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A heuristic method for the inventory routing and pricing problem in a supply chain

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A R T I C L E   I N F O

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A B S T R A C T

The inventory routing problem (IRP) in a supply chain (SC) is to determine delivery routes from suppliers to some geographically dispersed retailers and inventory policy for retailers. In the past, the pricing and demand decisions seem ignored and assumed known in most IRP researches. Since the pricing decision affects the demand decision and then both inventory and routing decisions, it should be considered in the IRP simultaneously to achieve the objective of maximal profit in the supply chain. In this paper, a mathematical model for the inventory routing and pricing problem (IRPP) is proposed. Since the solution for this model is an NP (non-polynomial) problem, a heuristic method, tabu search adopting different neighborhood search approaches, is used to obtain the optimal solution. The proposed heuristic method was compared with two other methods considering the IRPP separately. The experimental results indicate that the proposed method is better than the two other methods in terms of average profit.

1. Introduction

The inventory routing problem (IRP) in a supply chain (SC) is to determine delivery routes from suppliers to some geographically dispersed retailers and inventory policy for retailers. It is consisted of two sub-problems: inventory problem for retailers and vehicle routing problem (VRP) for suppliers. The IRP considering inventory and routing simultaneously has gained attentions since the coordination of the inventory and routing decisions between the supplier and retailers leads to a better overall performance (Vidal & Goetschalckx, 1997). According to the literature (Raa & Aghezzaf, 2009; Zhao, Wang, & Lai, 2007), the pricing and demand decisions seem ignored and assumed known in most IRP researches. Since the pricing decision affects the demand decision and then both inventory and routing decisions, it should be made in the IRP simultaneously to achieve the objective of maximal profit in the supply chain. For example, higher pricing causes lower demand and then lower order quantity and lower inventory. In contrast, lower pricing causes higher demand and then higher order quantity and higher inventory. Since the pricing decision is interrelated to inventory routing decisions, the profit may decrease when they are made separately. Hence, how to determine inventory, routing and price simultaneously becomes an important issue in supply chain management.

Because the inventory routing and pricing problem (IRPP) is a NP-hard problem (Since inventory routing decisions is a NP-hard problem (Lenstra & Rinnooy, 1981), the IRPP is more complex than the IRP.), a heuristic method is adopted to resolve this problem. Until now, there are few researches about IRPP. Hence, this paper presented a survey for two related areas: inventory routing problem and pricing problem, in the following. Bell, Dalberto, and Fisher (1983) adopted an optimization method to resolve the IRP. After that, some other optimization methods were developed to resolve the IRP (Anily & Federgruen, 2004; Dror & Ball, 1987; Gallego & Simchi-Levi, 1990; Kleywegt, Nori, & Savelsbergh, 2002; Qu, Bookbinder, & Iyogun, 1999; Yu, Chen, & Chu, 2008). Since the IRP is an NP-hard problem, heuristic methods are needed. Federgruen and Zipkin (1984) developed a nonlinear integer programming model and adopted an exchange method to resolve the IRP. Golden, Assad, and Dahl (1984) adopted an insertion method to resolve the IRP. Viswanathan and Mathur (1997) adopted a stationary nested joint replenishment policy heuristic (SNJRP) to resolve the IRP. The results show the method simultaneously making inventory and routing decisions is better than that making inventory and routing decisions separately. Campbell and Savelsbergh (2004) adopted a two-phase method to resolve the IRP. The first phase adopted an integer programming method to obtain the initial solution. The second phase adopted an insertion method to improve the initial solution. Gaur and Fisher (2004) adopted a randomized sequential matching algorithm (RSMA) to resolve the IRP. An insertion method was adopted to obtain the initial solution. Then a cross-over method was adopted to improve the initial solution. Sindhuchao, Romeijn, Akcali, and Boondiskulchok (2005) adopted a two-phase method for the IRP. The first phase adopted a column generation method to obtain the initial solution. The second phase adopted a very large-scale neighborhood search (VLSN) to improve the initial solution. Lee, Jung, and Lee (2006) adopted a tabu search method to resolve the IRP. Raa and Aghezzaf (2008) adopted a heuristic method to resolve the IRP. A column generation method was

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Nomenclature

c_{rjl}	distance from j to l for route r	R	route (or vehicle) number
A	ordering cost per order	N	retailer number
a_i	intercept value for the demand pattern of retailer i	T_r	replacement time of route r
b_i	slope of the demand pattern of retailer i	V_r	retailer set for route r ($1 \leqslant r \leqslant R$)
C	supplier capacity	q_{imax}	upper bound for demand of retailer i per day
v	vehicle capacity	q_{imin}	lower bound for demand of retailer i per day
ψ	vehicle dispatching cost	y_{imax}	upper bound for demand of retailer i in the planning period (=$w \times q_{imax}$)
w	working days in the planning period		
δ	production cost per unit	y_{imin}	lower bound for demand of retailer i in the planning period (=$w \times q_{imin}$)
cm	traveling cost per unit distance		
h	holding cost per period	q_i	demand of retailer i per day
i	index of retailers	y_i	demand of retailer i in the planning period (=$w \times q_i$)
r	index of routes (or vehicles)	p_i	sales price of retailer i in the planning period
j	index of retailers ($1 \leqslant j \leqslant N$) or supplier ($j = N + 1$)	X_{rjl}	1, if point j immediately precedes point l on route r; 0, otherwise
l	index of retailers ($1 \leqslant l \leqslant N$) or supplier ($l = N + 1$)		

adopted to find the initial solution. Then a saving heuristic method was adopted to improve the initial solution. Zhao et al. (2007) adopted a heuristic method to resolve the IRP. The initial solution was generated randomly. Then a tabu search method adopting the GENI neighborhood search was used to improve the initial solution. Zhao, Chen, and Zang (2008) adopted a variable large neighborhood search (VLNS) method to resolve the three-echelon (suppliers, distributors, retailers) IRP. The results show the proposed method is better than the tabu search method. In summary, tabu search (TS) adopting the GENI neighborhood search approach and VLNS have been adopted to find the optimal solution for the inventory routing problem effectively and efficiently (Gaur & Fisher, 2004; Lee et al., 2006; Zhao et al., 2007; Zhao et al., 2008). Hence, they will be adopted to resolve the IRP sub-problem in IRPP in this paper. As for the pricing problem, some researchers (Jung & Klein, 2006; Kotler, 1971; Lau & Lau, 2003; Ray, Gerchak, & Jewkes, 2005) determined the prices and demands using calculus according to the known demand function based on the maximal profit criterion. Nachiappan and Jawahar (2007) adopted a genetic algorithm (GA) method to find the prices and demands based on the maximal profit criterion in a supply chain. The pricing problem is a nonlinear integer programming (NIP) problem. Searching for the optimal solution is an NP problem. According to the literature (Costa & Oliveria, 2001; Exler, Antelo, Egea, Alonso, & Banga, 2008; Schlüter, Egea, & Banga, 2009; Yin & Wang, 2008), genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO) and tabu search (TS) have been adopted to resolve the NIP problem. Since tabu search is adopted to resolve the IRP sub-problem in IRPP mentioned above, if GA, PSO or ACO is adopted to resolve the pricing sub-problem in IRPP, the IRPP would be resolved separately by different methods. Hence, tabu search is adopted to resolve the IRPP simultaneously in this paper.

2. Model formulation for the inventory routing and pricing problem

2.1. Assumptions and notations

2.1.1. Assumptions

According to the literature survey, there are no other researches available for the IRPP. Hence, the used assumptions in this paper are selected from two related research areas: the inventory routing model (Raa & Aghezzaf, 2009; Zhao et al., 2007) and the pricing model (Nachiappan & Jawahar, 2007). The details are as follows:

A supplier serves retailers which are geographically dispersed in a given area.
A homogenous fleet of vehicles is considered with the same capacity.
A single product is considered and distributed to retailers.
Each retailer is served by exactly one vehicle.
The total demand on each route is less than or equal to the vehicle capacity.
Each route begins and ends at the same supplier.
No vehicle loading and unloading cost is considered.
No supplier ordering and inventory cost is considered.
Demand lies between a specific range and the validity of the assumption of linear demand function holds very well within this range.
The pricing can not be zero.

2.2. Model formulation

Before the model for the inventory routing and pricing problem is formulated, the relevant information is discussed first.

2.2.1. Revenue

Demand function defines the price and demand quantity relationship. The planning horizon is usually 1 year or half year. The demand function for retailer i: $p_i = a_i - b_i y_i$ (The linear demand function is the most popular in the related research (Lau & Lau, 2003; Nachiappan & Jawahar, 2007)). Since $y_i = w \times q_i$, the demand function becomes as follows: $p_i = a_i - b_i w q_i$. Hence, the revenue per day $p_i q_i = a_i q_i - b_i w q_i^2$.

2.2.2. Supply chain cost

2.2.2.1. Transportation cost. The transportation cost includes the traveling cost plus the vehicle dispatching cost, Ψ. The detailed computation is as follows: transportation cost per day = $\sum_{r=1}^{R}\sum_{j=1}^{N+1}\sum_{l=1}^{N+1}\frac{c_{rjl}\times cm+\psi}{T_r}$.

2.2.2.2. Production cost. The production cost includes material cost and manufacturing cost. The detailed computation is as follows: production cost per day = $\sum_{r=1}^{R}\sum_{i\in V_r}\delta \times q_i$.

2.2.2.3. Inventory cost. The inventory cost includes ordering cost and holding cost. The detailed computation for these costs is as follows: (1) ordering cost per day $= \sum_{r=1}^{R}\frac{A}{T_r}$. (2) holding cost per day $= \sum_{r=1}^{R}\sum_{i\in V_r}\frac{T_r \times q_i \times h}{2}$.

After the revenue and supply chain cost are discussed, the model for inventory routing and pricing problem is as follows:

$$\text{Max}\sum_{i=1}^{N}(a_i q_i - b_i w q_i^2) - \sum_{r=1}^{R}\left[\sum_{j=1}^{N+1}\sum_{l=1}^{N+1}\frac{c_{rjl}\times cm+\psi}{T_r}\times X_{rjl} + \sum_{i\in V_r}\delta\times q_i + \frac{A}{T_r} + \sum_{i\in V_r}\frac{T_r\times q_i\times h}{2}\right]$$

s.t.

$$q_{imin} \le q_i \le q_{imax}, \quad i = 1,\ldots,N \tag{1}$$

$$\sum_{i=1}^{N} w\times q_i \le C \tag{2}$$

$$\sum_{i\in V_r} q_i\times T_r \le v, \quad r = 1,\ldots,R \tag{3}$$

$$\sum_{r=1}^{R}\sum_{l=1}^{N+1} X_{rjl} = 1, \quad j = 1,\ldots,N+1 \tag{4}$$

$$\sum_{j\in V_r\cup\{N+1\}}\ \sum_{l\in V_r\cup\{N+1\}}\ \sum_{r=1}^{R} X_{rjl} \ge 1, \quad \forall(V_r,\overline{V}_r) \tag{5}$$

$$\sum_{j=1}^{N+1} X_{rjl} - \sum_{j=1}^{N+1} X_{rlj} = 0, \quad r = 1,\ldots,\ R, l = 1,\ldots,N+1 \tag{6}$$

$$q_i \geqslant 0,\ (\text{Nonnegative Constraint and Integer}), \quad i = 1,\ldots,N \tag{7}$$

$$X_{rjl} \in \{0,1\} \quad \forall j,\ \ l\in S_r,\ \ \forall r\in R \tag{8}$$

The goal of the objective function is to make the supply chain profit maximum. The constraint (1) indicates the demand per day for retailer i lies between q_{imin} and q_{imax}. The constraint (2) indicates the sum of retailer demand in the planning period must be less than or equal to supplier capacity C. The constraint (3) indi-

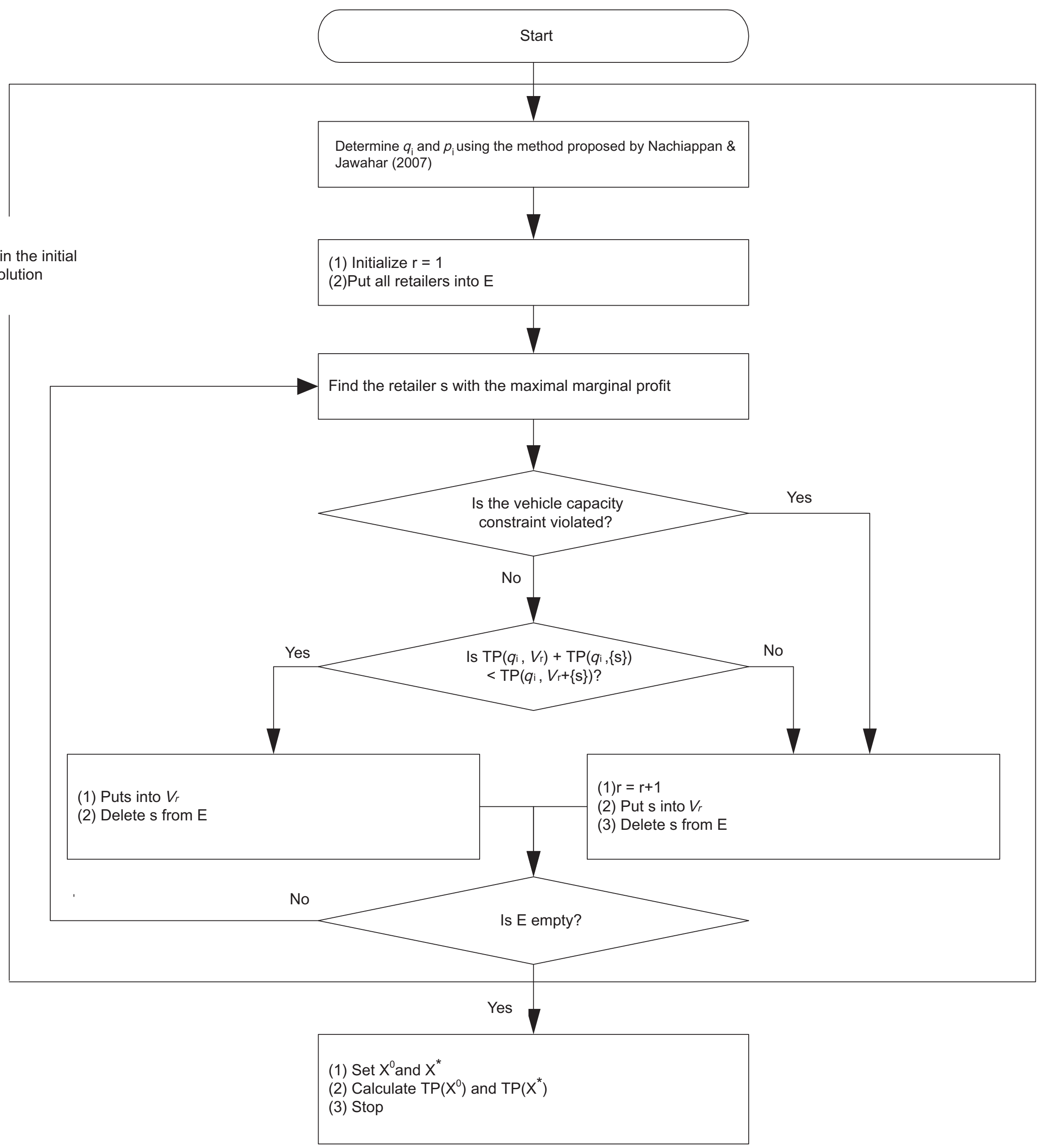

Fig. 1. The flowchart for obtaining the initial solution of the proposed method.

cates that the demand per day on route r multiplying replacement time of the route must be less than or equal to vehicle capacity. The constraint (4) indicates each retailer appears on only one route. The constraint (5) indicates each route begins and ends at the same supplier. The constraint (6) indicates that every point entered by the vehicle should be the same point the vehicle leaves. The constraint (7) indicates the demand per day must be a positive integer. The constraint (8) indicates the routing decision variable is in the term of 0 or 1.

3. The proposed method for the inventory routing and pricing problem

To resolve the IRPP, three major decisions: inventory, routing and pricing, need to be made. Because the IRPP is a NP-hard problem (Since both inventory routing problem and pricing problem are NP problem), this paper proposes a heuristic method to improve the initial solution through inventory routing improvement procedure and pricing improvement procedure. The inventory routing improvement procedure is based on tabu search adopting VLNS and GENI, to resolve the inventory routing problem. The neighborhood search approach for the vehicle routing improvement procedure is explained as follows: (1) The VLNS approach is to randomly select $k \times a$ retailers (k can be 1, 2, or 3 and $a = \lceil N/10 \rceil$). Insert the selected retailers into the same route or different routes based on the maximal profit criterion and generate a new solution. If the new solution is better than the optimal solution, the new solution is accepted as the optimal solution (Zhao et al., 2008), (2) The GENI approach is to successively attempt to insert retailer i into the route that the neighborhood retailer belongs to. If the number of routes into which the retailer is inserted is less than five, set up a new route with retailer i. If the total demands of the route in which retailer i is inserted are larger than vehicle capacity, divide the route into two parts so that the total distance of the two routes is as low as possible (Zhao et al., 2007). The pricing improvement procedure is based on tabu search adopting variable large neighborhood search (VLNS) to resolve the pricing problem. The VLNS approach for the pricing improvement procedure is explained as follows: (1) Randomly select $k \times a$ retailers ($a = \lceil N/10 \rceil$). (2) Randomly select an integer z_i (can be negative or positive) from $U[-\frac{(q_{imax}-q_{imin})}{2}, \frac{q_{imax}-q_{imin}}{2}]$ for each selected retailer i. Add the number z_i to the demand q_i of selected retailer i and generate a new solution. If the new solution is better than the optimal solution, the new solution is accepted as the optimal solution.

3.1. The proposed method for the inventory routing and pricing problem

The proposed method in this paper is to obtain the initial solution first. Then tabu search adopting different neighborhood search approaches is applied to improve the initial solution through two procedures: inventory routing improvement and pricing improvement. The detailed procedure is as follows:

Obtain the initial solution (Fig. 1). The initial solution for pricing and demand decisions.

- Step 1: Determine the daily demand q_i for retailer i and its corresponding price p_i using the method proposed by Nachiappan and Jawahar (2007).

The initial solution for inventory routing decisions.

- Step 2: (1) Initialize $r = 1$. (2) Put all retailers into E.
- Step 3: Find the retailer s in E with the maximal marginal profit (The marginal profit of a specific retailer $s = \mathrm{TP}(q_i, V_r + \{s\}) - \mathrm{TP}(q_i, V_r)$. $\mathrm{TP}(q_i, V_r + \{s\})$ and $\mathrm{TP}(q_i, V_r)$ can be calculated based on the objective function in Section 2.2).
- Step 4: Is the demand of $V_r + \{s\}$ > vehicle capacity? If yes, (1) $r = r + 1$, (2) put s into V_r, (3) delete s from E, (4) go to Step 6. Otherwise, go to Step 5.
- Step 5: Is $\mathrm{TP}(q_i, V_r) + \mathrm{TP}(q_i,\{s\}) < \mathrm{TP}(q_i, V_r + \{s\})$? If yes, (1) put s into V_r, (2) delete s from E. Otherwise, (1) $r = r + 1$, (2) put s into V_r, (3) delete s from E.
- Step 6: Is E empty? If yes, (1) set the initial solution $X^0 = (q_i, V_r)$ and the optimal solution $X^* = (q_i, V_r)$, (2) calculate $\mathrm{TP}(X^0)$ and $\mathrm{TP}(X^*)$, (3) stop. Otherwise, go to Step 3.

Improve the initial solution (Fig. 2).

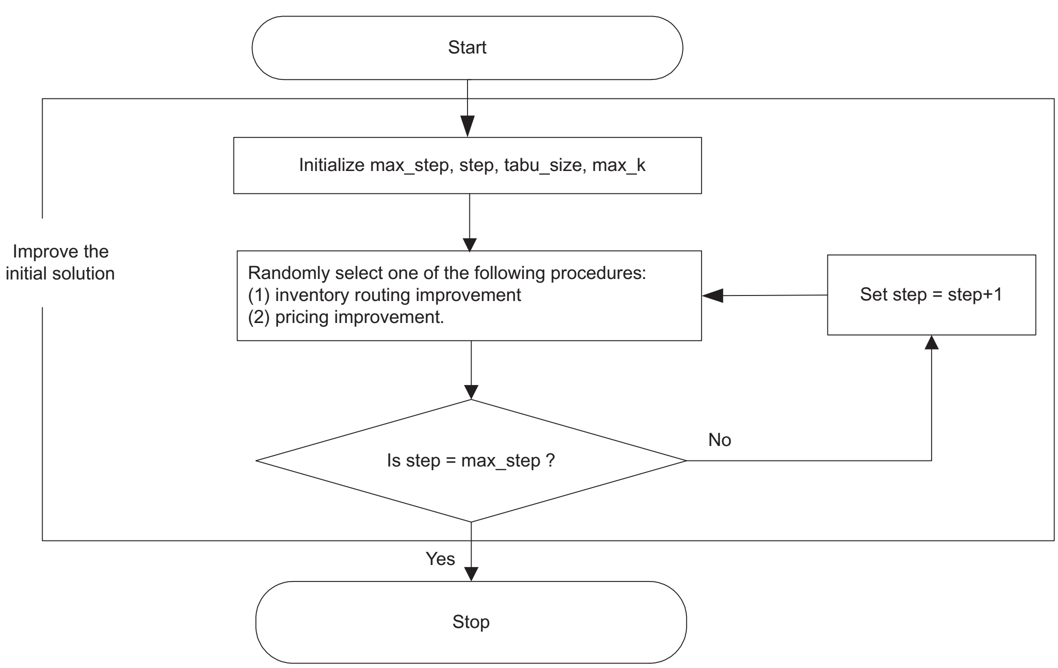

Fig. 2. The flowchart for improving the initial solution of the proposed method.

- Step 7: Initialize max_step (maximal recursive number of iteration), step, tabu_size (size of the tabu list), max_k.
- Step 8: Randomly select one of the following procedures: (1) inventory routing improvement (2) pricing improvement.
- Step 9: Is step = max_step? If yes, the best solution is obtained and stops. Otherwise, (1) set step = step + 1, (2) go to Step 8.

The detailed procedures for inventory routing improvement and pricing improvement are as follows:

Inventory routing improvement procedure (Fig. 3).

- Step 1: Generate a candidate move (from the initial solution X^0 to the candidate solution X^1) using one of the following neighborhood search approaches: (1) VLNS (2) GENI.
- Step 2: Is the vehicle capacity constraint violated? If yes, go to Step 1. Otherwise, go to Step 3.
- Step 3: Is the move in the tabu list (the tabu list is shared in vehicle routing improvement procedure and inventory control improvement procedure)? If yes, go to Step 1. Otherwise, (1) update $X^0 = X^1$, $\mathrm{TP}(X^0) = \mathrm{TP}(X^1)$, (2) update the tabu list.
- Step 4: Is $\mathrm{TP}(X^1) > \mathrm{TP}(X^*)$? If yes, (1) update $X^* = X^1$, $\mathrm{TP}(X^*) = \mathrm{TP}(X^1)$, (2) generate a candidate move using the last selected neighborhood search approach, (3) go to Step 2. Otherwise, stops.

Pricing improvement procedure (Fig. 4).

- Step 1: Initialize $k = 1$.
- Step 2: (1) Randomly select $k \times a$ retailers ($a = \lceil N/10 \rceil$). (2) Randomly select an integer z_i (can be negative or positive) from $U[-\frac{(q_{imax}-q_{imin})}{2}, \frac{q_{imax}-q_{imin}}{2}]$ for each selected retailer i. (3) Add the number z_i to the demand q_i of each corresponding retailer i. (4) Generate a candidate solution X^1.
- Step 3: Is the vehicle capacity constraint violated? If yes, go to Step 2. Otherwise, go to Step 4.
- Step 4: Is the move in the tabu list (the tabu list is shared in vehicle routing improvement procedure and inventory control improvement procedure)? If yes, go to Step 2. Otherwise, (1) update $X^0 = X^1$, $\mathrm{TP}(X^0) = \mathrm{TP}(X^1)$, (2) update the tabu list.
- Step 5: Is $\mathrm{TP}(X^1) > \mathrm{TP}(X^*)$? If yes, (1) update $X^* = X^1$, $\mathrm{TP}(X^*) = \mathrm{TP}(X^1)$, (2) set $k = 1$, (3) go to Step 2. Otherwise, go to Step 6
- Step 6: Is k = max_k? If yes, stops. Otherwise, (1) set $k = k + 1$, (2) go to Step 2.

3.2. The parameter setting for the proposed method

There are three parameters used in the proposed method: max_step (maximal outer recursive number of iteration), tabu_size (size of the tabu list) and max_k. max_step, tabu_size and max_k

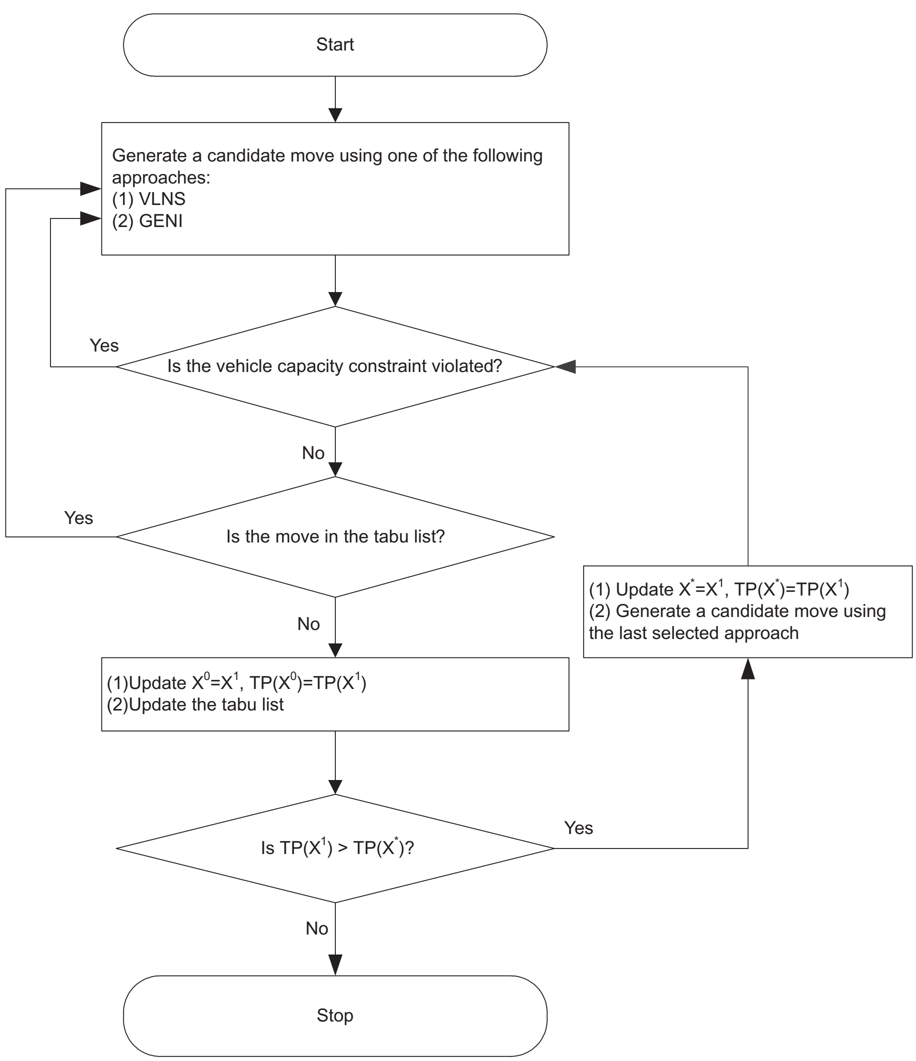

Fig. 3. The flowchart for the inventory routing improvement procedure.

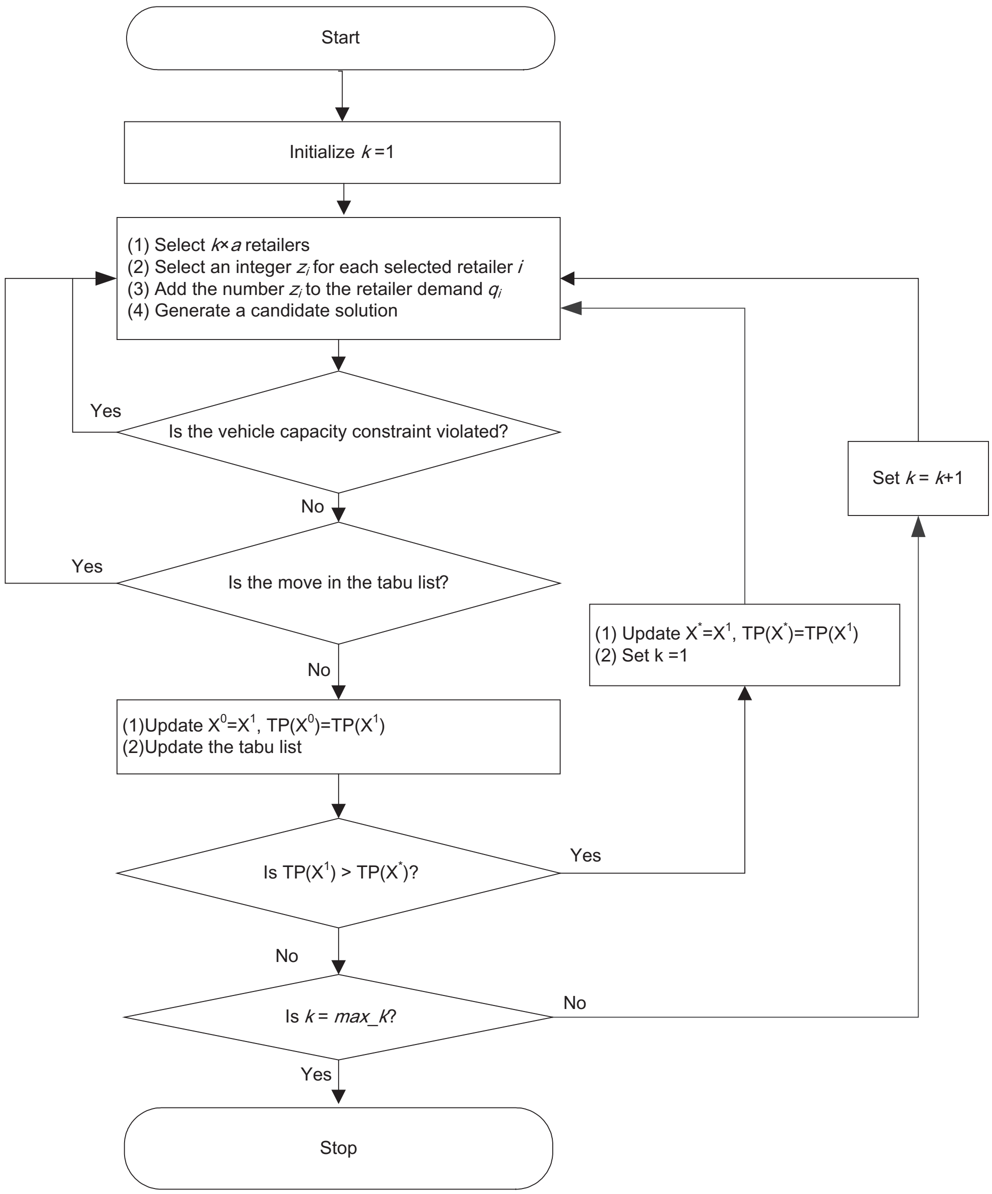

Fig. 4. The flowchart for the pricing improvement procedure.

are experimentally determined based on the maximal profit criterion (Please refer to the objective function in Section 2.2.). max_step is tried from 100 to 500 (100, 300, 500) when the size of retailer number is small (retailer number $N = 3$, 5) and from 1000 to 10,000 (1000, 4000, 7000, 10,000) when the size of retailer number is large (retailer number $N = 25$, 50, 100). tabu_size is tried from 4 to 13 (4, 7, 10, 13). max_k is tried from 1 to 3 (1, 2, 3).

4. Computational results and comparisons

Computational experiments were conducted to examine the computation effectiveness and efficiency of the proposed heuristic method (H1), based on the maximal average profit criterion, by comparing it with two other heuristic methods: H2 (The method makes the inventory, routing and pricing decisions separately. The pricing and demand decisions are made first based on the method proposed by Nachiappan and Jawahar (2007) (the best method until now) and then the inventory routing decisions are made based on the method proposed by Zhao et al. (2007). Please refer to Appendix A for the details), and H3 (The method makes the inventory, routing and pricing decisions separately. The pricing and demand decisions are made first based on the method proposed by Nachiappan and Jawahar (2007) (the best method until now) and then the inventory routing decisions are made based on the method proposed by Raa and Aghezzaf (2008). Please refer to Appendix B for the details). All heuristic methods were coded in C++ and run on a Core(TM)2 Duo 3G CPU with 2 GB RAM.

Table 1
The parameters and their values in small-sized problems.

Parameters	Values
a_i	$U[18, 21]$
b_i	$U[0.03, 0.08]$
Vehicle capacity	100
Supplier capacity	Infinity
Dispatching cost	25
Distance cost	1
Ordering cost	250
Holding cost	1
Production cost	2

For evaluating the proposed heuristic H1, the test problems are divided into two categories: small and larger size. For small-sized problems with up to five retailers, the solutions for H1, H2, and H3 are compared to the optimal solution yielded by enumeration search. A set of 40 tests classified in four different problem sizes (3 retailers × demand range [q_{imin} = 1, q_{imax} = 10], 3 retailers × demand range [q_{imin} = 1, q_{imax} = 20], 5 retailers × demand range [q_{imin} = 1, q_{imax} = 10], 5 retailers × demand range [q_{imin} = 1, q_{imax} = 20]) was designed to evaluate the performance of the proposed heuristic solutions versus the optimal solutions. Each problem instance contained 10 tests. According to the literature (Nachiappan & Jawahar, 2007; Raa & Aghezzaf, 2008; Raa & Aghezzaf, 2009; Zhao et al., 2007), the detailed settings for each test problem are as follows (Table 1).

As for the parameter setting of the proposed method H1, the values are determined based on the maximal average profit criterion after experiments as follows (Please refer to Section 3.2. for the details.): max_step = 500, tabu_size = 7, max_k = 3. Table 2 shows the average solution quality and average CPU times for H1, H2, H3 and optimal solutions. It can be seen that the solutions of H1 are the same as those optimal solutions and better than those of H2 and H3 in terms of average profit in different small-sized problems. In addition, according to the ANOVA and multiple comparisons, H1 is significantly better than H2 and H3 at level 0.05 (Please refer to Table 3.). The average CPU times are less than 3 s for H1, H2, and H3. However, the maximal average CPU time for obtaining optimal solutions is around 30,937 s. The larger the problem size, the larger the computational time for obtaining opti-

Table 2
Results for small-sized problems.

Retailer number	Demand range [q_{imin}, q_{imax}]	Optimal solution		H1		H2		H3	
		Average profit	CPU (s)	Average profit	CPU (s)	Average profit	CPU (s)	Average profit	CPU (s)
3	[1, 10]	73.06	2.04	73.06	0.89	64.76	1.56	16.04	0.11
5	[1, 10]	143.84	420.24	143.84	1.65	114.49	2.37	28.78	0.17
3	[1, 20]	73.16	19.03	73.16	1.72	67.46	1.64	16.98	0.12
5	[1, 20]	146.76	30937.41	146.76	2.71	116.48	2.32	29.79	0.19

Table 3
ANOVA and multiple comparisons for small-sized problems.

Source	ANOVA				
	Sum of squares	df	Mean square	*F*	Sig.
Dependent variable: profit					
Corrected model	154975.849[a]	11	14088.714	25.052	.000
Intercept	860816.350	1	860816.350	1530.657	.000
Method	**63002.198**	**2**	**31501.099**	**56.014**	**.000**
Number	86000.247	1	86000.247	152.921	.000
Range	178.643	1	178.643	.318	.574
Method * number	5746.493	2	2873.247	5.109	.008
Method * range	19.266	2	9.633	.017	.983
Number * range	12.599	1	12.599	.022	.881
Method * number * range	16.402	2	8.201	.015	.986
Error	60737.426	108	562.384		
Total	1076529.625	120			
Corrected total	215713.275	119			

Multiple comparisons				
Method (*I*)	Method (*J*)	Mean difference (*I*–*J*)	Std. error	Sig.
Tukey HSD				
H1	**H2**	**18.411475*****	**5.3027520**	**.002**
	H3	**55.122463*****	**5.3027520**	**.000**
H2	H1	−18.411475*	5.3027520	.002
	H3	36.710988*	5.3027520	.000
H3	H1	−55.122463*	5.3027520	.000
	H2	−36.710988*	5.3027520	.000

Based on observed means.
[a] R^2 = .718 (adjusted R^2 = .690).
* The mean difference is significant at the .05 level.

Table 4
Results for larger-sized problems.

Retailer number	Demand range [q_{imin}, q_{imax}]	H1		H2		H3	
		Average profit	CPU (s)	Average profit	CPU (s)	Average profit	CPU (s)
25	[1, 10]	743.24	4.93	577.90	4.67	505.42	2.01
50	[1, 10]	1438.44	56.10	1247.01	40.01	1068.97	3.92
100	[1, 10]	2947.81	429.86	2547.45	399.02	2084.67	9.06
25	[1, 20]	777.82	9.89	594.64	4.67	512.25	2.09
50	[1, 20]	1522.19	80.43	1285.17	40.27	1085.85	4.02
100	[1, 20]	3071.97	672.98	2585.59	400.53	2114.03	10.01

mal solutions. The heuristic methods are more efficient than the optimal procedure.

For larger-sized problems, the optimal solutions cannot be obtained in a reasonable time and there is no tight lower bound for this problem. The performance of the proposed heuristic method H1 is evaluated against the solutions of H2 and H3. A set of 60 tests classified in six different problem sizes (25 retailers × demand range [q_{imin} = 1, q_{imax} = 10], 50 retailers × demand range [q_{imin} = 1, q_{imax} = 10], 100 retailers × demand range [q_{imin} = 1, q_{imax} = 10], 25 retailers × demand range [q_{imin} = 1, q_{imax} = 20], 50 retailers × demand range [q_{imin} = 1, q_{imax} = 20], 100 retailers × demand range [q_{imin} = 1, q_{imax} = 20]) was designed to evaluate the performance of the heuristic solutions. Each problem instance contained 10 tests. The detailed settings are the same as those in small-sized problems. The values of parameters for the proposed method H1 are determined based on the maximal average profit criterion after experiments as follows (Please refer to Section 3.2. for the details.): max_step = 10,000, tabu_size = 10, max_k = 3. Table 4 shows the average solution quality and average CPU times for H1, H2, and H3 for larger-sized problems. It is found that H1 is much better than H2 and H3 in terms of average profit in all problems. In addition, according to the ANOVA and multiple comparisons, H1 is significantly better than H2 and H3 at level 0.05 (Please refer to Table 5.). Because H1 makes the inventory, routing and pricing decisions simultaneously using tabu search adopting different neighborhood search approaches based on the maximal average profit criterion, it can acquire the highest profit effectively. When the number of retailers or demand range increases, the average profits for H1, H2 and H3 increase. As for the computational time, the CPU times of H1, H2 and H3 are all less than 673 s. When the number of retailers or demand range increases, the CPU times for H1, H2 and H3 increase because the solution space increases and needs more time to find a better solution.

Besides the problem size, the following tests are used to compare the proposed method, H1, with H2 and H3. (1) The slope of demand function b_i ranges from 0.8 × b_i to 1.2 × b_i (0.8 × b_i, 1.0 × b_i, 1.2 × b_i). (2) The vehicle capacity v decreases from 100 to 50. (3) The supplier capacity C decreases from infinity to 500. The number of retailers is 100. The demand range is [q_{imin} = 1, q_{imax} = 20]. The other settings are the same as those in larger-sized problems. Fig. 5 shows the average solution quality and average CPU times for H1, H2, and H3 based on different slopes of demand function, vehicle capacities, and supplier capacities. It is found that H1 is better than H2 and H3 in all problem instances in terms of average profit. When the slope increases (from 0.8 × b_i to 1.2 × b_i), the average profits for H1, H2 and H3 decrease. When the vehicle capacity decreases (v is from 100 to 50), the average profits for H1, H2 and H3 decrease. When the supplier capacity C decreases, the average profits for H1, H2 and H3 decrease. The CPU time for H1 is affected by slope, vehicle capacity and supplier capacity. When the slope, vehicle capacity or supplier capacity increases, the average CPU time decreases. The CPU times for H2 and H3 are not affected by slope, vehicle capacity and supplier capacity.

5. Conclusions

In this paper, we have developed an effective heuristic method for the inventory routing and pricing problem. The proposed heuristic method making the inventory, routing and pricing decisions simultaneously is better than two other heuristic methods making the inventory, routing and pricing decisions separately based on the maximal average profit criterion. In addition, when the slope decreases, the vehicle capacity increases, or the supplier capacity increases, the average profits for all heuristic methods increase. When the slope, vehicle capacity or supplier capacity increases, the average CPU time of the proposed heuristic method decreases.

Table 5
ANOVA and multiple comparisons for larger-sized problems.

Source	ANOVA				
	Sum of squares	df	Mean Square	*F*	Sig.
Dependent variable: profit					
Corrected model	128287658.257[a]	17	7546332.839	781.985	.000
Intercept	384583159.863	1	384583159.863	39852.232	.000
Method	**6224430.637**	**2**	**3112215.319**	**322.502**	**.000**
Number	118700062.078	2	59350031.039	6150.116	.000
Range	68257.422	1	68257.422	7.073	.009
Method * number	3247304.619	4	811826.155	84.125	.000
Method * range	20491.305	2	10245.652	1.062	.348
Number * range	15997.215	2	7998.608	.829	.438
Method * number * range	11114.981	4	2778.745	.288	.885
Error	1563337.079	162	9650.229		
Total	514434155.199	180			
Corrected total	129850995.336	179			

Multiple comparisons

Method (*I*)	Method (*J*)	Mean difference (*I*–*J*)	Std. error	Sig.
Tukey HSD				
H1	**H2**	**210.653633***	**17.9352808**	**.000**
	H3	**455.083233***	**17.9352808**	**.000**
H2	H1	−210.653633*	17.9352808	.000
	H3	244.429600*	17.9352808	.000
H3	H1	−455.083233*	17.9352808	.000
	H2	−244.429600*	17.9352808	.000

Based on observed means.
[a] R^2 = .988 (adjusted R^2 = .987).
* The mean difference is significant at the .05 level.

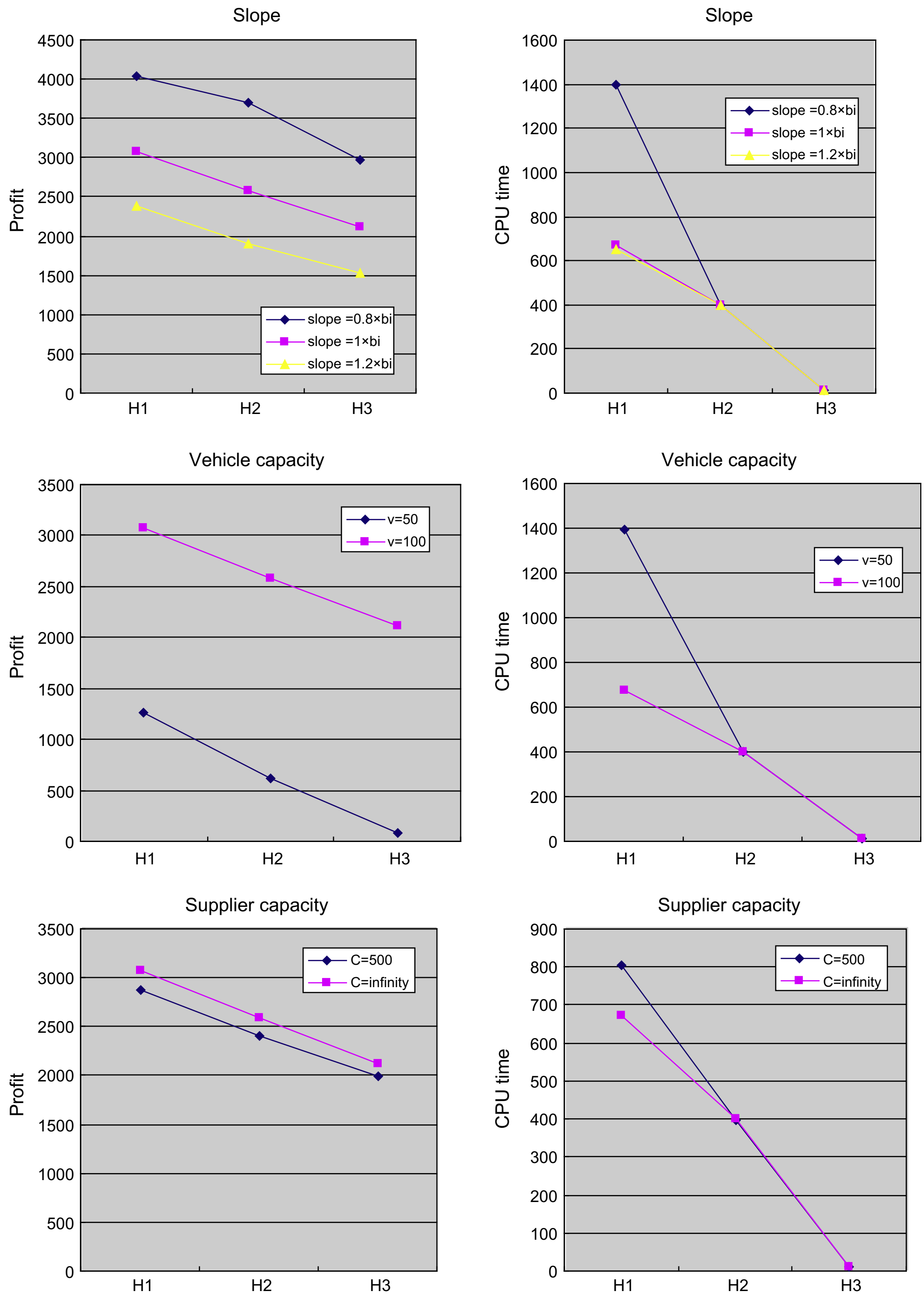

Fig. 5. The differences among H1, H2, and H3 under different slopes, vehicle capacities and supplier capacities in terms of average profit and CPU time.

Due to the limitations of this paper, some factors such as multiple products, different vehicle fleet, etc. are not considered. So considering these factors would help the inventory routing and pricing decisions made more realistically.

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Appendix A

The procedure to find the solution for the IRPP is as follows:

1. A genetic algorithm method proposed by Nachiappan and Jawahar (2007) is adopted to determine the demand and price for each retailer.
2. A heuristic method proposed by Zhao et al. (2007) for the IRP based on the known demands and prices is adopted. The detailed procedure is as follows:

(1) Generate the initial solution(s) for inventory routing decisions based on the known demands and prices.
(2) Use the GENI algorithm to successively attempt to insert vertex *i* into the route that the neighbored vertex belongs to. If the number of routes into which the vertex is inserted is less than five, set up a new route with vertex *i*. if the total demands of the route in which vertex *i* is inserted are larger than vehicle capacity, try to divide the route into two parts so that the total distance of the two routes is as low as possible.
(3) If a move leads to a solution better than the best one found by the search so far, perform it. Else perform the best non-tabu move of all the neighborhoods of the current solution.
(4) The tabu search procedure ends if Stop_step = $N * 5$.

Appendix B

The procedure to find the solution for the IRPP is as follows:

1. A genetic algorithm method proposed by Nachiappan and Jawahar (2007) is adopted to determine the demand and price for each retailer.
2. A heuristic method proposed by Raa and Aghezzaf (2008) for the IRP based on the known demands and prices is adopted. The detailed procedure is as follows:
   (1) A list is initialized with a separate distribution pattern for each retailer.
   (2) For each possible pair of distribution patterns, a new distribution pattern is constructed that combines the two. If this results in a saving (i.e. the reduced cost rate of the new distribution pattern is smaller than the sum of the reduced cost rates of the two constituent distribution patterns), this new distribution pattern is kept.
   (3) The distribution pattern combination from (2) with the largest saving is selected. The two constituent distribution patterns are removed from the list and replaced by the new distribution pattern combining them. If this distribution pattern has a negative reduced cost rate, it is added as a new route in the master-partitioning problem.
   (4) (2) and (3) are repeated as long as savings can be obtained by combining distribution patterns.

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