Genetic Algorithm based approach for the Multi Product Multi Period Inventory Routing Problem

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Abstract- **The Inventory Routing Problems (IRP) is an important component of Supply Chain Management. The IRP refers to the coordination of the inventory management and transportation. The solution in IRP gives the optimum vehicle routing while at the same time minimizes the transportation and inventory costs. The problem addressed is of the many-to-one type with finite horizon, multi-periods, multi-suppliers, single assembly plant, where a fleet of capacitated identical vehicles, housed at a depot, transport parts from the suppliers to meet the demand specified by the assembly plant for each period. We propose a hybrid genetic algorithm based on allocation first, route second method to determine an optimal inventory and transportation policy that minimizes the total cost. We introduce two new representations and design corresponding crossover and mutation operators. It is found that a simple representation produces very encouraging results.**

Keywords – **Inventory Routing, Genetic Algorithm**

I. INTRODUCTION

The coordination of the inventory management and transportation, known as the Inventory Routing Problem (IRP) is a critical component of the Supply Chain Management. The objective of IRP is to determine an optimal inventory and transportation policy that minimizes the total cost. The resulting inventory and transportation policies usually assign retailers to routes and obtain replenishment intervals and collection sizes for each retailer.

Among the first to study the inventory routing problem is Federgruen and Zipkin [1]. They approach the problem as a single day problem with a limited amount of inventory and the customers' demands are assumed to be a random variable. They represent the problem as a nonlinear integer program using a generalized Benders' decomposition approach. This approach has the attributes that for any assignment of customers to routes, the problem decomposes into a nonlinear inventory allocation problem which determines the inventory and shortage costs and a TSP for each vehicle considered which produces the transportation costs. However, not all customers will be visited every day as there are the inventory and shortage costs as well as the limited amount of inventory to be considered.

Chien et al. [2] is among the first to simulate a multiple period planning model based on a single period approach. This is achieved by passing some information from one period to the next through inter-period inventory flow. In their problem, there is a central depot with many customers around it. The supply capacities of the depot and the demand of the customers are fixed. An integer program is modeled using a Lagrangean dual ascent method to handle the allocation of the limited inventory available at the plant to the customers, the customer to vehicle assignments, and the routing.

Lee et al. [3] work on IRP which consists of multiple suppliers and an assembly plant in an automotive part supply chain. They address the problem as a finite horizon, multi-period, multi-supplier, single assembly plant part-supply network. The objective of their study is to minimize the total transportation and inventory cost over the planning horizon. The problem is divided into two sub-problems that is vehicle routing and inventory control. To solve these problems, a mixed integer programming model is proposed using a heuristic based on simulated annealing. The purpose of using the heuristic is to generate and evaluate alternative sets of vehicle routes while a linear program determines the optimum inventory levels for a given set of routes. In their work, Lee et al. also discover that the optimal solution is dominated by the transportation cost, regardless of the magnitude of the unit inventory carrying cost. Here, it is assumed that no backordering is allowed since any shortage of parts leads to excessively high costs at the assembly plant.

Ribeiro and Lourenço [4] investigate IRP model for two types of customers namely the vendor-managed inventory (VMI) customers and the customer managed inventory (CMI) customers. The former customers have a random demand and the distributor manages the stock at the customers' location. Meanwhile, the CMI type of customers has fixed demand and there are no inventory costs for the distributor. They analyzed both the integrated solutions and the non-integrated solutions. The result shows that the inventory and transportation management in an integration model yields a better performance. We refer the readers to Moin and Salhi [5] for a comprehensive review of the IRP.

In this study we consider a distribution network that is similar to Lee et. al [3]. The network consists of a depot, an assembly plant and N suppliers where each supplier supplies distinct product to the assembly plant. The problem addressed in this paper is based on a finite horizon, multi-period, multi-supplier, single warehouse, where a fleet of capacitated vehicles, housed at a depot, transports products from the suppliers to meet the demand specified by the assembly plant for each period. The vehicles return to the depot at the end of the trip. In this model, no backordering/backlogging is allowed. However, if the demand for more than one period is

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collected, then inventory is carried forward subject to product-specific holding cost incurred at the assembly plant. The objective is thus to minimize the overall transportation and inventory carrying costs over the planning horizon.

We propose a hybrid genetic algorithm based on allocation first, route second method to determine an optimal inventory and transportation policy that minimizes the total cost. We introduce two types of representations. The first is based on a $N \times T$ binary matrix, where N and T are the number of suppliers and the number of periods respectively. It determines which supplier that needs to be visited in each period. The second representation encodes a collection matrix that allocates the amount to be collected from each supplier in each period. To ensure that all the related constraints are not violated, a new crossover and mutation operators are introduced.

This paper is organized as follows. The first part introduces the general concept of IRP. The mathematical formulation is given in Section 2 whilst the genetic algorithms developed are introduced in Section 3. Results and discussions are presented in Section 4 and conclusion is given in Section 5.

II. Mathematical Formulation

The mathematical formulation for the inventory routing problem is given as follows [3]:

Min

$$V\sum_{i=0}^{m+1}\sum_{j=0}^{m+1}c_{ij}\left(\sum_{t=1}^{T}\sum_{k=1}^{J_t}x_{ijkt}\right)+\sum_{i=1}^{m}h_i\left(\sum_{t=0}^{T}s_{it}\right)+K\sum_{t=0}^{T}\sum_{k=1}^{J_t}y_{0kt} \qquad (1a)$$

s.t

$$0 \le a_{ikt} \le C \cdot y_{ikt}, \quad \forall i \in \{1,\ldots,m\}, \forall k, \forall t \qquad (1b)$$

$$\sum_k a_{ikt} = a_{it}, \quad \forall i \in \{1,\ldots,m\}, \forall t \qquad (1c)$$

$$\sum_i a_{ikt} \le C, \quad \forall k, \forall t \qquad (1d)$$

$$\sum_j x_{ijkt} = \sum_j x_{j,i,k,t} = y_{i,k,t}, \quad \forall i \in \{1,\ldots,m\}, \forall k, \forall t \qquad (1e)$$

$$u_{ikt} - u_{jkt} + m \cdot x_{ijkt} \le m-1, \quad \forall i,j \in \{1,\ldots,m\}, \forall k, \forall t \qquad (1f)$$

$$s_{it+1} = s_{it} + a_{it} - d_{it}, \quad \forall i \in \{1,\ldots,m\}, \forall t \qquad (1g)$$

$$y_{ikt}, x_{ijkt} \in \{0,1\}, \quad \forall i,j \in \{0,1,\ldots,m+1\}, \forall k, \forall t \qquad (1h)$$

Input Parameters

T	Periods in the planning horizon
C	Capacity of the truck
K	The fixed cost per trip
V	Travel cost per unit distance
c_{ij}	Travel distance between supplier i and j
h_i	Unit inventory carrying cost for supplier i
J_t	Upper bound on the number of trips needed in period t
a_{ikt}	Amount picked up by truck k from supplier i in period t
a_{it}	Total amount to be picked up from supplier i in period t
s_{it}	Inventory level of supplier i at the end of period t

0 – 1 Decision Variables

x_{ijkt}	= 1 if truck k visits supplier j immediately after supplier i in period t
y_{ikt}	= 1 if supplier i is visited by truck k in period t

Let $\{0,1,\ldots,m\}$ denotes the set of suppliers, where 'supplier 0' is the depot. For simplicity of terminology, a truck is assumed to perform one trip (route) in each period. However, this does not mean that the truck must not be used when it returns to the depot but will simply be given a different name so that 'truck' and 'trip' can be used interchangeably.

The objective function (1a) consists of fixed plus variable travel costs, and the inventory carrying cost. Meanwhile, constraint (1d) ensures that the truck capacity is not violated. Constraint (1f) serves as the sub-tour elimination constraint for each truck in each period. The inventory balance equation is given by constrain (1g). Constraint (1c) accounts for the split demand.

III. Genetic Algorithms for the IRP

Genetic algorithm (GA) is a well-known searching tool which strikes a remarkable balance between exploration and exploitation of the search space. It has been used successfully on optimization problems such as wire routing, scheduling, transportation problems and traveling salesman problems. Several attempts have been made to solve IRP using GA [5].

A. Binary Matrix Representation

In the first representation, a chromosome is a binary matrix of size $(N \times T - 1)$ where N is the number of suppliers while T defines the number of periods. A 1 at position (i,j) in the chromosome indicates that supplier i will be visited in period j. The amount to be collected depends on whether there will be collections in the subsequent period or not. Since backordering is not allowed, the total collection from supplier i in period j is the sum of all the demands in period $j, j+1, \ldots, k-1$ where the next collection will be made in period k. As the initial inventory, s_{i0} for $i = 1,2,\ldots,N$ is assumed to be zero, the values in the first column consist of all ones, thus ignored from the representation. However the algorithm can be adjusted accordingly if the initial inventory for part i is given.

We employ a two dimensional uniform crossover where a binary mask of size $(N \times T - 1)$ is generated randomly for each pair of parents. The position of the ones in the binary mask determines the values in the first parent that are transferred to the offspring. On the other hand, the zeros determine the values in the second parent that are transferred to the offspring.

B. Real Valued Representation

The chromosomes in the second representation encode the collection matrices (the amount to be collected) in the form of a 2-dimensional $N \times T$ matrix. The collection matrix is constructed using the first representation. This is known as the preprocessing step. Here, a binary matrix of size $N \times T$, where the elements in the first column are all ones is randomly generated. The amount to be collected from supplier i in period j is generated randomly in the interval of $\left(\sum_{l=j}^{k-1} d_{il}, \sum_{l=j}^{k} d_{il}\right)$ where d_{il} is the demand for product i in period l and the next collection for product i is in period k. This is to allow the flexibility of satisfying some demands if the transportation cost is reduced. Note that there will always have collection in the first period from all the suppliers as the initial inventory, s_{i0} for $i = 1,2,\ldots,N$ is assumed to be zero. Fig. 1 gives an example of the construction of a chromosome.

	Period				
N	1	2	3	4	5
1	1	0	0	1	0
2	1	1	1	0	0
3	1	1	0	0	1
4	1	0	1	0	0
5	1	0	1	1	1

Fig. 1 (a). The $5x5$ binary matrix

	Period				
N	1	2	3	4	5
1	4	2	4	4	4
2	2	2	2	2	2
3	2	1	2	2	2
4	4	1	4	4	4
5	2	1	2	2	2

Fig. 1 (b). The Demand Matrix

	Period				
N	1	2	3	4	5
1	11	0	0	7	0
2	2	2	6	0	0
3	2	6	0	0	1
4	7	0	0	0	0
5	4	0	1	2	2

Fig. 1 (c). The Collection Matrix

	Period				
N	1	2	3	4	5
1	7	5	1	4	0
2	0	0	4	2	0
3	0	5	3	1	0
4	3	2	8	4	0
5	2	1	0	0	0

Fig 1 (d). Inventory Matrix

Fig. 1. An example of the construction of a chromosome

A new crossover operator has to be designed to ensure that the resultant children do not violate either the demand or the vehicle's capacity constraints. Some study has been done by Abdelmaguid and Dessouky [6] for a different problem to investigate the effectiveness of using the vertical and horizontal breakdown in the crossover mechanism. We opted for a horizontal breakdown as the vertical breakdown will produce illegal solutions and a repair mechanism may be needed to rectify the problem. This is undesirable as it is time consuming to use a repair mechanism in GA.

Firstly, a mask of size (N x 1) is randomly generated. To create Child 1, for each customer i, $i = 1,2,3,\ldots,N$, the position of zeros in the binary mask indicates that the corresponding row is taken from Parent 1. The ones determine that the corresponding row is taken from the Parent 2. The complimentary mask will be used to construct the second child. Fig. 2 illustrates the crossover operator.

	Period				
N	1	2	3	4	5
1	11	0	0	7	0
2	2	2	6	0	0
3	2	6	0	0	1
4	7	0	10	0	0
5	4	0	1	2	2

Fig. 2 (a). Parent 1

	Period				
N	1	2	3	4	5
1	4	9	0	5	0
2	5	0	1	4	0
3	8	0	0	0	1
4	7	0	10	0	0
5	2	3	0	2	2

Fig. 2 (b). Parent 2

Mask = [0 1 1 0 1]

Fig. 2 (c). The Mask Vector

	Period				
N	1	2	3	4	5
1	11	0	0	7	0
2	5	0	1	4	0
3	8	0	0	0	1
4	7	0	10	0	0
5	2	3	0	2	2

Fig. 2 (d). Child 1

	Period				
N	1	2	3	4	5
1	4	9	0	5	0
2	2	2	6	0	0
3	2	6	0	0	1
4	7	0	10	0	0
5	4	0	1	2	2

Fig. 2 (e). Child 2

Fig. 2. Crossover Operator

We observe that this type of representation tends to produce higher inventory holding costs. Therefore we have designed a mutation operator that attempts to reduce the inventory holding cost.

The algorithm of the mutation process can be stated as follows:

STEP 1: Randomly determine the gene that will undergo mutation process. Let this gene be $gene(i, j)$ where i and j denote supplier and period respectively.

STEP 2: If $j \neq 1$, go to Step 3. Otherwise, set $q = 1$ where q is the number of periods before the next collection. If $gene\,(i, j+q) = 0$, $q = q+1$. Repeat the process until $gene\,(i, j+q) \neq 0$. Next, let $r = \sum_{j=1}^{j+q} a_{ij} - \sum_{j=1}^{j+q} d_{ij}$ where a_{ij} and d_{ij} is the collected amount and the demand in period j respectively. Generate the amount to be transferred (say V) randomly in the interval $(0, r)$. Let $gene\,(i, j) = gene\,(i, j) - V$ and $gene\,(i, j+q) = gene\,(i, j+q) + V$.

STEP 3: Set $p = 1$ where p is the number of periods since the last collection. If $gene\,(i, j-p) = 0$, $p = p+1$. Repeat the process until $gene\,(i, j-p) \neq 0$. Let $r = \sum_{j=j-p}^{j} a_{ij} - \sum_{j=j-p}^{j} d_{ij}$. Generate randomly $V \in (0, r)$. Calculate $gene\,(i, j-p) = gene\,(i, j-p) - V$ and $gene\,(i, j) = gene\,(i, j) + V$.

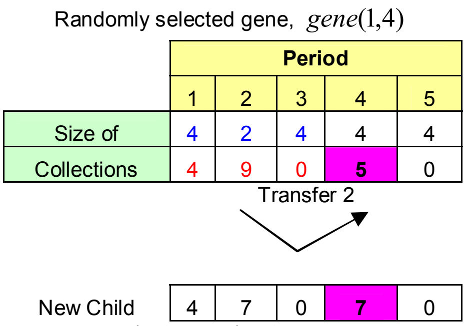

Fig. 3. Mutation Operator

The overall algorithm can formally be stated as follows:

STEP 1: Generate initial population using each type of representation.

STEP 2: For each period j, $j = 1, 2, \ldots, T$, arrange the suppliers around the depot. Sort the suppliers $\{s_1, s_2, \ldots, s_m\}$ in ascending order according to their angles. Let $s_{(i)}$ be the ith supplier after the sort. Set $i = 1$ and $k = 1$. Open a route $R_k = \{\ \}$ and set $Q_k = 0$, where Q_k is total pick-up quantity assigned to cluster k.

STEP 3: If $Q_k + d_{ij} \leq C$, assign $s_{(i)}$ to route R_k, set $Q_k = Q_k + d_{ij}$ and $a_{ik} = d_{ij}$. Otherwise set $a_{ik} = C - Q_k$ and $Q_k = C$. Set $k = k+1$ and open a new route $R_k = \{\ \}$, assign $s_{(i)}$ to R_k. Set $a_{ik} = d_{ij} - a_{ik-1}$ and $Q_k = a_{ik}$. If $i > m$, set $k = 1$ and go to Step 4. Otherwise, set $i = i+1$ and repeat Step 3.

STEP 4: Evaluate the total objective.

STEP 5: Perform crossover and mutation

STEP 6: Repeat Step 3 – Step 5 until the maximum number of generations is attained.

We note that STEPs 2 and 3 are the double sweep algorithm proposed in [3].

IV. RESULTS AND DISCUSSION

The algorithms were written in Microsoft Visual C++ with Genetic Algorithms Library (GAlib) to run the program. The algorithm were run on three data sets (taken from Lee et. al [3]), S12T14, S20T21 and S50T21, that comprises of (12 suppliers, 14 periods), (20 suppliers, 21 periods), and (50 suppliers, 21 periods). In our experiment, the number of generations, crossover rate and mutation rate are fixed at 300, 0.9 and 0.01 respectively for all the problems. Each data set will be tested using three different population sizes that are 50, 100 and 200. For each population size, the data set is executed five times. Finally, the best result for each population is collected together with the total inventory holding cost, total distance, number of trucks and the computation time.

Tables 1 and 2 tabulate the results for each data set. Generally, it can be said that the total cost decreases when the population size increases. Table 1 displays better results for all the data sets. However the difference is very significant for S50T21 data set.

TABLE 1
RESULTS FOR THE 3 DATA SETS USING FIRST REPRESENTATION

Data Set		Population Size		
		50	100	200
S12T14	Total Distance	4239.20	4803.50	4339.90
	Inventory Holding Cost	1371	612	858
	Total Cost	**5610.2**	**5415.5**	**5197.9**
	Number of Truck	127	138	136
S20T21	Total Distance	10453.0	10235.0	10742.0
	Inventory Holding Cost	3354	3603	2247
	Total Cost	**13807.0**	**13838.0**	**12989.0**
	Number of Truck	358	349	378
S50T21	Total Distance	19420.0	18908.0	22945.7
	Inventory Holding Cost	3460.0	3876.0	3662.0
	Total Cost	**22880.0**	**22784.0**	**22035.0**
	Number of Truck	632	615	604

TABLE 2
RESULTS FOR THE 3 DATA SETS USING SECOND REPRESENTATION

Data Set		Population Size		
		50	100	200
S12T14	Total Distance	4475.86	4610.36	4501.38
	Inventory Holding Cost	1173	840	939
	Total Cost	**5648.86**	**5450.36**	**5440.38**
	Number of Trucks	134	146	142
S20T21	Total Distance	11168.6	11004.9	10960.1
	Inventory Holding Cost	2304	2292	2142
	Total Cost	**13472.6**	**13296.9**	**13102.1**
	Number of Trucks	392	392	386
S50T21	Total Distance	24318.4	23905.4	22945.7
	Inventory Holding Cost	5252	5008	4803
	Total Cost	**29570.4**	**28913.4**	**27748.7**
	Number of Trucks	596	586	588

The best total cost in each generation has been collected for each data set with different population size. Fig. 4 and Fig. 5 show the convergence graphs for S50T21 using the first and second representations, respectively.

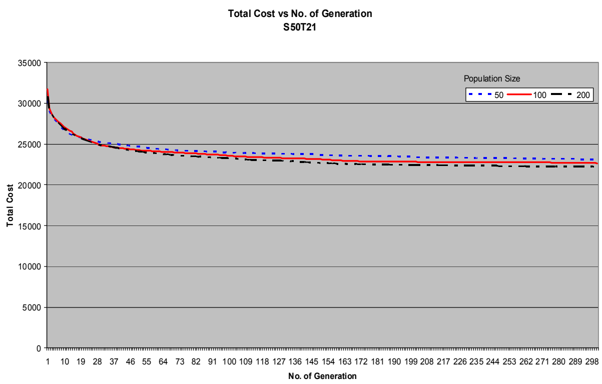

Fig. 4. Convergence graph for S50T21 using the first representation

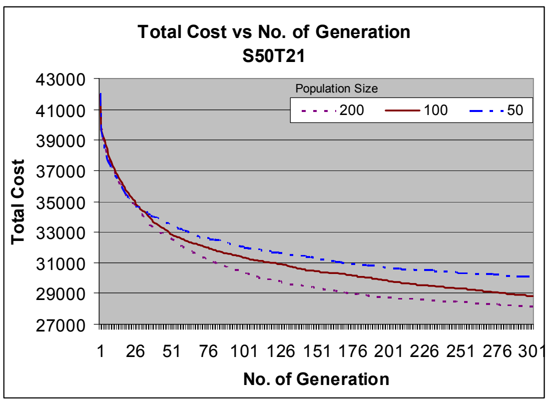

Fig. 5. Convergence graph for S50T21 using the second representation

V. CONCLUSION

Both transportation and inventory costs should be considered concurrently in the logistic planning functions as these two areas might lead to significant gains and more competitive distribution strategies. In this paper, we present a solution for Inventory Routing Problem using the modified Genetic Algorithms approach on a finite horizon, multi-period, and a multi product problem. We introduce two new representations and design new crossover and mutation operators to enhance the solutions. Although the simple representation produces better results in most problems, we believe that the second representation has more potential in finding better solution as it allows the algorithm to exploit the trade off between transportation and inventory holding costs. However the preprocessing stage needs to be refined so as to give a better starting initial population.

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