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A multi-objective healthcare inventory routing problem; a fuzzy possibilistic approach

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A R T I C L E   I N F O

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A B S T R A C T

This paper presents a new multi-objective mathematical model to address a Healthcare Inventory Routing Problem (HIRP) for medicinal drug distribution to healthcare facilities. The first part of objective function minimizes total inventory and transportation costs, while satisfaction is maximized by minimizing forecast error which caused by product shortage and the amount of expired drugs; Greenhouse Gas (GHG) emissions are also minimized. A demand forecast approach has been integrated into the mathematical model to decrease drug shortage risk. A hybridized possibilistic method is applied to cope with uncertainty and an interactive fuzzy approach is considered to solve an auxiliary crisp multi-objective model and find optimized solutions.

1. Introduction

In the last decades, the increasing number of published studies in the context of healthcare shows that healthcare issues are of considerable interest to both academia and practitioners (Rais and Viana, 2011; Kumar and Rahman, 2014). The healthcare world is faced with various challenges coming from different sides, such as manufacturing and supplying. In recent years (2007–2011) in the U.S., the frequency of drug shortages increased by about 40% while 13% of shortage causes were related to distribution and inventory policies as reported by U.S. department of health and human services (2012) (Fig. 1). In order to tackle distribution and inventory problems in healthcare, various related Operation Research (OR) problems can be applied. Among them, the Inventory Routing Problem (IRP) may lead to efficient tools that monitor and address both inventory and distribution issues. The integration of IRP (HIRP) models in healthcare leads to a decrease in the frequency of drug shortages in healthcare facilities by defining a set of facilities to which the drugs are to be delivered, a sequence of transportation routes, and the quantity of drugs to be delivered to each facility.

Global warming and the rise of environmental pollutants, particularly via GHG emissions, present a worldwide challenge. Transportation activity is one of the main sources of GHG emissions, while reducing GHG emissions is the main objective of the IRP. Via applications of the HIRP, priority is given to the GHG issue, which has wide effect on public health, by decreasing the amount of GHG released by vehicles during drug distribution.

With regard to the matters enumerated, the aim of this research is to present a new multi objective mathematical model for integrating the healthcare issues and inventory routing problem under uncertainty. The objective functions that are considered in the model, minimize the total cost, forecast error which caused by product shortage, amount of expired and GHG

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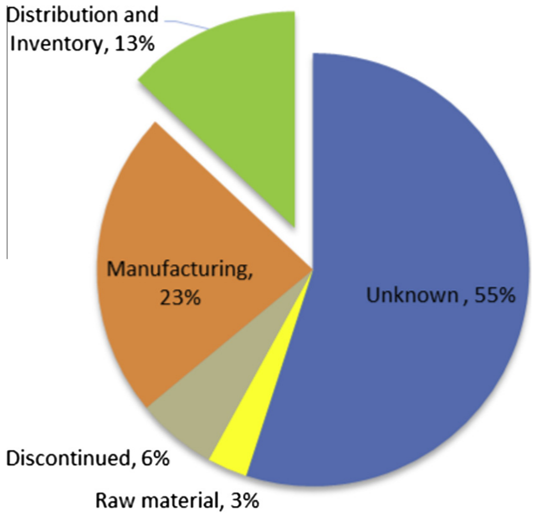


Fig. 1. Causes of drug shortage in U.S. (2011).

emission. Furthermore, a demand forecast approach is applied to decrease shortage risk. Then, an interactive fuzzy approach is considered due to lack of the information about some parameters.

The remainder of this paper is organized as follows: Section 2, reviews the related literature about IRP and presents research motivations and contributions; notation and the mathematical model are presented in Section 3; Section 4, is related to applied linearization methods; Section 5, elaborates on the solution and approach for the proposed mathematical model; Section 6, includes experimental results and analysis; concluding thoughts and future research options comprise Section 7.

2. Literature review

IRP was first introduced by Bell et al. (1983) by integrating decisions on inventory, vehicle scheduling, and gas distribution methods for the delivery of chemical products. Several researchers extended this study, such as Anily and Federgruen (1990), Dror and Ball (1987), Speranza and Ukovich (1996). Several concepts are studied by researchers in inventory routing problem; some authors consider cost or profit as objective functions (Archetti et al., 2007; Moin et al., 2011). Other studies focus on the minimization of traveling time (Li et al., 2014), a topic closely related to the distribution of perishable products or disaster situations and some others (Jozefowiez et al., 2008; Yu et al., 2010) consider other criteria such as service level and customer satisfaction in routing problems.

Due to the large number of studies related to the inventory routing problems, we limit ourselves to the mention some important studies here. Federgruen and Zipkin (1984) extended some methods for vehicle routing problem and combined the location and allocation delivery planning in their study. Golden et al. (1984) addressed the combined problem of vehicle routing and inventory problems and they used simulation model for investigation this combination. Berman and Larson (2001) investigated on IRP in gas industry. They minimized some related cost such as expected cost, costs of earliness, lateness, product shortfall, and cost of returning. The authors used stochastic dynamic programming to model the problem. Jaillet et al. (2002) investigated IRP for distribution some products such as heated oil from depot to large number of the customers. They considered cost approximation to minimize the total delivery costs. Kleywegt et al. (2002) applied Markov decision process to formulate the IRP and also an approximation method is considered to solve the model. Moin and Salhi (2006) classified the literature review of the IRPs. Also Andersson et al. (2010) categorized the IRPs according to the finite or infinite planning horizons, deterministic or stochastic demands, limited or unlimited number of vehicles and one or multiple customers visited. Moreover recently, Coelho et al. (2013) did a comprehensive survey on inventory routing problems. Custódio and Oliveira (2006) proposed a new model for the distribution of frozen foods by considering the interaction between retailer and supplier. Hsu et al. (2007) studied a stochastic vehicle routing problem with time windows for delivering perishable food from the distribution center. They considered cost minimization based on different costs, such as inventory, transportation, energy, and a penalty for violating time windows. Archetti et al. (2007) addressed mixed-integer linear programming in IRP while considering a maximum inventory level and a replenishment policy. The authors proposed a branch and cut algorithm to find optimal solutions. Savelsbergh and Song (2008) studied continuous moves in the IRPs for a closer approximation of real-life conditions. They also proposed a local search-embedded heuristic to find better solutions. Li et al. (2008) integrated replenishment procedure in IRPs without shortage consideration. Chen and Lin (2009) addressed multi-product and period IRP optimization models by considering stochastic demand and risk aversion. Abdelmaguid et al. (2009) considered multi-period IRP with backlogging. The authors subsequently proposed a new heuristic to solve the same problem. Ahmadi Javid and Azad (2010) proposed a new mixed-integer mathematical model in IRP and integrated some strategic decisions such as location allocation, capacity, inventory and routs. In their study, the customer's demand is assumed to be an uncertain parameter, and each distribution center maintains a safety stock. Huang and Lin (2010) proposed a model with an uncertain demand by combining multi-item inventory management and vehicle-routing optimization. An Ant Colony Optimization (ACO) algorithm was applied as the solving approach. Shiguemoto and Armentano (2010) addressed a multi-product model to coordinate production, inventory, and distribution by considering the time window for the distribution process. A tabu search procedure was developed to obtain optimal solutions. Zhao et al. (2010) applied the Markov Decision Process (MDP) to integrate inventory and transportation problem, considering various transportation methods. The authors also proposed an efficient solving approach for their proposed model.

Yu et al. (2010) integrated service level and inventory routing in a multi-period mathematical model. These authors developed a hybrid Lagrangian relaxation and partial linearization approach to solve a large-scale problem. Bard and Nananukul (2010) proposed a mixed-integer model in production, inventory, and routing to minimize production inventory and distribution costs. A combination of branch-and-price and column generation heuristic was developed to obtain better solutions. Siswanto et al. (2011) studied a ship inventory routing problem and integrated four strategic and operational decisions such as route selection, ship selection, loading and unloading activity procedures. They developed a one-step greedy heuristic and solved several instance problems to show the applicability of their model. Liu and Chen (2011) addressed a new mathematical model for the inventory routing and pricing problem. These authors applied the tabu search method and compared their results with those of two different heuristics to demonstrate the efficiency of their solving approach. Erdoğan and Miller-Hooks (2012) formulated a mixed integer linear mathematical model in Vehicle Routine Problem (VRP), considering green criteria. They also proposed two heuristic algorithms and prove efficiency of heuristic by some numerical examples. Stålhane et al. (2012) considered a large-scale ship routing and inventory management problem for the distribution of liquefied natural gas to maximize sales revenue. Coelho et al. (2012) focused on new consistency features and transshipment options in IRP. They assumed a homogeneous and a heterogeneous fleet for transportation. A branch-and-cut algorithm was also applied as an exact solution approach. Bertazzi et al. (2013) proposed a stochastic mathematical model by considering uncertainty in demand. These authors sought to minimize various inventory and transportation related costs. Mjirda et al. (2013) solved a multi-product IRP with a new two-phase Variable Neighborhood Search (VNS) metaheuristic. Shukla et al. (2013) proposed a methodology based on evolutionary algorithm to solve the inventory routing problem with stochastic demand. Amorim and Almada-Lobo (2014) proposed a multi-objective model for the distribution of highly perishable food. These authors contributed to our understanding of the trade-off between distribution scenarios and the cost of freshness. Coelho and Laporte (2014) investigated product age in IRP by considering two suboptimal selling priority policies. They also solved the proposed model using a branch-and-cut algorithm. Agra et al. (2014) studied a "short sea fuel oil distribution problem" to determine distribution policies while the routing and operating costs are minimized. Nolz et al. (2014) presented a multi-objective mathematical model and integrated social issues (the collection of pharmaceutical waste) with classical IRP. To the best of our knowledge, there are a few studies which tried to integrate healthcare and IRP. Hemmelmayr et al. (2008) formulated an integer programming model related to the distribution the blood products to hospitals by minimizing time traveling. The authors introduced a Variable Neighborhood Search (VNS) and extensive simulation approaches to handle stochastic product usage and convert it into a deterministic optimization problem. Also Liu et al. (2013) applied their research to a home healthcare problem by considering the pickup and delivery of medical devices in vehicle routing problems, a topic close to IRP.

2.1. Motivation and contribution

According to the previous section, there are few studies applied to healthcare issues in IRP. We find that healthcare is a neglected issue in the literature body of IRP. Since the shortage of medical products (drugs, blood products, etc.) has a direct effect on medical procedures, achieving reliable distribution and inventory is a major concern for healthcare facilities. Therefore, the main objective of our study is to develop a multi-objective mathematical model by integrating healthcare and a multi-product IRP. This paper contributes to the existing literature in five categories which are described as follows:

- Apply IRP methods to address a significant healthcare issue: drug distribution from the supplier to the healthcare facilities.
- In order to decrease the drug's shortage risk, we apply a smoothing approach in the mathematical formulation to forecast the demand for each period so as to reduce errors (the error of each period is equal to the amount of shortage in the previous period divided by the demand of the previous period) and increase the service level.
- Due to seasonal variations in demand for some products, we define the seasonal requirement factor as a coefficient for the forecast error caused by product shortage.
- A minimum usable time is defined for the delivery of drugs, so that drugs received by a hospital should have a minimum usable period before their expiration date.
- Since, in real life there are various uncertain parameters, we consider demand, transportation, and shortage cost to be fuzzy. In order to transfer the model to an equivalent auxiliary crisp model, a multi-objective possibilistic approach is developed. Furthermore, an interactive fuzzy method is applied to find an optimized solution.

3. Mathematical model

In this section, a new multi-objective mathematical model is proposed to reach a compromise between the total cost for drug (pharmaceutical) distribution, the satisfaction of the customer (hospital) and the total GHG emissions produced in transportation over the planning horizon. The first part of objective function covers costs related to the inventory (holding and shortage) and transportation issues. The second part of objective function regards customer satisfaction as it minimizes the amount of expiration and error in demand forecasting which caused by drug shortage over the planning horizon. Due to differences in the major units of the items considered for the satisfaction level, a weighting method (normalized weight) is applied to aggregate them as one objective (Cruz and Wakolbinger, 2008). The third part of objective function considers GHG emissions of the transportation vehicles. The proposed model facilitates tactical/operational-level decisions over the planning horizon, such as the set of hospitals to be visited in each period, the delivery sequence for each transportation mode, as well as the quantity of drugs delivered to each hospital in each period.

In this model, one supplier is responsible for distributing drugs with specific expiration dates to different hospitals. This task is done by various types of vehicles with different capacities, costs, and GHG emission factors. In view of the effect of drug availability on the quality of medical procedures, a safety stock (buffer amount) for each drug can be specified to decrease the likelihood of an inventory shortage. The main assumptions about the proposed model are as follows:

- In the logistic network, one supplier delivers drugs to several hospitals (medical facilities).
- The supplier uses a heterogeneous fleet, with vehicles that differ in capacity, cost, and amount of GHG emission, to distribute drugs to hospitals.
- Each drug has a specific expiration date and will be unusable thereafter.
- Hospital requests should be delivered within the specified time frame, before the expiration date of the drug requested by the hospital.
- The inventory capacity of each hospital for each drug is known.
- Due to the high degree of uncertainty in demand, a high shortage risk is imposed on the supplier. A penalty cost is considered for shortages.
- The supplier faces uncertainty in the demand for each drug, the cost of transportation, and the shortage penalty cost.
- The initial inventory level of each drug in each hospital is known.
- There is no inventory for the supplier, therefore drugs delivered to the hospitals are assumed to be fresh.
- The safety stock (buffer) for each drug is specified in advance.
- Due to an observed relation between time periods and drug consumption levels, a seasonal requirement factor is defined. This factor is a binary parameter and it is defined according to previous checklists and hospital reports.

3.1. Notation

The following notations are used in proposed model.

Sets	
M	set of customers, index for customer $(1, 2, \ldots, m)$
M'	set of customers and supplier, $M \cup \{0\}$
r	index for drug $(1, 2, \ldots, R)$
k	index for vehicle type $(1, 2, \ldots, K)$
t, t'	index for period $(1, 2, \ldots, T)$
Parameters	
R	number of types of drugs
K	number of types of vehicles
T	number of periods
I_{ir0}	initial inventory level of drug r in hospital i
h_{ir}	inventory holding cost at hospital i per unit of drug r
B_{irt}	safety stock (Buffer) for drug r in hospital i at period t
IC_{ir}	maximum inventory capacity of hospital i for drug r
$\tilde{\psi}_{ir}$	penalty cost of one unit of drug r for a shortage at hospital i
c_{ij}	distance between hospital i and j
$\tilde{v}_k$	transportation cost per unit distance for vehicle type k
cap_k	capacity of vehicle type k
$\tilde{D}_{irt}$	demand of hospital i for drug r at period t
L_r	shelf life of drug r
UMT_r	minimum usable time of drug r before expiration date, when the drug is delivered to the hospital
SR_{irt}	seasonal requirement factor for drug r in hospital i at period t

(*continued on next page*)

GH_k	GHG emission produced per unit distance by vehicle type k
λ_{ex}	normalized weighting factor of total amount of expired drugs
λ_{er}	normalized weighting factor of total amount of demand forecast error caused by product shortage
M	an arbitrarily large number
Note: $A = \sum_k cap_k$	
Variables	
x_{ijkt}	1 if hospital j is visited exactly after hospital i by vehicle type k at period t, otherwise 0
I_{irt}	inventory level of drug r in hospital i at the end of period t
y_{kt}	number of transportation type k at period t
$q_{irktt'}$	quantity of received drug r by hospital i through vehicle k at period t to be used at period t'
Q_{irkt}	total quantity of received drug r by hospital i through vehicle k at period t
SH_{irt}	amount of shortage for drug r in hospital i at the end of period t
φ_{irt}	demand forecast error in demand forecasting for drug r in hospital i at the end of period t
EX_{irt}	amount of expired drug r in the inventory of hospital i at the end of period t
z_{ikt}	1 if hospital i is served at period t by vehicle k, otherwise 0
w_{ikt}	a dummy variable to eliminate sub-tours

3.2. Mathematical formulation

$$Min \quad f_1 = \sum_{i\in M}\sum_{r\in R}\sum_{t\in T} h_{ir} I_{irt} + \sum_{(i,j)\in M'}\sum_{k\in K}\sum_{t\in T} \tilde{\upsilon}_k y_{kt} c_{ij} x_{ijkt} + \sum_{i\in M}\sum_{r\in R}\sum_{t\in T} \tilde{\psi}_{ir} SH_{irt} \tag{1}$$

$$Min \quad f_2 = \lambda_{er}\left(\sum_{i\in M}\sum_{r\in R}\sum_{t\in T}\varphi_{irt}\right) + \lambda_{ex}\left(\sum_{i\in M}\sum_{r\in R}\sum_{t\in T} EX_{irt}\right) \tag{2}$$

$$Min \quad f_3 = \sum_{(i,j)\in M'}\sum_{k\in K}\sum_{t\in T} GH_k y_{kt} c_{ij} x_{ijkt} \tag{3}$$

S.t.

$$I_{irt} - B_{irt} - SH_{irt} = I_{ir(t-1)} - \tilde{D}_{irt} + \sum_{k\in K} Q_{irkt} - EX_{irt} - SH_{ir(t-1)} - B_{ir(t-1)} \quad \forall i \in M, r, t \tag{4}$$

$$\tilde{D}_{ir(t+1)} = \tilde{D}_{irt} + \varphi_{irt} \times \left[\left(\sum_{k\in K} q_{irktt'}\right) - \tilde{D}_{irt}\right] \quad \forall t' = t, i \in M, r, t \tag{5}$$

$$\varphi_{irt} = SR_{irt} \times \left[\frac{1}{\tilde{D}_{ir(t-1)}} \times \left(SH_{ir(t-1)}\right)\right] \quad \forall i \in M, r, t \tag{6}$$

$$SH_{irt} \geqslant \sum_{k\in K}\sum_{t>t'} q_{irktt'} \quad \forall i \in M, r, t \tag{7}$$

$$(I_{irt} - B_{irt}) \times SH_{irt} = 0 \quad \forall i \in M, r, t \tag{8}$$

$$\sum_{t'\in T}\sum_{k\in K} q_{irktt'} \leqslant IC_{ir} - I_{ir(t-1)} \quad \forall i \in M, r, t \tag{9}$$

$$\sum_{k\in K}\sum_{t'>(t+L_r+UMT_r)} q_{irktt'} \leqslant EX_{irt} \quad \forall i \in M, r, t \tag{10}$$

$$\sum_{t'\in T} q_{irktt'} \leqslant Q_{irkt} \quad \forall i \in M, r, k, t \tag{11}$$

$$\sum_{i\in M}\sum_{r\in R}\sum_{t'\in T} q_{irktt'} \leqslant cap_k y_{kt} z_{0kt} \quad \forall k, t \tag{12}$$

$$\sum_{j\in M'} x_{ijkt} = \sum_{j\in M'} x_{jikt} = z_{ikt} \quad \forall i \in M', k, t \tag{13}$$

$$w_{ikt} - w_{jkt} + (|M| + 1)x_{ijkt} \leqslant |M| \quad \forall (i,j) \in M, k, t \tag{14}$$

$$x_{ijkt}, z_{ikt} \in \{0,1\} \quad \forall i,j \in M, t, k, i \neq j \tag{15}$$

$$\begin{matrix} q_{irktt'}, I_{irt}, EX_{irt}, SH_{irt}, y_{kt} \geqslant 0, \quad Integer \\ \varphi_{irt} \in [0,1] \end{matrix} \quad \forall i \in M, r, t, k \tag{16}$$

Objective function (2) represents the minimization of related costs, including holding cost, transportation cost, and shortage cost. The objective function (2) maximizes satisfaction by minimizing the amount of errors (caused by product shortage) in demand forecasting and the quantity of expired drugs. Objective function (3) considers GHG emission, an environmental criterion dependent on transportation type. Constraint (4) balances the inventory level in between periods. Equation (5) forecasts demand based on the previous demand and forecast error which is calculated based on product shortage. Eq. (6) calculates the demand forecast error in each period. Constraint (7) determines the amount of shortage for each product. Constraint (8) ensures that $I_{irt} - B_{irt}$ and SH_{irt} cannot be a positive value simultaneously. Constraint (9) guarantees that the inventory capacity of each hospital for each drug is not exceeded. Eq. (10) calculates the quantity of expired drugs. Eq. (11) determines the total quantity of delivered drugs. Constraint (12) ensures that the capacity of each vehicle should not be exceeded. This constraint also prevents making a tour without visiting the supplier (whenever the quantity of drug delivered to a hospital is positive, z_{ikt} should be 1, which means the supplier is visited). Eqs. (13) and (14) are for tour making and sub-tour elimination. Eqs. (15) and (16) state the decision variables.

Due to the sensitivity and dynamic nature of the HIRP, the system is faced with a high degree of uncertainty in real life (variable demand, transportation cost, and shortage penalty cost). To model the imprecise nature of uncertain parameters (lack of knowledge), the pattern sets of triangular fuzzy number $\tilde{n} = (n^p, n^m, n^o)$, where n^p is estimated as the most pessimistic value, n^m is estimated as the most likely value, and n^o is estimated as the optimistic value of $\tilde{n}$, represent possibility distributions. Therefore, to cope with uncertain parameters in this model, a fuzzy possibilistic programming method is developed as described in Section 5.

4. Linearization

The presented mathematical model is a Mixed-Integer Nonlinear Programming model (MINLP) due to some nonlinear equations. Linearization methods are now applied to obtain an equivalent linear mathematical model.

The multiple variables in constraint (5) made it nonlinear. Since two variables (φ_{irt} and $\sum_{k \in K} q_{irktt'}$) are continuous and integer, generally exact linearization is no longer possible. Therefore, a relaxation method (convex envelopes) is applied to cope with multilinear terms. This method was proposed by McCormick (1976) and can be explained as follows:

$$If\ W = x_1 x_2 \text{ and } x_1 \in [x_1^L, x_1^U], \quad x_2 \in [x_2^L, x_2^U] \tag{17}$$

Then the convex envelopes of the multilinear terms are added as four new constraints:

$$W \geqslant x_1^L x_2 + x_2^L x_1 - x_1^L x_2^L \tag{18}$$

$$W \geqslant x_1^U x_2 + x_2^U x_1 - x_1^U x_2^U \tag{19}$$

$$W \leqslant x_1^L x_2 + x_2^U x_1 - x_1^L x_2^U \tag{20}$$

$$W \leqslant x_1^U x_2 + x_2^L x_1 - x_1^U x_2^L \tag{21}$$

According to the above, in order to convert multiples of the two variables $(\varphi_{irt} \times \sum_{k \in K} q_{irktt'})$, the nonnegative variable $E_{irtt'}$ is assumed, and the lower and upper bounds of the two variables are:

$$\varphi_{irt} \in [0,1] \tag{22}$$

$$\sum_k q_{irktt'} \in [0, A] \tag{23}$$

A reformulated constraint (5) and four new constraints are then presented as follows:

$$\tilde{D}_{ir(t+1)} = \tilde{D}_{irt} + \varphi_{irt} \times \tilde{D}_{irt} + E_{irtt'} \quad \forall t' = t, i, r, t \tag{24}$$

$$E_{irtt'} \geqslant 0 \quad \forall i, r, t, t' \tag{25}$$

$$E_{irtt'} \geqslant \sum_k q_{irktt'} + [A \times (\varphi_{irt} - 1)] \quad \forall i, r, t, t' \tag{26}$$

$$E_{irtt'} \leqslant A \times \varphi_{irt} \quad \forall i, r, t, t' \tag{27}$$

$$E_{irtt'} \leqslant A \times \sum_{k} q_{irktt'} \quad \forall i, r, t, t' \tag{28}$$

Also, in order to linearize constraint (8) due to two multiple integer variables, a new binary variable F_{irt} and two new constraints replace the previous one as follows:

$$F_{irt} = \begin{cases} 1 & I_{irt} = 0 \\ 0 & SH_{irt} = 0 \end{cases} \quad \forall i, r, t \tag{29}$$

$$SH_{irt} - (B_{irt} \times SH_{irt}) \leqslant M \times F_{irt} \quad \forall i, r, t \tag{30}$$

$$I_{irt} - (B_{irt} \times SH_{irt}) \leqslant M \times (1 - F_{irt}) \quad \forall i, r, t \tag{31}$$

Finally, the multiple integer and binary variables in constraint (12) and objectives (2) and (3) made them nonlinear. It can be linearized as follows:

$$\sum_{i \in M} \sum_{r \in R} \sum_{t' \in T} q_{irktt'} \leqslant cap_k y_{kt} \quad \forall k, t \tag{32}$$

$$\begin{aligned} &\tfrac{z_{0kt}}{M} \leqslant y_{kt} \leqslant z_{0kt} M \quad \forall k, t \\ &\tfrac{x_{ijkt}}{M} \leqslant y_{kt} \leqslant x_{ijkt} M \quad \forall i, k, t \end{aligned} \tag{33}$$

5. Solution method

To solve the present multi-objective HIRP model, a two-phase approach is applied. In the first phase, by hybridizing two possibilistic methods (Jiménez et al., 2007; Arenas Parra et al., 2005) the equivalent auxiliary crisp model is obtained. In the second phase, an efficient fuzzy multi-objective method, i.e., Torabi and Hassini (TH) (Torabi and Hassini, 2008) is applied to solve the auxiliary crisp multi-objective programming model and find the optimized solution.

5.1. The auxiliary crisp multi-objective model

According to the above explanation, the main HIRP model is converted to the equivalent crisp model in the first phase. There are several methods in the literature to introduce imprecise coefficients in possibilistic models, see (Jiménez, 1996; Lai and Hwang, 1993, 1992; Inuiguchi and Ramík, 2000). An efficient hybridized possibilistic method by Jiménez et al. (2007) and Arenas Parra et al. (2005) is applied here to transform the present model into the auxiliary crisp multi-objective integer linear programming model (MOILP). With this method, the expected interval and expected value is defined for a fuzzy number, as first presented by Heilpern (1992), Jiménez (1996), Yager (1981), and Dubois and Prade (1987), respectively.

If $\tilde{n}$ is a triangular fuzzy number, the membership function $\mu_{\tilde{n}}$ will be defined thus:

$$\mu_{\tilde{n}}(x) = \begin{cases} f_n(x) = \frac{x - n^p}{n^m - n^p} & \text{if } n^p \leqslant x \leqslant n^m \\ 1 & \text{if } x = n^m \\ g_n(x) = \frac{n^o - x}{n^o - n^m} & \text{if } n^m \leqslant x \leqslant n^o \\ 0 & \text{if } x \leqslant n^p,\ x \geqslant n^o \end{cases} \tag{34}$$

Then, the expected interval (EI) and expected value (EV) of $\tilde{n}$ are calculable (Jiménez, 1996) as follows:

$$EI(\tilde{n}) = [E_1^n, E_2^n] = \left[\int_0^1 f_n^{-1}(x)dx, \int_0^1 g_n^{-1}(x)dx\right] = \left[\frac{1}{2}(n^p + n^m), \frac{1}{2}(n^m + n^o)\right] \tag{35}$$

$$EV(\tilde{n}) = \frac{E_1^n + E_2^n}{2} = \frac{n^p + 2n^m + n^o}{4} \tag{36}$$

Eqs. (35) and (36) are usable for a trapezoidal fuzzy number, as well. As per (Jiménez, 1996), for any pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is bigger than $\tilde{b}$ is introduced in Eq. (37):

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \frac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \tag{37}$$

If $\mu_M(\tilde{a}, \tilde{b}) \geqslant \alpha$, it implies that $\tilde{a}$ is greater or equal to $\tilde{b}$ at least in a degree of preference α, which will be denoted by $\tilde{a} \geqslant_{\alpha} \tilde{b}$.

On the other hand, according to a definition presented by Arenas Parra et al. (2005), for any pair of fuzzy number $\tilde{a}$ and $\tilde{b}$, when $\tilde{a}$ and $\tilde{b}$ are indifferent in a degree α, the following relationships hold simultaneously:

$$\tilde{a} \geqslant_{\alpha/2} \tilde{b}, \quad \tilde{a} \leqslant_{\alpha/2} \tilde{b} \tag{38}$$

Eq. (38) can be rewritten as follows:

$$\frac{\alpha}{2} \leqslant \mu_M(\tilde{a}, \tilde{b}) \leqslant 1 - \frac{\alpha}{2} \tag{39}$$

Now, assume the following mathematical model in which $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n), \tilde{a} = [\tilde{a}_{ij}]_{s\times n}$ and $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_s)^t$ are trapezoidal or triangular fuzzy numbers:

$$\begin{aligned}
&\min \quad z = \tilde{c}^t x \\
&s.t. \\
&\quad \tilde{a}_{ij} x \geqslant \tilde{b}_i \quad \forall i = 1, \ldots, k, \quad j = 1, \ldots, n \\
&\quad \tilde{a}_{ij} x = \tilde{b}_i \quad \forall i = k+1, \ldots, s, \quad j = 1, \ldots, n \\
&\quad x \geqslant 0
\end{aligned} \tag{40}$$

If $\min_{i=1,\ldots,s}\{\mu_M(\tilde{a}_i x, \tilde{b}_i)\} = \alpha$, where $\tilde{a}_i = (\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{in}),\ x \in \Re^n$ (decision vector) will be feasible in a degree α (Jiménez et al., 2007). According to equations (37)–(39), constraints $\tilde{a}_i x \geqslant \tilde{b}$ and $\tilde{a}_i x = \tilde{b}$ can be rewritten:

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \geqslant \alpha \quad \forall i = 1, \ldots, k \tag{41}$$

$$\frac{\alpha}{2} \leqslant \frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \leqslant 1 - \frac{\alpha}{2} \qquad \forall i = k+1, \ldots, s \tag{42}$$

Then, these equations will be equivalent to:

$$\left[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right]x \geqslant \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \qquad \forall i = 1, \ldots, k \tag{43}$$

$$\begin{aligned}
&\left[(1-\tfrac{\alpha}{2})E_2^{a_i} + \tfrac{\alpha}{2}E_1^{a_i}\right]x \geqslant \tfrac{\alpha}{2}E_2^{b_i} + (1-\tfrac{\alpha}{2})E_1^{b_i} \qquad \forall i = k+1, \ldots, s \\
&\left[\tfrac{\alpha}{2}E_2^{a_i} + (1-\tfrac{\alpha}{2})E_1^{a_i}\right]x \leqslant (1-\tfrac{\alpha}{2})E_2^{b_i} + \tfrac{\alpha}{2}E_1^{b_i} \qquad \forall i = k+1, \ldots, s
\end{aligned} \tag{44}$$

According to the ranking method which is presented by Jiménez (1996), when vector x is presented as a feasible vector, then the vector x^0 can be presented as an optimal solution at least in degree $\frac{1}{2}$, as opposed to vector x for model (40), if it is verified for all $\tilde{c}^t x \geqslant_{1/2} \tilde{c} x^0$. Therefore, the previous equation can be expressed thus:

$$\frac{E_2^{c^t x} - E_1^{c^t x}}{2} \geqslant \frac{E_2^{c^t x^o} + E_1^{c^t x^o}}{2} \tag{45}$$

Finally, according to above explanation of EV and EI, model (40) can be transformed into an equivalent crisp α-parametric linear model as follows:

$$\begin{aligned}
&\min \quad EV(\tilde{c})x \\
&s.t. \\
&\quad \left[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}\right]x \geqslant \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \quad \forall i = 1, \ldots, k \\
&\quad \left[(1-\tfrac{\alpha}{2})E_2^{a_i} + \tfrac{\alpha}{2}E_1^{a_i}\right]x \geqslant \tfrac{\alpha}{2}E_2^{b_i} + (1-\tfrac{\alpha}{2})E_1^{b_i} \quad \forall i = k+1, \ldots, s \\
&\quad \left[\tfrac{\alpha}{2}E_2^{a_i} + (1-\tfrac{\alpha}{2})E_1^{a_i}\right]x \leqslant (1-\tfrac{\alpha}{2})E_2^{b_i} + \tfrac{\alpha}{2}E_1^{b_i} \quad \forall i = k+1, \ldots, s \\
&\quad x \geqslant 0
\end{aligned} \tag{46}$$

Therefore, the main HIRP model will be transformed into the equivalent auxiliary crisp multi-objective model as follows:

$$Min \quad f_1 = \sum_{i\in M}\sum_{r\in R}\sum_{t\in T} h_{ir} I_{irt} + \sum_{(i,j)\in M'}\sum_{k\in K}\sum_{t\in T}\left(\frac{\upsilon_k^p + 2\upsilon_k^m + \upsilon_k^o}{4}\right)c_{ij} y_{kt} + \sum_{i\in M}\sum_{r\in R}\sum_{t\in T}\left(\frac{\psi_{ir}^p + 2\psi_{ir}^m + \psi_{ir}^o}{4}\right)SH_{irt} \tag{47}$$

$$Min \quad f_2 = \lambda_{er}(\sum_{i\in M}\sum_{r\in R}\sum_{t\in T}\varphi_{irt}) + \lambda_{ex}(\sum_{i\in M}\sum_{r\in R}\sum_{t\in T}EX_{irt}) \tag{48}$$

$$Min \quad f_3 = \sum_{(i,j)\in M'}\sum_{k\in K}\sum_{t\in T}GH_k y_{kt} c_{ij} \tag{49}$$

S.t.

$$I_{irt} - B_{irt} - SH_{irt} \leqslant I_{ir(t-1)} - \left[\left(\frac{\alpha}{2}\right)\times\frac{D^m_{irt}+D^o_{irt}}{2} + \left(1-\frac{\alpha}{2}\right)\times\frac{D^p_{irt}+D^m_{irt}}{2}\right] + \sum_{k\in K}Q_{irkt} - EX_{irt} - SH_{ir(t-1)} - B_{ir(t-1)} \quad \forall i\in M, r, t \tag{50}$$

$$I_{irt} - B_{irt} - SH_{irt} \geqslant I_{ir(t-1)} - \left[\left(1-\frac{\alpha}{2}\right)\times\frac{D^m_{irt}+D^o_{irt}}{2} + \left(\frac{\alpha}{2}\right)\times\frac{D^p_{irt}+D^m_{irt}}{2}\right] + \sum_{k\in K}Q_{irkt} - EX_{irt} - SH_{ir(t-1)} - B_{ir(t-1)} \quad \forall i\in M, r, t \tag{51}$$

$$\left[\left(\frac{\alpha}{2}\right)\times\frac{D^m_{ir(t+1)}+D^o_{ir(t+1)}}{2} + \left(1-\frac{\alpha}{2}\right)\times\frac{D^p_{ir(t+1)}+D^m_{ir(t+1)}}{2}\right] \leqslant (1+\varphi_{irt})\left[\left(\frac{\alpha}{2}\right)\times\frac{D^m_{irt}+D^o_{irt}}{2} + \left(1-\frac{\alpha}{2}\right)\times\frac{D^p_{irt}+D^m_{irt}}{2}\right] + E_{irtt'} \quad \forall t'=t, i, r, t \tag{52}$$

$$\left[\left(1-\frac{\alpha}{2}\right)\times\frac{D^m_{ir(t+1)}+D^o_{ir(t+1)}}{2} + \left(\frac{\alpha}{2}\right)\times\frac{D^p_{ir(t+1)}+D^m_{ir(t+1)}}{2}\right] \geqslant (1+\varphi_{irt})\left[\left(1-\frac{\alpha}{2}\right)\times\frac{D^m_{irt}+D^o_{irt}}{2} + \left(\frac{\alpha}{2}\right)\times\frac{D^p_{irt}+D^m_{irt}}{2}\right] + E_{irtt'} \quad \forall t'=t, i, r, t \tag{53}$$

$$E_{irtt'} \geqslant \sum_k q_{irktt'} + [A\times(\varphi_{irt}-1)] \quad \forall i, r, t, t' \tag{54}$$

$$E_{irtt'} \leqslant A\times\varphi_{irt} \quad \forall i, r, t, t' \tag{55}$$

$$E_{irtt'} \leqslant A\times\sum_k q_{irktt'} \quad \forall i, r, t, t' \tag{56}$$

$$\varphi_{irt} \leqslant \frac{SR_{irt}\times SH_{ir(t-1)}}{\left[\left(\frac{\alpha}{2}\right)\times\frac{D^m_{ir(t-1)}+D^o_{ir(t-1)}}{2} + \left(1-\frac{\alpha}{2}\right)\times\frac{D^p_{ir(t-1)}+D^m_{ir(t-1)}}{2}\right]} \quad \forall i\in M, r, t \tag{57}$$

$$\varphi_{irt} \geqslant \frac{SR_{irt}\times SH_{ir(t-1)}}{\left[\left(1-\frac{\alpha}{2}\right)\times\frac{D^m_{ir(t-1)}+D^o_{ir(t-1)}}{2} + \left(\frac{\alpha}{2}\right)\times\frac{D^p_{ir(t-1)}+D^m_{ir(t-1)}}{2}\right]} \quad \forall i\in M, r, t \tag{58}$$

$$SH_{irt} \geqslant \sum_{k\in K}\sum_{t>t'} q_{irktt'} \quad \forall i\in M, r, t \tag{59}$$

$$SH_{irt} - (B_{irt}\times SH_{irt}) \leqslant M\times F_{irt} \quad \forall i, r, t \tag{60}$$

$$I_{irt} - (B_{irt}\times SH_{irt}) \leqslant M\times(1-F_{irt}) \quad \forall i, r, t \tag{61}$$

$$\sum_{t'\in T}\sum_{k\in K} q_{irktt'} \leqslant IC_{ir} - I_{ir(t-1)} \quad \forall i\in M, r, t \tag{62}$$

$$\sum_{k\in K}\sum_{t'>(t+L_r+UMT_r)} q_{irktt'} \leqslant EX_{irt} \quad \forall i\in M, r, t \tag{63}$$

$$\sum_{t'\in T} q_{irktt'} \leqslant Q_{irkt} \quad \forall i\in M, r, k, t \tag{64}$$

$$\sum_{i\in M}\sum_{r\in R}\sum_{t'\in T} q_{irktt'} \leqslant cap_k y_{kt} \quad \forall k,t \tag{65}$$

$$\begin{aligned}&\tfrac{z_{0kt}}{M} \leqslant y_{kt} \leqslant z_{0kt} M \quad \forall k,t\\ &\tfrac{x_{ijkt}}{M} \leqslant y_{kt} \leqslant x_{ijkt} M \quad \forall i,k,t\end{aligned} \tag{66}$$

$$\sum_{j\in M'} x_{ijkt} = \sum_{j\in M'} x_{jikt} = z_{ikt} \quad \forall i \in M', k, t \tag{67}$$

$$w_{ikt} - w_{jkt} + (|M|+1)x_{ijkt} \leqslant |M| \quad \forall (i,j) \in M, k, t \tag{68}$$

$$x_{ijkt}, z_{ikt}, F_{irt} \in \{0,1\} \quad \forall i,j \in M, t, k, i \neq j \tag{69}$$

$$\left.\begin{aligned}&q_{irktt'}, I_{irt}, EX_{irt}, SH_{irt}, y_{kt}, E_{irtt'} \geqslant 0, Integer\\ &\varphi_{irt} \in [0,1]\end{aligned}\right. \quad \forall i \in M, r, t, k \tag{70}$$

5.2. The fuzzy solution approach

There are various methods to solve multi-objective linear programming (MOLP) problems in the literature, but a fuzzy programming approach is used by a large group of researchers which allows maximization of the satisfaction degree for each objective function. The main advantage of this approach is that it helps the decision maker (DM) obtain an efficient solution according to his/her preference, or the relative importance of each objective. Zimmermann (1978) was the first person to propose a fuzzy approach for solving a MOLP by the max–min approach. However, the solutions yielded by this approach are not unique and efficient (Lai and Hwang, 1992). Therefore, Lai and Hwang (1992) proposed an augmented min–max approach to remove this deficiency. Recently, Selim and Ozkarahan (2006) proposed a new approach, according to a modification to aggregation functions suggested by Werners (1988). In this research, the TH (Torabi and Hassini, 2008) method is applied to find an optimized, efficient solution as follows:

Step 1: Define the possibility distribution (triangular or trapezoidal) for ambiguous parameters and formulate the multi-objective possibilistic mixed-integer linear programming (MOPMILP) for the HIRP problem.

Step 2: Transform fuzzy constraints to crisp ones by defining a minimum acceptable feasibility degreeα, and convert the original fuzzy objective into crisp ones according to the expected values of the ambiguous parameters.

Step 3: Specify the β-positive ideal solution (β-*PIS*) and the β-negative ideal solution (β-*NIS*) for each objective function. $(Y_1^{\beta-PIS}, x_1^{\beta-PIS})$, $(Y_2^{\beta-PIS}, x_2^{\beta-PIS})$ and $(Y_3^{\beta-PIS}, x_3^{\beta-PIS})$ could be obtained as β-positive ideal solutions by solving the crisp MOMILP model for each objective function. Then, related β-negative ideal solutions could be estimated thus:

$$Y_1^{\beta-NIS} = \max\{Y_1(x_2^{\beta-PIS}), Y_1(x_3^{\beta-PIS})\} \tag{71}$$

$$Y_2^{\beta-NIS} = \max\{Y_2(x_1^{\beta-PIS}), Y_2(x_3^{\beta-PIS})\} \tag{72}$$

$$Y_3^{\beta-NIS} = \max\{Y_3(x_1^{\beta-PIS}), Y_3(x_2^{\beta-PIS})\} \tag{73}$$

Step 4: Linear membership objective functions are presented as follows:

$$\mu_1(x) = \begin{cases} 1 & if\ Y_1 < Y_1^{\beta-PIS} \\ \frac{Y_1^{\beta-NIS} - Y_1}{Y_1^{\beta-NIS} - Y_1^{\beta-PIS}} & if\ Y_1^{\beta-PIS} \leqslant Y_1 \leqslant Y_1^{\beta-NIS} \\ 0 & if\ Y_1 > Y_1^{\beta-NIS} \end{cases} \tag{74}$$

$$\mu_2(x) = \begin{cases} 1 & if\ Y_2 < Y_2^{\beta-PIS} \\ \frac{Y_2^{\beta-NIS} - Y_2}{Y_2^{\beta-NIS} - Y_2^{\beta-PIS}} & if\ Y_2^{\beta-PIS} \leqslant Y_2 \leqslant Y_2^{\beta-NIS} \\ 0 & if\ Y_2 > Y_2^{\beta-NIS} \end{cases} \tag{75}$$

$$\mu_3(x) = \begin{cases} 1 & if\ Y_3 < Y_3^{\beta-PIS} \\ \frac{Y_3^{\beta-NIS} - Y_3}{Y_3^{\beta-NIS} - Y_3^{\beta-PIS}} & if\ Y_3^{\beta-PIS} \leqslant Y_3 \leqslant Y_3^{\beta-NIS} \\ 0 & if\ Y_3 > Y_3^{\beta-NIS} \end{cases} \tag{76}$$

Step 5: The equivalent crisp MOMILP will transform into a single objective MILP by the TH aggregation method. The aggregation function is presented as follows:

$$\begin{aligned} \max \quad & \lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_h \theta_h \mu_h(x) \\ s.t. \quad & \\ & \lambda_0 \leqslant \mu_h(x), \quad h = 1,2,3 \\ & x \in F(x), \quad \lambda_0 \text{ and } \lambda \in [0,1] \end{aligned}, \tag{77}$$

where $\mu_h(x)$ denotes the satisfaction degree of the hth objective function for vector x, and also λ_0 defines the lower bound of the satisfaction degree for objectives $(\lambda_0 = \min_h\{\mu_h(x)\})$. Moreover, θ_h represents the relative importance, which is defined by the decision maker according to his/her preference, that $\sum_h \theta_h = 1, \quad \theta_h > 0$. Also, γ determines the coefficient of compensation for each objective, which can be unbalanced, and balances the optimized solution.

Step 6: Determine θ_h (relative importance) and γ (coefficient of compensation), then solve proposed auxiliary MILP model (77). The decision maker is able to change the values of α, γ and θ_h in order to obtain the optimized, efficient solution.

6. Experimental results

To verify the efficiency of the proposed model and solution approach, first five numerical examples with different sizes are generated. Then to illustrate the applicability and usefulness of the proposed mathematical model, a case study problem is applied in continue.

6.1. Generated numerical examples

The numerical examples (Table 1) are generated according to the information which is provided in Table 2. It is noteworthy that some of the generated parameters are determined by the subject matter experts and the others are taken from previous study (Coelho and Laporte, 2014). The triangular fuzzy parameter is generated based on the proposed method by Lai and Hwang (1992). In this method, the value of three prominent points (n^p: most pessimistic value, n^m: most possible value, and n^o: optimistic value of $\tilde{n}$) are estimated as shown below. First, the value of the most likely point (n^m) is generated randomly according to the distribution presented in Table 2, which means that the most likely value is considered equivalent to the crisp value. The pessimistic and optimistic values are then calculated by the following equations:

$$\begin{aligned} n^o &= (1+d_1)n^m \\ n^p &= (1-d_2)n^m \end{aligned} \quad (d_1, d_2) \sim \mathit{Uniform}(0.2, 0.8)$$

Also, the related transportation information (vehicle capacity and produced GHG emission) for each type of vehicle is presented in Table 3. The GHG emission of each vehicle is calculated according to the Carbonify website (Carbon dioxide emissions calculator).

In order to set the relative importance ($\theta = (\theta_1, \theta_2, \theta_3)$) for the objectives of the problem, the Analytical Hierarchy Process (AHP) is included as a one of the Multi Criteria Decision Making (MCDM) techniques. According to AHP, relative importance is defined as $\theta_2 > \theta_1 > \theta_3$ and $\theta = (0.35, 0.5, 0.15)$. Firstly, all the test problems are solved with different α levels, while a low value of 0.4 is set for γ. Since the second objective has higher relative importance, the low value is considered for γ so as to yield unbalanced solutions. (Table 4). It is noteworthy, in order to take into account the randomness effect, 5 instances are generated for problem No. 1, 7 instances for problems No. 2, 3 and 10 instances for problems No. 4, 5 and the average results of them are reported. Then the worst and best case performance of the algorithm in terms of the value of each objective function and CPU time are demonstrated in Table 5 ($\alpha = 0.6$ and $\gamma = 0.4$). Secondly, for all test problems, a sensitivity analysis is done by changing the value of γ while $\alpha = 0.9$ (Table 6).

The possibilistic model are coded in optimization software GAMS 23.5, Solver CPLEX and run for all test problems on a PC Pentium Corei5, 2.27 GHZ with 4.0 GB RAM. The mean optimality gap is less than 2% for all size of the numerical examples.

As results of Table 6 show, the TH method is very sensitive against γ variations, and it could provide suitable solutions according to the priority of the decision makers. In other words, setting a higher value for γ results in balanced solutions that pay attention to the minimum satisfaction degree of the objectives (λ_0); on the other hand, the lower value of γ yields unbalanced solutions that give more attention to the importance of the objectives.

Table 1
Size of numerical examples.

Problem No.	Customer	Product	Vehicle type	Period
1	10	1	3	7
2	15	3	3	8
3	20	5	3	8
4	30	5	3	9
5	35	7	3	10

Table 2
Test problem generation.

Parameter	Corresponding random distribution		
I_{ir0} (Box)	$\sim$ *Uniform* (0, 5)		
h_{ir} (€)	$\sim$ *Uniform* (100, 300)		
B_{irt} (Box)	$\sim$ *Uniform* (2, 6)		
$\tilde{\psi}_{ir}$ (€)	$\sim$ *Uniform* (100, 150)		
c_{ij}^{*} [a] (km)	$\sim$ *Uniform* (10, 30)		
$\tilde{D}_{irt}$ (Box)	$\sim$ *Uniform* (10, 30)		
UMT_r (Day)	$\sim$ *Uniform* (2, 4)		
IC_{ir} (Box)	$R \times \max\{\tilde{D}_{irt}\}$ [b]	$R \sim$ *Uniform* (2, 4)	
L_r (Month)	$S \times \lvert T \rvert$ [c]	$S \sim$ *Uniform* (0.5, 0.75)	
$\tilde{v}_k$ (Box)	S[d] $\sim$ *Uniform* (100, 250)	M[d] $\sim$ *Uniform* (250, 400)	L[d] $\sim$ *Uniform* (400, 500)

[a] The distance between two points is calculated by "minimum length path".
[b] (Coelho and Laporte, 2014).
[c] $\lvert T \rvert$ = Number of periods (The periods are considered monthly).
[d] S = Small, M = Medium, L = Large.

Table 3
Information of vehicle type.

Vehicle type	Capacity	GHG emission (kg/km)
Small	90	0.413
Medium	150	0.531
Large	300	0.884

Table 4
Results based on α-level variations.

Problem No.	α-level	Y_1	Y_2	Y_3	$\mu(Y_1)$	$\mu(Y_2)$	$\mu(Y_3)$	CPU time (s)
1	0.5	441,469	92.88	247.42	0.94	0.97	0.93	623
	0.6	443,857	93.23	248.76	0.93	0.96	0.92	659
	0.7	446,598	96.05	250.95	0.92	0.96	0.91	631
	0.8	447,365	101.26	249.36	0.92	0.88	0.88	611
	0.9	447,917	102.94	251.87	0.92	0.92	0.90	687
	1	430,162	104.32	250.32	0.87	0.92	0.90	671
2	0.5	2,127,296	397.09	1031.94	0.94	0.96	0.89	1815
	0.6	2,129,857	399.15	1074.87	0.94	0.95	0.91	1807
	0.7	2,145,089	405.38	1059.06	0.93	0.95	0.91	1882
	0.8	2,132,067	419.39	1063.74	0.90	0.95	0.92	1854
	0.9	2,139,892	420.85	1069.98	0.92	0.91	0.90	1883
	1	2,127,104	423.63	1077.05	0.92	0.92	0.93	1869
3	0.5	4,795,246	832.65	2436.32	0.95	0.98	0.92	3175
	0.6	4,805,209	834.12	2601.64	0.95	0.98	0.91	3423
	0.7	4,823,017	839.65	2956.65	0.86	0.97	0.90	3097
	0.8	4,814,904	845.20	2448.33	0.90	0.95	0.94	3164
	0.9	4,814,052	846.96	2440.90	0.90	0.95	0.94	3369
	1	4,795,195	850.05	2831.05	0.91	0.95	0.88	3529
4	0.5	9,484,095	1405.70	5708.03	0.91	0.94	0.88	4612
	0.6	9,496,832	1423.05	5832.98	0.90	0.94	0.86	4373
	0.7	9,494,051	1439.86	5807.39	0.91	0.94	0.90	4159
	0.8	9,507,691	1457.29	5782.67	0.91	0.92	0.90	4398
	0.9	9,514,008	1460.07	5827.53	0.87	0.92	0.87	4071
	1	9,518,142	1487.59	5905.71	0.85	0.91	0.86	4429
5	0.5	24,131,705	2938.04	11197.95	0.93	0.98	0.90	7301
	0.6	24,200,981	2980.61	11860.25	0.91	0.96	0.87	7252
	0.7	24,271,014	3071.03	13856.11	0.90	0.96	0.91	7389
	0.8	25,180,976	3184.71	11268.91	0.90	0.94	0.92	7091
	0.9	24,226,751	3220.77	12907.48	0.87	0.93	0.90	7515
	1	24,130,564	3242.09	13635.76	0.92	0.93	0.88	7186

In order to verify the impact of the consideration of GHG emission and forecast error caused by product shortage in the proposed HIRP, three sensitivity analyses are conducted for problem No. 3. First, we investigate the variation of the first part of objective function (total cost value) and third part of objective function (total GHG emission) against an increase in the relative importance of GHG emission (θ_3), while the relative importance for the second part of objective function (θ_2) is fixed and equal to 0.1 (Fig. 2).

Table 5
Best and worst case performance ($\alpha = 0.6$, $\gamma = 0.4$).

Problem No.	Worst case			Best case			Worst CPU time (s)	Best CPU time (s)
	Y_1	Y_2	Y_3	Y_1	Y_2	Y_3		
1	445,711	93.44	249.62	442,210	92.90	247.55	673	639
2	2,141,804	400.32	1079.28	2,120,807	397.58	1068.98	1858	1802
3	4,854,657	841.59	2630.32	4,760,816	827.78	2573.79	3546	3309
4	9,580,666	1432.05	5881.63	9,428,543	1415.95	5793.60	4401	4346
5	24,517,929	3009.79	11998.91	23,956,865	2954.05	11708.52	7491	7129

Table 6
Sensitivity analysis on γ-value.

Problem No.	α-level	γ-value	Y_1	Y_2	Y_3	$\mu(Y_1)$	$\mu(Y_2)$	$\mu(Y_3)$
1	0.9	0.1–0.3	454,076	94.17	279.14	0.91	0.90	0.89
		0.4	451,701	100.93	261.16	0.93	0.89	0.87
		0.5–0.9	448,596	103.48	252.10	0.91	0.85	0.90
2	0.9	0.1, 0.2	2,150,764	372.08	1094.83	0.90	0.94	0.84
		0.3, 0.4	2,139,892	420.85	1069.98	0.92	0.91	0.90
		0.5	2,139,892	421.76	1071.53	0.92	0.90	0.89
		0.6–0.9	2,141,067	422.96	1069.98	0.78	0.81	0.90
3	0.9	0.1, 0.2	4,930,514	828.04	2480.92	0.89	0.96	0.85
		0.3, 0.4	4,879,019	839.71	2458.15	0.92	0.92	0.90
		0.6–0.8	4,821,989	846.11	2463.90	0.93	0.90	0.87
		0.9	4,821,989	849.39	2458.15	0.93	0.88	0.90
4	0.9	0.1	9,614,391	1391.34	5913.09	0.90	0.93	0.82
		0.2–0.4	9,566,019	1433.08	5895.13	0.85	0.90	0.90
		0.5–0.7	9,566,019	1461.28	5857.34	0.85	0.89	0.88
		0.8, 0.9	9,512,033	1495.69	5829.81	0.88	0.87	0.91
5	0.9	0.1, 0.2	25,302,968	2912.04	14968.01	0.90	0.96	0.81
		0.3–0.5	24,226,751	3220.77	12907.48	0.87	0.93	0.90
		0.6, 0.7	24,209,163	3227.41	12885.09	0.83	0.79	0.92
		0.8, 0.9	24,209,163	3227.41	12912.43	0.83	0.79	0.89

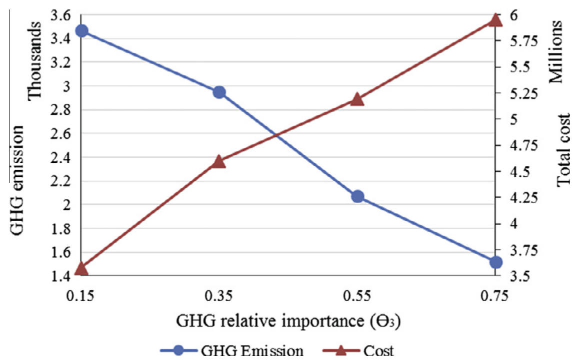


Fig. 2. Trade-off between GHG emission and total cost for problem No.3.

Fig. 2 shows that when the relative importance of GHG emission is increased, the value of amount of produced GHG emission is decreased, while the total cost is increasing. This trend illustrates the importance of considering GHG emissions in the proposed HIRP model. This happens because increasing in importance of GHG emission, change the configuration of the fleet. This means model attempts to decrease the number of the transportation by using the large size of vehicles instead of several small vehicles and deliver more products.

The second analysis is related to the impact of shortage cost (per unit) variation in total forecast error (Fig. 3) which shows that total forecast error (caused by product shortage) is decreased by increasing shortage cost (per unit). This is because when the shortage cost (per unit) increases, the model attempts to decrease the amount of shortage. Since the amount of shortage has a direct relation with forecast error (Eq. (6)), the decreasing amount of shortage leads to a reduction in total forecast error. The third analysis is investigated the effect of the inventory holding cost (per unit) variation in value of second part of objective function (Fig. 4). This analysis shows that increase in the inventory holding cost (per unit) caused

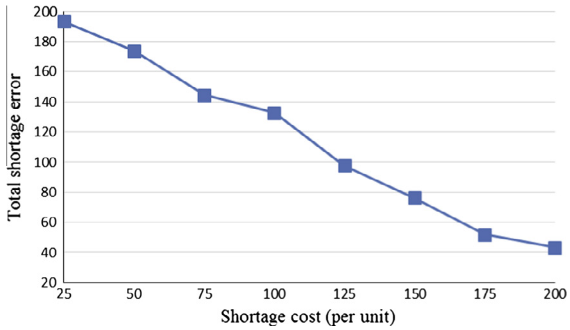


Fig. 3. Value of total forecast error (which is caused by shortage) against shortage cost (per unit) for problem No. 3.

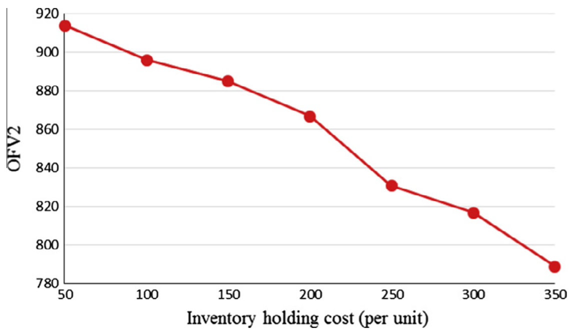


Fig. 4. Value of second objective function against inventory holding cost (h_{ir}) for problem No. 3.

decrease in second objective function value. This is because when the inventory holding cost (per unit) increases, the inventory level will be decreased which leads to a reduction in quantity of the expired products and second objective function value. It is noteworthy, in this situation model attempts to send more vehicles to satisfy the demand (the relative importance for these three analysis is $\theta = (0.35, 0.5, 0.15)$).

Also, another sensitivity analysis is performed to study the interaction of a varying number of transports and variations in the total cost of components in problem No. 3. The results show that increasing the number of transports causes the total cost of shortages to decrease. The results also show an increase in the total transportation cost, while the total inventory cost increases until an optimum number of transports is reached, with the total inventory cost collapsing beyond that point. This implies that before collapse point, the amount of forecast error is high due to the high value of shortage. Therefore the model attempts to increase the number of transportation and the quantity of delivered drugs (increase of inventory) to reach optimum forecast error but after this point due to low value of forecast error, by increasing number of transportation, less quantities are delivered in each visit so the inventory cost is decreased (Figs. 5 and 6).

6.2. Case study and computational results

In the following a real case from the region Rhône-Alpes, France is applied to show that theoretical framework which is explained in previous section can easily be implemented. A pharmaceutical supplier distributes 2 types of products to 12

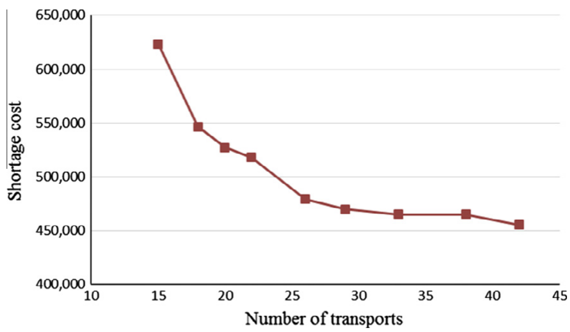


Fig. 5. Number of transports against shortage cost for problem No. 3.

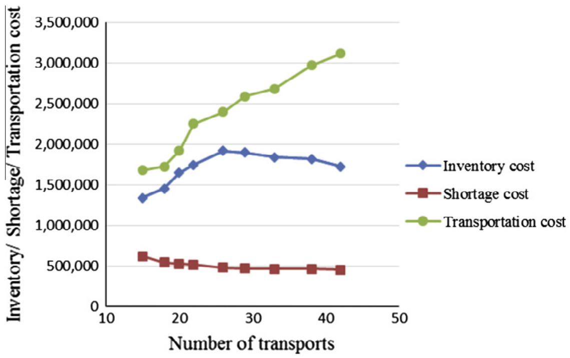


Fig. 6. Number of transports against inventory, shortage, and transportation costs for problem No. 3.

Table 7
Information of vehicle type (caste study).

Vehicle type	Capacity	GHG emission (kg/km)
Small	80	0.387
Medium	170	0.619
Large	290	0.752

Table 8
Triangular sets for uncertain parameters (case study).

Parameter	Corresponding values		
h_{ir} (€)	Product A: 65		Product B: 80
$\tilde{\psi}_{ir}$ (€)		(45, 56, 60)[a]	
$\tilde{D}_{irt}$ (Box)		(5, 27, 35)	
$\tilde{v}_k$ (Box)	S[b]: (8, 10, 15)	M[b]: (19, 22, 27)	L[b]: (32, 36, 41)

[a] $\tilde{n} = (n^p, n^m, n^o)$.
[b] S = Small, M = Medium, L = Large.

Table 9
The summary of results.

Problem	α-level	Y_1	Y_2	Y_3	$\mu\,(Y_1)$	$\mu\,(Y_2)$	$\mu\,(Y_3)$	CPU time (s)
Case study	0.6	245,988	417.32	1794.51	0.92	0.96	0.89	1721

Note: $\gamma - value$ = 0.4.
θ = (0.35, 0.5, 0.15).

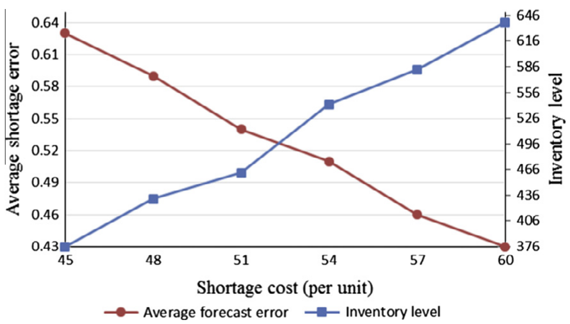
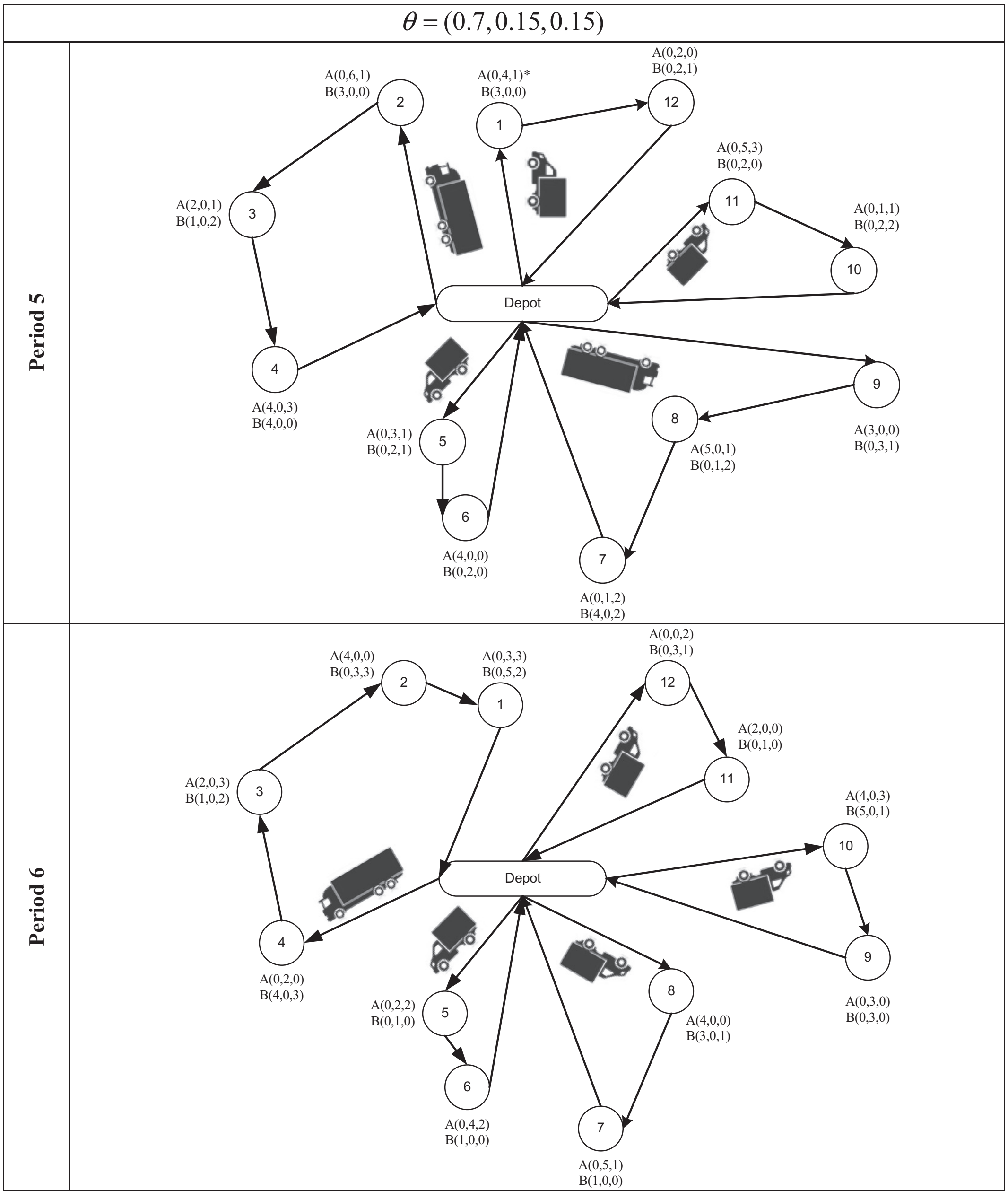


Fig. 7. Trade-off between shortage cost (per unit), inventory level and forecast error (which is caused by shortage) (φ_{irt}).

customers in a planning horizon with 12 periods (periods are monthly). In the current condition there are three sizes of vehicles with different capacity, cost and GHG emission (Table 7).

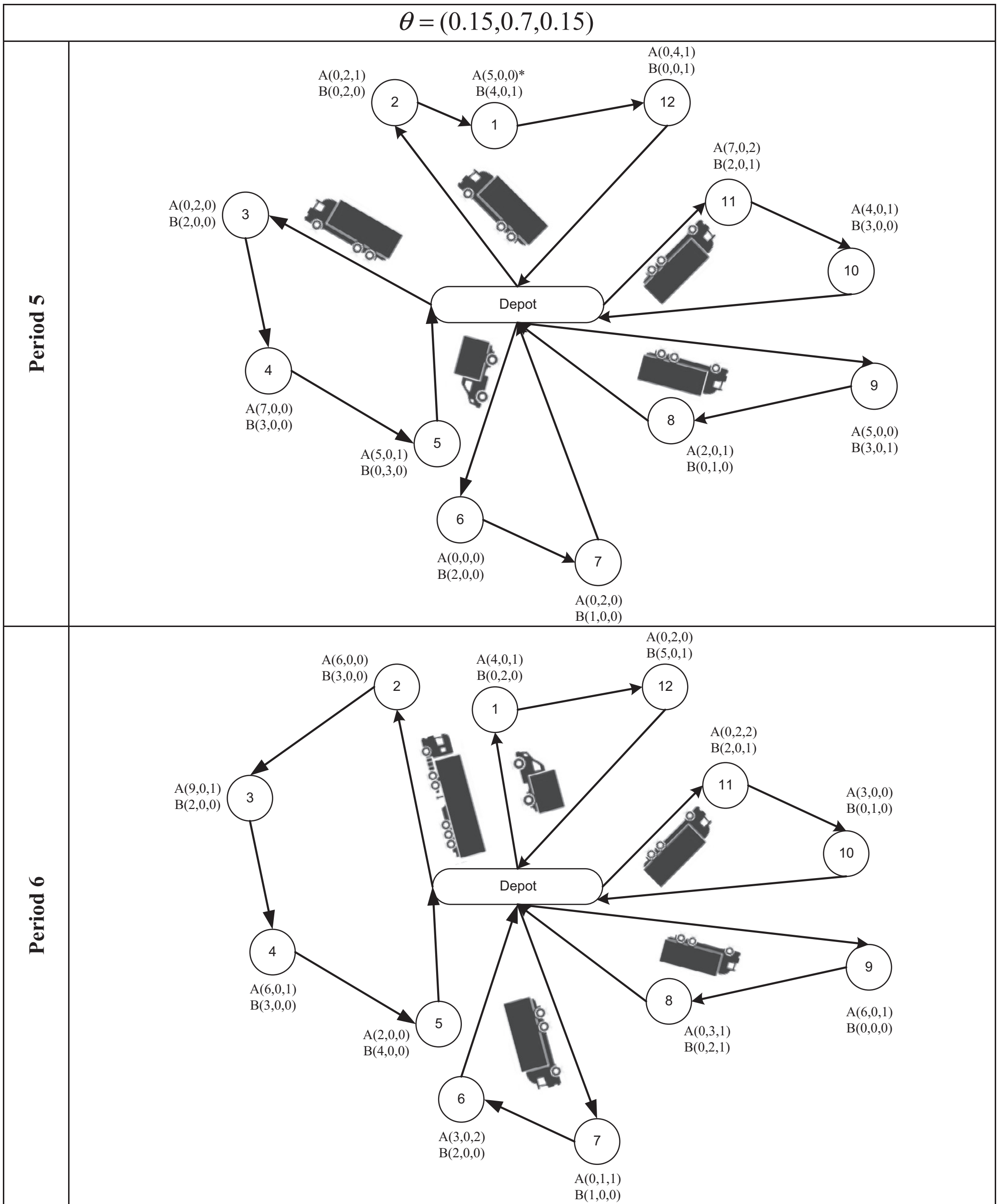
The amount of demand for each customer, shortage and variable transportation costs are highly variable and uncertain from one period to the other. Due to lack of the information about uncertain parameters the sets of triangular fuzzy number



Small Vehicle	3	*Small Vehicle*	4
Medium Vehicle	2	*Medium Vehicle*	1
Large Vehicle	0	*Large Vehicle*	0
Total inventory level	33	*Total inventory level*	31
Total amount of shortage	36	*Total amount of shortage*	35
Total amount of expired	25	*Total amount of expired*	29

* *i.e., (3,2,1) means: ("inventory level", "amount of shortage", "amount of expired")*

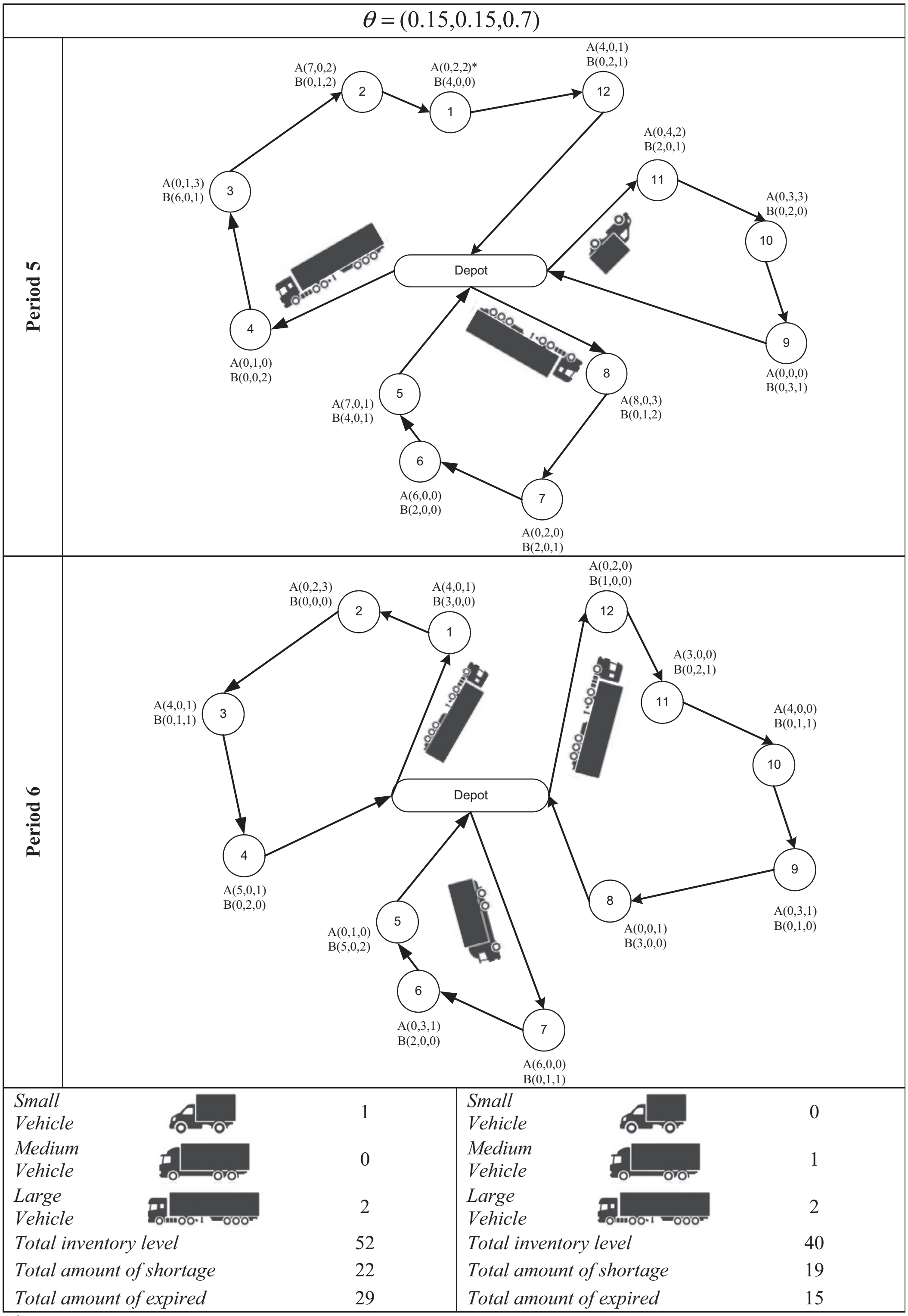
Fig. 8. Schematic of configuration for two periods with cost priority.



Small Vehicle	1	*Small Vehicle*	1
Medium Vehicle	4	*Medium Vehicle*	3
Large Vehicle	0	*Large Vehicle*	1
Total inventory level	55	*Total inventory level*	61
Total amount of shortage	16	*Total amount of shortage*	13
Total amount of expired	11	*Total amount of expired*	13

* *i.e., (3,2,1) means: ("inventory level", "amount of shortage", "amount of expired")*

Fig. 9. Schematic of configuration for two periods with customer satisfaction priority.



* *i.e., (3,2,1) means: ("inventory level", "amount of shortage", "amount of expired")*

Fig. 10. Schematic of configuration for two periods with GHG emission priority.

is applied which is shown in Table 8. In reality, some parameters such as the importance of the goods transportation, weather conditions, road condition and variation of fuel cost make the transportation cost as an uncertain parameter (Cui and Sheng, 2012). Also in this case, since importance of product availability is variable in different periods shortage cost is presented as uncertain parameter (*e.g.* the importance of the cold drugs are high in winter periods, therefore the cost of the shortage in these periods are more than the others).

The minimum usable time of drug before expiration date (UMT_r) for both of drug A and B is 0.8 period also shelf life (L_r) are estimated 3 periods for drug A and 2.4 periods for drug B. Finally Table 9, demonstrates the summaries of the computational results for the presented case.

A sensitivity analysis is conducted in order to study the impact of shortage cost (per unit) variation on the value of inventory level and average forecast error in optimal point (Fig. 7). As shown in Fig. 7, when the shortage cost (per unit) is increased, the inventory level is increased while forecast error is decreased. This happens because when the shortage cost increases, the model attempts to decrease the amount of shortage by sending more products to the customers which is causes an increase in the inventory level. Also, since amount of shortage has direct relation with forecast error (Eq. (6)), this decrease in amount of shortage (because of the increase in shortage cost) leads to a reduction in forecast error. As a managerial insight, this sensitivity analysis shows that a 25% increase in shortage cost leads to a 42% increase of inventory level and a 33% decrease in forecast error.

In the following, in order to validate each of objectives' effects on the obtained solution, the value of relative importance ($\theta = (\theta_1, \theta_2, \theta_3)$) for each objective function is adjusted for better demonstration of this effect; a schematic of a configuration for two periods (periods 5 and 6) of the solution are depicted in Figs. 8–10. It should be mentioned that the values of α, γ for all three combinations of the vector θ_h are same ($\alpha = 0.6$, $\gamma = 0.4$).

Fig. 8, demonstrates the influences of the cost in the model by increasing the related importance value (θ_1), while two other objectives have low and equal effects on the solution ($\theta_1 = 0.7$ and $\theta_2 = \theta_3 = 0.15$).

Fig. 9, shows the influences of customer satisfaction in the model by increasing the related importance value (θ_2), while the two other objectives have low and equal effects on the solution ($\theta_2 = 0.7$ and $\theta_1 = \theta_3 = 0.15$).

And finally, in Fig. 10 the influence of the GHG emission in the model is demonstrated by increasing the related importance value (θ_3), while the two other objectives have low and equal effects on the solution ($\theta_3 = 0.7$ and $\theta_1 = \theta_2 = 0.15$).

The analysis represents that in the cost priority situation (Fig. 8) the configuration of the vehicles has lower transportation cost compared with the others because, in this priority, the model attempts to decrease the length of each tour and select more small vehicles which leads to lower transportation cost. In this configuration, the utilization of a greater number

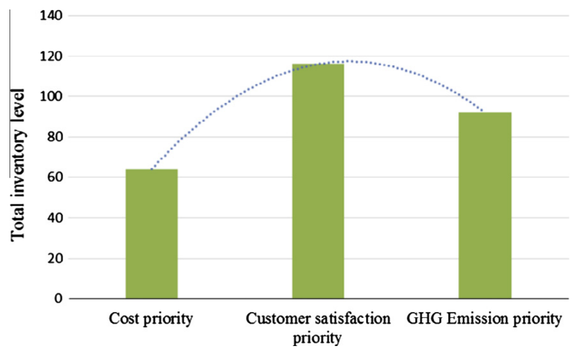


Fig. 11. Impact of variation of objective function importance in total inventory level.

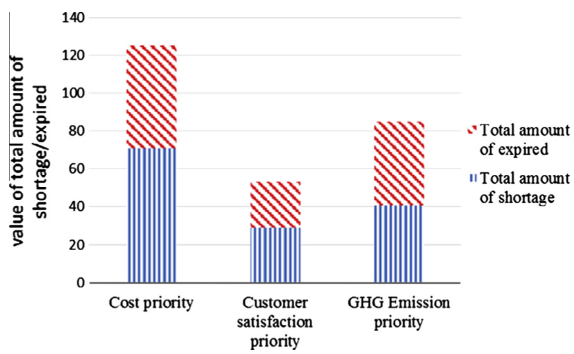


Fig. 12. Impact of variation of objective function importance in total amount of expired and shortage.

of small vehicles makes for an increase in GHG emission; but, on the other hand, in the GHG emission priority, the utilization of a greater number of large vehicles decreases the GHG emissions without concern for cost (Fig. 10).

To further clarify the impact of each objective function, the variation of three variables—inventory level, amount of shortage, and amount of expired—for each situation is analyzed (Figs. 11 and 12).

Fig. 11 shows that the value of total inventory level is a minimum in cost priority because the critical impact of holding cost in first objective and moreover, as illustrated in Fig. 12, the customer satisfaction priority has a low value of the amount of shortage and expired. It is noteworthy that the amount of expired has a direct effect in the second objective and also the forecast error as other components of the second objective are calculated by considering the value of shortage.

7. Conclusion

This study presented a new IRP mathematical model to tackle distribution and inventory issues related to healthcare. First, we proposed a multi-objective mathematical model in which the first part of objective function minimizes costs related to inventory and transportation, the second part of objective maximizes customer satisfaction by minimizing the amount of errors in demand forecasting and the quantity of expired drugs, and the third part of objective minimizes the amount of GHG emissions produced by various types of vehicles during distribution. Because of a high degree of uncertainty in real-life conditions, we increased the capability of the proposed model to deal with uncertain parameters with respect to demand, shortage, and transportation costs. Therefore, a possibilistic fuzzy approach was applied to transform the fuzzy mathematical model to an equivalent crisp model, and an efficient fuzzy approach was applied to find both balanced and unbalanced solutions according to the DM's priorities. The results and sensitivity analysis illustrate the strength of the proposed solving approach in handling uncertain parameters.

To continue our current research, some other related transportation, inventory, and healthcare issues can be considered, such as traffic, availability of facilities (time window), and route availability in disaster conditions. Various inaccurate parameters and possibilistic approaches might be applied and also a heuristic can be developed to decrease solving time in addition.

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