Expert Systems With Applications 53 (2016) 149–159

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

A genetic algorithm for the vendor-managed inventory routing problem with lost sales

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ARTICLE INFO

Keywords:
Genetic algorithm
Vendor-managed inventory
Inventory routing problem
Mixed-integer linear programming model
Logistics

ABSTRACT

This paper proposes a genetic algorithm (GA) for the inventory routing problem with lost sales under a vendor-managed inventory strategy in a two-echelon supply chain comprised of a single manufacturer and multiple retailers. The proposed GA is inspired by the solving mechanism of CPLEX for the optimization model of the problem. The proposed GA determines replenishment times and quantities and vehicle routes in a decoupled manner, while maximizing supply chain profits. The proposed GA is compared with the optimization model with respect to the effectiveness and efficiency in various test problems. The proposed GA finds solutions in a short computational time that are very close to those obtained with the optimization model for small problems and solutions that are within 3.2% of those for large problems. Furthermore, sensitivity analysis is conducted to investigate the effects of several problem parameters on the performance of the proposed GA and total profits.

1. Introduction

Vendors and their customers often adopt vendor-managed inventory (VMI) strategies to improve the profitability of the vendors' brand for both customers and vendors through collaboration. With VMI, vendors are responsible for all decisions regarding customer inventory management. As a result, control of replenishment decisions resides with the vendors instead of their customers. In many cases of VMI, the inventory is owned by the vendor until it is sold by the customer. Accordingly, vendors require their customers to supply them with information about product sales, current inventory levels, dates for receipt of goods, and dead stock and returns through an EDI or other electronic network, so that the information is up-to-date at all times. With VMI, the vendors are able to send the products to the customers at an earlier stage and will be charged for inventory carrying costs. On the other hand, the increased costs are mitigated by a reduced bullwhip effect, increased sales, freight consolidation, and aggregate production planning. This leads to higher levels of production efficiency and much lower transportation costs, which constitute a significant portion of overall supply chain costs. Many researchers have studied the potential benefits of the VMI strategy according to various aspects of supply chain management (Chen & Wei, 2012; Choudhary, Shankar, Dey, Chaudhary, & Thakur, 2014; Savaşaneril & Erkip, 2010; Zachariassen et al., 2014).

In the existing literature, the class of problems in which inventory management and vehicle routing problems are integrated into a unified framework is often referred to as the inventory routing problem (IRP). Effective implementation of the IRP is critical, especially in a VMI environment. Some surveys on the IRP are shown in the works of Bertazzi, Savelsbergh, and Speranza (2008), and Coelho, Cordeau, and Laporte (2012). Several real-world applications of the IRP are surveyed in the work of Coelho et al. (2012). Although the IRP has been extensively researched over the last two decades, some important characteristics inherent to VMI are rarely treated in a complex way in the literature. First, under VMI, intentional lost sales are often allowed (as opposed to full backorders for customer stock-outs) when the supply chain cost per product exceeds the unit profit in the supply chains of grocery or consumer products. Therefore, it is more sensible to set a goal of maximizing supply chain profits through the collaboration of customers and vendors when lost sales are allowed, rather than a goal of minimizing total costs. It is notable that the problem of establishing a network of agents (or retailers) under VMI, which is an extended version of the IRP, sets a goal that maximizes total profits while allowing lost sales (Rabbani, Baghersad, & Jafari, 2013). Second, the customer's storage space for an item is pre-assigned under VMI. Thus, the delivery quantity from the vendor cannot exceed the available storage space. Third, under VMI, customers are likely to set due times (e.g., store opening times) or time windows for delivery based on their operational preferences.

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http://dx.doi.org/10.1016/j.eswa.2016.01.041

Considering the characteristics synthetically, we define the vendor-managed inventory routing problem with lost sales (VMIRPL) as follows. The VMIRPL determines optimal replenishment times and quantities for customers and vehicle routes that maximize supply chain profits during a planning horizon in a VMI environment, while allowing lost sales at customers and while restricting transportation capacity, storage space at customers, and due times for delivery. Several costs are incurred in the VMIRPL. Vendors incur production costs, inventory carrying costs (their own and the inventory carrying costs of customers), and fixed and variable transportation costs. Customers incur purchasing and space costs. Central to the VMIRPL is trade-off analysis, which, in turn, leads to a concept of weighing total costs against the satisfaction of demand at customers. For instance, transportation costs due to consolidated replenishments and the direct effects on inventory carrying costs at customers are in conflict with each other. Profit loss and supply chain costs due to lost sales are also in conflict. In a VMI environment, the problem becomes one of balancing the combined conflicting costs so that they are collectively optimized.

This paper proposes a genetic algorithm (GA) for the VMIRPL in a two-echelon supply chain comprised of a single manufacturer and multiple retailers. The proposed GA is compared with the optimization model with respect to effectiveness and efficiency in various test problems. Furthermore, sensitivity analysis is conducted to investigate the effects of several problem parameters on the performance of the proposed GA and total profits. Evolutionary algorithms are a versatile and effective approach to finding solutions to problems with high levels of complexity or interdependency (Simon, 2013). In recent years, GAs, the most popular type of evolutionary algorithms, have been advocated for solving IRPs.

The remainder of this paper is organized as follows. Section 2 reviews previous studies on IRPs. Section 3 describes and presents a mathematical formulation for the VMIRPL. Section 4 proposes a new GA, which includes an improvement process for the VMIRPL. Section 5 conducts computational experiments and sensitivity analysis to evaluate the performance of the proposed GA. Finally, Section 6 draws conclusions and presents topics for future research.

2. Literature review

Focus on the IRP in current supply chain research has increased during the last two decades. The IRP is non-deterministic polynomial-time hard (NP-hard), thus, many effective heuristics have been developed. A number of exact algorithms are also available to address the IRP in the context of supply chain management and logistics. The solutions may be classified into three categories with respect to methodology, including mathematical models, evolutionary algorithms, and search algorithms. This section briefly reviews the existing literature for each category, concentrating mainly on the articles published in recent years for the IRP with deterministic demands at customers.

The first category uses mathematical programming, combined in some cases with primal heuristics. In general, the problem size to be solved by the methods in this category is quite limited although they provide very good solutions. Abdelmaguid, Dessouky, and Ordóñez (2009) proposed a constructive heuristic approach to obtain an approximate solution for the IRP with backlogging, wherein three mixed-integer programming formulations are solved to compute the delivery quantity, determine consolidations of deliveries, and compute transportation costs, respectively. Archetti, Bertazzi, and Laporte (2007) introduced a mixed-integer linear programming model for the IRP with order-up-to level inventory policies at retailers, in which a single vehicle is available. That research presents an exact branch-and-cut algorithm, while strengthening the linear relaxation of the model with new additional valid inequalities. Archetti, Bertazzi, Hertz, and Speranza (2012) subsequently extended their previous work, including maximum level policies at retailers and presenting a hybrid heuristic that combines a tabu search scheme for improvement processes with mixed-integer programming models. Tabu search algorithms have been proved to be very effective for many vehicle routing problems (Gendreau, Hertz, & Laporte, 1994). Bard and Nananukul (2009) developed a two-step procedure that first estimates daily delivery quantities and next solves a vehicle routing problem for each day. As part of the methodology, a linear program is used to determine the days on which it is necessary to make at least some deliveries to avoid stock-outs. Coelho et al. (2012) presented a mixed-integer linear programming model for the IRP that allows transshipments, either from the supplier to customers or between customers. Making the first contribution to the IRP with multi-products is the work of Coelho and Laporte (2013). They presented mixed-integer linear programs for the problem and developed a branch-and-cut algorithm to obtain exact solutions. Coelho and Laporte (2014) proposed an exact branch-and-cut algorithm to solve the IRP for perishable products. They also modeled and solved exactly two variants of the IRP defined by applying two selling priority policies, in which the retailer sells with higher priority older and fresher items, respectively. Cordeau, Laganà, Musmanno, and Vocaturo (2015) proposed a three-phase heuristic for the IRP with multi-products, which determines replenishment plans and vehicle routes in the first two phases, respectively, and finds better solutions with an integrated mixed-integer linear programming model incorporating those decisions in the third phase. Rusdiansyah and Tsao (2005) presented a solution procedure for the IRP with time windows encountered in vending machine supply chains under a VMI scheme, which uses some properties derived from a mixed-integer linear programming model. Ramkumar, Subramanian, Narendran, and Ganesh (2012) considered the IRP with multiple commodities and multiple sources (or manufacturers) in a two-echelon supply chain network, with a provision for buffer stocks at warehouses, and proposed a mixed-integer linear programming for the problem. Their work may be recognized as a significant leap in the state of the art on the IRP literature. Engineer, Furman, Nemhauser, Savelsbergh, and Song (2012) developed a branch-and-cut algorithm for a complex single-product maritime IRP with varying storage capacity at discharge ports and production/consumption rates at facilities. The resulting mixed-integer pricing problem was solved exactly and efficiently using a dynamic program. Michel and Vanderbeck (2012) developed a truncated branch-and-cut algorithm combined with local search heuristics, yielding both primal solutions and dual bounds for the IRP with pickups at customers. That research shows that a mathematical programming-based approach can be a viable option to obtain solutions for large-scale problems. Modeling the IRP with deteriorating products was proposed by Mirzaei and Seifi (2015). They constructed a mixed-integer non-linear programming model, taking into account the cost of lost sales as a linear and an exponential function of the inventory age to optimally solve the problem. Niakan and Rahimi (2015) addressed a healthcare IRP for medical drug distribution to healthcare facilities. They developed an interactive fuzzy approach to solve the mathematical model with multiple objectives, including minimizing total distribution costs, minimizing total greenhouse gas emissions, and maximizing customer satisfaction.

The second category performs optimization or learning tasks with the ability to evolve over generations. Chi, Ersoy, Moskowitz, and Ward (2007) proposed an approach for the IRP that uses machine learning and a GA for determining the shipping frequency and order-up-to inventory levels in a simulated VMI system. That research confirms that the models resulting from the proposed machine learning algorithms have a strong power of predicting supply chain performance measures, such as shortages and average inventory, comparable to that of traditional design of experiments (DOE) methods. Abdelmaguid and Dessouky (2006) introduced a GA approach for the IRP, which allows for backorders and partial deliveries and represents transportation costs as a non-linear function. In the GA improvement phase, two random neighborhood search mechanism are involved. Esparcia-Alcázar et al. (2009)) developed a two-level methodology for the IRP. The top level uses a GA to obtain delivery patterns for each shop, while the bottom level solves the vehicle routing problem to obtain transport costs associated with a particular set of delivery patterns. Huang and Lin (2010) proposed a modified ant colony evolutionary algorithm for the IRP, in which daily visits to each customer are feasible due to fleet resources. Li, Chu, and Chen (2011) addressed the IRP in a three-level distribution system and presented a decomposition solution approach based on a fixed partition policy wherein the retailers are partitioned into disjoint sets, for which a GA is used to find a near-optimal fixed partition. Liao, Hsieh, and Lai (2011) formulated a distribution network problem that integrates the effects of facility location, distribution, and inventory issues under VMI, and presented a multi-objective evolutionary algorithm based on a non-dominated sorting GA for solving the problem. Moin, Salhi, and Aziz (2011) proposed a hybrid GA based on the allocation-first and route-second strategy for the IRP with multi-products and multi-suppliers, in which an assembly plant is supplied from many distinct suppliers. Cho, Lee, and Lee (2014) proposed a GA for the IRP with time-dependent travel time, which makes simultaneous decisions on inventory control and vehicle routing. That research uses an adaptive genetic operator for the automatic setting of genetic parameter values. Shukla, Tiwari, and Ceglarek (2013) presented an algorithm portfolio methodology for the IRP based on evolutionary algorithms such as the memetic algorithm, GA with chromosome differentiation, age-specific GA, and gender-specific GA. They designed the algorithm portfolios on the basis of a parallel run of various algorithms. The searching capabilities of the evolutionary algorithms could be further improved through communications among several algorithms.

The third category finds good solutions through the efficient search of solution space, sometimes using lower bounds computed. Search algorithms are often used as tools for improving an initial solution obtained in the first phase of solution approaches for the IRP. Zhao, Chen, and Zang (2008) proposed a variable large neighborhood search algorithm for the IRP in a three-echelon logistics system consisting of a supplier, a central warehouse, and a group of retailers. In the algorithm, the inventory decision of each member and the routing decisions are made simultaneously. Zachariadis, Tarantilis, and Kiranoudis (2009) presented an integrated local search method for a periodic IRP, which applies two innovative search operators for jointly tackling the inventory and transportation aspects of the problem, termed insertion and removal operators. The routing quality of the distribution system is further improved by a tabu search procedure. Benoist, Gardi, and Jeanjean (2011) proposed a pure and direct local search heuristic for the IRP, using a surrogate objective function based on lower bounds computed and following the three-layer methodology for improving the performance of a local search. Liu and Lee (2011) proposed a two-phase heuristic for the IRP. The initial solution found in the first phase is improved in the second phase using a variable neighborhood tabu search while adopting different search strategies. Chan, Speranza, and Bertazzi (2013) proposed two new classes of partition-based periodic policies, which are applicable to the IRP with a constraint of a minimum inter-shipment time insofar as retailers are partitioned into sets and vehicles can serve either all the retailers of a set or all the retailers of two or more sets. Coelho et al. (2012) proposed a hybrid heuristic for the IRP with transshipments, which determines delivery quantities and transshipment moves through a network flow algorithm and determines vehicle routes by using an adaptive large neighborhood search algorithm, under two different inventory policies. Li, Chen, Sivakumar, and Wu (2014) presented a tabu search algorithm to solve the IRP with the hours-of-service regulations of a company in the gasoline distribution industry, using lower bounds obtained via the Lagrangian relaxation technique. Mjirda, Jarboui, Macedo, Hanafi, and Mladenović (2014) proposed a two-phase heuristic for the multi-supplier multi-product IRP with the allowance of split delivery. The initial solution, obtained by solving a vehicle routing problem using a variable neighborhood search in the first phase, is iteratively improved by using a variable neighborhood descent and a variable neighborhood search in the second phase, while applying a linear programming formulation to determine the quantities to collect from each supplier after each local move. Popović, Vidiović, and Radivojević (2012) developed a variable neighborhood search heuristic for the multi-product IRP in fuel delivery systems with multi-compartment vehicles. A typical constraint in the problem is that one vehicle can visit up to three stations as a consequence of the vehicle compartments structure. Vonolfen, Affenzeller, Beham, Lengauer, and Wagner (2013) presented a simulation-based optimization approach for the IRP, the objectives of which are the minimization of total costs and maximization of resource usage while maintaining a given service level. Qin, Miao, Ruan, and Zhang (2014) proposed a decoupled solution approach for the periodic IRP, which first solves the inventory problem by using a local search method and next solves the routing problem by using a tabu search method. Vansteenwegen and Mateo (2014) presented an iterated local search heuristic to deal with the single-vehicle cyclic IRP, in which a single vehicle can make multiple trips so that the division of customers and trips is optimized. The heuristic could be used to calculate upper and lower bounds on the number of trips for a given selection of customers in order to speed up exact solution approaches for the problem. Mirzaei and Seifi (2015) devised an efficient heuristic based on simulated annealing and a tabu search for the IRP for perishable goods, taking into account the cost of lost sales as a non-linear function of the inventory age.

Table 1 shows a summary of the solutions for the IRP mentioned in this section with respect to several factors, such as goal, solution approach, number of products, number of vendors, time horizon, time window, stock-outs, inventory policy, storage space, and vehicle type. Solution approaches define ways of solving inventory replenishment and transportation problems. In general, an integrated approach generates better solutions than a decoupled approach for this type of planning. Drawbacks to the integrated approach, however, include limited solvable problem size and long computation time primarily due to vehicle routing. Inventory policies define pre-established rules to replenish customers. Under a maximum level policy, the replenishment quantity is flexible but bounded by conditions, such as the storage space available at each customer. Under an order-up-to level policy, whenever a customer is visited, the replenishment quantity is that to fill pre-specified inventory level. As shown in Table 1, no solution treats synthetically the important characteristics inherent to the operation of supply chains in a VMI environment, such as profit maximization, lost sales, limited storage space, and delivery due times which are incorporated in the VMIRPL. This paper presents an effective solution for the VMIRPL, using a genetic algorithmic approach.

3. Problem description and mathematical formulation

The VMIRPL in this study consists of a single manufacturer supplying a single item to multiple retailers. The manufacturer plans inventory replenishments for retailers based on demand forecasted by the manufacturer over a planning period. The manufacturer allows stock-outs at retailers, which are considered lost sales (in

Table 1
Summary of the solutions for the IRP mentioned in the the literature review.

Methodology	Reference	Goal: Cost Min.	Goal: Profit Max.	Approach: Decoupled	Approach: Integrated	Product: Single	Product: Multiple	Vendor: Single	Vendor: Multiple	Time horizon: Finite	Time horizon: Infinite	Time window: Inclusive	Time window: Non-inclusive	Stock-outs: Lost sales	Stock-outs: Backlogging	Stock-outs: Non-negative	Inventory policy: Max. level	Inventory policy: Order-up-to level	Storage space: Limited	Storage space: Unlimited	Vehicle type: Homogeneous	Vehicle type: Heterogeneous
Mathematical models	Abdelmaguid et al. (2009)	√		√		√		√		√			√		√		√		√			√
	Archetti et al. (2007)	√			√	√		√		√			√			√		√	√		√	
	Archetti et al. (2012)	√			√	√		√		√			√			√	√	√	√		√	
	Bard and Nananukul (2009)		√	√		√		√		√			√			√		√	√		√	
	Campbell & Savelsbergh (2004)	√		√		√		√		√			√			√	√		√		√	
	Coelho and Laporte (2013)	√			√		√	√		√			√			√	√		√		√	
	Coelho and Laporte (2014)		√		√	√		√		√			√			√	√		√		√	
	Coelho et al. (2012)	√		√		√		√		√			√			√	√	√	√		√	
	Cordeau et al. (2015)	√		√			√	√		√			√			√	√		√		√	
	Engineer et al. (2012)	√		√		√			√	√			√			√	√		√			√
	Michel and Vanderbeck (2012)	√			√	√		√		√			√			√		√	√		√	
	Niakan and Rahimi (2015)	√			√		√	√		√			√		√		√		√			√
	Ramkumar et al. (2012)	√			√		√		√	√			√			√	√			√	√	
	Rusdiansyah and Tsao (2005)	√			√	√		√		√		√				√	√			√	√	
Evolutionary algorithms	Abdelmaguid and Dessouky (2006)	√			√	√		√		√			√		√		√		√		√	
	Chi et al. (2007)		√	√			√	√			√		√	√				√		√	√	
	Cho et al. (2014)	√			√	√		√		√			√		√		√		√		√	
	Esparcia-Alcázar et al. (2009))	√		√			√	√		√			√		√		√		√		√	
	Li et al. (2011)	√			√		√	√			√		√		√		√		√		√	
	Huang and Lin (2010)	√		√		√		√			√		√			√	√			√	√	
	Liao et al. (2011)	√			√	√		√			√		√	√			√			√	√	
	Moin et al. (2011)	√			√		√		√	√			√			√	√			√	√	
	Shukla et al. (2013)	√			√	√		√			√		√			√	√			√	√	
Search algorithms	Benoist et al. (2011)	√			√	√			√	√			√	√			√			√		√
	Chan et al. (2013)	√		√		√		√			√		√			√	√				√	
	Coelho et al. (2012)	√		√		√		√		√			√			√	√	√	√		√	
	Li et al. (2014)	√		√		√		√			√	√				√		√	√		√	
	Liu and Lee (2011)	√			√	√		√		√		√		√			√			√	√	
	Mizraei & Seifi (2015)	√		√		√		√		√			√			√	√		√		√	
	Mjirda et al. (2014)	√		√		√		√		√			√			√	√			√	√	
	Popović et al. (2012)	√			√		√	√		√			√			√	√		√		√	
	Qin et al. (2014)	√		√		√		√		√			√			√	√			√	√	
	Vansteenwegen and Mateo (2014)	√			√	√		√			√		√			√	√			√	√	
	Vonolfen et al. (2013)	√			√		√	√			√		√	√			√		√		√	
	Zachariadis et al. (2009)	√			√	√		√		√			√			√	√			√	√	
	Zhao et al. (2008)	√			√	√		√			√		√			√	√		√		√	

other words, no revenue). The manufacturer uses homogeneous trucks for delivery. The vehicles must leave the manufacturer (or a central depot) at the same time in the morning and must return after performing deliveries to assigned retailers by the retailers' opening times. The fleet size is given but can potentially be reduced as necessary. The storage space at retailers is limited and the space costs are fixed. The manufacturer is assumed to have a sufficient amount of supply to cover all the replenishment quantities throughout the planning horizon.

A mixed-integer linear programming model can be constructed for the VMIRPL, which simultaneously determines the optimal replenishment times and quantities and vehicle routes, while maximizing the supply chain profit over a planning horizon T. Virtual depots whose number is equal to that of available vehicles are created. Their locations are identical to that of the manufacturer. They correspond to returning depots for each of the available vehicles, respectively. Thus, all vehicles start from the manufacturer and each vehicle must return to its specified virtual depot after performing deliveries to assigned retailers. When a vehicle goes back to one of the virtual depots right after departing the manufacturer, no route is constructed and so no vehicle is required. The use of virtual depots allows us to construct the model without specifying which vehicle visits a retailer, because the virtual depots distinguish the vehicle routes (Malandraki and Daskin, 1992). The model is formulated as follows:

Indices

$$i, j \begin{cases} \text{manufacturer}\, i, j = 0 \\ \text{retailer}\, i, \ j = 1, \ \ldots, \ R \\ \text{virtual depot}\, i, \ j = R+1, \ \ldots, \ R+V \end{cases}$$

t period t=1,2, …T

Input parameters

V	Number of available vehicles
D_{it}	Customer demand at retailer i in period t
G	Vehicle capacity (expressed as the number of products)
O_i	Storage space at retailer i (expressed as the number of products)
p	Sale price
c	Production cost
K	Fixed vehicle cost ($/vehicle)
v	Variable vehicle cost ($/travel-time unit)
q_i	Space cost of retailer i
h_1	Manufacturer's inventory carrying cost
h_2	Retailer's inventory carrying cost
a_{ij}	Travel time from retailer i to j
W_j	Opening time of retailer j
u_i	Unloading time at retailer i
IIM	Manufacturer's initial inventory
M	Very big number (e.g., 999,999)

Auxiliary variables

I_{it}	Inventory level at the end of period t for retailer i
b_{it}	Lost sales at period t for retailer i
g_{it}	Quantity loaded on the vehicle upon arrival at retailer or virtual depot i in period t (g_{0t} quantity loaded on the vehicle when departing the manufacturer in period t)
e_{it}	Vehicle arrival time at retailer or virtual depot i in period t
$y_{it} = 1$	If a vehicle visits retailer i for replenishment in period t, 0 otherwise

Decision variables

δ_{it}	Replenishment quantity to retailer i at the beginning of period t
$x_{ijt} = 1$	If a vehicle moves from retailer i to retailer or virtual depot j in period t, 0 otherwise

$$Max\ Z = (p-c)\sum_{i=1}^{R}\sum_{t=1}^{T}(D_{it}-b_{it}) - \sum_{t=1}^{T}h_1 I_{0t} - \frac{1}{2}\sum_{i=1}^{R}\sum_{t=1}^{T}h_2(I_{i,\,t-1}+\delta_{it}+I_{it}) - T\sum_{i=1}^{R}q_i O_i - K\sum_{i=1}^{R}\sum_{t=1}^{T}x_{0it} - v\sum_{i=0}^{R}\sum_{j=1}^{R+V}\sum_{t=1}^{T}a_{ij}x_{ijt} \quad (1)$$

s.t.

$$I_{0t} = I_{0,\,t-1} - \sum_{i=1}^{R}\delta_{it} \quad t=1,\ldots,T \quad (2)$$

$$I_{i,\,t-1} + \delta_{it} - D_{it} + b_{it} = I_{it} \quad i=1,\ldots,R;\, t=1,\ldots,T \quad (3)$$

$$I_{i,\,t-1} + \delta_{it} \le O_i \quad i=1,\ldots,R;\, t=1,\ldots,T \quad (4)$$

$$\delta_{it} \le M\cdot y_{it} \quad i=1,\ldots,R;\, t=1,\ldots,T \quad (5)$$

$$\sum_{\substack{i=0\\ i\ne j}}^{R} x_{ijt} = y_{jt} \quad j=1,\ldots,R;\, t=1,\ldots,T \quad (6)$$

$$\sum_{\substack{j=1\\ i\ne j}}^{R+V} x_{ijt} = y_{it} \quad i=1,\ldots,R;\, t=1,\ldots,T \quad (7)$$

$$\sum_{j=1}^{R+V} x_{0jt} = V \quad t=1,\ldots,T \quad (8)$$

$$\sum_{i=0}^{R} x_{ijt} = 1 \quad j=R+1,\ldots,R+V;\, t=1,\ldots,T \quad (9)$$

$$\begin{aligned} & e_{jt} - e_{it} - M\cdot x_{ijt} \ge a_{ij} + u_i - M \quad i=0,\ldots,R;\, j=1,\ldots,R \\ & +V;\, i\ne j;\, t=1,\ldots,T \\ & e_{jt} - e_{it} + M\cdot x_{ijt} \le a_{ij} + u_i + M \end{aligned} \quad (10)$$

$$e_{jt} \le W_j \quad j=1,\ldots,R;\, t=1,\ldots,T \quad (11)$$

$$\begin{aligned} & g_{it} - g_{jt} - M\cdot x_{ijt} \ge \delta_{it} - M \quad i=0,\ldots,R;\, j=1,\ldots,R \\ & +V;\, i\ne j;\, t=1,\ldots,T \\ & g_{it} - g_{jt} + M\cdot x_{ijt} \le \delta_{it} + M \end{aligned} \quad (12)$$

$$g_{0t} \le G \ t=1,\ldots,T \quad (13)$$

$$\begin{aligned} & I_{i0}=0,\ I_{00}=IIM, b_{i0}=0,\ I_{iT}=0,\ b_{iT}=0, I_{it}\ge 0, b_{it}\ge 0 \\ & x_{ijt}=\{0,1\}, y_{it}=\{0,\ 1\},\ g_{it}\ge 0,\ e_{it}\ge 0, \delta_{it}\ge 0 \quad i=0,\ldots,R; \\ & j=1,\ldots,R+V;\, t=1,\ldots,T \end{aligned} \quad (14)$$

Constraint (2) computes the manufacturer's inventory level. Constraint (3) computes the retailer's inventory level and lost sales. Constraint (3) is the inventory balance equation for retailers. Constraint (4) limits the retailer's storage space. Constraint (5) controls the inclusion of retailers to vehicle routes in a certain period. Constraints (6)–(9) determine the routes of V available vehicles, each departing the manufacturer and returning to one of the virtual depots. Constraints (6) and (7) imply that a retailer has to be visited by only one vehicle of inbound and outbound trips when the retailer is replenished in period t, ensuring route continuity. Constraint (8) ensures that a vehicle departs from the manufacturer and goes to one of the retailers. When $x_{0jt}=1$ for $j=R+1, R+2,\ldots, R+V$, no route is constructed and so no vehicle is required. Constraint (9) ensures that a vehicle returns to one of the virtual depots after leaving one of the retailers. It is required to determine the last path of a route. When $x_{0jt}=1$ for $j=R+1, R+2, \ldots, R+V$, no route is constructed and so no vehicle is required. Constraint (10) computes the vehicle arrival time at the retailer and the vehicle return time to the virtual depot. When there is no vehicle to arrive at retailer i in period t, $e_{it}=0$. Constraint (11) limits the vehicle arrival time at the retailer to its opening time. Constraint (12) computes the quantity loaded on the vehicle when it arrives at the retailer and virtual depot. Constraint (13) limits the total amount delivered in a given period to the available vehicle capacity, ensuring the vehicle capacity when departing the manufacturer. Constraint (14) sets manufacturer and retailers' initial inventories and enforces integrality and non-negativity conditions on the variables.

4. Proposed GA

When we use CPLEX, an optimization software package, to solve the optimization model presented in Section 3, it turns out that CPLEX first determines the replenishment times and quantities for the retailers and next constructs the vehicle routes for every period. This approach is adopted because the sum of the lost sales and the fixed transportation costs affected by the replenishment plan is much higher than the variable transportation costs affected by vehicle routes. Additionally, it turns out that the solutions generated by CPLEX prefer to replenish the retailers' inventories in advance as much as possible. This reduces the lost sales and number of delivery vehicles. However, when the consolidation causes unnecessarily high transportation or inventory carrying costs, CPLEX produces solutions with just-in-time deliveries.

Based on the CPLEX solving mechanism, we propose a GA that includes an improvement process for the VMIRPL. The evolutionary process of the GA is continued until the termination condition is satisfied, incorporating a cyclical process with successive operations of selection, crossover, mutation, and improvement. Over the evolutionary process, we continually generate new populations consisting of the m best chromosomes in the current population of size N, together with the N–m best offspring obtained by genetic operations.

4.1. Chromosome representation

A chromosome is represented by two two-dimensional matrices. One matrix represents the inventory replenishment times and quantities, and the other matrix represents the vehicle routes. An example of a chromosome representation is shown in Fig. 1 The genes in matrix (a) represent the replenishment quantities for retailer i in period t. The rows of matrix (b) represent the vehicle routes for delivery to the retailers in period t, where 0 denotes the manufacturer. For instance, two vehicles are required in period 2, the routes of which are 0-3-0 and 0-1-2-0.

4.2. Generation of initial population

Notations (supplement)

$\theta(i)$	Route including retailer i
$\Phi_{\theta(i)}$	Set of retailers in route $\theta(i)$
ET_{it}	Estimated transportation cost for retailer i in period t, computed as the sum of a portion of the fixed transporta-

$i \backslash t$	1	2	3	4
1	21	25	0	7
2	24	15	22	14
3	0	46	33	9
4	11	0	24	16

(a) inventory replenishments

t	routes				
1	0	1	4	0	2
2	0	3	0	1	2
3	0	2	4	0	3
4	0	1	4	2	3

(b) vehicle routes

Fig. 1. Example of two two-dimensional matrices for a chromosome representation.

tion cost, $K(\delta_{it}/\sum_{j\in\Phi_{\theta(i)}}\delta_{jt})$, and variable transportation cost

RS_{it} Remaining storage space at retailer i in period t ($RS_{it} = O_i - I_{i,t-1} - \delta_{it}$)

$RV_{\theta(i),\,t}$ Remaining vehicle capacity for route $\theta(i)$ in period t

- Phase I: Initial assignment of replenishments

The procedure is described as follows:

1. Set the replenishment quantity for retailer i in period t equal to their forecasted demand in that period, but not exceeding the capacity at this retailer: $\delta_{it} = \min\{D_{it}, O_i\}$, for all i and t. Then, construct vehicle routes in each period using the r-savings algorithm (Cordeau, Gendreau, Laporte, Potvin, & Semet, 2002).
2. Test the condition $(p-c)\ \delta_{it} \leq ET_{it}$ for all i and t, which compares the roughly estimated profits and transportation costs resulting from the replenishment of δ_{it} to retailer i. Change δ_{it} to 0 with 50% probability for retailers and periods satisfying the condition, and revise vehicle routes.
3. If the number of vehicles required exceeds the available quantity in a period, then select the routes equal to the excess number of vehicles based on the smallest shipment-first rule, and change replenishment quantities to 0 for all the retailers included in those routes.

- Phase II: Consolidation of replenishments to prior periods

Cost savings are attempted by transferring replenishments to prior periods for consolidation, in consideration of vehicle capacity and retailer storage space. A cost saving is simply estimated by subtracting the increased inventory carrying cost from the reduced transportation cost. The concept of consolidation is derived from the work of Abdelmaguid and Dessouky (2006). The procedure is described as follows:

1. for $i = 1$ to R
   for $t = 1$ to $T-1$
   for $\Delta t_i = 1$ to $T-t$
   Compute the cost savings expected by transferring the whole or part of the replenishment in $t+\Delta t_i$ to t for retailer i by using $\eta_i = ET_{i,t+\Delta t_i} - \Delta t_i(h_2 - h_1)MQ$ where $MQ = \min(RV_{\theta(i),t}, \delta_{i,t+\Delta t_i}, RS_{it})$. If $\eta_i > 0$, then execute the transfer by setting $\delta_{it} = \delta_{it} + MQ$ and $\delta_{i,t+\Delta t_i} = \delta_{i,t+\Delta t_i} - MQ$, and update the remaining vehicle capacity and remaining storage space as $RV_{\theta(i),t} = RV_{\theta(i),t} + MQ$ and $RS_{it} = RS_{it} + MQ$. Revise vehicle routes in period $t+\Delta t_i$, while vehicle routes in period t remain unchanged.
   next Δt_i
   next t
   next i
2. Set $\delta_{it} = 0$ for all i and t if the expected profit is less than or equal to the estimated transportation cost for δ_{it} (that is, if $(p-c)\ \delta_{it} \leq ET_{it}$), and revise vehicle routes for the corresponding periods.

4.3. Fitness evaluation and selection

For the selection process, a roulette-wheel method is applied using the relative fitness of a chromosome. The use of relative fitness clarifies the superiority of chromosomes. This normalization makes it possible to adjust the selection behavior from fitness-proportional selection to pure random selection, widening the intervals between the selection probabilities of chromosomes. The relative fitness of chromosome k is calculated as follows:

$$f'_k = \frac{f_k - f_{\min}}{f_{\max} - f_{\min}}, \tag{15}$$

where $f_{\min}$ and $f_{\max}$ are the minimum and maximum supply chain profits of chromosomes in the population, respectively. The supply chain profit of a chromosome is computed using the objective function (1).

4.4. Crossover

Pairs of parents are selected for crossover in the population according to the crossover rate p_c. Then, for each pair of parents, common rows (or retailers) are selected according to the probability p_c^{row}. The crossover procedure exchanges the genes (or replenishment plans) in pairs of parents in the same row to produce offspring. It is followed by the repair operation to maintain restrictions regarding the transportation capacity after revising vehicle routes. The repair operation is executed for periods in which the number of vehicles required exceeds the available quantity. The repair operation distributes the excess replenishment quantity over the available transportation capacity in a period to other periods where lost sales have occurred, taking into account vehicle capacity and the storage space of retailers in those periods. The repair procedure for period t', in which the number of vehicles required exceeds the available quantity, is described as follows:

Notation (supplement)

1. $NV_{t'}$ Number of vehicles required in t'
2. $\Phi_{t'}$ Set of routes arranged in ascending order of shipment quantity in t'

1. Form a set $\Pi_{t'}$ consisting of retailers included in the top $NV_{t'} - V$ routes in $\Phi_{t'}$.
2. For all retailers $i \in \Pi_{t'}$ do the following:
   for $t = 1$ to T $(t \neq t')$
   Determine the quantity for transferring to t from t' by using $\Psi_{it} = Min(RV_{\theta(i),t}, b_{it}, RS_{it}, \delta_{it'})$ and execute the transfer by setting $b_{it} = b_{it} - \Psi_{it}$ and $\delta_{it'} = \delta_{it'} - \Psi_{it}$.
   next t
   If $\delta_{it'}$ is still positive, then try to distribute that quantity to prior periods, expecting an increase in inventories at retailers. The inventories are later used to meet demand at retailer i in period t'. Otherwise, stop.
   for $t = t'-1$ to 1

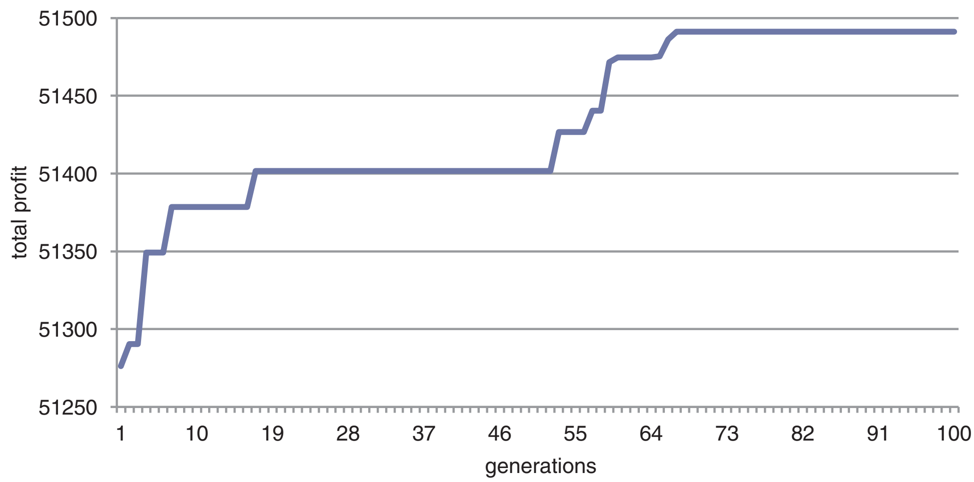

Fig. 2. Convergence behavior of the proposed GA over 100 generations.

Determine the quantity for transferring to t from t' by using $\Psi_{it} = Min(RV_{\theta(i),t}, RS_{it}, \delta_{it'})$ and execute the transfer by setting $\delta_{it} = \delta_{it} + \Psi_{it}$ and $\delta_{it'} = \delta_{it'} - \Psi_{it}$.
next t
If $\delta_{it'}$ is still positive, then give up that quantity by setting $\delta_{it'} = 0$. Otherwise, stop.

3. Update inventory levels for all retailers in Π_t.

4.5. Mutation

Pairs of parents are selected according to the mutation rate p_m. The mutation uses a single vertical cutoff-point method, which selects one common cutoff-point randomly between two adjacent periods in the parents, and exchanges the randomly selected right or left parts (or genes) of a pair of parents to produce offspring. To ensure restrictions regarding the storage space at retailers, the repair operation follows. The mutation causes the solution quality to deteriorate because it simultaneously changes the replenishment plans of the parent with those of another parent for all retailers in the selected periods. On the other hand, the mutation expands the search space for good solutions. The repair procedure is described as follows:

1. Let t' be the period immediately after the vertical cutoff-point and do the following:
for i=1 to R
for t=t' to T
If $I_{i,t-1} + \delta_{it} > O_i$, then reduce the replenishment quantity to maintain restrictions regarding the storage space of retailers by setting $\delta_{it} = O_i - I_{i,t-1}$. Update the inventory level and lost sales as $I_{it} = Max(I_{i,t-1} + \delta_{it} - D_{it}, 0)$ and $b_{it} = Max(D_{it} - I_{i,t-1} - \delta_{it}, 0)$, respectively.
next t
next i
2. When inventories exist at retailers at the end of period T, attempt to reduce replenishment quantities in reverse order of periods, starting from period T.
for i=1 to R
If $I_{iT} > 0$, then set $\kappa = I_{iT}$.
for t=T to 1
If $\kappa > 0$, then reduce the replenishment quantity at retailer i in period t by $Min(\delta_{it}, \kappa)$, update as $\kappa = Max(\kappa - \delta_{it}, 0)$ at the same time, and also update the inventory level and lost sales. Otherwise, move out of for-loop.
next t
next i

Table 2
Inventory replenishment plan for retailer 11 obtained using the proposed GA.

Period	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Sum
Demand	24	35	38	19	30	20	21	39	33	13	24	40	26	33	395
Replenishment quantity	54	0	80	0	7	0	21	57	30	0	37	65	0	19	370
Inventory level	30	0	42	23	0	0	0	18	15	2	15	40	14	0	199
Lost sales	0	5	0	0	0	20	0	0	0	0	0	0	0	0	25

3. Revise vehicle routes.

4.6. Solution improvement

After the genetic operations, solution improvement is attempted for all the offspring by reducing lost sales and transportation costs. The lost sales in a period are reduced by increasing replenishment quantities over prior periods in reverse order, starting from the given period itself. Transportation costs are reduced by consolidating replenishments to prior periods. The improvement procedure is described as follows:

1. Attempt to reduce lost sales while checking the feasibility of increasing replenishment quantities in prior periods with respect to the vehicle capacity and retailer's storage capacity.
for i=1 to R
for t=1 to T
for $\Delta t = 0, \ldots, t-1$
If $b_{it} = 0$, then move out of for-loop. Otherwise, increase the replenishment quantity in period $t - \Delta t$ by ϖ where ϖrepresents the amount of the feasible increase determined by $Min(b_{it}, RV_{\theta(i),t-\Delta t}, RS_{i,t-\Delta t})$, and decrease lost sales at retailer i in period t by ϖ. Revise vehicle routes in $t - \Delta t$ if necessary. If $NV_{t-\Delta t} > V$ in revised routes, then do not increase the replenishment quantity in period $t - \Delta t$. Lost sales in period t also remain unchanged.
next Δt
next t
next i
2. Transfer replenishments to prior periods for consolidation, while applying the Phase II procedure in Section 4.2.

5. Computational experiments

This section validates the proposed GA by solving an example, and then evaluates the performance of the proposed GA

Table 3
Comparison of the optimization model and proposed GA on 27 test problems.

Problem		Solution	Revenue	Manufacturer		Retailer		Transportation		Profit	Error rate (%)	Computation time of GA (sec)
RxT	Rep			Production cost	Inventory carrying cost	Storage space cost	Inventory carrying cost	Fixed cost	Variable cost			
6×6	1	Opt model GA	24570H	17199	223	58	370	900	858	4962	0.4	9
			24510H	17157	210	58	403	900	840	4942		
	2	Opt model GA	27750	19425	282	58	372	1000	1011	5602	0.6	9
			27750	19425	259	58	433	1000	1009	5566		
	3	Opt model GA	26310	18417	270	58	335	900	890	5440	1.0	7
			26310	18417	241	58	412	900	894	5388		
6×9	1	Opt model GA	40530	28371	721	86	609	1400	1321	8022	1.4	13
			40530	28371	691	86	689	1400	1382	7911		
	2	Opt model GA	36420	25494	591	86	601	1300	1290	7058	1.7	13
			36390	25473	555	86	693	1300	1346	6937		
	3	Opt model GA	38190	26733	656	86	571	1300	1244	7600	0.9	11
			38190	26733	609	86	696	1300	1234	7532		
6×12	1	Opt model GA	52500	36750	1271	115	789	1800	1751	10024	2.6	19
			52290	36603	1158	115	1057	1900	1697	9760		
	2	Opt model GA	50640	35448	1208	115	854	1700	1781	9534	2.7	16
			50640	35448	1143	115	1026	1800	1830	9278		
	3	Opt model GA	54180	37926	1357	115	840	1900	1715	10327	1.6	19
			54180	37926	1265	115	1086	1900	1725	10163		
9×6	1	Opt model GA	37680	26376	378	86	519	1300	1233	7788	1.3	12
			37680	26376	326	86	660	1300	1247	7685		
	2	Opt model GA	38370	26859	417	86	481	1300	1313	7914	1.6	13
			38370	26859	319	86	741	1300	1280	7785		
	3	Opt model GA	41640	29148	400	86	584	1400	1437	8585	1.9	17
			41610	29127	382	86	628	1400	1569	8418		
9×9	1	Opt model GA	59640	41748	1033	130	828	2100	1897	11904	1.6	23
			59640	41748	987	130	951	2100	2007	11717		
	2	Opt model GA	58080	40656	1023	130	800	2000	1906	11565	1.8	20
			57960	40572	935	130	1027	2000	1943	11353		
	3	Opt model GA	59340	41538	1010	130	980	2000	2034	11648	2.5	18
			59280	41496	958	130	1114	2100	2124	11358		
9×12	1	Opt model GA	83400	58380	2108	173	1080	2900	2700	16059	1.9	30
			83400	58380	1980	173	1423	2900	2786	15758		
	2	Opt model GA	78600	55020	2029	173	1064	2700	2568	15046	2.6	28
			78600	55020	1886	173	1444	2700	2716	14661		
	3	Opt model GA	79800	55860	1932	173	1088	2800	2748	15199	3.2	29
			79260	55482	1753	173	1484	2800	2857	14711		
12×6	1	Opt model GA	53790	37653	538	115	720	1800	1756	11208	1.8	24
			53400	37380	483	115	837	1800	1774	11011		
	2	Opt model GA	52590	36813	541	115	677	1800	1725	10919	1.8	24
			52590	36813	455	115	906	1800	1774	10727		
	3	Opt model GA	53790	37653	510	115	738	1800	1599	11375	1.6	25
			53550	37485	471	115	823	1800	1665	11191		
12×9	1	Opt model GA	76350	53445	1291	173	1059	2600	2490	15292	3.2	32
			74820	52374	1164	173	1332	2500	2476	14801		
	2	Opt model GA	77670	54369	1362	173	1090	2600	2472	15604	2.9	37
			77340	54138	1208	173	1466	2700	2511	15144		
	3	Opt model GA	78570	54999	1347	173	1064	2700	2362	15925	2.2	33
			78240	54768	1218	173	1371	2700	2443	15567		
12×12	1	Opt model GA	105630	73941	2558	230	1513	3600	3280	20508	2.4	47
			105360	73752	2406	230	1876	3600	3484	20012		
	2	Opt model GA	100860	70602	2417	230	1483	3400	3236	19492	3.1	44
			100560	70392	2285	230	1780	3500	3492	18881		
	3	Opt model GA	102990	72093	2552	230	1349	3500	3125	20141	2.2	47
			102810	71967	2548	230	1332	3600	3431	19702		

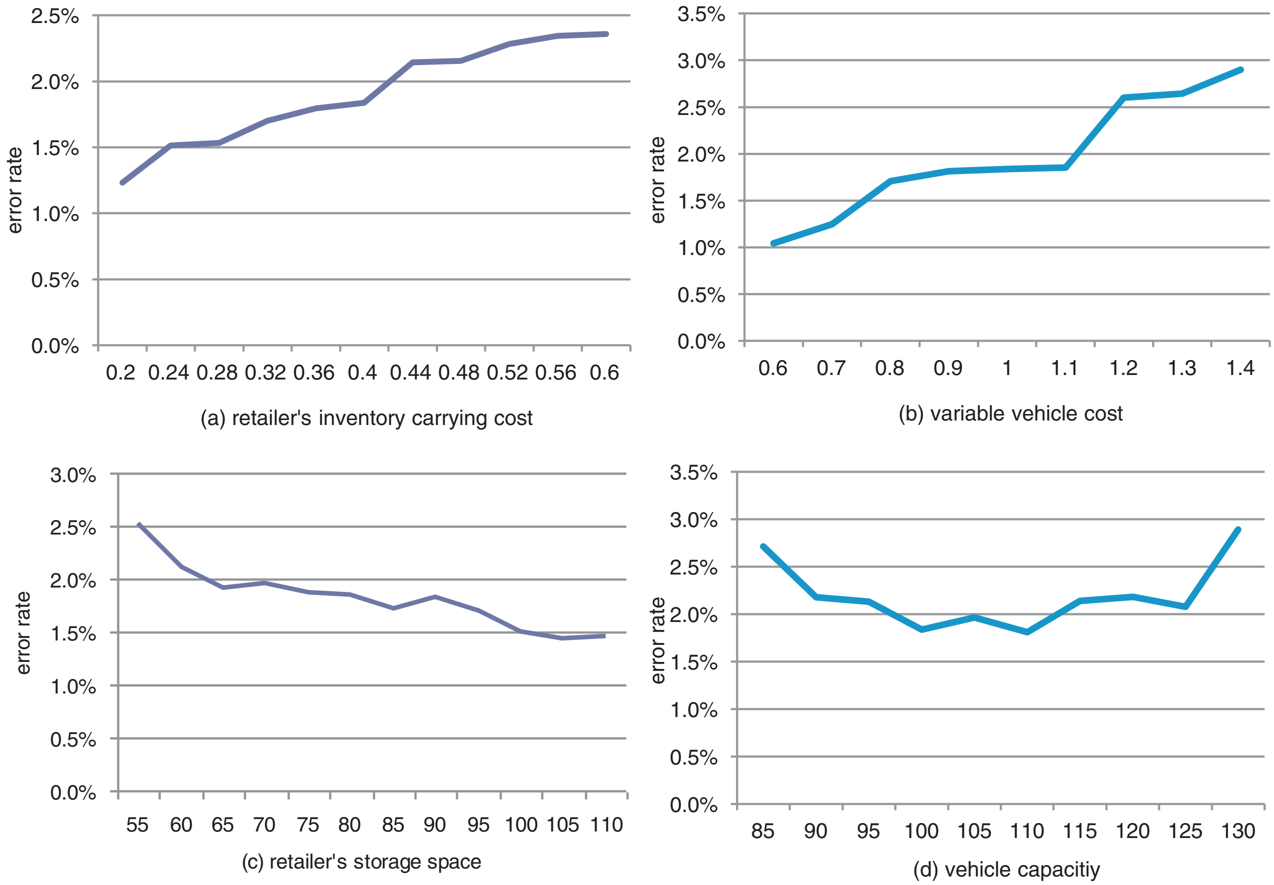

Fig. 3. Sensitivity analysis of the effectiveness of the proposed GA for four problem parameters.

by comparing its solutions with the solutions obtained by solving the optimization model presented in Section 2 in randomly generated test problems. Furthermore, this section presents the results of sensitivity analyses for some problem parameters. The optimization models were solved using CPLEX 9.1, while setting the tolerance for stopping iterations at 5% and limiting the computation time to 10 h. All the experiments were executed on a Pentium IV PC (AMD Athlon 64 × 2 Dual Core Processor 4200+, CPU 2.21 GHz, 3GB RAM).

5.1. Example

We construct an example consisting of a single manufacturer and 25 retailers, which is inspired from the case studies of Vonolfen et al. (2013) regarding supermarket distribution systems in Austria. Because the case studies do not provide all the necessary data, we arbitrarily set the problem parameters for the example as follows: $T = 14$, $c = 21$, $p = 30$, $h_1 = 0.15$, $h_2 = 0.4$, $q_i = 0.02$, $K = 100$, $v = 1$, $V = 5$, $G = 130$, $W_j = 200$, $u_i = 10$, and $O_i = 80$. The coordinates of retailer locations are generated from a uniform distribution of [0, 100]. The customer demands at retailers are generated from a uniform distribution of [10, 40]. The manufacturer's location is set at (50, 50). The travel time between two locations is assumed to be the Euclidean distance between the locations. We set the genetic parameters for the example as follows: $N = 60$, $m = 5$, $p_c = 0.4$, $p_c^{row} = 0.5$, and $p_m = 0.1$. The parameters are obtained through repetitive pilot runs.

The proposed GA obtains the best solution with a total profit of 51,491 over 100 generations (total revenue = 256,650; total cost = 205,159; total demand = 8,759 units; and total lost sales = 204 units). Lost sales are equivalent to approximately 2.3% of the total demand, because the inventory is not replenished when expected transportation costs are higher than sales profits, even though transportation capacity and retailer's storage space are available. Fig. 2 shows the convergence behavior of the proposed GA over 100 generations. The solution converges at 70 generations, which is relatively fast. Table 2 shows the inventory replenishment plan for retailer 11.

5.2. Performance evaluation of the proposed GA

A total of 27 test problems were constructed by changing the planning horizon and the number of retailers, together with their locations and demands. The problem size was limited due to the significant computations required by the optimization models to obtain optimal solutions. The input data, including the genetic parameters, were set to be the same as the input data in the example except that $G = 100$, $W = 180$, and $V = \lfloor R/3 \rfloor$.

Table 3 compares the proposed GA with the optimization model with respect to costs, total profit and computation time on the test problems. The error rate of the proposed GA was computed with the following equation: 100∗(total profit obtained using the optimization model - total profit obtained using the proposed GA)/total profit obtained using the optimization model. The proposed GA resulted in very good solutions with error rates of 0.4–3.2% on the test problems, requiring very short computation times of approximately 7–47 s. Conversely, the optimization model required approximately 5–10 h for computations. The CPLEX gap for the optimal model appeared as 3.4–5% on the test problems. Notably, the solution quality of the proposed GA was slightly improved when the evolution was continued over 100 generations.

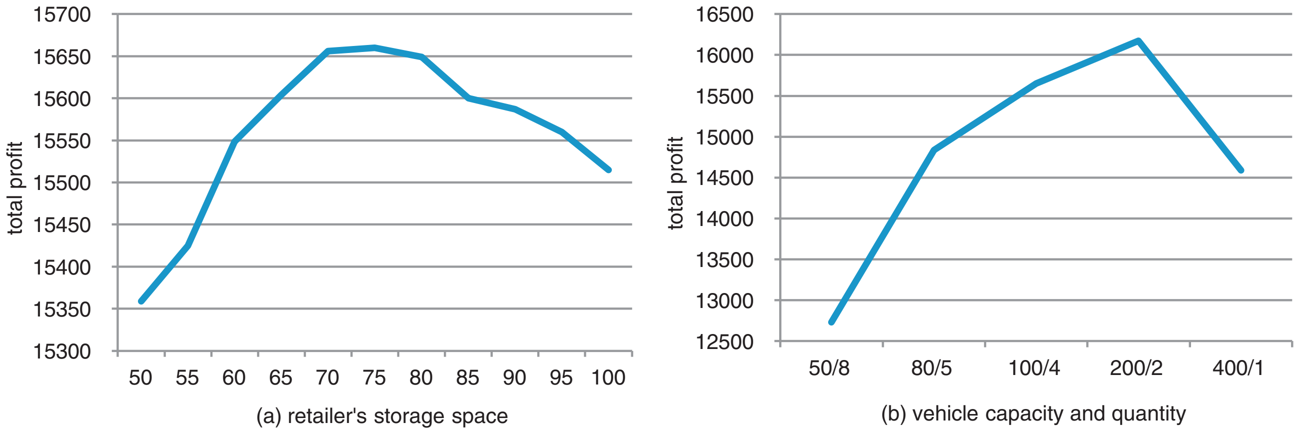

Fig. 4. Sensitivity analysis of total profit for two parameters related to resource capacity.

A sensitivity analysis was conducted to evaluate the effects of the problem parameters on the performance of the proposed GA. The following four problem parameters were selected: retailer's inventory carrying cost, variable vehicle cost, retailer's storage space, and vehicle capacity. The first replication of the test problem with $R=12$ and $T=9$ was selected as the base problem. The fixed vehicle cost K of capacity G was estimated by the equation $K=54.3+0.457\,G$. This was obtained with $r^2=0.929$ through a regression analysis using the commercial data for sales prices, salvage values, and lifetimes of trucks available in the market. For instance, the fixed vehicle cost of capacity 100 was estimated as 100.

Fig. 4 shows variations in the error rate of the proposed GA for the four problem parameters. The t-test at $\alpha=0.01$ showed that the solution quality of the proposed GA was significantly affected by those parameters. The error rate increased with increases in the retailer's unit inventory carrying costs because the proposed GA focuses on consolidation of replenishments to prior periods, resulting in higher inventory levels at retailers. In contrast, the optimization model searches for solutions with just-in-time deliveries for retailers whenever necessary, resulting in lower inventory levels at retailers. The error rate increased with increases in variable vehicle costs because the proposed GA finds good solutions for vehicle routing, while the optimization model is designed to find optimal solutions. The error rate decreased as the retailer's storage space or vehicle capacity increased, because high storage space or high vehicle capacity facilitates the consolidations of replenishments for the proposed GA, contributing to improvement in solution quality. However, excessive consolidations of replenishments due to excessively large storage space or vehicle capacity causes the solution quality of the proposed GA to deteriorate, as shown in (c) and (d) of Fig. 3.

Additional sensitivity analysis was conducted to evaluate the effects of the problem parameters relative to the resource capacity on the total supply chain profits. Retailer storage space and vehicle capacity were selected as the problem parameters. The total transportation capacity per period, which was determined by multiplying the number of available vehicles by vehicle capacity, was set at 400 for all the problems. Because the optimization model was unable to solve several problems with large values for the two parameters, the proposed GA was employed to obtain the total profits. Fig. 4 shows variations in total profits for the two parameters, which are in the convex shape, meaning that their optimal values exist. According to Fig. 4, the maximum total profit was obtained when the retailer's storage space was 75 and the number of available vehicles with a unit capacity of 200 was 2.

When the retailer's storage space was too small, transportation costs and lost sales increased due to frequent small replenishments. Conversely, when the retailer's storage space was too large, the retailer's space costs and inventory carrying costs increased. When the vehicle capacity was too small (and thus the number of available vehicles increased), the fixed and variable transportation costs increased. Conversely, when the vehicle capacity was too large (and thus the number of available vehicles decreased), the lost sales increased due to an insufficient number of vehicles operating on a schedule of tight delivery due times. The vehicles with excessively large capacity showed a relatively low shipment rate. Therefore, a retailer's storage space and vehicle capacity, together with the number of available vehicles, should be treated as important decision variables that can significantly impact supply chain profits in the VMIRPL.

6. Conclusions

This paper proposes an effective GA that includes an improvement process for the VMIRPL, which determines replenishment times and quantities and vehicle routes that maximize total profits in retail supply chains. The proposed GA treats synthetically important characteristics inherent to the operation of supply chains in a VMI environment, such as profit maximization, lost sales, limited storage space, and delivery due times which are rarely treated in a complex way in the literature. In the current study, an example was solved to validate the proposed GA. The performance of the proposed GA was evaluated by comparing its solution with the solution obtained by solving a mixed-integer linear programming model for optimal solutions using 27 test problems with different planning horizons and a number of retailers with different locations and demands.

The proposed GA demonstrated solutions that are very similar to those obtained with the optimization model for small problems and solutions that remain within 3.2% of those obtained using the optimization model for large problems, requiring a much shorter computation time of 7–47 s. Sensitivity analysis indicated that the error rate of the proposed GA increased gradually with increases in the retailer's unit inventory carrying costs or variable vehicle costs and decreased gradually as the retailer's storage space or vehicle capacity increased. However, overly large storage space or vehicle capacity caused the solution quality of the proposed GA to deteriorate due to excessive consolidation of replenishments. Further sensitivity analysis indicated that supply chain profits increased gradually, and then decreased gradually after reaching the peak point as the retailer's storage space or vehicle capacity increased, resulting in a convex shape. This result indicates that the two parameters related to resource capacity should be treated as important decision variables that can significantly impact total supply chain profits in the VMIRPL. The proposed GA, which maximizes the total supply chain profits under VMI, is also applicable to the problem of establishing a network of agents (or retailers).

In terms of future research, it is necessary to develop effective approximation methods to determine the storage space of retailers and vehicle capacity (or the number of available vehicles) along with replenishments and vehicle routes that maximize supply chain profit. Effective approximation methods may include relaxed algorithms based on an optimization model of the extended problem or GAs incorporating additional genes for retailer storage space and vehicle capacity into a chromosome.

Acknowledgments

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (NRF-2013R1A1A2005302).

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