynolds numbers
De Rosis et al. (2018) J Turbul

Advanced LBM for high-Re flows



Advanced LBM for high-Re flows



Theory and implementation
might be **cumbersome** (especially in 3D...)

$$\begin{aligned}
 \widehat{g}_{26} = & \frac{\omega_{26}}{8} \left[-\widehat{\eta}'_{xxyyzz} + 2 \left(u_x \widehat{\eta}'_{xyyzz} + u_y \widehat{\eta}'_{xxyz} + u_z \widehat{\eta}'_{xxyyz} \right) - \left(u_x^2 \widehat{\eta}'_{yyzz} \right. \right. \\
 & + u_y^2 \widehat{\eta}'_{xxzz} + u_z^2 \widehat{\eta}'_{xxyy} \left. \left. \right) - 4 \left(u_x u_y \widehat{\eta}'_{xyzz} + u_x u_z \widehat{\eta}'_{xyyz} + u_y u_z \widehat{\eta}'_{xxyz} \right) \right. \\
 & + 2 \left(u_x^2 u_y \widehat{\eta}'_{yzz} + u_x u_y^2 \widehat{\eta}'_{xzz} + u_x^2 u_z \widehat{\eta}'_{yyz} + u_x u_z^2 \widehat{\eta}'_{xyy} + u_y^2 u_z \widehat{\eta}'_{xxz} \right. \\
 & \left. \left. + u_y u_z^2 \widehat{\eta}'_{xxy} \right) + 8 u_x u_y u_z \widehat{\eta}'_{xyz} - \left(u_y^2 u_z^2 \widehat{\eta}'_{xx} + u_x^2 u_z^2 \widehat{\eta}'_{yy} + u_x^2 u_y^2 \widehat{\eta}'_{zz} \right) \right. \\
 & \left. - 4 u_x u_y u_z \left(u_z \widehat{\eta}'_{xy} + u_y \widehat{\eta}'_{xz} + u_x \widehat{\eta}'_{yz} \right) + 5 \rho u_x^2 u_y^2 u_z^2 + \right. \\
 & \left. \widetilde{\kappa}_{xx} \widetilde{\kappa}_{yy} \widetilde{\kappa}_{zz} + u_x u_y u_z \left(u_x u_y \widehat{\sigma}'_z + u_x u_z \widehat{\sigma}'_y + u_y u_z \widehat{\sigma}'_x \right) \right] \\
 & + (-4 u_x u_y - 6 u_x u_y u_z^2) \widehat{g}_4 + (-4 u_x u_z - 6 u_x u_y^2 u_z) \widehat{g}_5 \\
 & + (-4 u_y u_z - 6 u_x^2 u_y u_z) \widehat{g}_6 + \left(\frac{1}{2} u_x^2 - \frac{1}{2} u_y^2 + \frac{3}{4} u_x^2 u_z^2 - \frac{3}{4} u_y^2 u_z^2 \right) \widehat{g}_7 \\
 & + \left(\frac{1}{2} u_x^2 + \frac{1}{2} u_y^2 - u_z^2 - \frac{3}{4} u_y^2 u_z^2 - \frac{3}{4} u_x^2 u_z^2 + \frac{3}{2} u_x^2 u_y^2 \right) \widehat{g}_8 \\
 & + \left(-u_x^2 - u_y^2 - u_z^2 - \frac{3}{4} u_x^2 u_y^2 - \frac{3}{4} u_x^2 u_z^2 - \frac{3}{4} u_y^2 u_z^2 - 1 \right) \widehat{g}_9 \\
 & + (3 u_x u_y^2 + 3 u_x u_z^2 + 4 u_x) \widehat{g}_{10} + (3 u_x^2 u_y + 3 u_y u_z^2 + 4 u_y) \widehat{g}_{11} \\
 & + (3 u_x^2 u_z + 3 u_y^2 u_z + 4 u_z) \widehat{g}_{12} + (u_x u_z^2 - u_x u_y^2) \widehat{g}_{13} \\
 & + (u_y u_z^2 - u_x^2 u_y) \widehat{g}_{14} + (u_y^2 u_z - u_x^2 u_z) \widehat{g}_{15} + 8 u_x u_y u_z \widehat{g}_{16} \\
 & + \left(-\frac{1}{2} u_x^2 - \frac{1}{2} u_y^2 - \frac{1}{2} u_z^2 - 1 \right) \widehat{g}_{17} + \left(u_x^2 - \frac{1}{2} u_y^2 - \frac{1}{2} u_z^2 \right) \widehat{g}_{18} \\
 & + \left(\frac{1}{2} u_y^2 - \frac{1}{2} u_z^2 \right) \widehat{g}_{19} - 4 u_y u_z \widehat{g}_{20} - 4 u_x u_z \widehat{g}_{21} - 4 u_x u_y \widehat{g}_{22} + 2 u_x \widehat{g}_{23} \\
 & + 2 u_y \widehat{g}_{24} + 2 u_z \widehat{g}_{25}.
 \end{aligned}$$

Advanced LBM for high-Re flows

Central-moments-based LBM

Central moments are defined as:

$$k_{x^a y^b z^c} = \sum_i f_i \bar{c}_{xi}^a \bar{c}_{yi}^b \bar{c}_{zi}^c$$

Key idea:

$$\bar{c}_{xi} = c_{xi} - u_x$$

$$\bar{c}_{yi} = c_{yi} - u_y$$

$$\bar{c}_{zi} = c_{zi} - u_z$$

Advanced LBM for high-Re flows

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Build a basis \bar{T} : $k_{x^a y^b z^c} \leftrightarrow f_i$

$$\bar{T}(0) = [1, \dots, 1]$$

$$\bar{T}(1) = \bar{c}_{xi} \quad \bar{T}(2) = \bar{c}_{yi} \quad \bar{T}(3) = \bar{c}_{zi}$$

$$\bar{T}(4) = \bar{c}_{xi} \bar{c}_{yi} \quad \bar{T}(5) = \bar{c}_{xi} \bar{c}_{zi} \quad \bar{T}(6) = \bar{c}_{yi} \bar{c}_{zi}$$

$$\bar{T}(7) = \bar{c}_{xi}^2 - \bar{c}_{yi}^2 \quad \bar{T}(8) = \bar{c}_{xi}^2 - \bar{c}_{zi}^2$$

$$\bar{T}(9) = \bar{c}_{xi}^2 + \bar{c}_{yi}^2 + \bar{c}_{zi}^2 \quad \bar{T}(10) = \bar{c}_{xi} \bar{c}_{yi}^2 + \bar{c}_{xi} \bar{c}_{zi}^2$$

...

$$\bar{T}(24) = \bar{c}_{xi}^2 \bar{c}_{yi} \bar{c}_{zi}^2 \quad \bar{T}(25) = \bar{c}_{xi}^2 \bar{c}_{yi}^2 \bar{c}_{zi}$$

$$\bar{T}(26) = \bar{c}_{xi}^2 \bar{c}_{yi}^2 \bar{c}_{zi}^2$$

D3Q27

Advanced LBM for high-Re flows

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...

$$\bar{T}(24) = \bar{c}_{xi}^2 \bar{c}_{yi} \bar{c}_{zi}^2$$

$$\bar{T}(25) = \bar{c}_{xi}^2 \bar{c}_{yi}^2 \bar{c}_{zi}$$

$$\bar{T}(26) = \bar{c}_{xi}^2 \bar{c}_{yi}^2 \bar{c}_{zi}^2$$

D3Q27

Algorithm of computation

1. Compute pre-collision central moments:

$$k = \bar{T} f$$

2. Evaluate equilibrium state:

$$k^{(0)} = \bar{T} f^{(0)}$$

3. Collide central moments:

$$k_i^* = (1 - \omega_i)k_i + \omega_i k_i^{(0)}$$

4. Reconstruct post-collision populations:

$$f^* = \bar{T}^{-1} k^*$$

De Rosis (2017) Phys Rev E

Advanced LBM for high-Re flows

Some practical aspects

- Moments $k_{0\dots 3}^*$ are collision invariants
- Only 2nd-order moments $k_{4\dots 8}^*$ are related to the fluid kinematic viscosity, so $\omega = (3\nu + 0.5)^{-1}$
- Higher-order moments refer to non-hydrodynamic ghost modes, so $\omega = 1$
- We need to collide only few moments!

Post-collision central moments:

$$\begin{aligned} k_0^* &= \rho & k_9^* &= \rho \\ k_4^* &= (1 - \omega)k_4 & k_{17}^* &= \rho c_s^2 \\ k_5^* &= (1 - \omega)k_5 & k_{18}^* &= \rho c_s^4 \\ k_6^* &= (1 - \omega)k_6 & k_{26}^* &= \rho c_s^6 \\ k_7^* &= (1 - \omega)k_7 \\ k_8^* &= (1 - \omega)k_8 \end{aligned}$$

De Rosis & Luo (2019) Phys Rev E

- Equilibrium distributions:

$$\begin{aligned} f_i^{(0)} = w_i \rho [1 + \frac{\xi_i \cdot u}{c_s^2} + \frac{1}{2c_s^4} \mathcal{H}_i^{(2)} : uu \\ + \frac{1}{2c_s^6} (\mathcal{H}_{ixxy}^{(3)} u_x^2 u_y + \mathcal{H}_{ixxz}^{(3)} u_x^2 u_z + \mathcal{H}_{ixyy}^{(3)} u_x u_y^2 \\ + \mathcal{H}_{ixzz}^{(3)} u_x u_z^2 + \mathcal{H}_{iyzz}^{(3)} u_y u_z^2 + \mathcal{H}_{iyyz}^{(3)} u_y^2 u_z \\ + \mathcal{H}_{ixyz}^{(3)} u_x u_y u_z) \\ + \frac{1}{4c_s^8} (\mathcal{H}_{ixxyy}^{(4)} u_x^2 u_y^2 + \mathcal{H}_{ixxzz}^{(4)} u_x^2 u_z^2 + \mathcal{H}_{iyyzz}^{(4)} u_y^2 u_z^2 \\ + 2(\mathcal{H}_{ixyzz}^{(4)} u_x u_y u_z^2 + \mathcal{H}_{ixyyz}^{(4)} u_x u_y^2 u_z + \mathcal{H}_{ixxyz}^{(4)} u_x^2 u_y u_z)) \\ + \frac{1}{4c_s^{10}} (\mathcal{H}_{ixxyzz}^{(5)} u_x^2 u_y u_z^2 + \mathcal{H}_{ixxyyz}^{(5)} u_x^2 u_y^2 u_z \\ + \mathcal{H}_{ixyyzz}^{(5)} u_x u_y^2 u_z^2) \\ + \frac{1}{8c_s^{12}} \mathcal{H}_{ixxyyzz}^{(6)} u_x^2 u_y^2 u_z^2], \end{aligned}$$

Malaspinas (2015) arXiv
Coreixas et al. (2017) Phys Rev E

Equilibrium central moments:

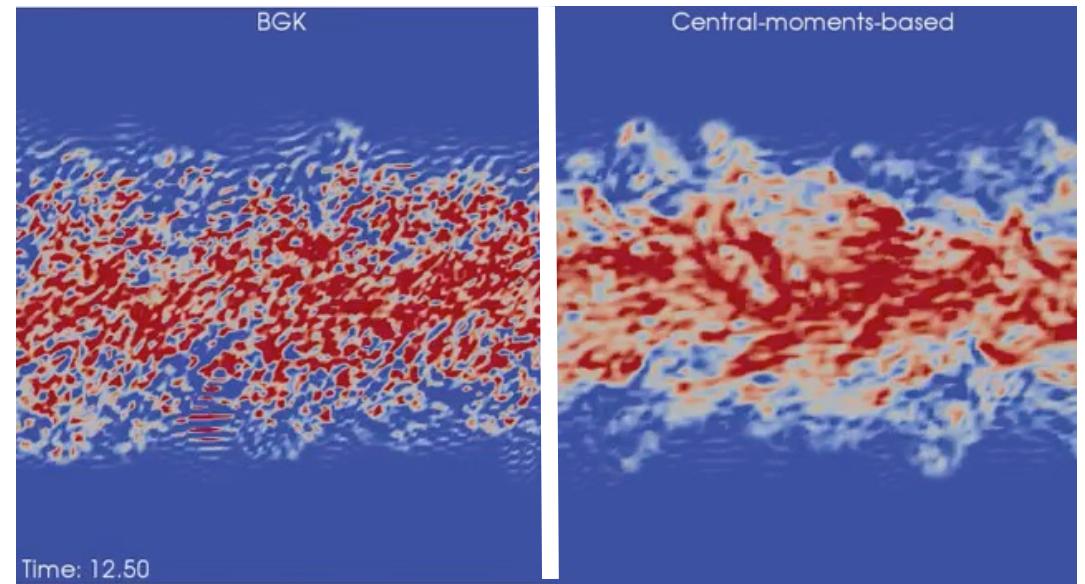
$$\begin{aligned} k_0^{(0)} &= \rho, & k_9^{(0)} &= \rho, \\ k_{17}^{(0)} &= \rho c_s^2, & k_{18}^{(0)} &= \rho c_s^4, & k_{26}^{(0)} &= \rho c_s^6 \end{aligned}$$

Key features

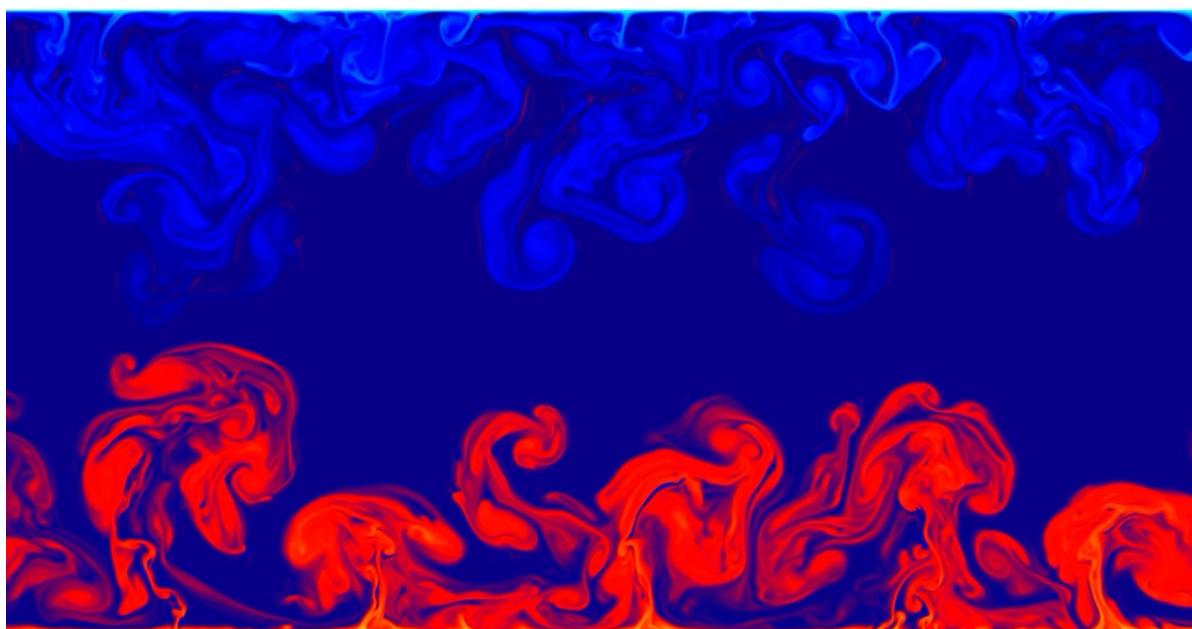
Properties:

- 2^o-order accurate
- Improved stability
- Intelligible derivations
- Easy implementation
- Very general framework

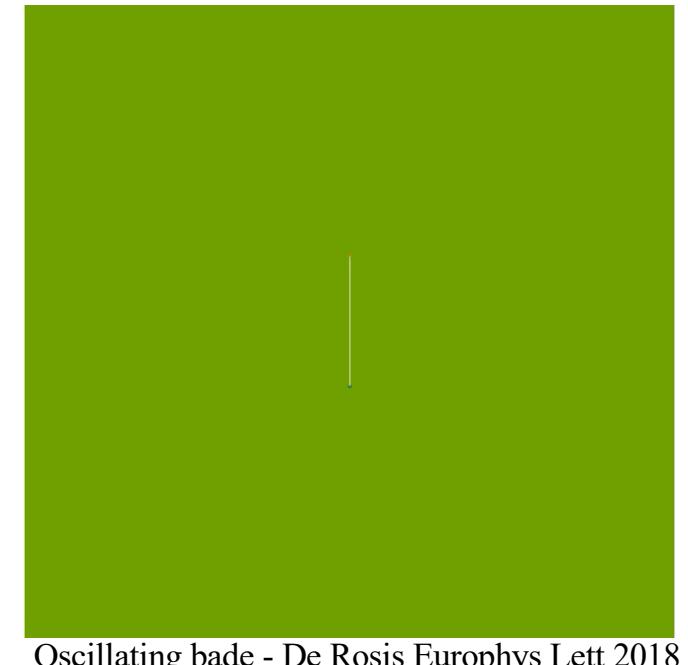
De Rosis
Comput Meth Appl M 2017



High-Re high-Sc scalar transport by BGK (left) and CMs (right)



Rayleigh-Benard convection at $\text{Ra}=10^{11}$



Oscillating bade - De Rosis Europhys Lett 2018

Advanced LBM for high-Re MHD flows

Central moments for Dellar's 2D MHD

$$f_i^{(0)} = w_i \rho \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u} \cdot \mathbf{u}}{2c_s^2} \right] \\ + \frac{w_i}{2c_s^4} \left[\frac{1}{2} |\mathbf{c}_i|^2 |\mathbf{B}|^2 - (\mathbf{c}_i \cdot \mathbf{B})^2 \right]$$

Build a basis \bar{T} : $k_x^a y^b \leftrightarrow f_i$

$\bar{T}(0) = [1, \dots, 1]$	$\bar{T}(2) = \bar{c}_{yi}$	$\bar{T}(3) = \bar{c}_{xi}^2 + \bar{c}_{yi}^2$
$\bar{T}(1) = \bar{c}_{xi}$	$\bar{T}(5) = \bar{c}_{xi} \bar{c}_{yi}$	$\bar{T}(6) = \bar{c}_{xi}^2 \bar{c}_{yi}$
$\bar{T}(4) = c_{xi}^2 - \bar{c}_{yi}^2$	$\bar{T}(7) = \bar{c}_{xi} \bar{c}_{yi}^2$	$\bar{T}(8) = \bar{c}_{xi}^2 \bar{c}_{yi}^2$

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$$k_i^* = (1 - \omega_i) k_i + \omega_i k_i^{(0)}$$

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Advanced LBM for high-Re MHD flows

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$$+ \frac{w_i}{2c_s^4} \left[\frac{1}{2} |\mathbf{c}_i|^2 |\mathbf{B}|^2 - (\mathbf{c}_i \cdot \mathbf{B})^2 \right]$$

Build a basis \bar{T} : $k_x^a y^b \leftrightarrow f_i$

$$\begin{aligned} \bar{T}(0) &= [1, \dots, 1] \\ \bar{T}(1) &= \bar{c}_{xi} & \bar{T}(2) &= \bar{c}_{yi} & \bar{T}(3) &= \bar{c}_{xi}^2 + \bar{c}_{yi}^2 \\ \bar{T}(4) &= \bar{c}_{xi}^2 - \bar{c}_{yi}^2 & \bar{T}(5) &= \bar{c}_{xi} \bar{c}_{yi} & \bar{T}(6) &= \bar{c}_{xi}^2 \bar{c}_{yi} \\ \bar{T}(7) &= \bar{c}_{xi} \bar{c}_{yi}^2 & \bar{T}(8) &= \bar{c}_{xi}^2 \bar{c}_{yi}^2 \end{aligned}$$

D2Q9 post-collision central moments:

$$k_0^* = \rho$$

$$k_1^* = 0$$

$$k_2^* = 0$$

$$k_3^* = 2\rho c_s^2$$

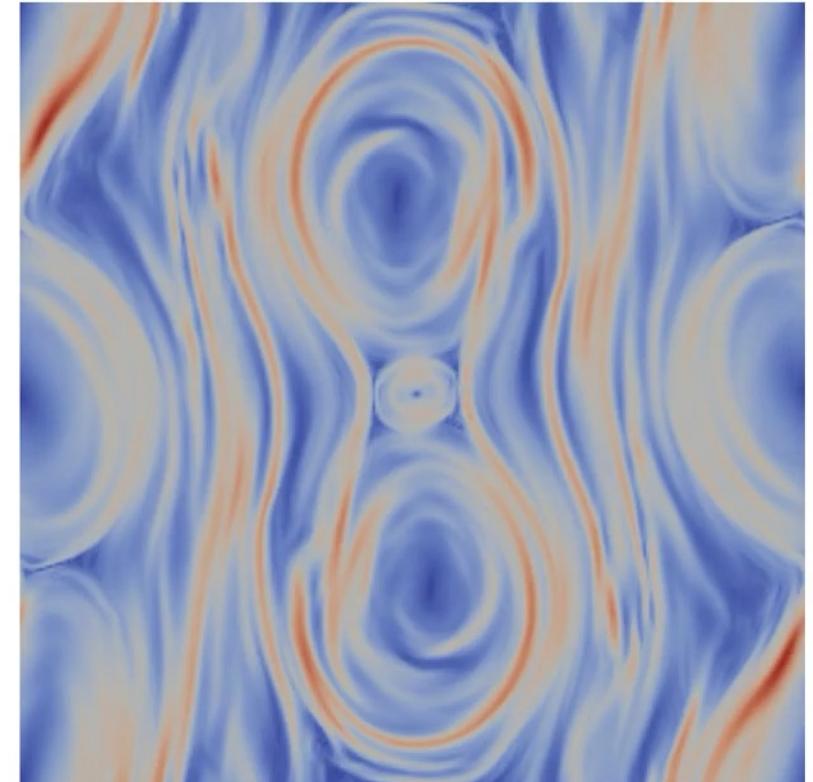
$$k_4^* = (1 - \omega_4)k_4 + \omega_4(B_y^2 - B_x^2)$$

$$k_5^* = (1 - \omega_5)k_5 + \omega_5(-B_x B_y)$$

$$k_6^* = \frac{u_y}{2} (B_x^2 - B_y^2) + 2u_x B_x B_y$$

$$k_7^* = \frac{u_x}{2} (B_y^2 - B_x^2) + 2u_y B_x B_y$$

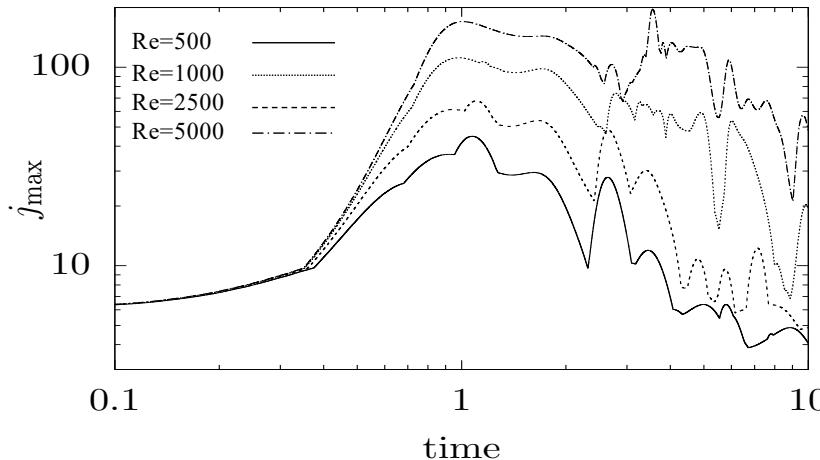
$$k_8^* = \rho c_s^4 + \frac{u_x^2 - u_y^2}{2} (B_x^2 - B_y^2) - 4u_x u_y B_x B_y$$



2D OT vortex: magnetic field

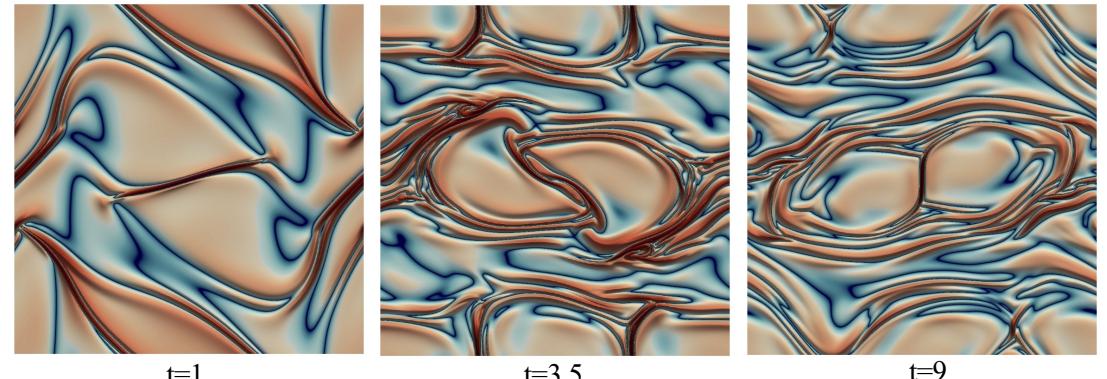
Advanced LBM for high-Re MHD flows

Stability is enhanced!

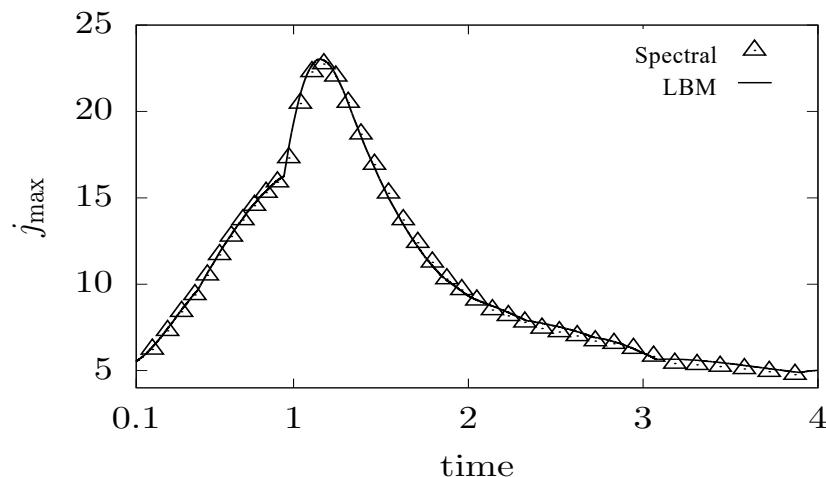


2D OT vortex: Maxima of the electric current

De Rosis et al. (2018) J Turbul

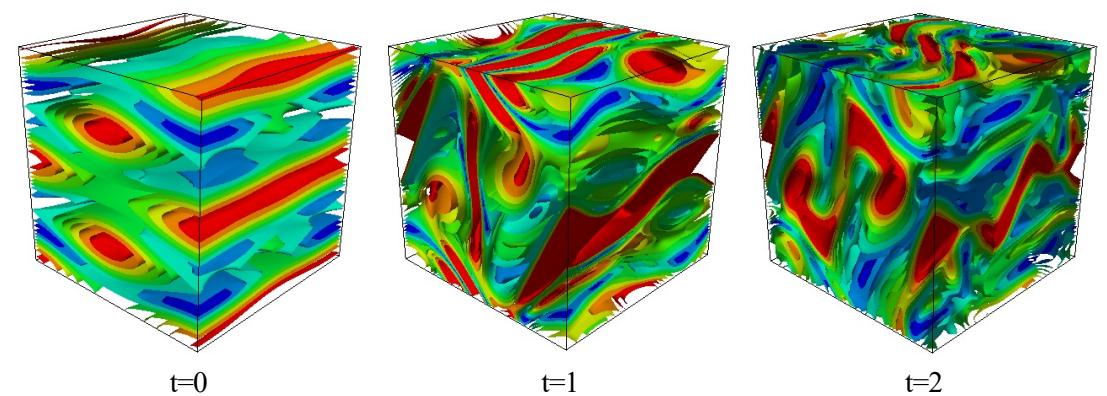


2D OT vortex: electric current at $Re=5000$



3D OT vortex: Maxima of the electric current at $Re=100$

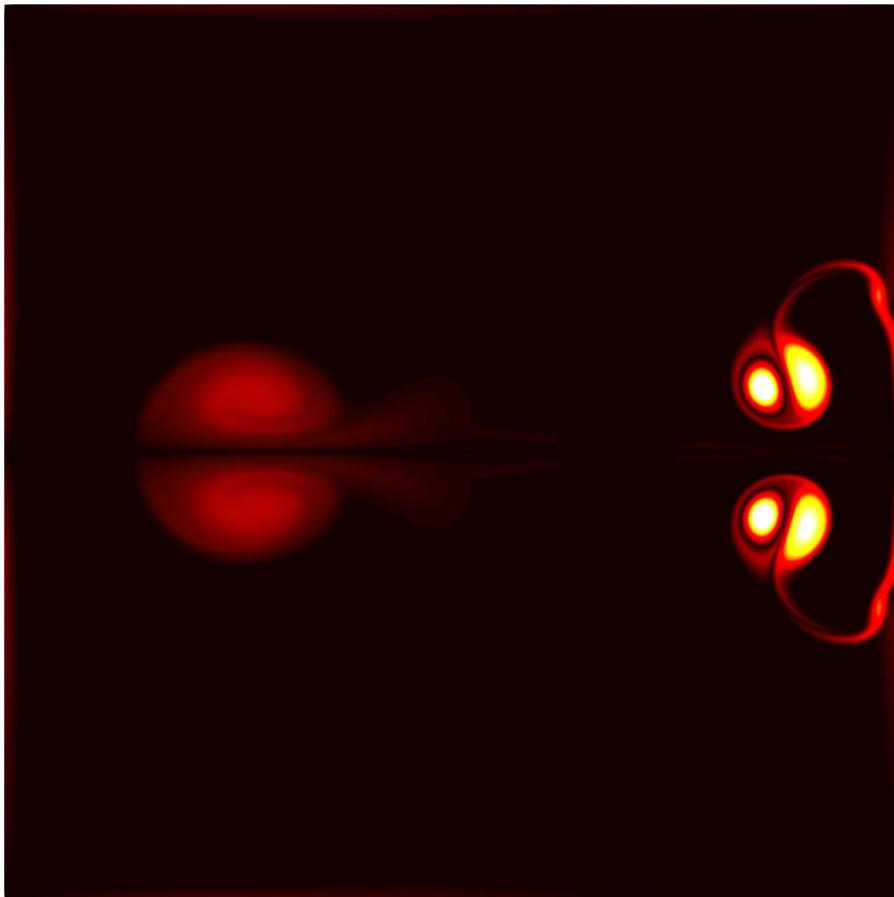
De Rosis (2017) Phys Rev E Kaleidoscope



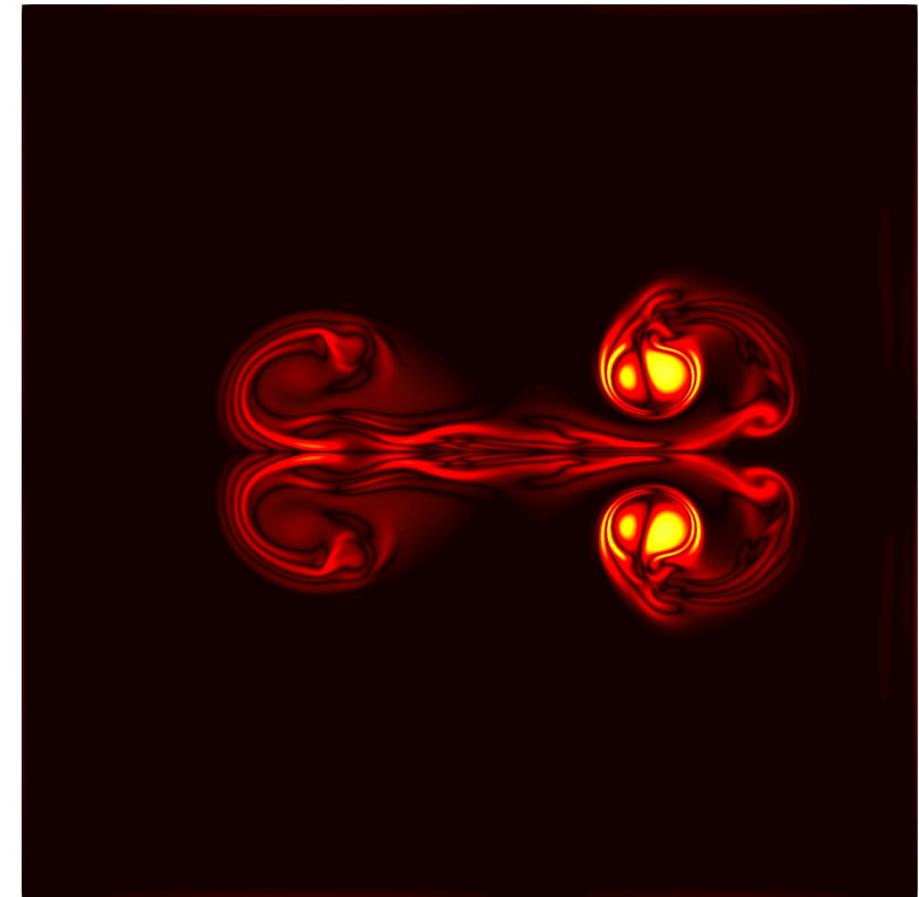
3D OT vortex: electric current at $Re=100$

MHD physics: vortex-wall collision

De Rosis & Skillen
Physics of Fluids 2022



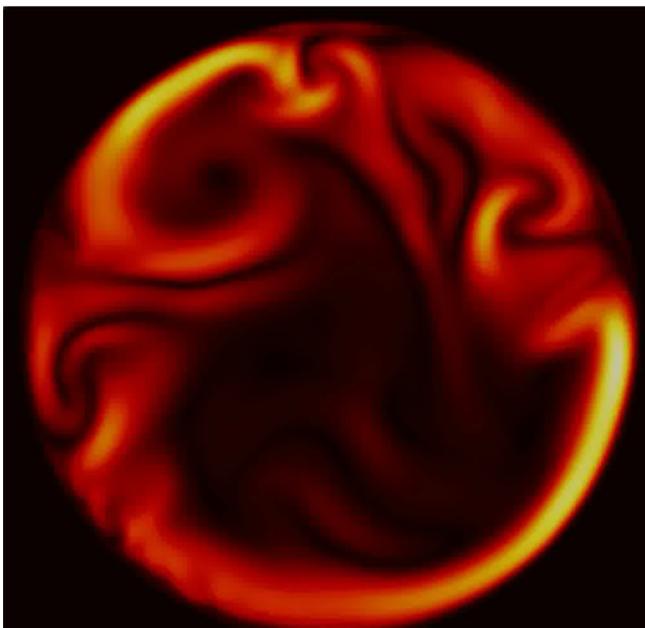
Without magnetic field



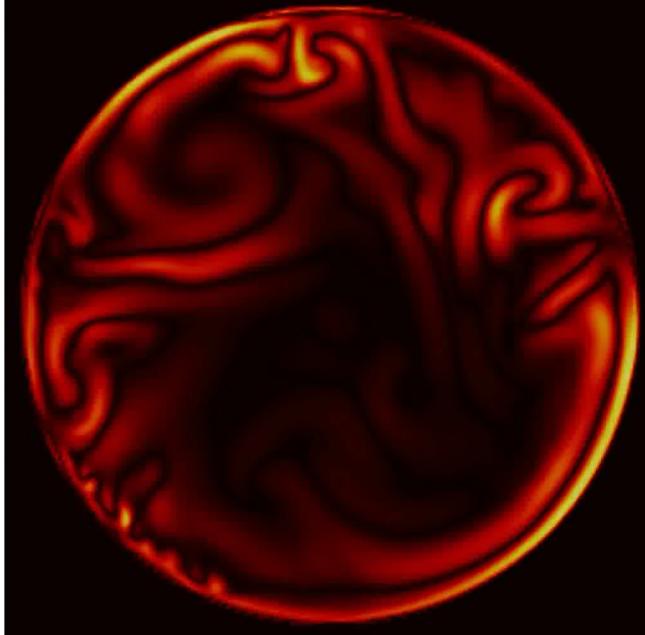
With magnetic field

MHD application: flow in rotating cavities

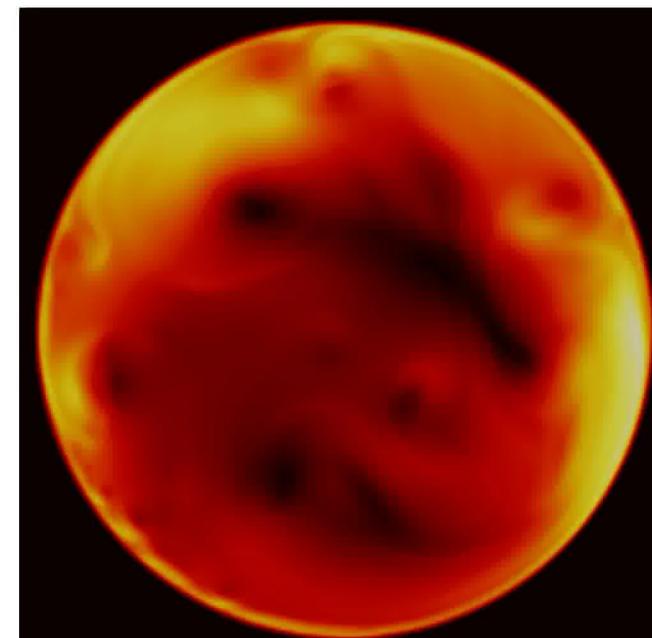
Magnetic field



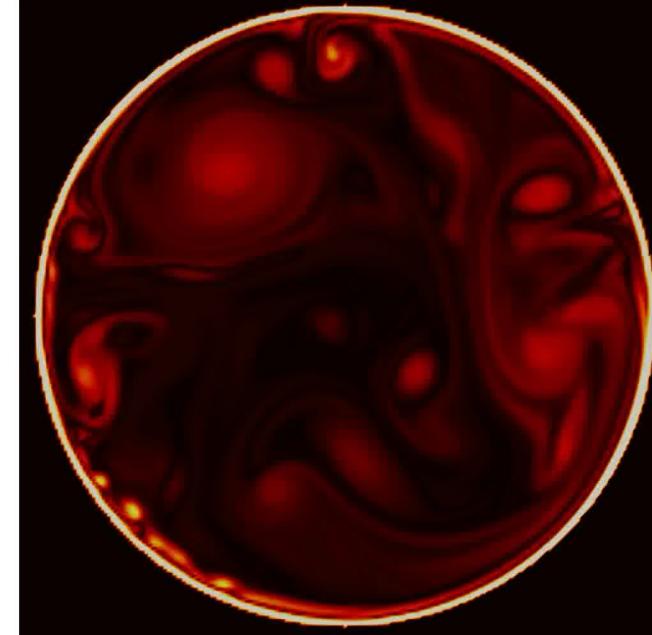
Current density



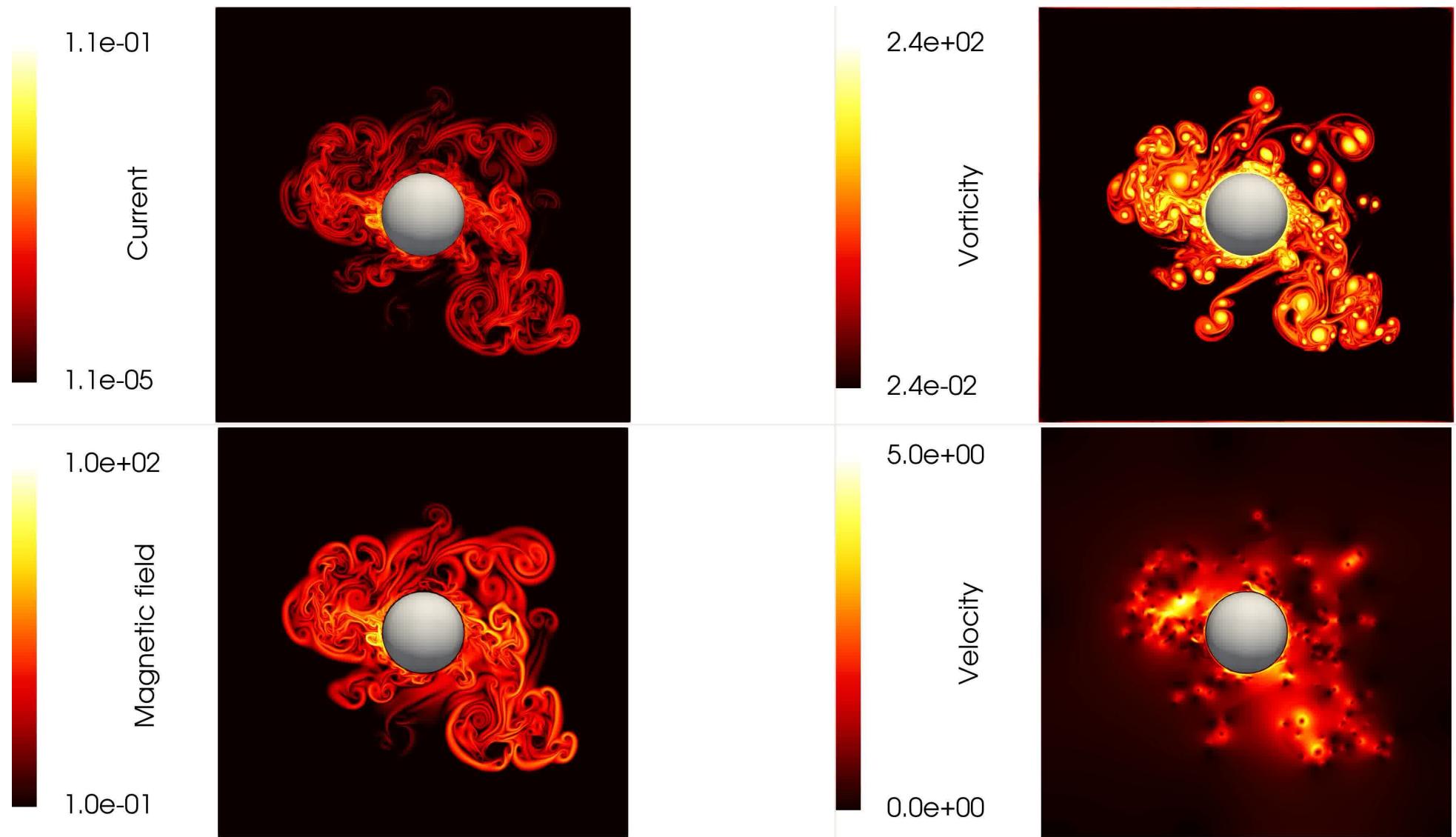
Flow velocity



Flow vorticity



MHD application: flow detachment



Advanced LBM for multicomponent flows

Macroscopic

$$\begin{aligned} \nabla \cdot \mathbf{u} &= 0, \\ \rho[\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u}] &= -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{F}, \\ \partial_t \phi + \nabla \cdot \phi \mathbf{u} &= \nabla \cdot M \left[\nabla \phi - \frac{\nabla \phi}{|\nabla \phi|} \frac{1 - 4(\phi - \phi_0)^2}{\xi} \right] \end{aligned}$$

Kinetic

$$\partial_t f + \mathbf{c} \cdot \nabla f = \frac{f^{(0)} - f}{\tau}$$

De Rosis & Enan (2021)
Phys Fluids

$$\partial_t g + \mathbf{c}' \cdot \nabla g = \frac{g^{(0)} - g}{\tau_\phi}$$

De Rosis & Tafuni (2022)
Comput Aid Civ Inf

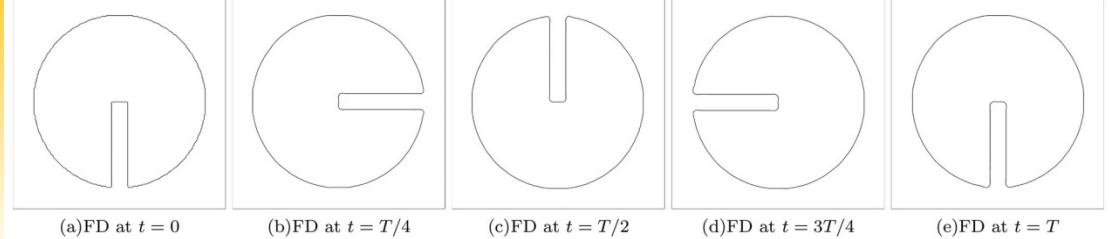
$$f_i^\star = f_i + \frac{1}{\tau + 1/2} (f_i^{eq} - f_i) + F_i,$$

$$g_i^\star = g_i + \frac{1}{\tau_\phi + 1/2} (g_i^{eq} - g_i) + G_i,$$

$$f_i^{eq} = w_i \left[\tilde{p} + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2} \right],$$

$$g_i^{eq} = w_i \phi \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2} \right]$$

Zalesak disk



$$u_x = -U_0 2\pi \left(\frac{y}{L_0} - 0.5 \right), \quad u_y = U_0 2\pi \left(\frac{x}{L_0} - 0.5 \right).$$

TABLE III. Zalesak disk: relative error at different Pe. Reproduced by Zu *et al.*, “Phase-field lattice Boltzmann model for interface tracking of a binary fluid system based on the Allen-Cahn equation,” with permission from Phys. Rev. E **102**, 053307 (2020). Copyright 2020 APS Publishing

Model	Pe			
	80	400	800	4000
FD	0.0593	0.0590	0.0558	0.0505
Mom	0.0567	0.0482	0.0483	0.0501
Ref. 52 (Zu <i>et al.</i>)	0.1226	0.1186	0.1170	0.1194
Ref. 52 (Geier <i>et al.</i>)	0.1307	0.1224	0.1209	0.1472
Ref. 52 (Wang <i>et al.</i>)	0.1307	0.1224	0.1209	0.1471

2D Rayleigh-Taylor instability

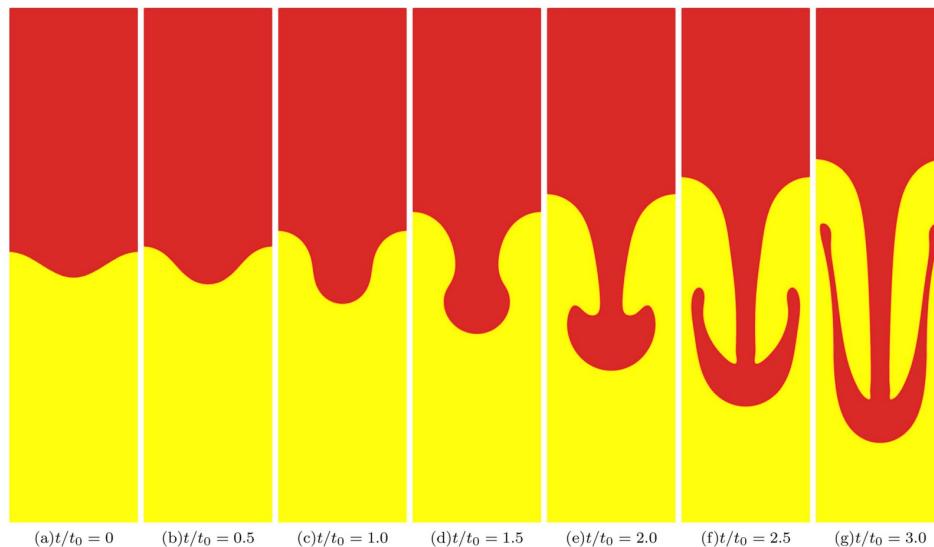


FIG. 12. Two-dimensional Rayleigh-Taylor instability at $Re = 256$: evolution of the interface at (a) $t/t_0 = 0$, (b) $t/t_0 = 0.5$, (c) $t/t_0 = 1.0$, (d) $t/t_0 = 1.5$, (e) $t/t_0 = 2.0$, (f) $t/t_0 = 2.5$ and (g) $t/t_0 = 3.0$. Red and yellow colors correspond to heavy and light fluids, respectively.

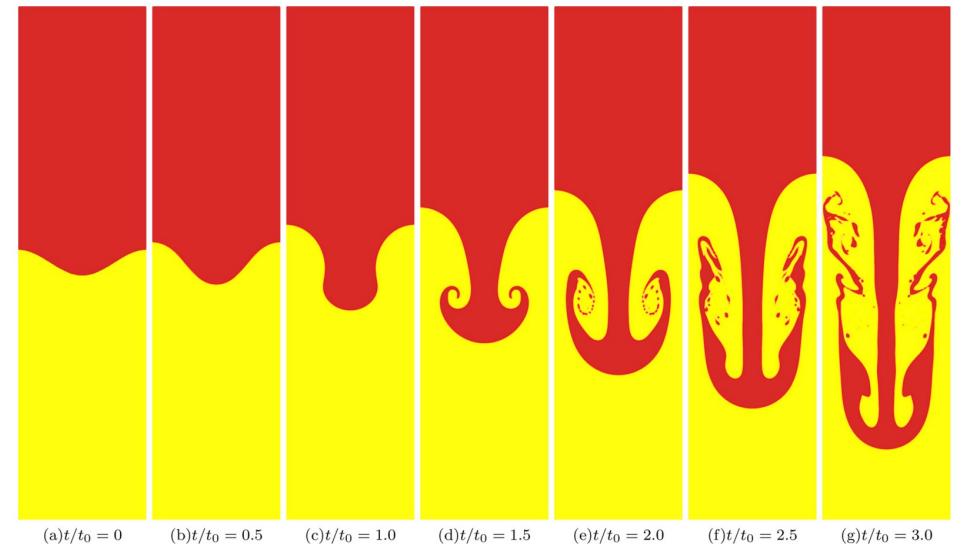
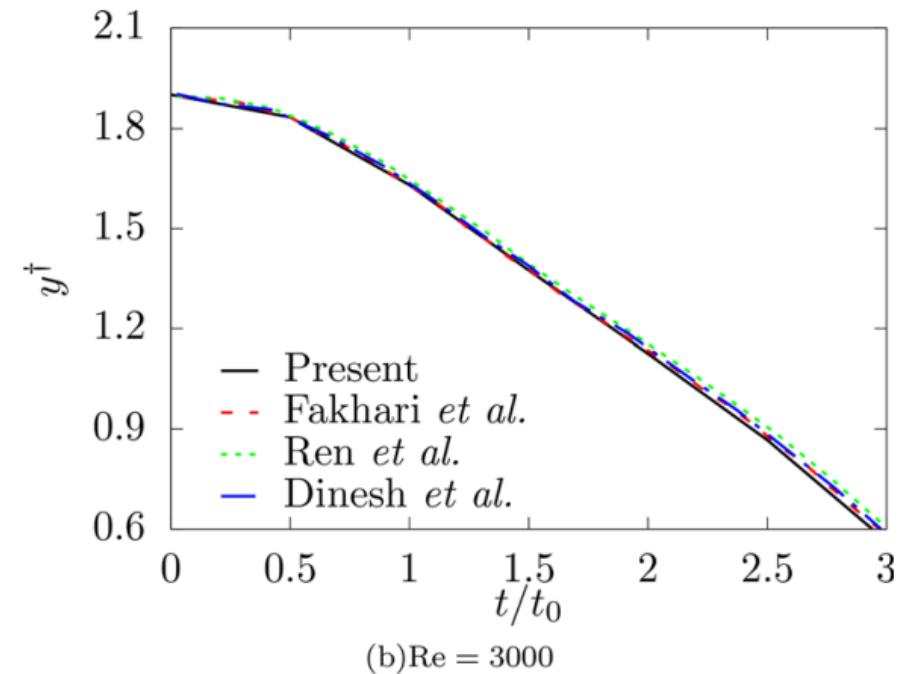
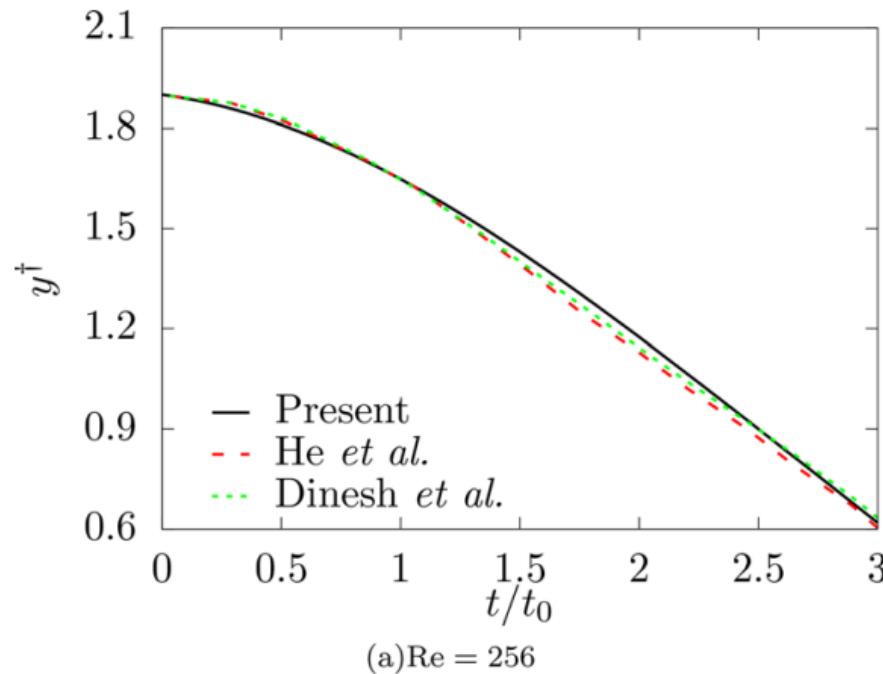
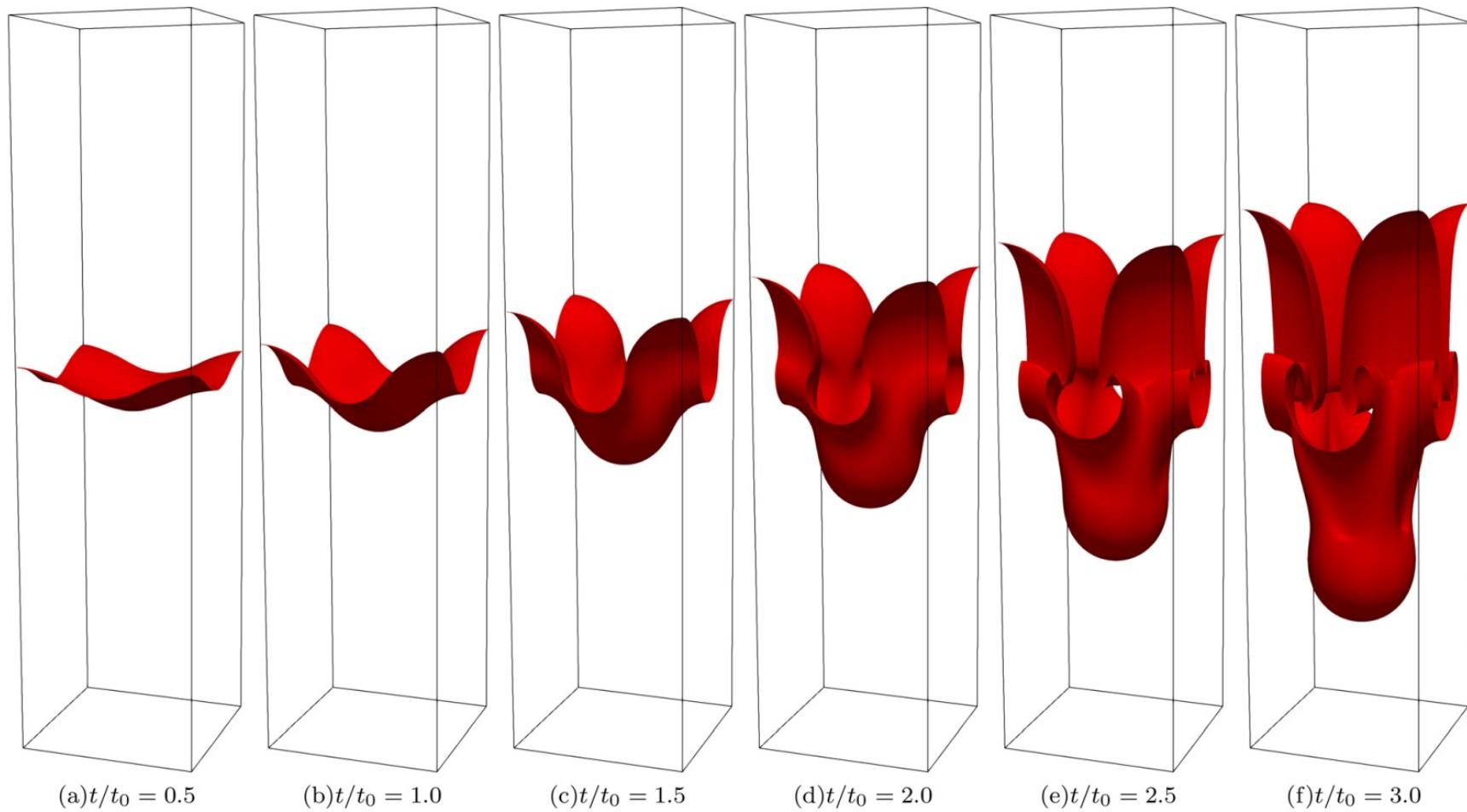


FIG. 13. Two-dimensional Rayleigh-Taylor instability at $Re = 3000$: evolution of the interface at (a) $t/t_0 = 0$, (b) $t/t_0 = 0.5$, (c) $t/t_0 = 1.0$, (d) $t/t_0 = 1.5$, (e) $t/t_0 = 2.0$, (f) $t/t_0 = 2.5$ and (g) $t/t_0 = 3.0$. Red and yellow colors correspond to heavy and light fluids, respectively.



3D Rayleigh-Taylor instability



t/t_0	FD	Mom	Ref. 74	Ref. 62	Ref. 28	Ref. 94	Ref. 95	Ref. 96
0.0	1.898	1.898	1.897	1.897	1.895	1.887	1.888	1.904
0.5	1.858	1.850	1.897	1.897	1.864	1.839	1.860	1.869
1.0	1.741	1.711	1.753	1.753	1.763	1.744	1.755	1.776
1.5	1.553	1.504	1.592	1.591	1.587	1.555	1.569	1.618
2.0	1.304	1.256	1.381	1.378	1.357	1.312	1.325	1.396
2.5	1.001	0.988	1.126	1.121	1.085	1.022	1.037	1.149
3.0	0.648	0.711	0.844	0.791	0.788	0.712	0.740	0.863

Outline

- Lattice Boltzmann method (LBM): what is it?
- LBM at UoM:
 - Advanced LBM for high-Reynolds flows
 - Beyond fluid mechanics: LBM for reaction-diffusion equations

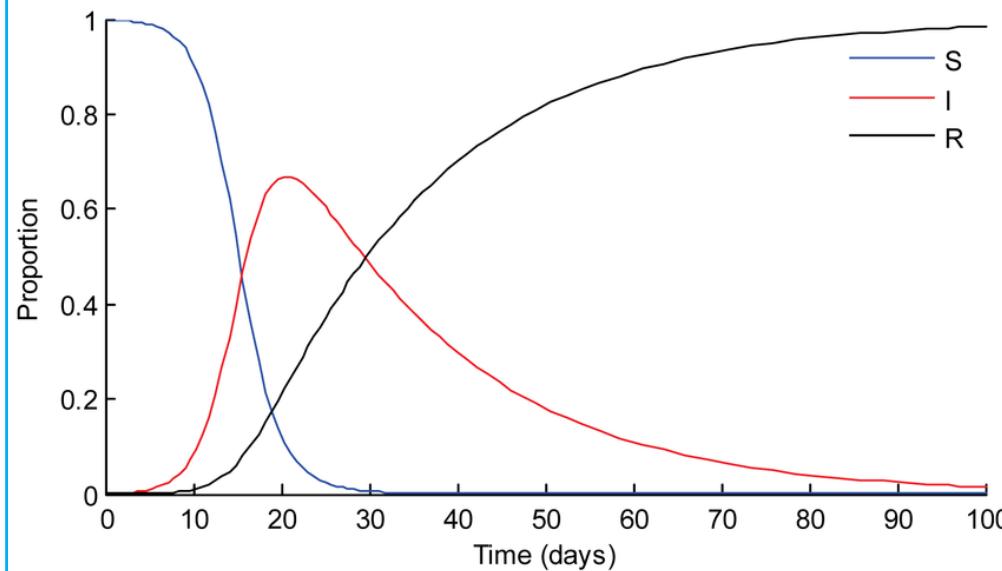
LBM for reaction-diffusion equations

A reaction-diffusion process: spread of epidemics

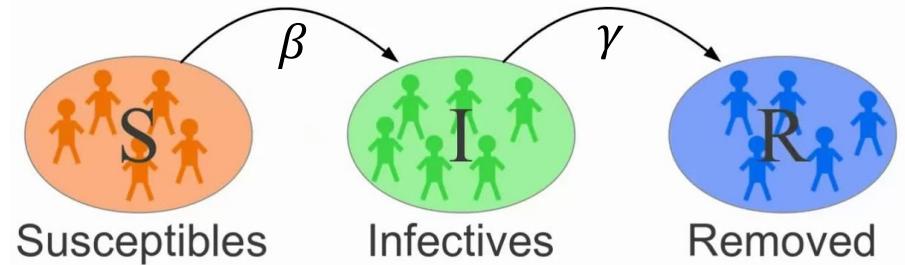
$$\frac{dS}{dt} = -\beta SI + d_S \nabla^2 S$$

$$\frac{dI}{dt} = \beta SI - \gamma I + d_I \nabla^2 I$$

$$\frac{dR}{dt} = \gamma I$$



The model in a nutshell



LBM for reaction-diffusion equations

A reaction-diffusion process: spread of epidemics

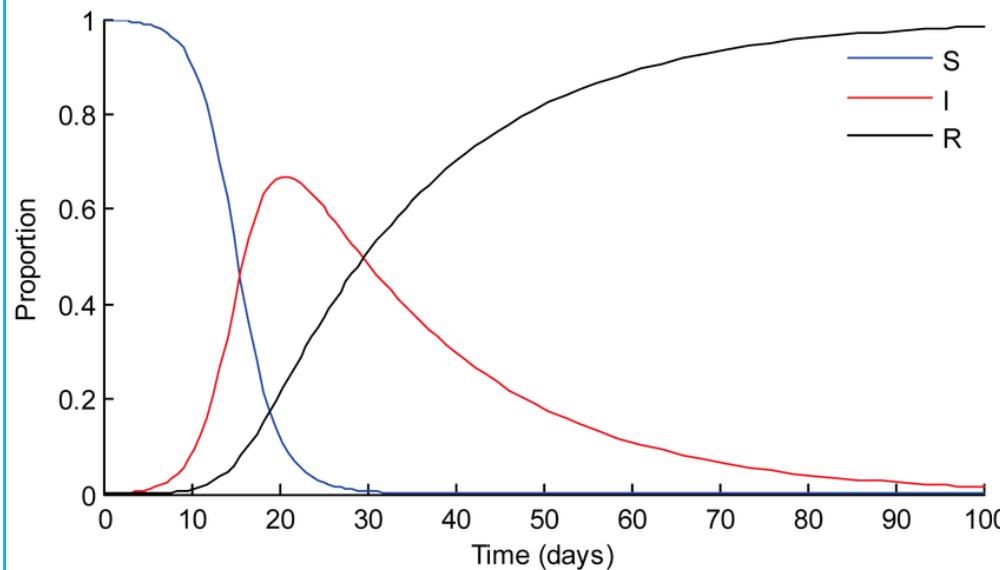
$$\frac{dS}{dt} = -\beta SI + d_S \nabla^2 S$$

Reactive terms

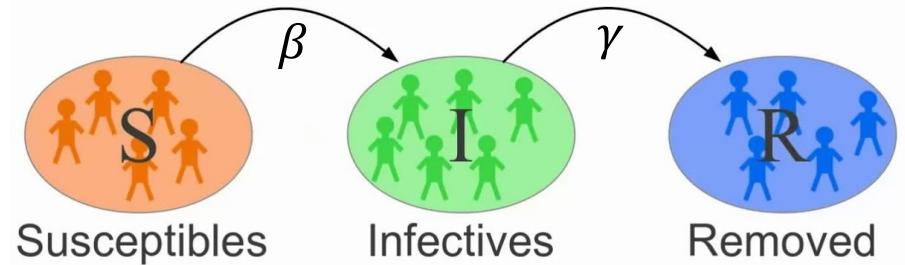
$$\frac{dI}{dt} = \beta SI - \gamma I + d_I \nabla^2 I$$

Non-reactive
(diffusive) terms

$$\frac{dR}{dt} = \gamma I$$



The model in a nutshell



LBM for reaction-diffusion equations

A reaction-diffusion process: spread of epidemics

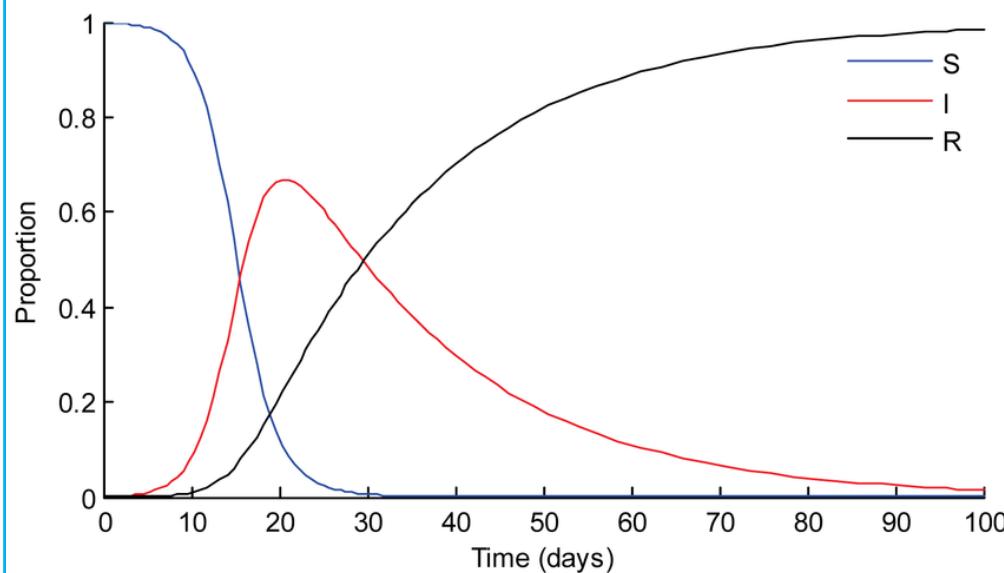
$$\frac{dS}{dt} = -\beta SI + d_S \nabla^2 S$$

Reactive terms

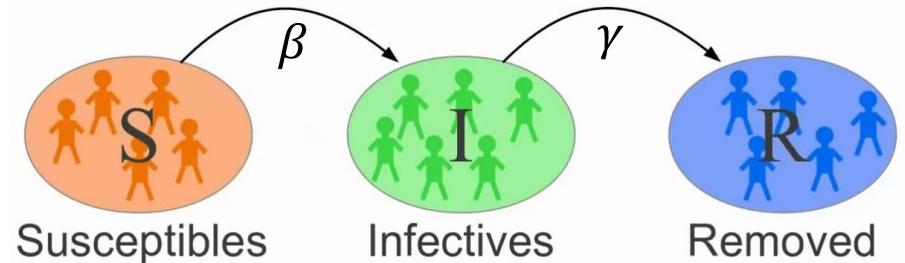
$$\frac{dI}{dt} = \beta SI - \gamma I + d_I \nabla^2 I$$

Non-reactive
(diffusive) terms

$$\frac{dR}{dt} = \gamma I$$



The model in a nutshell



Wait...can we do it by BGK LBM?

Yes! 2 reaction-diffusion equations (+1 an ODE)!

$$f_i^k(\mathbf{x} + \mathbf{c}_i, t + \Delta t) = f_i^k(\mathbf{x}, t) + \Omega_{i,\text{NR}}^k(\mathbf{x}, t) + \Omega_{i,\text{R}}^k(\mathbf{x}, t)$$

Non-reactive (diffusive) part:

$$\Omega_{i,\text{NR}}^k = \frac{1}{\tau^k} (f_{i,\text{eq}}^k - f_i^k)$$

$$f_{i,\text{eq}}^k = w_i \rho^k$$

$$d^k = \frac{1}{3} \left(\tau^k - \frac{1}{2} \right)$$

$$\rho^k = \sum_i f_i^k$$

LBM for reaction-diffusion equations

Wait...can we do it by BGK LBM?

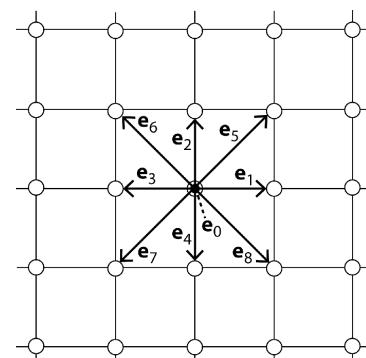
$$f_i^k(\mathbf{x} + \mathbf{c}_i, t + \Delta t) = f_i^k(\mathbf{x}, t) + \Omega_{i,\text{NR}}^k(\mathbf{x}, t) + \Omega_{i,\text{R}}^k(\mathbf{x}, t)$$

Reactive (non-diffusive) part:

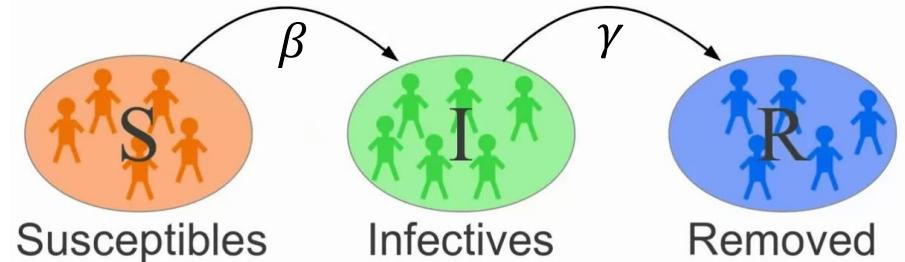
$$\Omega_{i,\text{R}}^S = w_i \left(-\frac{\beta \rho^S \rho^I}{\rho^N} \right),$$

$$\Omega_{i,\text{R}}^I = w_i \left(\frac{\beta \rho^S \rho^I}{\rho^N} - \gamma \rho^I \right),$$

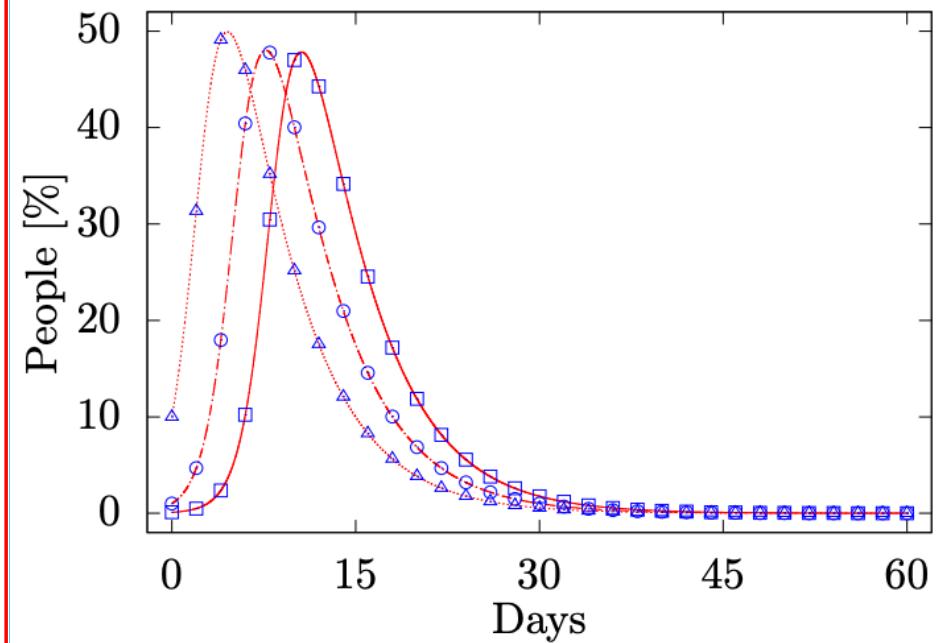
We are now in the position to solve the problem by the BGK LBM!



The model in a nutshell



Validation



LBM for reaction-diffusion equations

Wait...can we do it by BGK LBM?

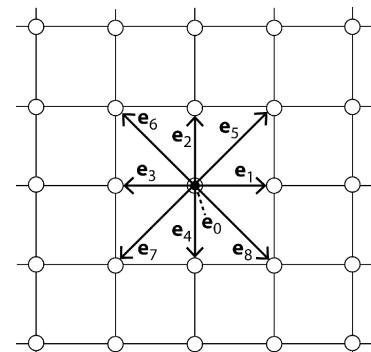
$$f_i^k(\mathbf{x} + \mathbf{c}_i, t + \Delta t) = f_i^k(\mathbf{x}, t) + \Omega_{i,\text{NR}}^k(\mathbf{x}, t) + \Omega_{i,\text{R}}^k(\mathbf{x}, t)$$

Reactive (non-diffusive) part:

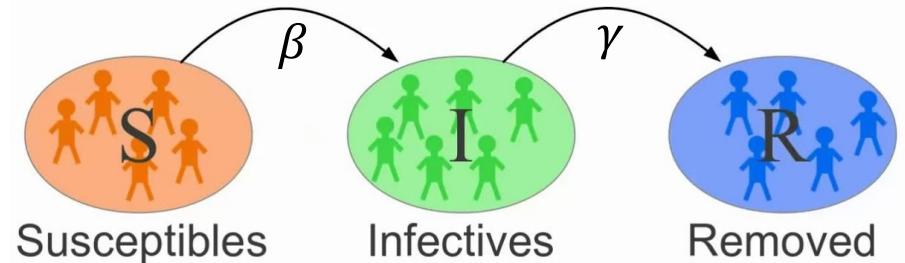
$$\Omega_{i,\text{R}}^S = w_i \left(-\frac{\beta \rho^S \rho^I}{\rho^N} \right),$$

$$\Omega_{i,\text{R}}^I = w_i \left(\frac{\beta \rho^S \rho^I}{\rho^N} - \gamma \rho^I \right),$$

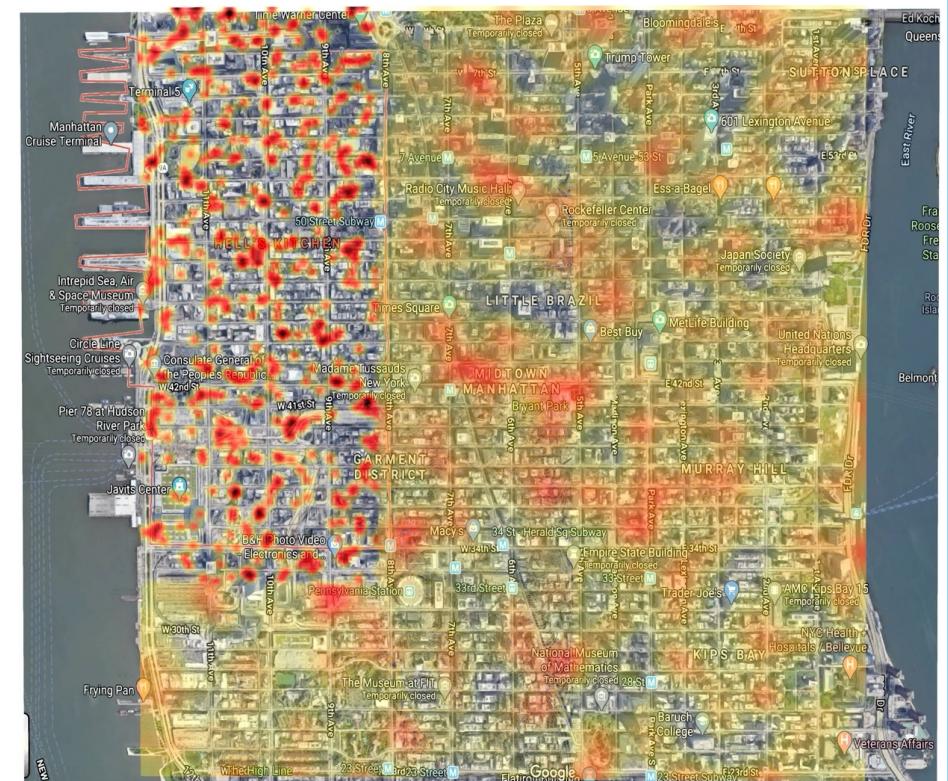
We are now in the position to solve the problem by the BGK LBM!



The model in a nutshell



An epidemic in Midtown Manhattan



Thanks for your attention!

```
printf(" Questions? ");  
if(question)  
    if(haveAnswer)  
        giveAnswer();  
    else  
        pretendTheQuestionIsIllPosed();  
    end  
else  
    printf(" Thank you! ");  
end
```