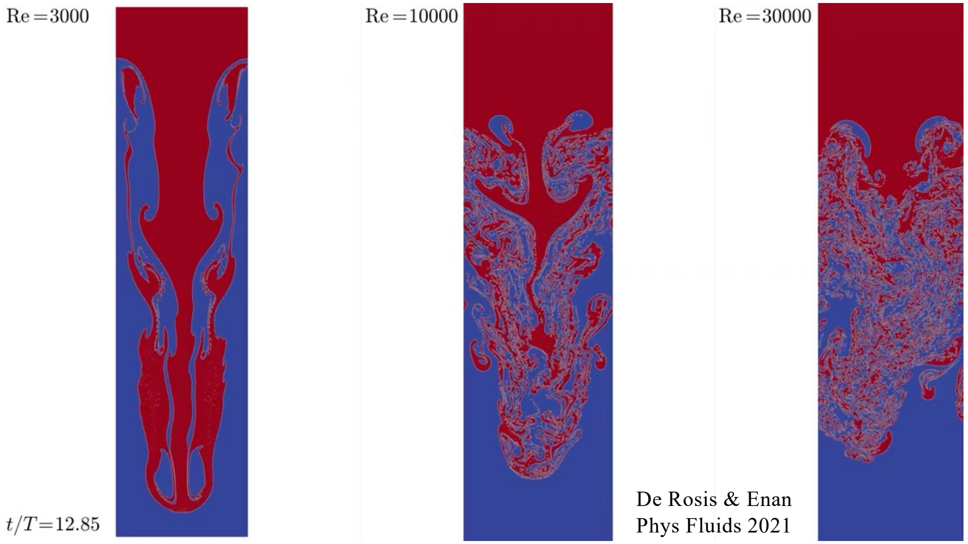
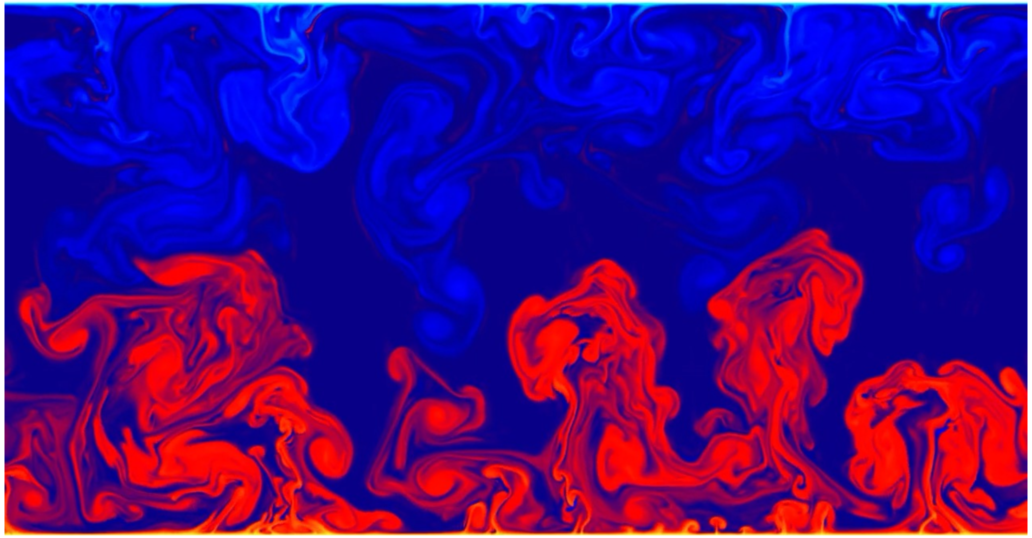


De Rosis & Enan
Phys Fluids 2021



Fluid dynamics and beyond: opportunities with the lattice Boltzmann method

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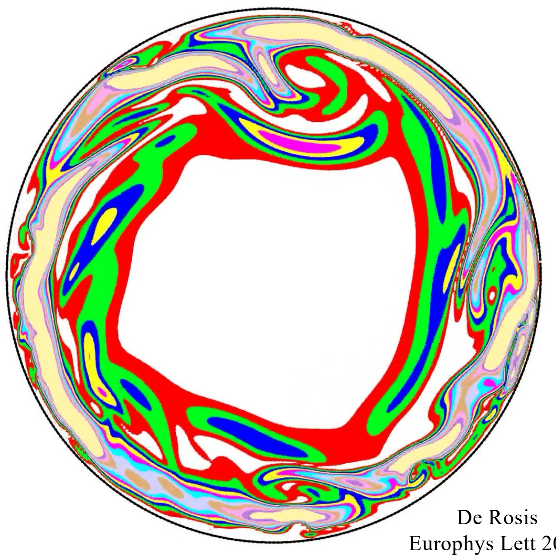
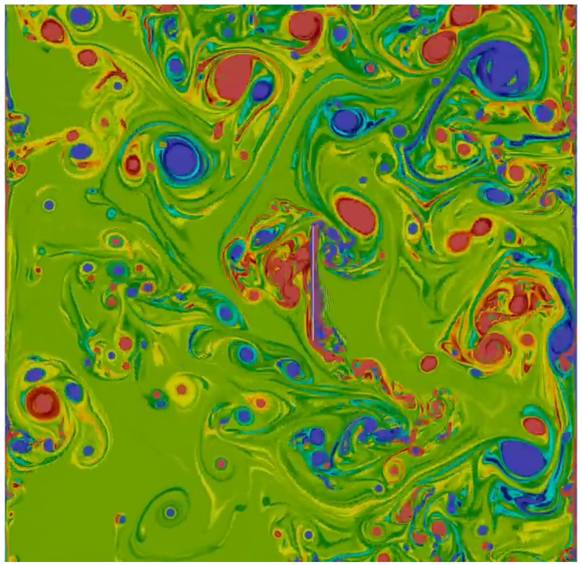
UK FLUIDS NETWORK



De Rosis
Europhys Lett 2018

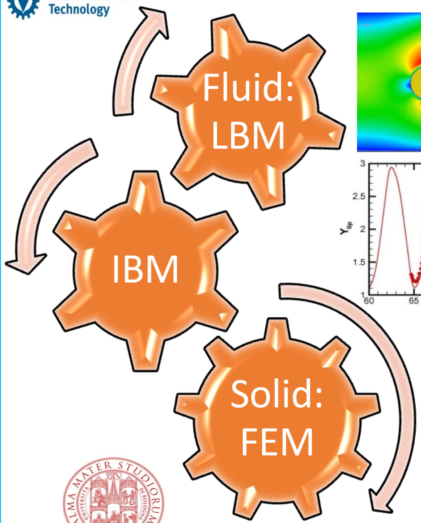


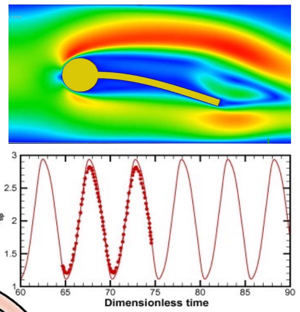
Multiphysics modelling by LBM

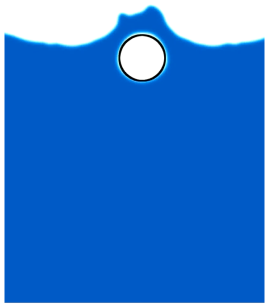
THEORY OF LBM & NUMERICAL SIMULATIONS

Fluid-structure interaction

Technion Israel Institute of Technology · NYU·poly POLYTECHNIC INSTITUTE OF NYU







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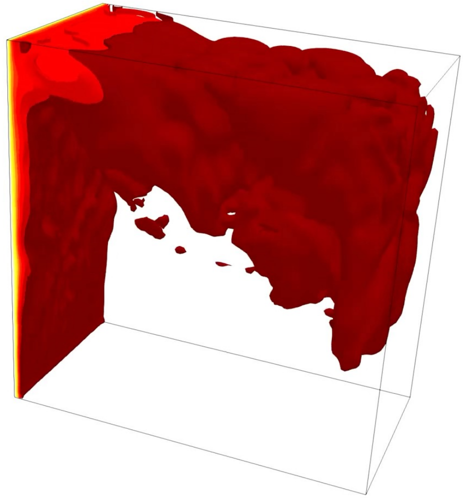
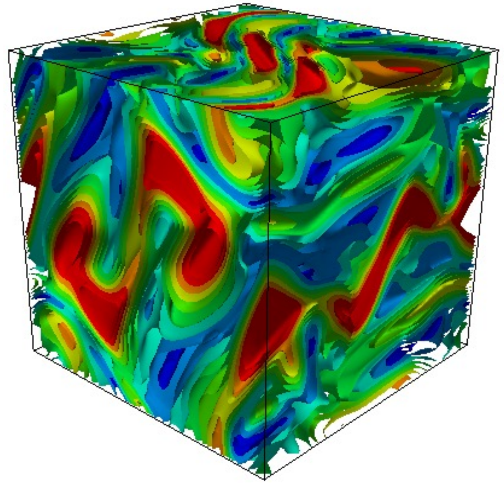
De Rosis & Tafuni (2022)
Comput Aid Civ Inf

Turbulence & Magnetohydrodynamics

ÉCOLE CENTRALE LYON



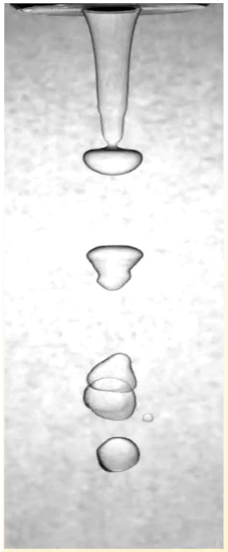
Natural convection

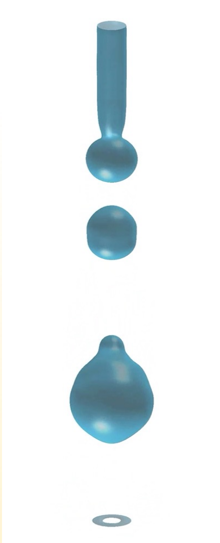


3D MHD vortex: electric current
De Rosis (2017) Phys Rev E Kaleid.

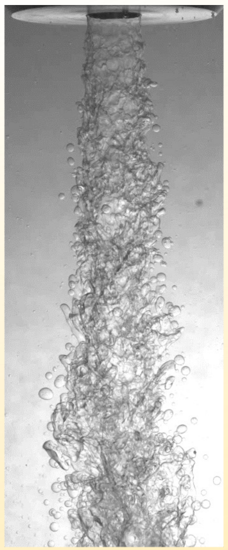
Multiphase/component flows

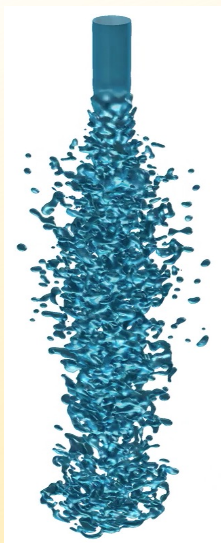
Electric Ant Lab
Scientific Modeling & Simulations



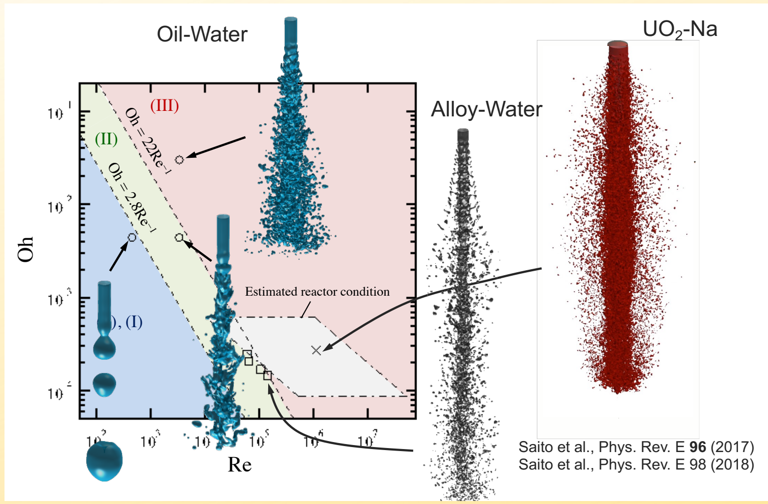


Varicose break-up





Atomisation



Liquid jet break-up
Saito, De Rosis et al. (2018) Phys Rev E Kaleidoscope

Magnetohydrodynamics modelling

Macroscopic

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\frac{\nabla p}{\rho} + \nu\Delta\boldsymbol{u} + \frac{\boldsymbol{j}\times\boldsymbol{B}}{\rho}$$

$$\partial_t \boldsymbol{B} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{B} = (\boldsymbol{B}\cdot\nabla)\boldsymbol{u} + \lambda\Delta\boldsymbol{B}$$

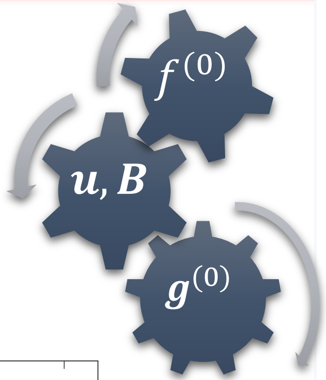
$$\nabla\cdot\boldsymbol{u} = 0, \nabla\cdot\boldsymbol{B} = 0$$

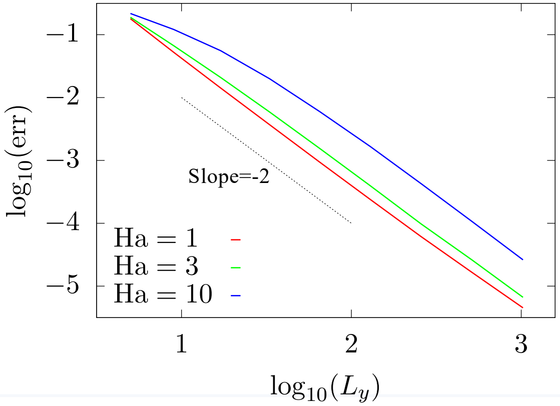
Kinetic

Dellar (2002) J Comput Phys

$$\partial_t f + \boldsymbol{c}\cdot\nabla f = \frac{f^{(0)} - f}{\tau}$$

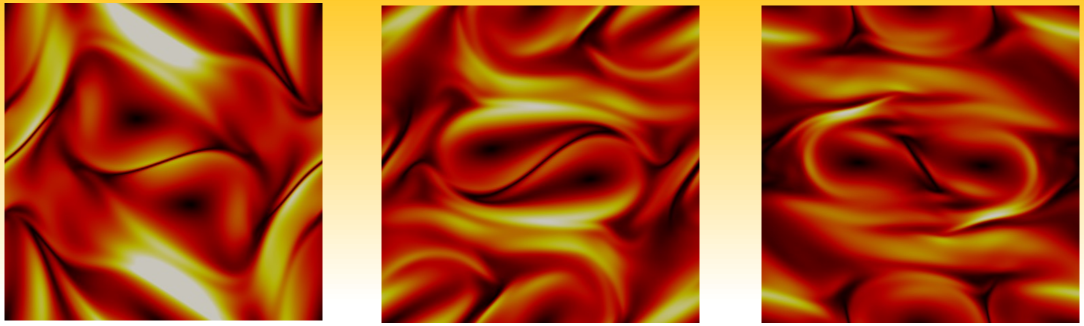
$$\partial_t \boldsymbol{g} + \boldsymbol{c}'\cdot\nabla \boldsymbol{g} = \frac{\boldsymbol{g}^{(0)} - \boldsymbol{g}}{\tau_m}$$





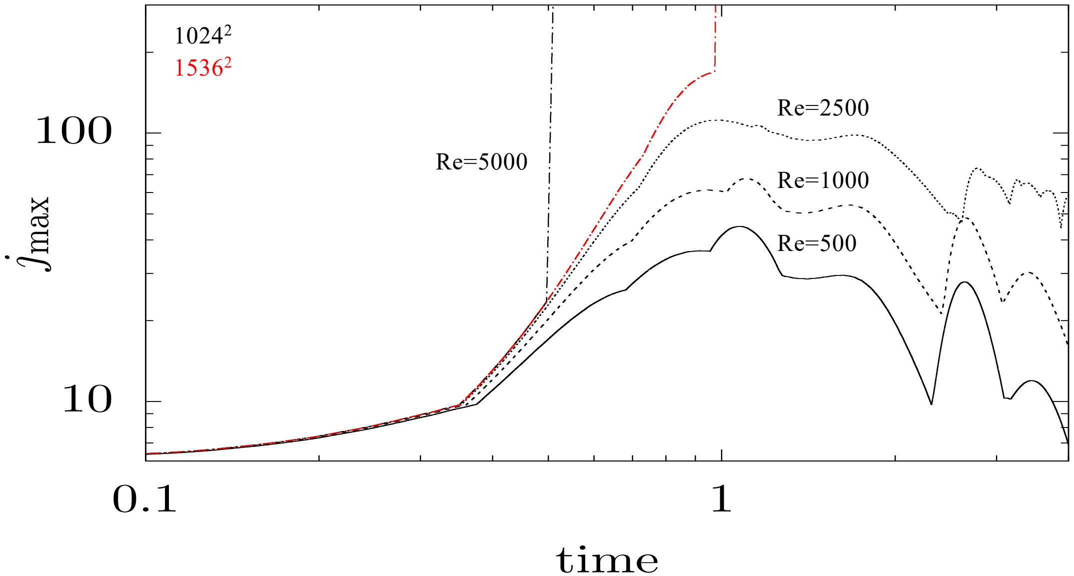
Hartmann flow: convergence analysis

A tough problem: 2D Orszag-Tang vortex



t=0.5 t=1 t=1.5

Evolution of the magnetic field at Re=2500



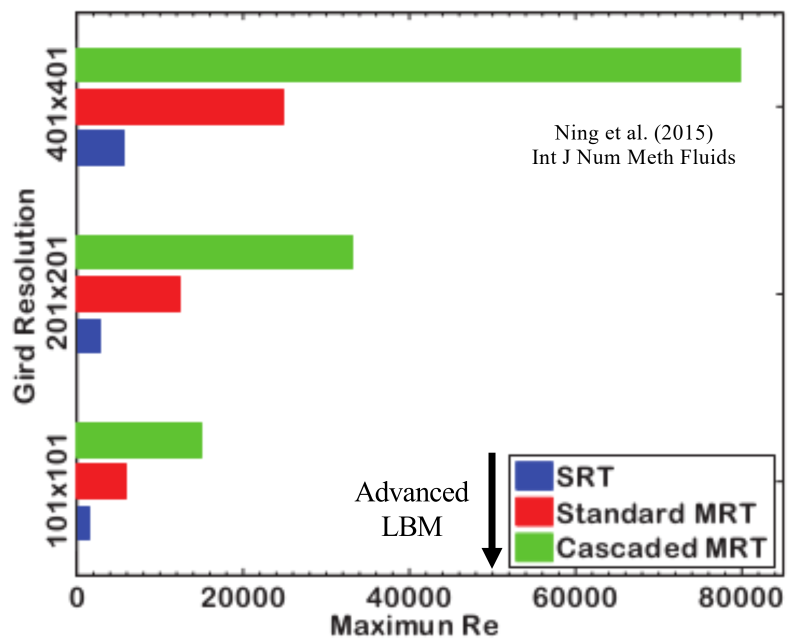
Maxima of the electric current at different Reynolds numbers

De Rosis et al. (2018) J Turbul

Advanced LBM for high-Re flows

Cascaded LBM

Geier et al. (2006) Phys Rev E

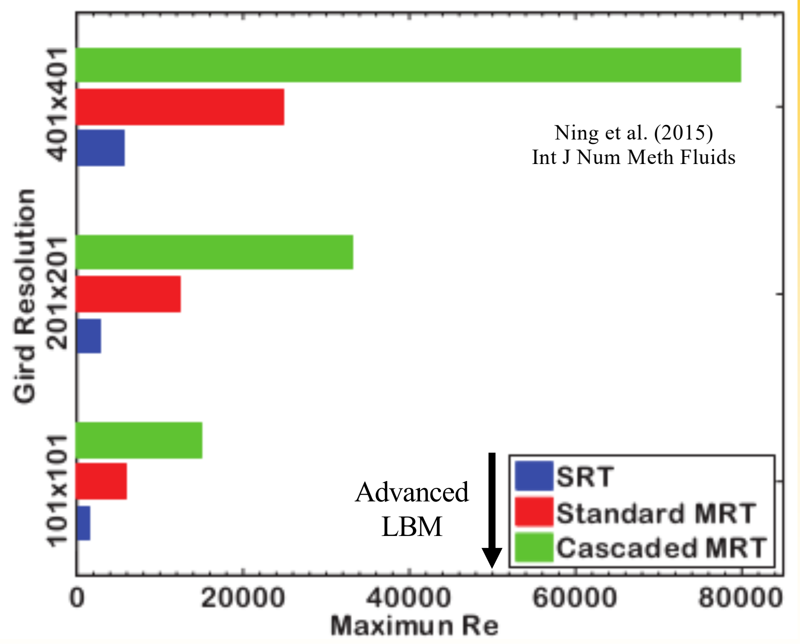


Lid-driven cavity: Maximum attenaible Reynolds number by different LB models and grid resolutions

Advanced LBM for high-Re flows

Cascaded LBM

Geier et al. (2006) Phys Rev E



Lid-driven cavity: Maximum attenaible Reynolds number by different LB models and grid resolutions

Theory and implementation might be cumbersome (especially in 3D…)

$$
\begin{aligned}
\widehat{g}_{26} = \frac{\omega_{26}}{8} \Big[& -\widehat{\overline{\eta}}'_{xxyyzz} + 2\left(u_x \widehat{\overline{\eta}}'_{xyyzz} + u_y \widehat{\overline{\eta}}'_{xxyzz} + u_z \widehat{\overline{\eta}}'_{xxyyz}\right) - \Big(u_x^2 \widehat{\overline{\eta}}'_{yyzz} \\
& + u_y^2 \widehat{\overline{\eta}}'_{xxzz} + u_z^2 \widehat{\overline{\eta}}'_{xxyy}\Big) - 4\left(u_x u_y \widehat{\overline{\eta}}'_{xyzz} + u_x u_z \widehat{\overline{\eta}}'_{xyyz} + u_y u_z \widehat{\overline{\eta}}'_{xxyz}\right) \\
& +2\Big(u_x^2 u_y \widehat{\overline{\eta}}'_{yzz} + u_x u_y^2 \widehat{\overline{\eta}}'_{xzz} + u_x^2 u_z \widehat{\overline{\eta}}'_{yyz} + u_x u_z^2 \widehat{\overline{\eta}}'_{xyy} + u_y^2 u_z \widehat{\overline{\eta}}'_{xxz} \\
& + u_y u_z^2 \widehat{\overline{\eta}}'_{xxy}\Big) + 8 u_x u_y u_z \widehat{\overline{\eta}}'_{xyz} - \left(u_y^2 u_z^2 \widehat{\overline{\eta}}'_{xx} + u_x^2 u_z^2 \widehat{\overline{\eta}}'_{yy} + u_x^2 u_y^2 \widehat{\overline{\eta}}'_{zz}\right) \\
& -4 u_x u_y u_z \left(u_z \widehat{\overline{\eta}}'_{xy} + u_y \widehat{\overline{\eta}}'_{xz} + u_x \widehat{\overline{\eta}}'_{yz}\right) + 5\rho u_x^2 u_y^2 u_z^2 + \\
& \widetilde{\overline{\kappa}}_{xx} \widetilde{\overline{\kappa}}_{yy} \widetilde{\overline{\kappa}}_{zz} + u_x u_y u_z \left(u_x u_y \widehat{\sigma}'_z + u_x u_z \widehat{\sigma}'_y + u_y u_z \widehat{\sigma}'_x\right)\Big] \\
& + \left(-4 u_x u_y - 6 u_x u_y u_z^2\right) \widehat{g}_4 + \left(-4 u_x u_z - 6 u_x u_y^2 u_z\right) \widehat{g}_5 \\
& + \left(-4 u_y u_z - 6 u_x^2 u_y u_z\right) \widehat{g}_6 + \left(\frac{1}{2} u_x^2 - \frac{1}{2} u_y^2 + \frac{3}{4} u_x^2 u_z^2 - \frac{3}{4} u_y^2 u_z^2\right) \widehat{g}_7 \\
& + \left(\frac{1}{2} u_x^2 + \frac{1}{2} u_y^2 - u_z^2 - \frac{3}{4} u_y^2 u_z^2 - \frac{3}{4} u_x^2 u_z^2 + \frac{3}{2} u_x^2 u_y^2\right) \widehat{g}_8 \\
& + \left(-u_x^2 - u_y^2 - u_z^2 - \frac{3}{4} u_x^2 u_y^2 - \frac{3}{4} u_x^2 u_z^2 - \frac{3}{4} u_y^2 u_z^2 - 1\right) \widehat{g}_9 \\
& + \left(3 u_x u_y^2 + 3 u_x u_z^2 + 4 u_x\right) \widehat{g}_{10} + \left(3 u_x^2 u_y + 3 u_y u_z^2 + 4 u_y\right) \widehat{g}_{11} \\
& + \left(3 u_x^2 u_z + 3 u_y^2 u_z + 4 u_z\right) \widehat{g}_{12} + \left(u_x u_z^2 - u_x u_y^2\right) \widehat{g}_{13} \\
& + \left(u_y u_z^2 - u_x^2 u_y\right) \widehat{g}_{14} + \left(u_y^2 u_z - u_x^2 u_z\right) \widehat{g}_{15} + 8 u_x u_y u_z \widehat{g}_{16} \\
& + \left(-\frac{1}{2} u_x^2 - \frac{1}{2} u_y^2 - \frac{1}{2} u_z^2 - 1\right) \widehat{g}_{17} + \left(u_x^2 - \frac{1}{2} u_y^2 - \frac{1}{2} u_z^2\right) \widehat{g}_{18} \\
& + \left(\frac{1}{2} u_y^2 - \frac{1}{2} u_z^2\right) \widehat{g}_{19} - 4 u_y u_z \widehat{g}_{20} - 4 u_x u_z \widehat{g}_{21} - 4 u_x u_y \widehat{g}_{22} + 2 u_x \widehat{g}_{23} \\
& + 2 u_y \widehat{g}_{24} + 2 u_z \widehat{g}_{25}.
\end{aligned}
$$

Advanced LBM for high-Re flows

Central-moments-based LBM

Central moments are defined as:

$$k_{x^a y^b z^c} = \sum_i f_i \bar{c}_{xi}^a \bar{c}_{yi}^b \bar{c}_{zi}^c$$

Key idea:

$$\bar{c}_{xi} = c_{xi} - u_x$$
$$\bar{c}_{yi} = c_{yi} - u_y$$
$$\bar{c}_{zi} = c_{zi} - u_z$$

Advanced LBM for high-Re flows

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$\bar{T}(9) = \bar{c}_{xi}^2 + \bar{c}_{yi}^2 + \bar{c}_{zi}^2$ $\bar{T}(10) = \bar{c}_{xi}\bar{c}_{yi}^2 + \bar{c}_{xi}\bar{c}_{zi}^2$
…
$\bar{T}(24) = \bar{c}_{xi}^2\bar{c}_{yi}\bar{c}_{zi}^2$ $\bar{T}(25) = \bar{c}_{xi}^2\bar{c}_{yi}^2\bar{c}_{zi}$
$\bar{T}(26) = \bar{c}_{xi}^2\bar{c}_{yi}^2\bar{c}_{zi}^2$

D3Q27

Advanced LBM for high-Re flows

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…
$\bar{T}(24) = \bar{c}_{xi}^2\bar{c}_{yi}\bar{c}_{zi}^2$ $\bar{T}(25) = \bar{c}_{xi}^2\bar{c}_{yi}^2\bar{c}_{zi}$
$\bar{T}(26) = \bar{c}_{xi}^2\bar{c}_{yi}^2\bar{c}_{zi}^2$

D3Q27

Algorithm of computation

1. Compute pre-collision central moments:

$$k = \bar{T} f$$

2. Evaluate equilibrium state:

$$k^{(0)} = \bar{T} f^{(0)}$$

3. Collide central moments:

$$k_i^* = (1 - \omega_i) k_i + \omega_i k_i^{(0)}$$

4. Reconstruct post-collision populations:

$$f^* = \bar{T}^{-1} k^*$$

De Rosis (2017) Phys Rev E

Advanced LBM for high-Re flows

Some practical aspects

- Moments $k^*_{0\ldots3}$ are collision invariants
- Only 2nd-order moments $k^*_{4\ldots8}$ are related to the fluid kinematic viscosity, so $\omega = (3\nu + 0.5)^{-1}$
- Higher-order moments refer to non-hydrodynamic ghost modes, so $\omega = 1$
- We need to collide only few moments!

Post-collision central moments:

$$k^*_0 = \rho \qquad k^*_9 = \rho$$
$$k^*_4 = (1-\omega)k_4 \qquad k^*_{17} = \rho c_s^2$$
$$k^*_5 = (1-\omega)k_5 \qquad k^*_{18} = \rho c_s^4$$
$$k^*_6 = (1-\omega)k_6 \qquad k^*_{26} = \rho c_s^6$$
$$k^*_7 = (1-\omega)k_7$$
$$k^*_8 = (1-\omega)k_8$$

De Rosis & Luo (2019) Phys Rev E

- Equilibrium distributions:

$$\begin{aligned}
f_i^{(0)} = w_i\rho[1 &+ \frac{\boldsymbol{\xi}_i\cdot\boldsymbol{u}}{c_s^2} + \frac{1}{2c_s^4}\mathcal{H}_i^{(2)} : \boldsymbol{uu} \\
&+ \frac{1}{2c_s^6}(\mathcal{H}^{(3)}_{ixxy}u_x^2u_y + \mathcal{H}^{(3)}_{ixxz}u_x^2u_z + \mathcal{H}^{(3)}_{ixyy}u_xu_y^2 \\
&+ \mathcal{H}^{(3)}_{ixzz}u_xu_z^2 + \mathcal{H}^{(3)}_{iyzz}u_yu_z^2 + \mathcal{H}^{(3)}_{iyyz}u_y^2u_z \\
&+ \mathcal{H}^{(3)}_{ixyz}u_xu_yu_z) \\
&+ \frac{1}{4c_s^8}(\mathcal{H}^{(4)}_{ixxyy}u_x^2u_y^2 + \mathcal{H}^{(4)}_{ixxzz}u_x^2u_z^2 + \mathcal{H}^{(4)}_{iyyzz}u_y^2u_z^2 \\
&+ 2(\mathcal{H}^{(4)}_{ixyzz}u_xu_yu_z^2 + \mathcal{H}^{(4)}_{ixyyz}u_xu_y^2u_z + \mathcal{H}^{(4)}_{ixxyz}u_x^2u_yu_z)) \\
&+ \frac{1}{4c_s^{10}}(\mathcal{H}^{(5)}_{ixxyzz}u_x^2u_yu_z^2 + \mathcal{H}^{(5)}_{ixxyyz}u_x^2u_y^2u_z \\
&+ \mathcal{H}^{(5)}_{ixyyzz}u_xu_y^2u_z^2) \\
&+ \frac{1}{8c_s^{12}}\mathcal{H}^{(6)}_{ixxyyzz}u_x^2u_y^2u_z^2],
\end{aligned}$$

Malaspinas (2015) arXiv

Coreixas et al. (2017) Phys Rev E

Equilibrium central moments:

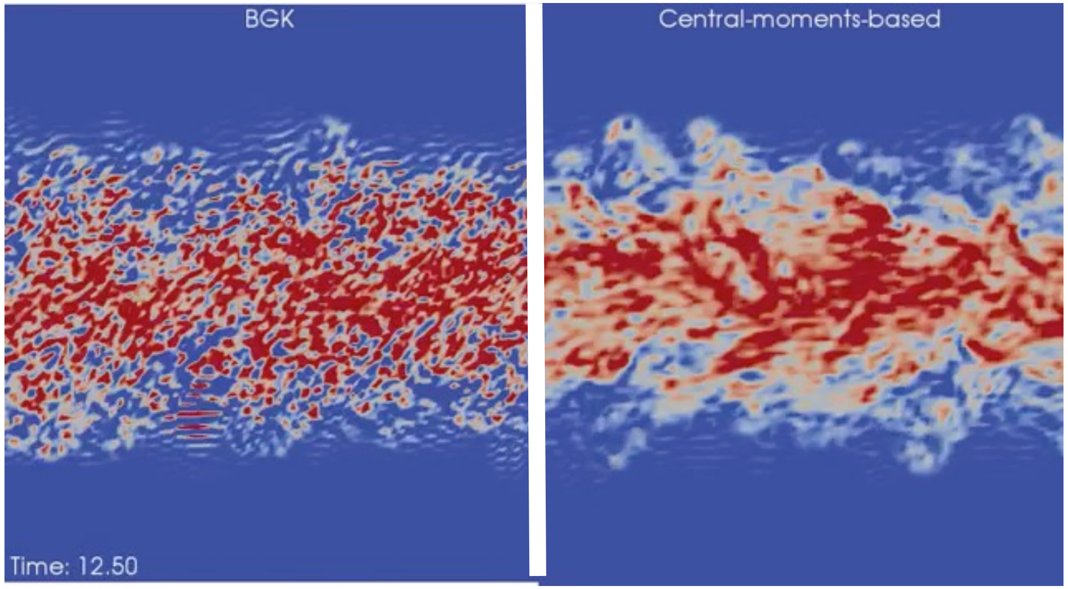
$$k_0^{(0)} = \rho, \qquad k_9^{(0)} = \rho,$$
$$k_{17}^{(0)} = \rho c_s^2, \qquad k_{18}^{(0)} = \rho c_s^4, \qquad k_{26}^{(0)} = \rho c_s^6$$

Key features

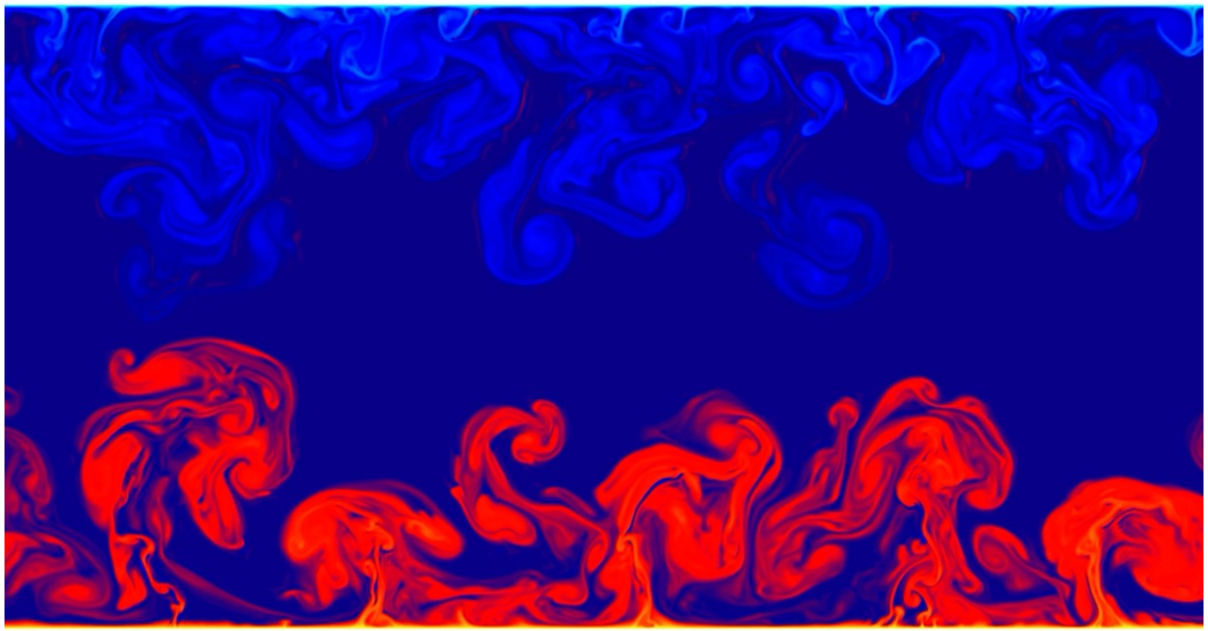
Properties:

De Rosis
Comput Meth Appl M 2017

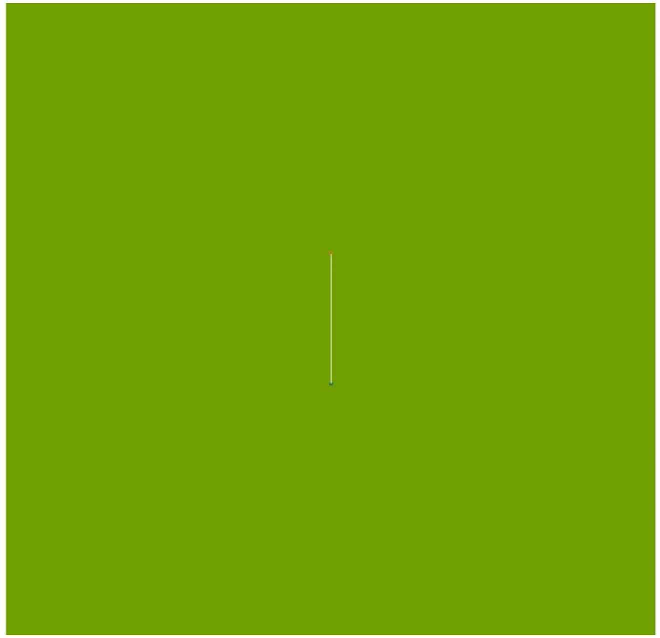
- 2°-order accurate
- Improved stability
- Intelligible derivations
- Easy implementation
- Very general framework



High-Re high-Sc scalar transport by BGK (left) and CMs (right)



Rayleigh-Benard convection at Ra=10^{11}



Oscillating bade - De Rosis Europhys Lett 2018

Advanced LBM for high-Re MHD flows

Central moments for Dellar's 2D MHD

$$f_i^{(0)} = w_i \rho \left[1 + \frac{\boldsymbol{c_i} \cdot \boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{c_i} \cdot \boldsymbol{u})^2}{2c_s^4} - \frac{\boldsymbol{u} \cdot \boldsymbol{u}}{2c_s^2} \right] + \frac{w_i}{2c_s^4} \left[\frac{1}{2} |\boldsymbol{c_i}|^2 |\boldsymbol{B}|^2 - (\boldsymbol{c_i} \cdot \boldsymbol{B})^2 \right]$$

1. Compute pre-collision central moments:

$$k = \bar{T} f$$

2. Evaluate equilibrium state:

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3. Collide central moments:

$$k_i^* = (1 - \omega_i) k_i + \omega_i k_i^{(0)}$$

4. Reconstruct post-collision populations:

$$f^* = \bar{T}^{-1} k^*$$

Build a basis $\bar{T}$: $k_{x^a y^b} \leftrightarrow f_i$

$\bar{T}(0) = [1,\ldots,1]$

$\bar{T}(1) = \bar{c}_{xi}$ $\bar{T}(2) = \bar{c}_{yi}$ $\bar{T}(3) = \bar{c}_{xi}^2 + \bar{c}_{yi}^2$

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Advanced LBM for high-Re MHD flows

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$$f_i^{(0)} = w_i\rho\left[1 + \frac{\boldsymbol{c_i}\cdot\boldsymbol{u}}{c_s^2} + \frac{(\boldsymbol{c_i}\cdot\boldsymbol{u})^2}{2c_s^4} - \frac{\boldsymbol{u}\cdot\boldsymbol{u}}{2c_s^2}\right] + \frac{w_i}{2c_s^4}\left[\frac{1}{2}|\boldsymbol{c_i}|^2|\boldsymbol{B}|^2 - (\boldsymbol{c_i}\cdot\boldsymbol{B})^2\right]$$

D2Q9 post-collision central moments:

$$k_0^* = \rho$$
$$k_1^* = 0$$
$$k_2^* = 0$$
$$k_3^* = 2\rho c_s^2$$
$$k_4^* = (1-\omega_4)k_4 + \omega_4(B_y^2 - B_x^2)$$
$$k_5^* = (1-\omega_5)k_5 + \omega_5(-B_xB_y)$$
$$k_6^* = \frac{u_y}{2}(B_x^2 - B_y^2) + 2u_xB_xB_y$$
$$k_7^* = \frac{u_x}{2}(B_y^2 - B_x^2) + 2u_yB_xB_y$$
$$k_8^* = \rho c_s^4 + \frac{u_x^2 - u_y^2}{2}(B_x^2 - B_y^2) - 4u_xu_yB_xB_y$$

Build a basis $\bar{T}$: $k_{x^a y^b} \leftrightarrow f_i$

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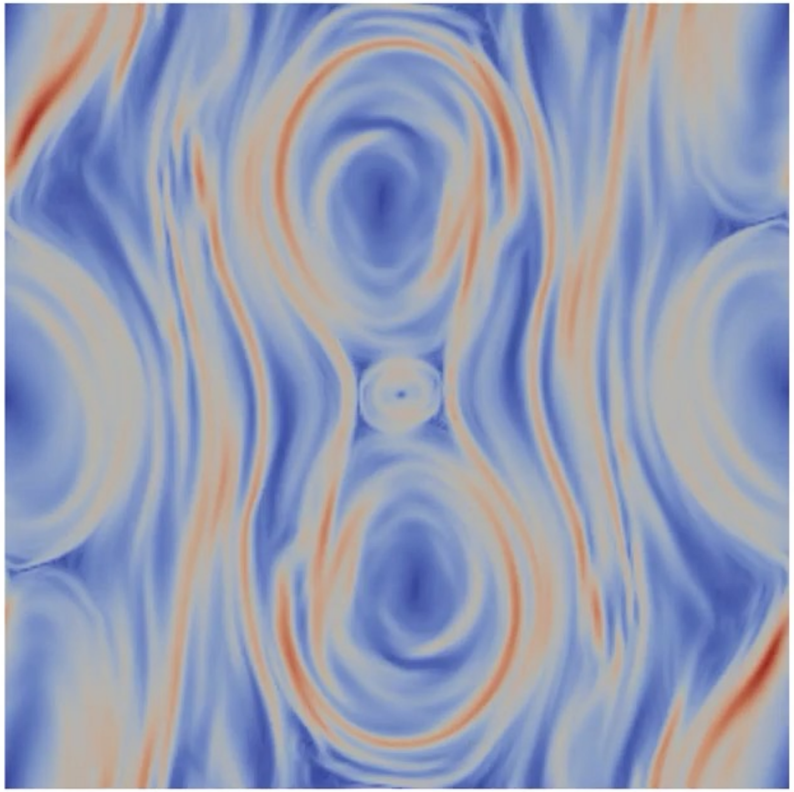
$\bar{T}(1) = \bar{c}_{xi}$ $\quad$ $\bar{T}(2) = \bar{c}_{yi}$ $\quad$ $\bar{T}(3) = \bar{c}_{xi}^2 + \bar{c}_{yi}^2$

$\bar{T}(4) = \bar{c}_{xi}^2 - \bar{c}_{yi}^2$ $\quad$ $\bar{T}(5) = \bar{c}_{xi}\bar{c}_{yi}$ $\quad$ $\bar{T}(6) = \bar{c}_{xi}^2\bar{c}_{yi}$

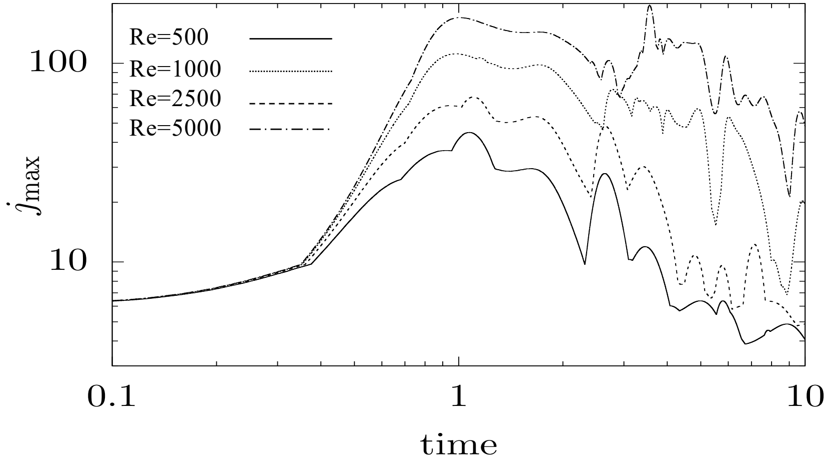
$\bar{T}(7) = \bar{c}_{xi}\bar{c}_{yi}^2$ $\quad$ $\bar{T}(8) = \bar{c}_{xi}^2\bar{c}_{yi}^2$



2D OT vortex: magnetic field

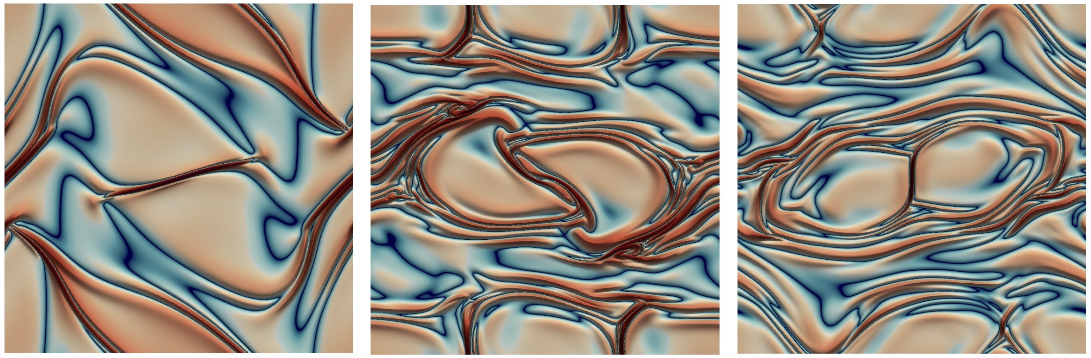
Advanced LBM for high-Re MHD flows

Stability is enhanced!



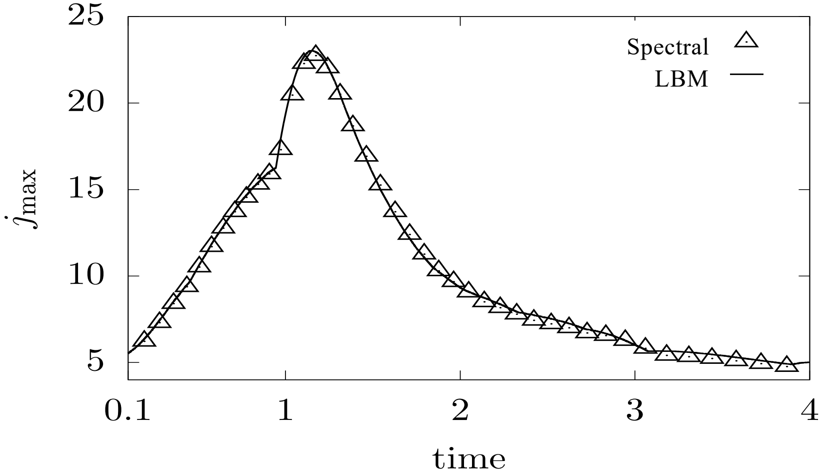
2D OT vortex: Maxima of the electric current

De Rosis et al. (2018) J Turbul



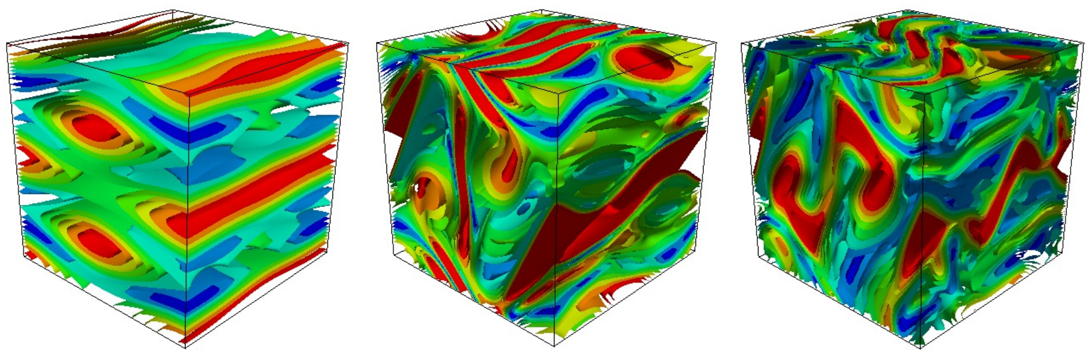
t=1 t=3.5 t=9

2D OT vortex: electric current at Re=5000



3D OT vortex: Maxima of the electric current at Re=100

De Rosis (2017) Phys Rev E Kaleidoscope

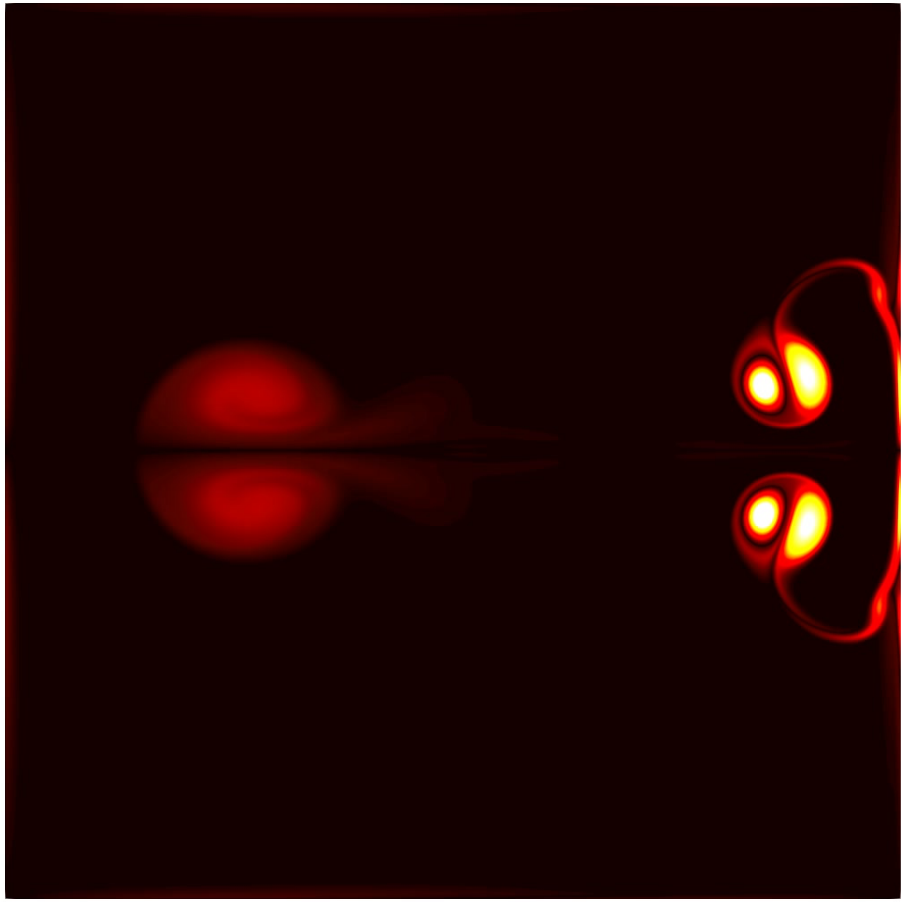


t=0 t=1 t=2

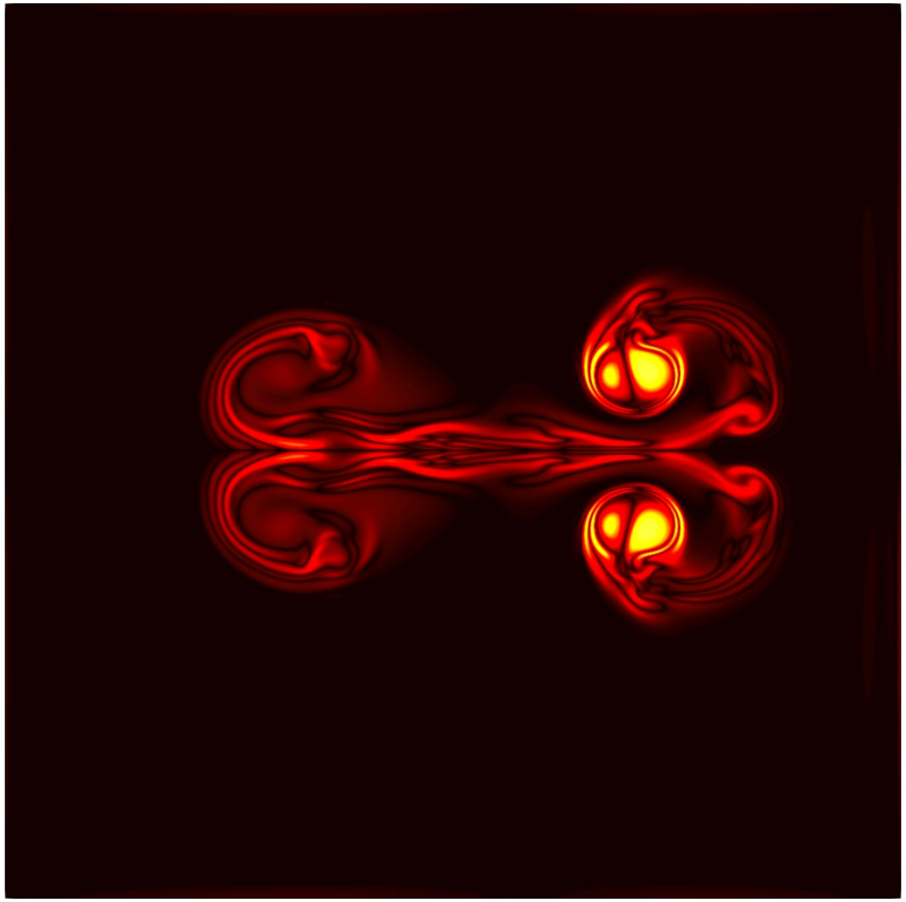
3D OT vortex: electric current at Re=100

MHD physics: vortex-wall collision

De Rosis & Skillen
Physics of Fluids 2022



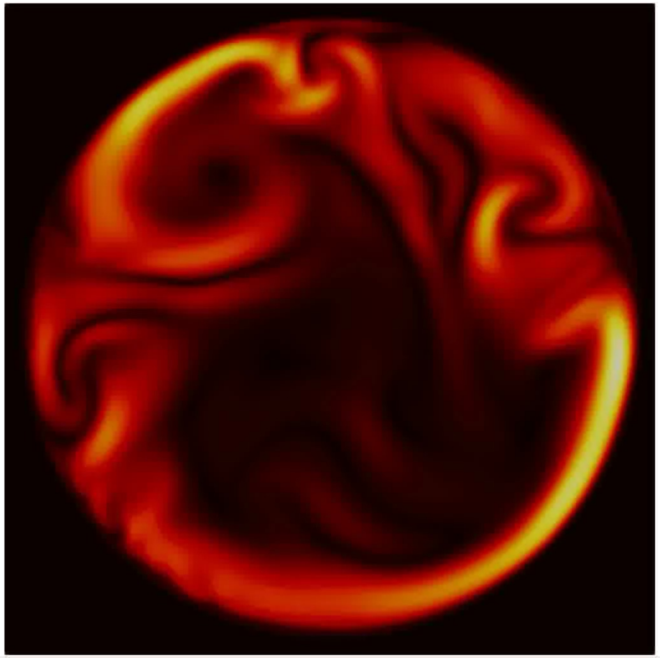
Without magnetic field



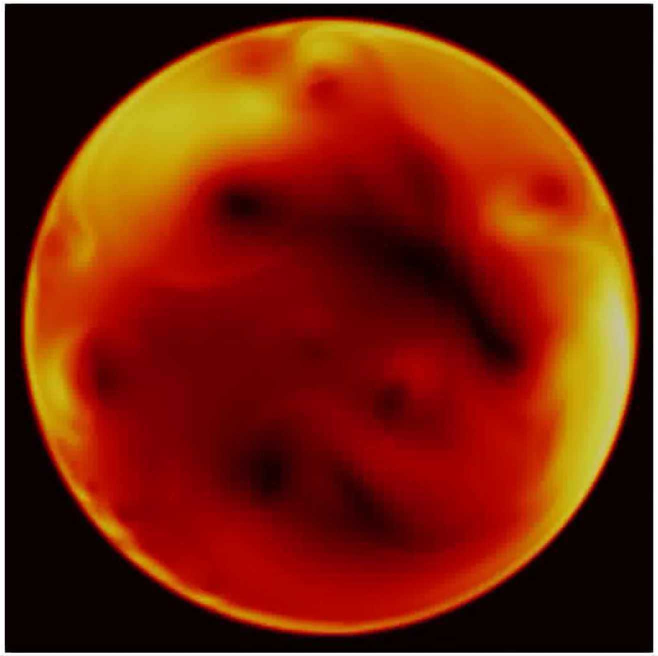
With magnetic field

MHD application: flow in rotating cavities

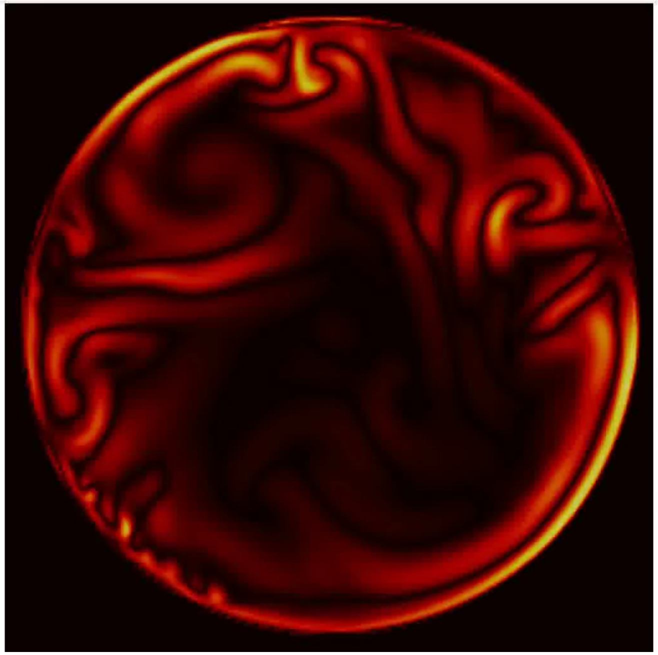
Magnetic field



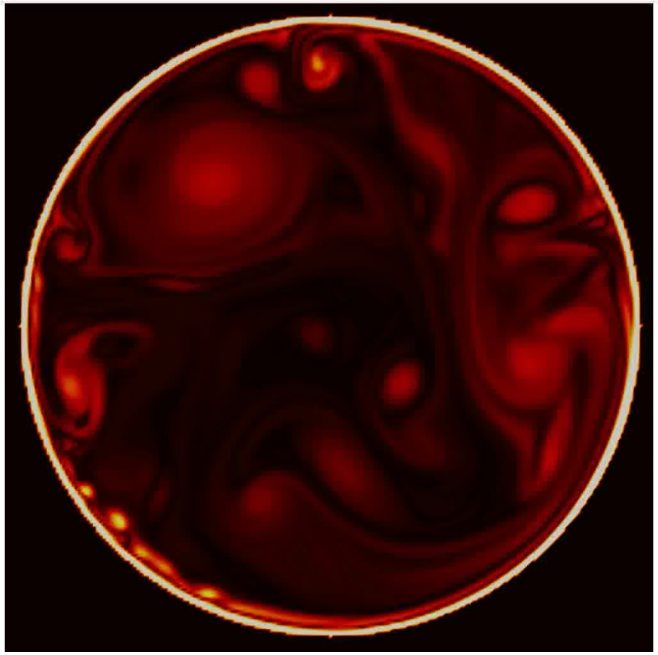
Flow velocity



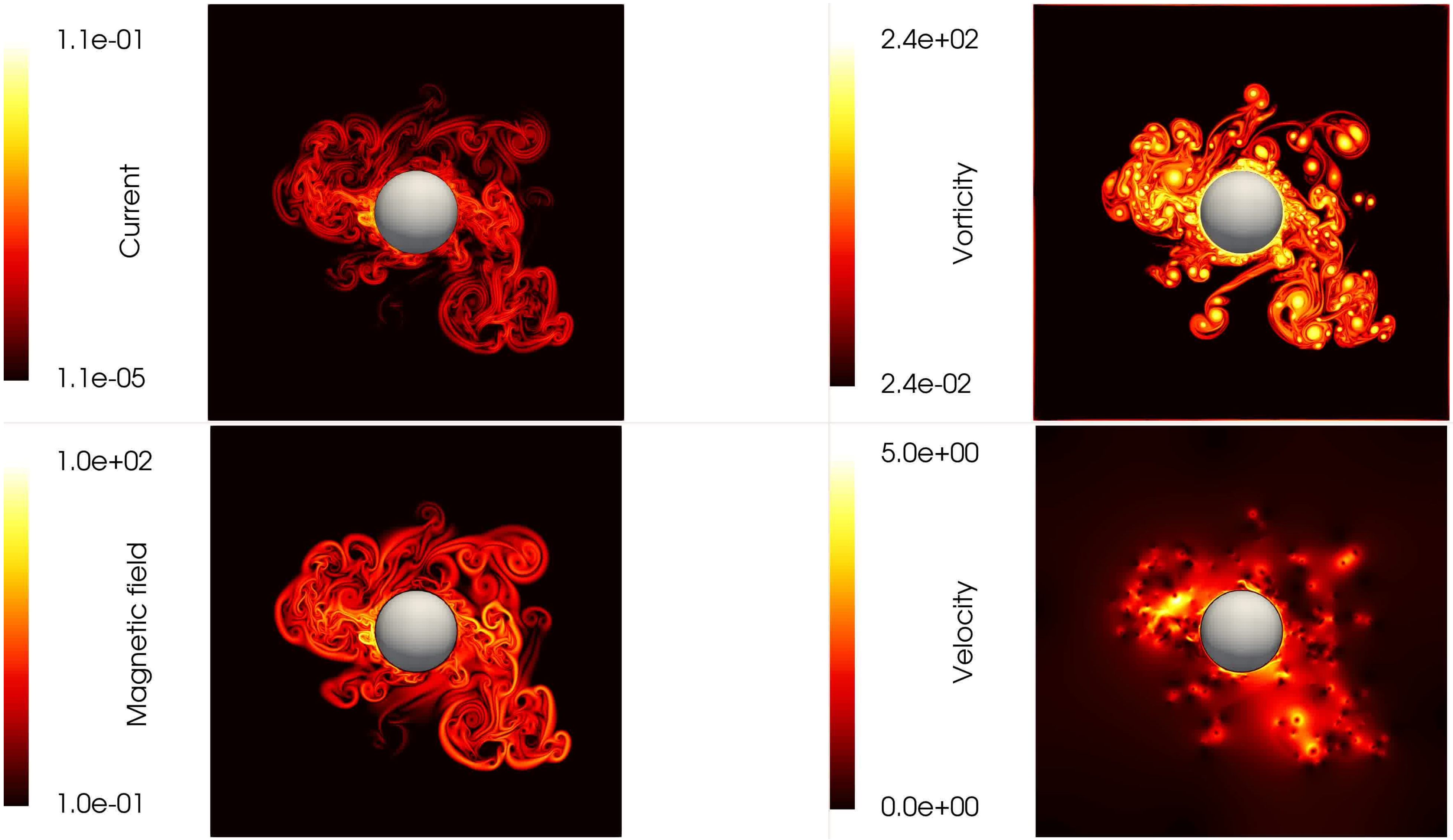
Current density



Flow vorticity



MHD application: flow detachment



Advanced LBM for multicomponent flows

Macroscopic

$$\nabla \cdot \mathbf{u} = 0,$$

$$\rho[\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u}] = -\nabla p + \mu \nabla^2 \mathbf{u} + \boldsymbol{F}$$

$$\partial_t \phi + \nabla \cdot \phi \mathbf{u} = \nabla \cdot M \left[\nabla \phi - \frac{\nabla \phi}{|\nabla \phi|} \frac{1 - 4(\phi - \phi_0)^2}{\xi} \right]$$

Kinetic

$$\partial_t f + \boldsymbol{c} \cdot \nabla f = \frac{f^{(0)} - f}{\tau}$$

De Rosis & Enan (2021) Phys Fluids

De Rosis & Tafuni (2022) Comput Aid Civ Inf

$$\partial_t g + \boldsymbol{c}' \cdot \nabla g = \frac{g^{(0)} - g}{\tau_\phi}$$

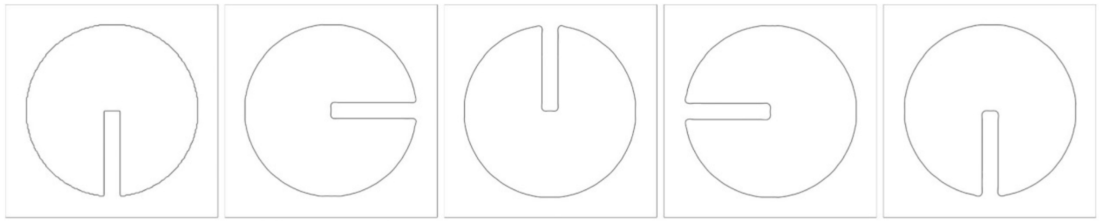
$$f_i^\star = f_i + \frac{1}{\tau + 1/2}\left(f_i^{eq} - f_i\right) + F_i,$$

$$g_i^\star = g_i + \frac{1}{\tau_\phi + 1/2}\left(g_i^{eq} - g_i\right) + G_i,$$

$$f_i^{eq} = w_i \left[\tilde{p} + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2} \right],$$

$$g_i^{eq} = w_i \phi \left[1 + \frac{\mathbf{c}_i \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{c}_i \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u}^2}{2c_s^2} \right]$$

Zalesak disk



(a)FD at $t = 0$ (b)FD at $t = T/4$ (c)FD at $t = T/2$ (d)FD at $t = 3T/4$ (e)FD at $t = T$

$$u_x = -U_0 2\pi \left(\frac{y}{L_0} - 0.5 \right), \qquad u_y = U_0 2\pi \left(\frac{x}{L_0} - 0.5 \right).$$

TABLE III. Zalesak disk: relative error at different Pe. Reproduced by Zu *et al.*, "Phase-field lattice Boltzmann model for interface tracking of a binary fluid system based on the Allen-Cahn equation," with permission from Phys. Rev. E **102**, 053307 (2020). Copyright 2020 APS Publishing

	Pe			
Model	80	400	800	4000
FD	0.0593	0.0590	0.0558	0.0505
Mom	0.0567	0.0482	0.0483	0.0501
Ref. 52 (Zu *et al.*)	0.1226	0.1186	0.1170	0.1194
Ref. 52 (Geier *et al.*)	0.1307	0.1224	0.1209	0.1472
Ref. 52 (Wang *et al.*)	0.1307	0.1224	0.1209	0.1471

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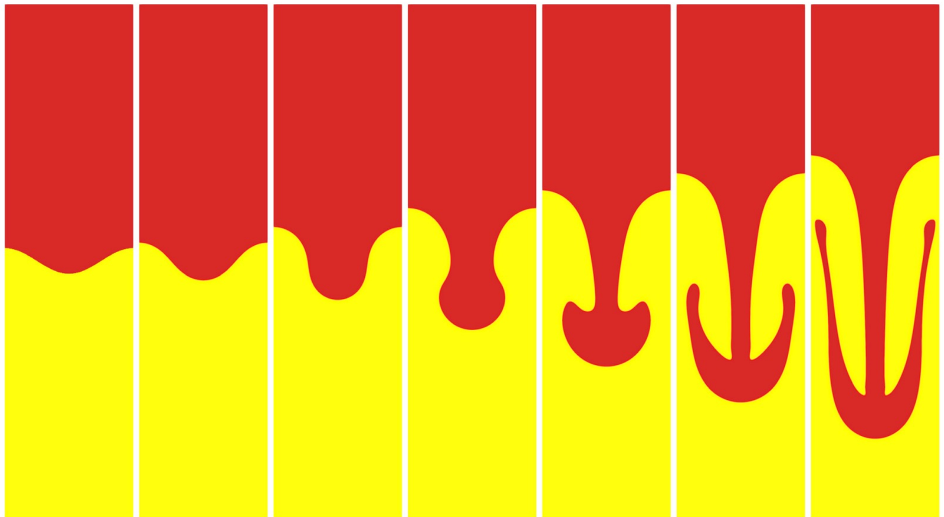
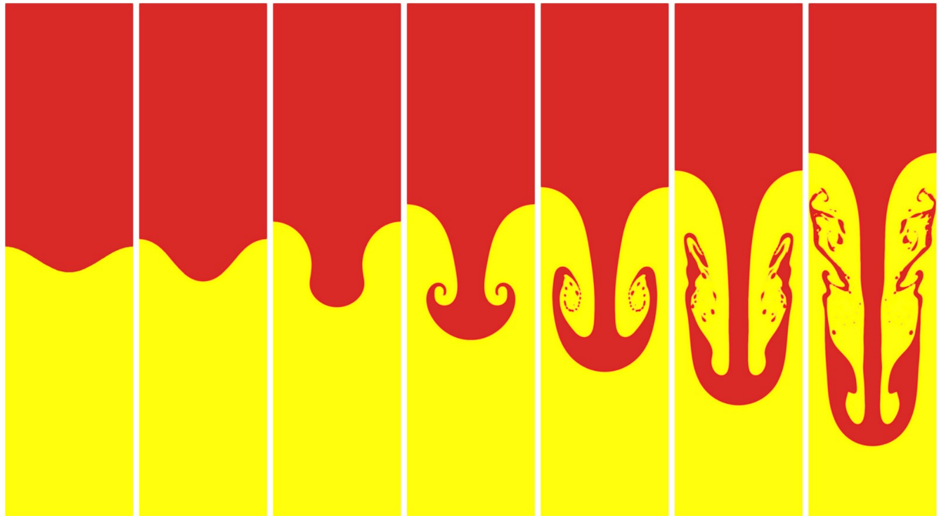
2D Rayleigh-Taylor instability



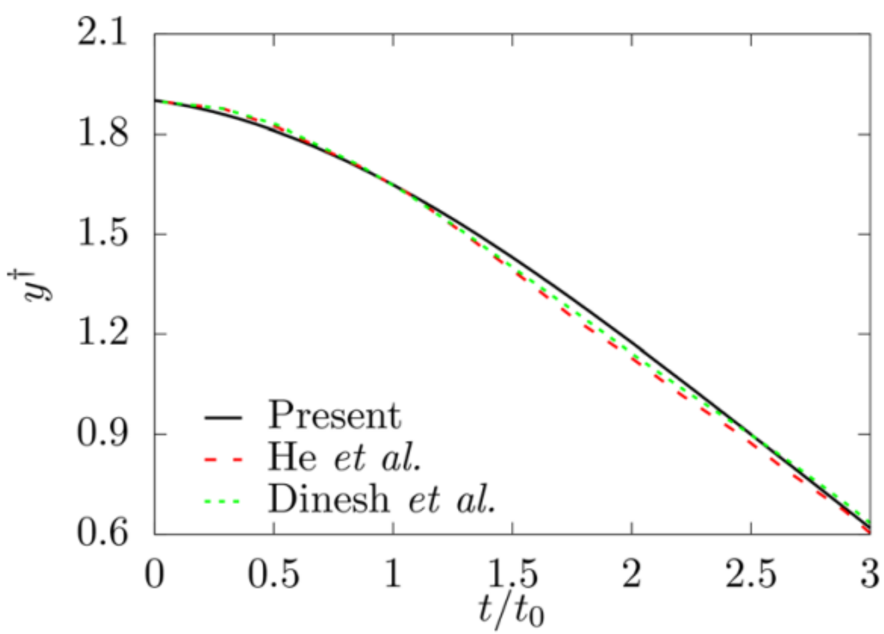
(a)$t/t_0 = 0$ (b)$t/t_0 = 0.5$ (c)$t/t_0 = 1.0$ (d)$t/t_0 = 1.5$ (e)$t/t_0 = 2.0$ (f)$t/t_0 = 2.5$ (g)$t/t_0 = 3.0$

FIG. 12. Two-dimensional Rayleigh–Taylor instability at Re = 256: evolution of the interface at (a) $t/t_0 = 0$, (b) $t/t_0 = 0.5$, (c) $t/t_0 = 1.0$, (d) $t/t_0 = 1.5$, (e) $t/t_0 = 2.0$, (f) $t/t_0 = 2.5$ and (g) $t/t_0 = 3.0$. Red and yellow colors correspond to heavy and light fluids, respectively.

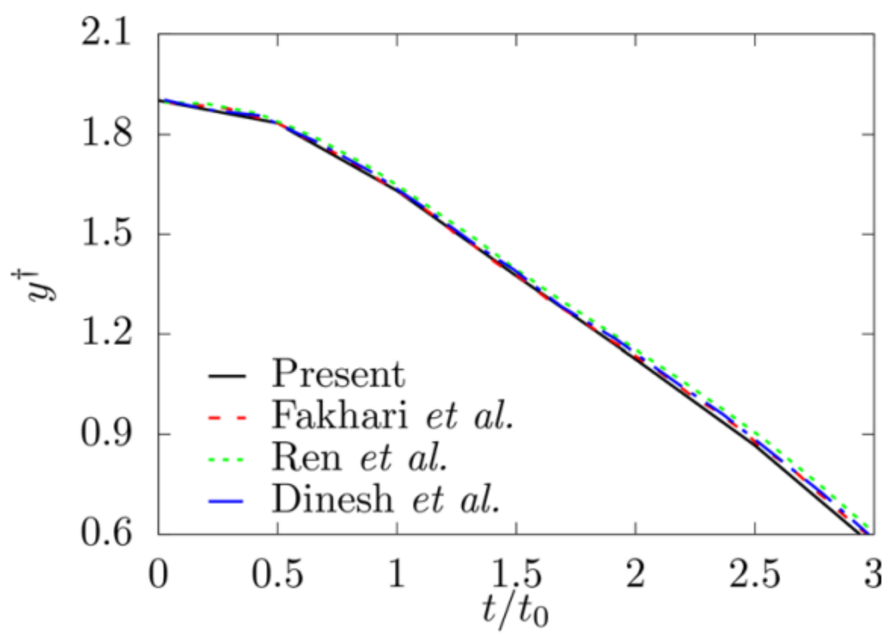


(a)$t/t_0 = 0$ (b)$t/t_0 = 0.5$ (c)$t/t_0 = 1.0$ (d)$t/t_0 = 1.5$ (e)$t/t_0 = 2.0$ (f)$t/t_0 = 2.5$ (g)$t/t_0 = 3.0$

FIG. 13. Two-dimensional Rayleigh–Taylor instability at Re = 3000: evolution of the interface at (a) $t/t_0 = 0$, (b) $t/t_0 = 0.5$, (c) $t/t_0 = 1.0$, (d) $t/t_0 = 1.5$, (e) $t/t_0 = 2.0$, (f) $t/t_0 = 2.5$ and (g) $t/t_0 = 3.0$. Red and yellow colors correspond to heavy and light fluids, respectively.



(a)Re = 256



(b)Re = 3000

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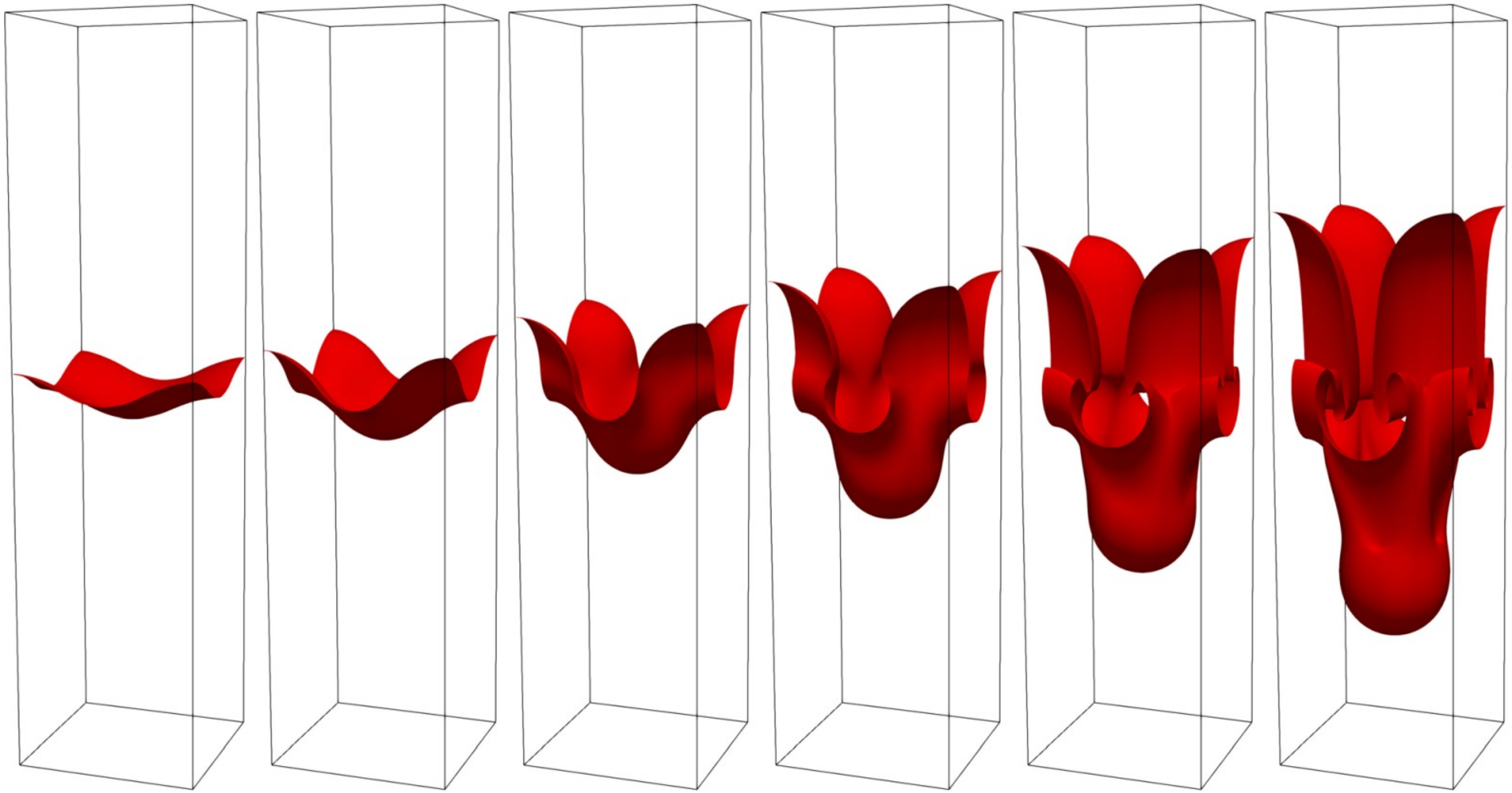
Phys Fluids

3D Rayleigh-Taylor instability



(a) $t/t_0 = 0.5$ (b) $t/t_0 = 1.0$ (c) $t/t_0 = 1.5$ (d) $t/t_0 = 2.0$ (e) $t/t_0 = 2.5$ (f) $t/t_0 = 3.0$

t/t_0	FD	Mom	Ref. 74	Ref. 62	Ref. 28	Ref. 94	Ref. 95	Ref. 96
0.0	1.898	1.898	1.897	1.897	1.895	1.887	1.888	1.904
0.5	1.858	1.850	1.897	1.897	1.864	1.839	1.860	1.869
1.0	1.741	1.711	1.753	1.753	1.763	1.744	1.755	1.776
1.5	1.553	1.504	1.592	1.591	1.587	1.555	1.569	1.618
2.0	1.304	1.256	1.381	1.378	1.357	1.312	1.325	1.396
2.5	1.001	0.988	1.126	1.121	1.085	1.022	1.037	1.149
3.0	0.648	0.711	0.844	0.791	0.788	0.712	0.740	0.863

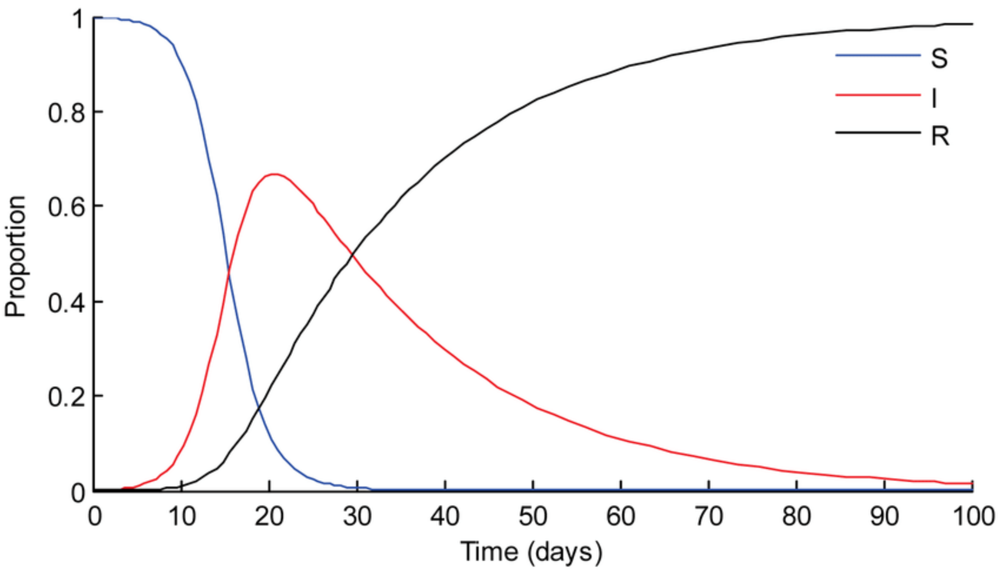
LBM for reaction-diffusion equations

A reaction-diffusion process: spread of epidemics

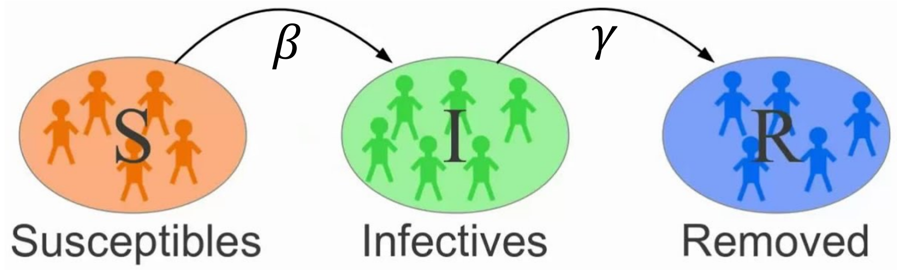
$$\frac{dS}{dt} = -\beta SI + \; d_S \nabla^2 S$$

$$\frac{dI}{dt} = \beta SI - \gamma I \; + \; d_I \nabla^2 I$$

$$\frac{dR}{dt} = \gamma I$$



The model in a nutshell



LBM for reaction-diffusion equations

A reaction-diffusion process: spread of epidemics

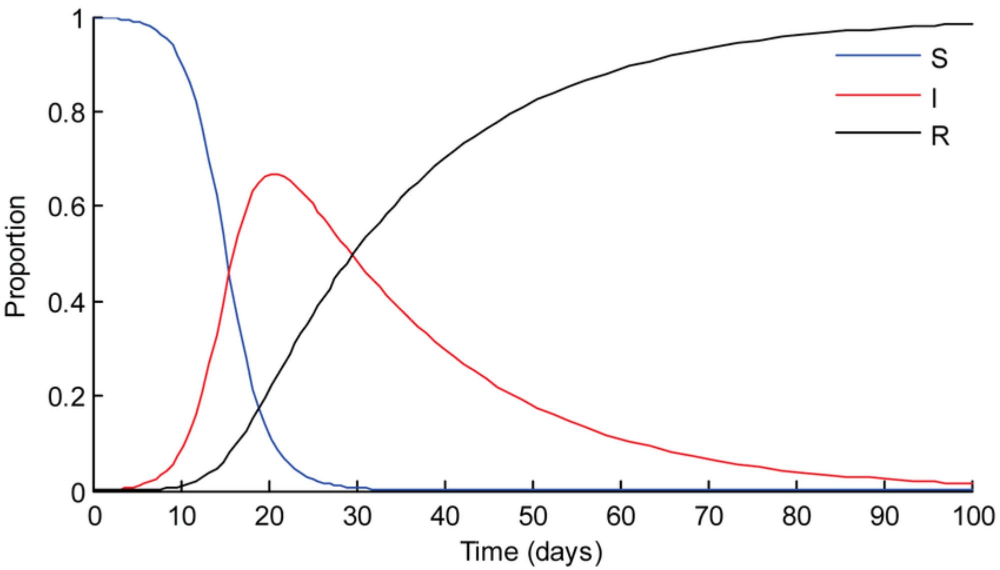
$$\frac{dS}{dt} = -\beta SI + d_S \nabla^2 S$$

$$\frac{dI}{dt} = \beta SI - \gamma I + d_I \nabla^2 I$$

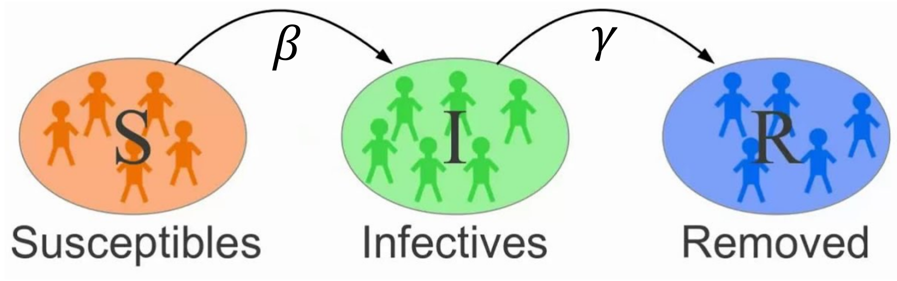
$$\frac{dR}{dt} = \gamma I$$

Reactive terms

Non-reactive (diffusive) terms



The model in a nutshell



LBM for reaction-diffusion equations

A reaction-diffusion process: spread of epidemics

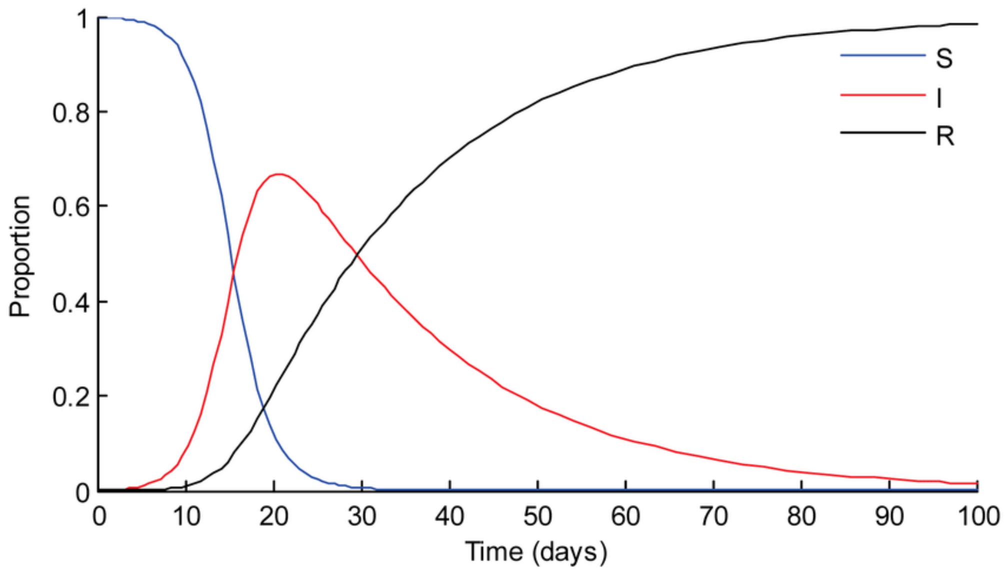
$$\frac{dS}{dt} = -\beta SI + d_S \nabla^2 S$$

$$\frac{dI}{dt} = \beta SI - \gamma I + d_I \nabla^2 I$$

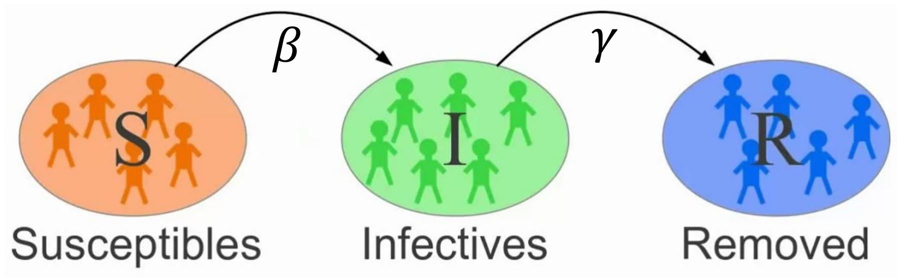
$$\frac{dR}{dt} = \gamma I$$

Reactive terms

Non-reactive (diffusive) terms



The model in a nutshell



Wait…can we do it by BGK LBM?

Yes! 2 reaction-diffusion equations (+1 an ODE)!

$$f_i^k(\mathbf{x}+\mathbf{c}_i, t+\Delta t) = f_i^k(\mathbf{x}, t) + \Omega_{i,\mathrm{NR}}^k(\mathbf{x}, t) + \Omega_{i,\mathrm{R}}^k(\mathbf{x}, t)$$

Non-reactive (diffusive) part:

$$\Omega_{i,\mathrm{NR}}^k = \frac{1}{\tau^k}\left(f_{i,eq}^k - f_i^k\right)$$

$$f_{i,eq}^k = w_i \rho^k$$

$$d^k = \frac{1}{3}\left(\tau^k - \frac{1}{2}\right)$$

$$\rho^k = \sum_i f_i^k$$

LBM for reaction-diffusion equations

Wait…can we do it by BGK LBM?

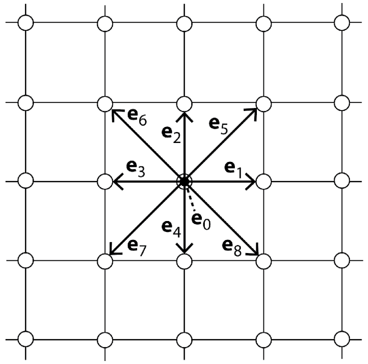
$$f_i^k(\mathbf{x}+\mathbf{c}_i, t+\Delta t) = f_i^k(\mathbf{x}, t)+\Omega_{i,\mathrm{NR}}^k(\mathbf{x}, t)+\Omega_{i,\mathrm{R}}^k(\mathbf{x}, t)$$

Reactive (non-diffusive) part:

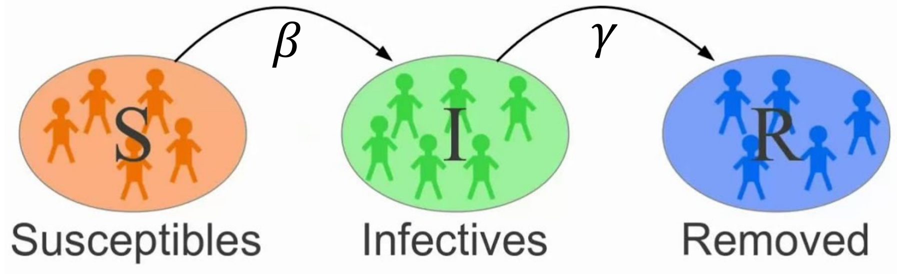
$$\Omega_{i,\mathrm{R}}^{\mathcal{S}} = w_i\left(-\frac{\beta\rho^{\mathcal{S}}\rho^{\mathcal{I}}}{\rho^N}\right),$$

$$\Omega_{i,\mathrm{R}}^{\mathcal{I}} = w_i\left(\frac{\beta\rho^{\mathcal{S}}\rho^{\mathcal{I}}}{\rho^N}-\gamma\rho^{\mathcal{I}}\right),$$

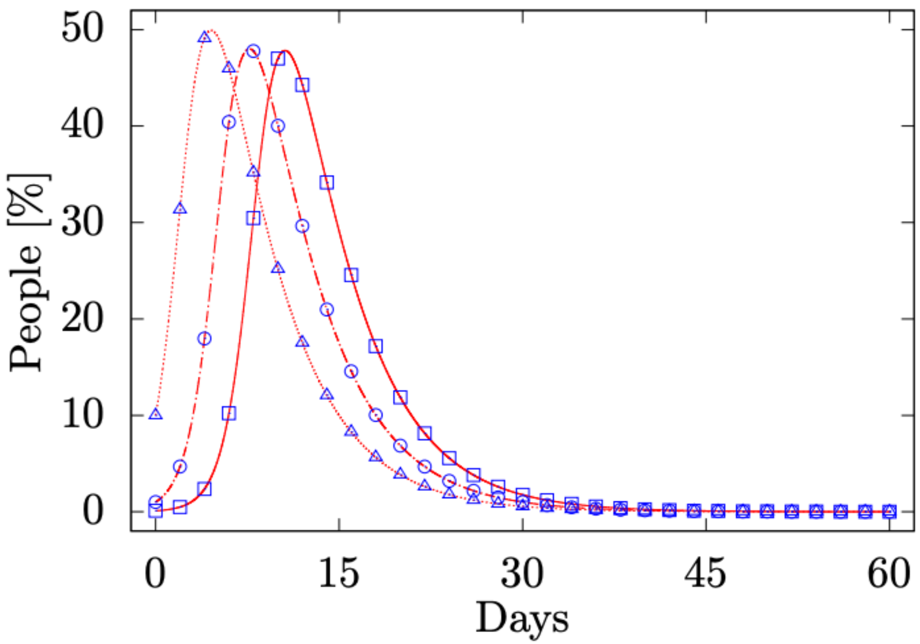
We are now in the position to solve the problem by the BGK LBM!



The model in a nutshell



Validation



LBM for reaction-diffusion equations

Wait…can we do it by BGK LBM?

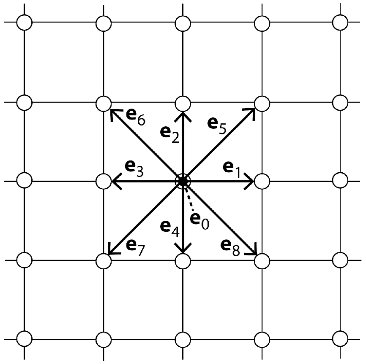
$$f_i^k(\mathbf{x}+\mathbf{c}_i, t+\Delta t) = f_i^k(\mathbf{x},t)+\Omega^k_{i,\mathrm{NR}}(\mathbf{x},t)+\Omega^k_{i,\mathrm{R}}(\mathbf{x},t)$$

Reactive (non-diffusive) part:

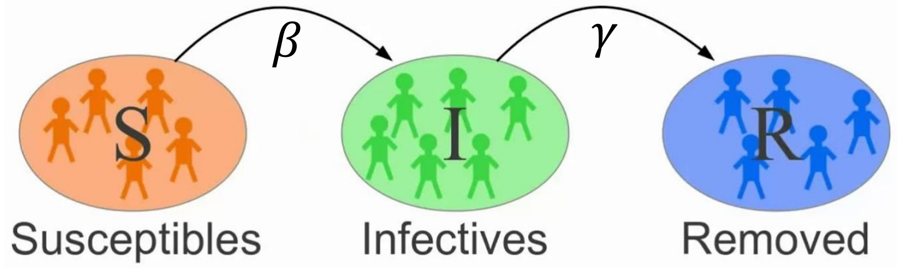
$$\Omega^{\mathcal{S}}_{i,\mathrm{R}} = w_i\left(-\frac{\beta\rho^{\mathcal{S}}\rho^{\mathcal{I}}}{\rho^N}\right),$$

$$\Omega^{\mathcal{I}}_{i,\mathrm{R}} = w_i\left(\frac{\beta\rho^{\mathcal{S}}\rho^{\mathcal{I}}}{\rho^N} - \gamma\rho^{\mathcal{I}}\right),$$

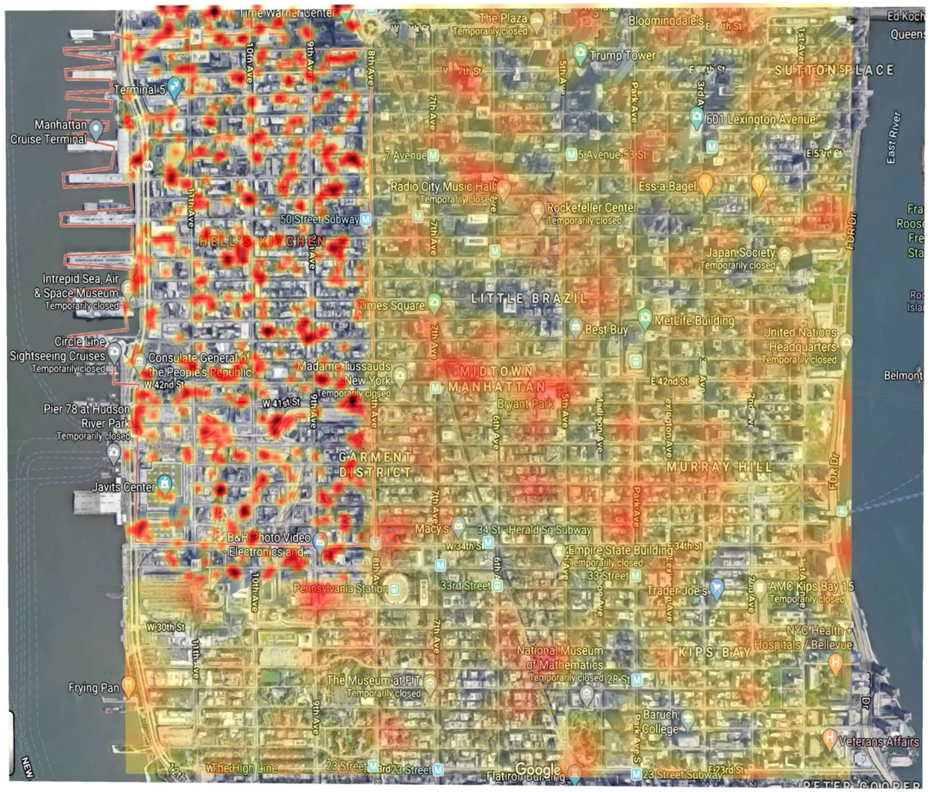
We are now in the position to solve the problem by the BGK LBM!



The model in a nutshell



An epidemic in Midtown Manhattan



Thanks for your attention!

```
printf(“ Questions? ”);
if(question)
    if(haveAnswer)
        giveAnswer();
    else
        pretendTheQuestionIsIllPosed();
    end
else
    printf(“ Thank you! ”);
end
```