Profit Maximization in Inventory Routing Problems

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Abstract - **The inventory routing problem (IRP) deals with the transportation of one product from a producer to multiple consumers, which have given demands and inventory capacities, over a discrete time horizon. The traditional goal of the IRP is to minimize the combination of inventory and transportation costs, while avoiding stock outs at customers. This paper proposes two variations of the IRP with profit maximization. First, when the market situation allows prices to be adjusted, the problem involves finding an optimal balance of volume and margin according to a demand function. Second, when prices are fixed, unit production costs depend on the production volume, which can be adjusted to maximize the profit. Both variations lead to non-linear models, which are linearized and then tested on a selection of standard benchmark instances. Computational results show that considering profit maximization instead of cost minimization leads to different decisions, generating a larger revenue and profit.**

Keywords – **Price setting, production costs, distribution planning**

I. INTRODUCTION

Companies may consider a holistic view of their supply chain, including and integrating business processes starting with raw materials extraction, via production stages, and to transportation activities reaching the end customers. Supply chain planning aims to integrate management activities and to coordinate flows of products and information to increase the competitiveness and maximize the overall gain. Inventory management, routing, and scheduling are pivotal elements of the management activities, and making integrated decisions can increase the total benefit for the complete supply chain.

Inventory routing problems (IRPs) are generally formulated in terms of minimizing costs, including inventory holding costs and distribution costs. This may be suitable for certain processes, but a supply chain may be better off focusing on profit maximization: The minimization of costs may lead to reduced revenues, and thereby reduced profits.

When trying to maximize the overall profit in a supply chain, the problem of profit sharing arises. To avoid dealing with this added complexity, this paper will consider the situation where a single company is controlling both a central production facility and warehouses where the goods are sold to end customers, as well as the transportation between the two. If this company is a price setter (as in the textbook situation referred to as a monopoly), the company can maximize profits by modifying its prices. If the company is a price taker (such as in the textbook situation referred to as a perfect market), the company must consider the price as fixed. However, it may choose to modify its production rates, and thereby let some demand be covered by the competing actors in the market.

While some of the literature on IRPs do consider profit maximization, the market mechanisms controlling the prices and demand have not been included [2]. Among studies including profit maximization, [8] considers a penalty cost for unsatisfied demand, while [2, 6, 10, 11, 14] consider a revenue partly based on optional deliveries but with a fixed unit price. The production stage is considered in [11, 14], but with a fixed unit production cost. For recent survey articles on IRPs, see [3, 9, 13]. It is also notable that the literature on production routing problems only considers cost minimization, covering the total production, setup, inventory, and routing costs [1].

In summary, previous research has not considered in depth the option of adjusting prices or allowing unit production costs to vary with the production rate. This work aims to introduce profit maximization for inventory routing problems by taking into account different market types. The contributions of this article are: 1) the introduction of the inventory routing problem with profit maximization for a price maker; 2) the introduction of the inventory routing problem with variable unit production costs; 3) linearizing both of the above models with standard techniques, allowing us solve the problem with mixed-integer linear programming solvers; 4) analyses of the computational results for small, randomly generated instances show the potential gain in considering profit maximization instead of cost minimization.

II. PROBLEM FORMULATION

The basic IRP considers the distribution of one product from one facility where the product is produced, to a set of facilities where the product is consumed. The consumption facilities have given, deterministic demands over a given time horizon. In the following, we refer to the production facility as the supplier and the consumption facilities as customers, even though we consider all facilities to be part of the same company. The customers have limited storages and are serviced by a fleet of identical, capacitated vehicles. All vehicles start and end their routes at the supplier.

In this paper, it is assumed that the supplier has a limited storage and that not all the demand has to be

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satisfied. The goal is to maximize profit, given limited resources. The details of the profit maximization depend on the market situation in which the company maneuvers.

A. IRP with cost minimization

We first present a path-flow based mathematical model corresponding to the case where the company is minimizing total costs, the production quantities are predetermined, and all demand has to be met.

Let 0 denote the supplier, and consider a set N of customers. The planning horizon $T = \{1, \dots, \overline{T}\}$ consists of $\overline{T}$ days. The supplier's inventory has a maximum level U^s. The unit holding cost for the supplier is h^s, the initial inventory level is B^0, and the production rate in each time period is r_t^s. A function $f(r_t^s)$ defines the unit production costs. Each customer $i \in N$ consumes an amount r_i of the product in each time period $t \in T$. The maximum inventory level for customer i is U_i, and the customer has an initial inventory level of I_i^0, as well as a given holding cost per unit of h_i. Variables B_t and I_{it} define the inventory levels at the end of time period t at the supplier and customers, respectively. A homogeneous fleet of vehicles is used to ship the product, each vehicle having a capacity of Q units. The number of available vehicles is n. Each vehicle must perform deliveries by using a route from a set of routes $K = \{1, 2, \dots, k\}$, where each route is associated with a cost c_k. To indicate whether a customer i is served on a given route k, the binary parameter a_{ik} is set to 1 iff the customer is visited on the route.

A vehicle can perform at most one route in each time period. To model this, binary variables Y_{kt} are used, with Y_{kt} =1 iff route k is used at time t. The quantity of product transported to customer i in time period t with route k is denoted by X_{ikt}. It is assumed that all deliveries are made before the consumption in a period takes place. Model 1, specified below, can be used to minimize the combined transportation and inventory holding costs:

$$min \sum_{t\in T}\sum_{k\in K} c_k\, Y_{kt} + \sum_{i\in N}\sum_{t\in T} h_i\, I_{it} + \sum_{t\in T} h^s\, B_t \ + \sum_{t\in T} f(r_t^s) r_t^s, \qquad (1.1)$$

subject to

$$\sum_{i\in N} X_{ikt} \le Q\, Y_{kt},\ \ t\in T, k\in K \qquad (1.2)$$

$$X_{ikt} \le Q\, a_{ik} Y_{kt},\ \ t\in T, i\in N, k\in K \qquad (1.3)$$

$$\sum_{k\in K} Y_{kt} \le n,\ t\in T \qquad (1.4)$$

$$I_{i0} = I_i^0,\ \ i\in N \qquad (1.5)$$

$$I_{it} = \ I_{i,t-1} + \sum_{k\in K} X_{ikt} - r_i\ , i\in N, t\in T \qquad (1.6)$$

$$\sum_{k\in K} X_{ikt} \le U_i - I_{i,t-1}, i\in N, t\in T \qquad (1.7)$$

$$B_t = \ B_{t-1} + r_t^s - \sum_{i\in N}\sum_{k\in K} X_{i,k,t}, t\in T \qquad (1.8)$$

$$B_{t-1} + r_t^s \le U^s, t\in T \qquad (1.9)$$

$$B_0 = B^0, \qquad (1.10)$$

$$I_{it} \ \ge 0, i\in N, t\in T \qquad (1.11)$$

$$B_t \ \ge 0, t\in T \qquad (1.12)$$

$$X_{ikt} \ \ge 0, i\in N, k\in K, t\in T \qquad (1.13)$$

$$Y_{kt} \ \in \{0,1\},\ \ k\in K, t\in T \qquad (1.14)$$

The objective function (1.1) states that the sum of the transportation costs, inventory holding costs at customers, inventory holding costs at the supplier, and production costs should be minimized. Constraints (1.2) are included to ensure that the amount transported on each route is not violating the capacity of the vehicle. Constraints (1.3) specify that a customer can only receive goods in a time period if it is visited by a route and this route is used in the particular time period. Constraints (1.4) state that the number of routes per time period is limited by the total number of vehicles available. Constraints (1.5) forces the inventory levels at all customers to be initialized. The inventory levels at subsequent periods are defined for each customer by constraints (1.6), and constraints (1.7) make sure that the inventory levels do not violate the inventory limits. Constraints (1.8) determine the supplier's inventory level, which is equal to the previous time period's inventory level plus the quantity produced in the current time period minus the amount delivered to customers. Furthermore, constraints (1.9) state that the inventory level at the supplier cannot violate the inventory capacity. Constraint (1.10) sets the initial inventory level of the supplier. Finally, the non-negativity and integrality restrictions are given by constraints (1.11)–(1.14).

B. IRP with profit maximization and lost sales

Now, consider variant of the former problem where profit is maximized. It is assumed that the supplier can obtain a revenue of p_i for each unit of product that customer i receives. That is, p_i is the unit price at that customer. Demands must not all be satisfied, and it is allowed to have partially lost demands. For each unit of unsatisfied demand, to reflect the dissatisfaction of the customers, a penalty of b_i is incurred. Variables C_{it} are introduced to identify the amount of product consumed by each customer in each time period, as the amount consumed can be lower than the demand. Model 2 can be stated as follows:

$$\max \sum_{i\in N}\sum_{k\in K}\sum_{t\in T} p_i X_{ikt} - \sum_{t\in T}\sum_{k\in K} c_k\, Y_{kt} - \sum_{i\in N}\sum_{t\in T} h_i\, I_{it} - \ \sum_{t\in T} h^s\, B_t - \sum_{i\in N}\sum_{t\in T} b_i(r_i - C_{it}) - \sum_{t\in T} f(r_t^s) r_t^s, \qquad (2.1)$$

subject to

(1.2)–(1.5), (1.7)–(1.14),

$$I_{it} = \ I_{i,t-1} + \sum_{k\in K} X_{ikt} - C_{it}, i\in N, t\in T \qquad (2.2)$$

$$C_{it} \le r_i, i\in N, t\in T \qquad (2.3)$$

$$C_{it} \ \ge 0, i\in N, t\in T \qquad (2.4)$$

The objective function (2.1) states that the total revenue, less the sum of transportation costs, inventory holding costs, penalties for the unsatisfied demand, and production costs, should be maximized. The revenue is calculated from the amount of product transported, rather than the amount consumed, under the assumption that the entire amount of product shipped will eventually be consumed. The inventory balance at customers is controlled by constraints (2.2). Here, the inventory level is given by the previous period's inventory level plus the amount of product received minus the amount of product consumed. Constraints (2.3) states that the consumed amount of product is limited by the demand of the customer. Constraints (2.4) define the domain of the new consumption variables. The rest of the constraints are directly adapted from model 1.

C. IRP with profit maximization for a price setter

The third model considers a company that can adjust the prices (the revenue obtained at the facilities referred to as customers) to maximize profit. A variable P_i is introduced to represent the unit revenue. The function $r_i = f(P_i)$ describes the relationship between the unit price and the demand. The new objective function (3.1) now contains the product of the unit revenue and the amount shipped. Compared to model 2, we have the same constraints except that (2.3) is replaced by constraints (3.2), now having a function on the right hand side. Furthermore, the domain of the new variable for the unit revenue is set by constraints (3.3):

$$\max \sum_{i\in N}\sum_{k\in K}\sum_{t\in T} P_i X_{ikt} - \sum_{t\in T}\sum_{k\in K} c_k\, Y_{kt}$$
$$- \sum_{i\in N}\sum_{t\in T} h_i\, I_{it} - \sum_{t\in T} h^s\, B_t \quad (3.1)$$
$$- \sum_{i\in N}\sum_{t\in T} b_i (f(P_i) - C_{it}) - \sum_{t\in T} f(r_t^s) r_t^s,$$

subject to

$$(1.2)–(1.5), (1.7)–(1.14), (2.2), (2.4),$$

$$C_{it} \le f(P_i),\ \ i \in N, t \in T \quad (3.2)$$

$$P_i \ge 0,\ \ i \in N \quad (3.3)$$

We consider a demand function that has the form $f(P_i) = bP_i + d$, for two parameters b and d [7]. The demand function does not lead to complications in constraints (3.2), as it is. However, the first term of the objective function (3.1) is no longer linear, and, being the product of the price and the shipped quantity, becomes non-separable. We first introduce a new variable $Z_i = \sum_{k\in K, t\in T} X_{ikt}$ for each $i \in N$, corresponding to the total amount of product received of a customer in all time periods. We then need to linearize the term $\sum_{i\in N} P_i Z_i$. This is linearized according to [15]: First, introduce two new sets of variables W_{1i} and W_{2i}. Second, the new variables W_{1i} and W_{2i} are related to P_i and Z_i by setting $W_{1i} = \frac{1}{2}(P_i + Z_i)$ and $W_{2i} = \frac{1}{2}(P_i - Z_i)$. Given $l^P \le P_i \le u^p$ and $l^Z \le Z_i \le u_i^Z$, the bounds on W_{1i} and W_{2i} can be expressed as $\frac{1}{2}(l^P + l^Z) \le W_{1i} \le \frac{1}{2}(u^P + u_i^Z)$, and $\frac{1}{2}(l^P - u_i^Z) \le W_{2i} \le \frac{1}{2}(u^P - l^Z)$.

The term $\sum_{i\in N} P_i Z_i$ in the objective function is then replaced by $\sum_{i\in N}(W_{1i}^2 - W_{2i}^2)$, only contains non-linear functions of a single variable, and is hence a separable function. Piecewise linear approximations are used to eliminate these nonlinearities, using what is known as the λ-formulation [15]. Our implementation explicitly models the resulting special ordered set of type 2 (SOS2) restrictions. As $\sum_{i\in N}(-W_{2i}^2)$ is a concave function (which is maximized), the corresponding SOS2 constraints are omitted, whereas to model $\sum_{i\in N} W_{1i}^2$ binary variables are used to make sure that at most two consecutive breakpoints are selected of the piecewise linear approximation.

D. IRP with variable unit production costs

Our final IRP considers a market where the company cannot influence prices, being a price-taker, but where profit can still be maximized by taking into account variable production costs. A function $f(R_t^s)$ describes how the unit production costs vary with production rates. The production quantity is described by the variables R_t^s.

$$\max \sum_{i\in N}\sum_{k\in K}\sum_{t\in T} p_i X_{ikt}$$
$$-(\sum_{t\in T}\sum_{k\in K} c_k\, Y_{kt} + \sum_{i\in N}\sum_{t\in T} h_i\, I_{it} \quad (4.1)$$
$$+ \sum_{t\in T} h^s\, B_t) - \sum_{i\in N}\sum_{t\in T} b_i (r_i - C_{it})$$
$$- \sum_{t\in T} f(R_t^S) R_t^S,$$

subject to

$$(1.2)–(1.5), (1.7), (1.10)–(1.14), (2.2)–(2.4),$$

$$B_t = B_{t-1} + R_t^S - \sum_{i\in N}\sum_{k\in K} X_{i,k,t},\ t \in T \quad (4.2)$$

$$B_{t-1} + R_t^S \le U^s,\ t \in T \quad (4.3)$$

$$R_t^S \ge 0,\ t \in T \quad (4.4)$$

In the objective function, the term $\sum_{t\in T} f(R_t^S) R_t^s$ represents the production costs, which are equal to the unit production costs multiplied by the amount produced. In this work we consider a function of the form $f(R_t^s) = eR_t^s + d + m/R_t^s$ for some constants e, d, and m [7]. If we multiply this function by the amount produced, total production costs can be written as $f(R_t^S)R_t^s = e{R_t^s}^2 + dR_t^s + m$. Then, the term $\sum_{t\in T}(e{R_t^s}^2 + dR_t^s + m)$ in the objective function is a separable non-linear function that we linearize. As the non-linear functions each contain a single variable, the linearization can be done using the λ-formulation method of piecewise linear approximation as for the previous model. In this case we will minimize the value of a convex function, and no binary adjacency variables are required.

III. COMPUTATIONAL EXPERIMENTS

All computational tests were performed on a 2.50 GHz Intel Core i5-6500T CPU with 16 GB of RAM using AMPL/CPLEX 12.7.00. Test instances were adopted from [4], with parameters set to make solutions from the different models comparable. The demand function is $f(P_i) = -2.5P_i + 113$, where P_i is the unit price. The penalty for unsatisfied demand is 0.2 p_i. For a price setting company, for which the price is a decision variable, this penalty is set to the absolute value of the penalty used by the other models. This assumption simplifies the model and avoids further nonlinearities in the objective function. The function describing the average costs is $f(R_t^s) = 0.0005R_t^s + 2 + \frac{3}{R_t^s}$, where R_t^s is the production rate.

We consider instances with 5, 10, and 15 customers, and 3 or 6 periods. The models take as an input the set of possible vehicle routes. As is common for some applications of IRPs [12], we consider routes of limited length. With this, we find that models A, B, and D can be solved to optimality within a minute for instances with up to 15 customers, six periods, and routes limited to two visits. Model C is harder to solve, and leaves an optimality gap of 10 % after 48 minutes for the largest instance considered.

The computational time for solving model C is related to the number of breakpoints used to linearize the revenue function. For an instance with three periods and five customers, the runtimes when using 5, 10, and 15 breakpoints were 0.4, 0.8, and 1.6 seconds, respectively. At the same time, the error in the estimation of the profit was 16, 3, and 2 %.

The runtimes also increase significantly when allowing routes with more visits. With routes limited to three visits instead of two, models B, C, and D are able to heighten the profits by increasing the quantity of product delivered. This implies that one should not limit the route lengths, unless there are technological reasons for using short routes. For model A, when increasing from two to three visits per route, the total cost is decreased, but so is the associated profits.

To illustrate the behavior of the models, Table 1 presents detailed results for two instances solved by all four models. The same behavior was observed for other instances. The number of break points (for models C and D) is five, and the maximum number of customers per route is two.

Model A minimizes costs, and it is therefore optimal to transport only the quantity of product that is required to satisfy the total demand in the time horizon considered. Consequently, the customers' levels of inventory near the end of the time horizon become close to zero. This aspect of the model is a drawback, as customers will require an additional service right after the end of the planning horizon. Transportation costs in the following planning horizon may therefore become unnecessarily high. When using model B, on the other hand, customers are provided with high levels of inventory at the end of the time horizon, as the model increases the revenue by upping the quantity transported to the customers. This means, however, that fewer deliveries are required in the next time periods. The transportation costs and the holding costs at customers may go up as a result. Nevertheless, the costs increase less than the revenue, since the profit per unit also increases.

In model C, for a price setting company, the amount of product distributed and consumed is less than in the other models. The reason is that the model chooses to increase the unit sales revenue, resulting in a lower demand. As the consumption rate is lower, the given production rate leads to an increase of the inventory holding costs at both the supplier and the customers. On the other hand, there is an increase in the profit per unit shipped and in the total profit. Model C leads to a penalty for unsatisfied demand. This may happen if the upper limit on the price is less than the otherwise best price, or if the model selects a price and a corresponding demand such that the demand cannot be completely satisfied in all periods.

In model D, where the unit production cost is variable and the company is a price taker, the quantity of product transported equals that of model B. Therefore, we again observe high levels of inventory at customers at the end of the planning horizon. However, the costs of production and inventory at the production facility are significantly lower, as the production rate can be adjusted, and the model induces the company to produce no more than that which is required. This leads to a situation where the supplier experiences a zero inventory at the end of the time horizon, and has to increase the production in the next planning horizon. However, high levels of inventory at customers give the supplier the possibility to start production in the beginning of the next planning horizon. In addition, the production rate chosen by model D leads to lower production costs per unit than the fixed costs incurred with the fixed production rate in model B.

TABLE 1
DETAILED RESULTS FOR TWO INSTANCES, WITH RESPECTIVELY 5 CUSTOMERS AND 3 TIME PERIODS, AND 10 CUSTOMERS AND 6 TIME PERIODS

	5 customers, 3 time periods				10 customers, 6 time periods			
Criterion	Model A	Model B	Model C	Model D	Model A	Model B	Model C	Model D
Profit	3249.7	9989.1	12476.3	11394.3	27022.2	38023.5	52873.5	45131.3
Revenue	6644.0	14719.2	17038.5	14719.2	48360.8	60032.0	76495.8	60032.0
Costs	3394.3	4730.2	4562.2	3324.9	21338.6	22008.5	23622.3	14900.7
Transportation costs	1445.7	2791.3	2504.5	2791.3	7907.4	8542.7	9118.2	8542.7
Holding costs at customers	85.8	194.3	206.3	187.9	746.5	865.7	1340.9	865.7
Holding costs at the supplier	639.9	521.7	608.2	200.3	3837.0	3752.4	4313.6	1009.7
Penalty	0.00	0.00	20.40	0.00	0.00	0.00	1.91	0.00
Production costs	1222.9	1222.9	1222.9	145.5	8847.7	8847.7	8847.7	4482.6
Produced amount	579	579	579	67	3810	3810	3810	1961
Transported quantity of product	262	577	473	577	2939	3544	2862	3544
Consumed quantity of product	579	579	421	579	3810	3810	2860	3810
Profit per unit transported	12.4	17.3	26.4	19.8	9.2	10.7	18.5	12.7

IV. CONCLUDING REMARKS

This paper presented different models for IRPs with profit maximization, taking into account different market conditions and corresponding ways of maximizing profits. The models were linearized and solved using a commercial solver.

Solving the IRP with profit maximization provides decisions, for every time period, regarding the amount to deliver to each customer and corresponding routes for a fleet of vehicles. In addition, one of the models provided the ability to set prices, to find the optimal mixture of prices and demands, while another model considered an adjustable production rate, considering variable unit production costs, to increase profitability. The developed models provide increased opportunities, and may assist decision makers in companies to make improved decisions by considering additional elements in the planning processes.

The computational experiments performed to test the different models reveal a potential problem with multi-period IRP models. This is also a valid criticism of the standard cost minimization model: the planning horizon is relatively short, and end-of-horizon effects are ignored. In the standard cost minimization variant of the IRP, this result in empty (or near empty) inventories at the customers at the end of the planning horizon. When being able to adjust the production volumes, and maximizing profits, it results in empty inventories at the supplier. In the case that a profit is calculated based on the amount of goods shipped, it results in full inventories at the customers, or possibly that too much goods is delivered compared to what would be reasonable in practice.

One suggestion has been to target the logistic ratio to improve the end-of-horizon problems [5]. While this may be one possible solution, future research on multi-period IRPs may need to actively address and adjust the consequences of the end-of-horizon effects. This appears to be equally valid when profit maximization is used, as when cost minimization is the only focus.

This work considered the consequences of introducing profit maximization in IRPs from both a modelling perspective and from the perspective of studying the resulting changes in the structure of optimal solutions. Therefore, only relatively small instances were solved. To solve larger instances, heuristic solution methods must be developed, addressing the additional decisions such as determining the best price and production volume.

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