Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
A		2302480

Small fungi plays a big role in the carbon cycle

summary

The carbon cycle has drawn more people's attention in recent years due to climate change and the greenhouse effect. One of the significant components in this cycle is the decomposition of plant material and woody fibers. Among several factors, fungi play a vital role in the decomposition process. In order to find out what determines the decomposition speed of fungi and what the interaction is among different kinds of fungi, we establish several models to examine.

In order to discover the decomposition rate when there are various kinds of fungus, we build **Decomposition rate with multiple species of fungi model** to solve this problem. We will first build a dynamic system to calculate fungus density at a specific time. To simplify this problem, we choose three typical fungi to simulate the procession, which are Tyromyces chioneus(T), Pycnoporus sanguineus(P), and Xylobolus subpileatus(X). We will use textbfCompetitive Lotka–Volterra equations to simulate the competitive process and reach the final result. After that, we will take the decomposition capacity of each type of fungi into consideration so that we can get the total decomposition rate within a certain time period.

The interaction between various kinds of fungi will affect the density of each kind of fungi. In the short-term, because there is a sufficient nutrient in the environment, there is no apparent interest conflict between groups of fungus. Most fungi may have a suitable environment and increase for a while. However, as time goes by, the nutrient will decrease to the extent that it cannot support all fungi to survive. In this case, we heritage our previous model and develop it into **Fungi trend model** after considering the daily temperature and moisture in reality. We choose another five fungi which are popular worldwide which are, Armillaria_gallica_FP102531(a.gal1.s), Armillaria_gallica_FP102535(a.gal3.s), Hyphodontia_crustosa(h.crust.n), Lentinus_crinitus(l.crin.s), Phellinus_hartigii(p.har.n) to build our model

Environments such as temperature and moisture are varied across the world. Because these factors will affect the hyphal extension rate of fungi, they have an impact on the density of each fungus when there are several kinds of fungi that coexist. We still choose the fungi we use in model II to analyze this problem to keep the consistency. To find out what will happen under different environments(arid, semi-arid, temperate, arboreal, and tropical rain forests), we extract the temperature data of corresponding cities. To simulate the moisture conditions, we choose to apply **Gamma distribution** $X \sim \Gamma(k, \theta)$. After applying our model to each condition, we found that although after one month, Phellinus_hartigii_DMR94(p.har.n) will dominate the environment, the distribution in the beginning ten days is different. All result graphs are shown in Appendix 1.

Biodiversity is an important topic nowadays. It can not only create a world with abundant species but also does benefit the whole ecosphere. The diversity of fungi will also increase the efficiency of decomposing the wood fiber. From our previous model, we found that the decomposition rate depends on the fungi density and corresponding speed of each fungus. Also, temperature and moisture do play an important role in the interaction among fungi. In this model, we try to find the decomposition efficiency considering all elements we mentioned before to examine the importance of biodiversity.

Key words: Competitive Lotka–Volterra equations, Gamma distribution, Biodiversity, Sensitivity analysis

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1 Introduction

1.1 Problem Background

Carbon cycle is the process in which carbon atoms continually travel from the atmosphere to the Earth and then back into the atmosphere. The balance of carbon cycle is critical for the live beings on the earth. There are several tunnels for carbon to back into the atmosphere, among which the decomposition of wood plays an important part.

Wood decay is caused by any species of fungus that digests moist wood, causing it to rot. The decomposition speed of different fungus is determined by both intrinsic (e.g., tree species properties) and environmental (e.g., temperature, moisture) factors. Among the associated organisms, wood decay fungi are essential and extremely important, being the only forms of life capable of degrading wood to its initial constituents.

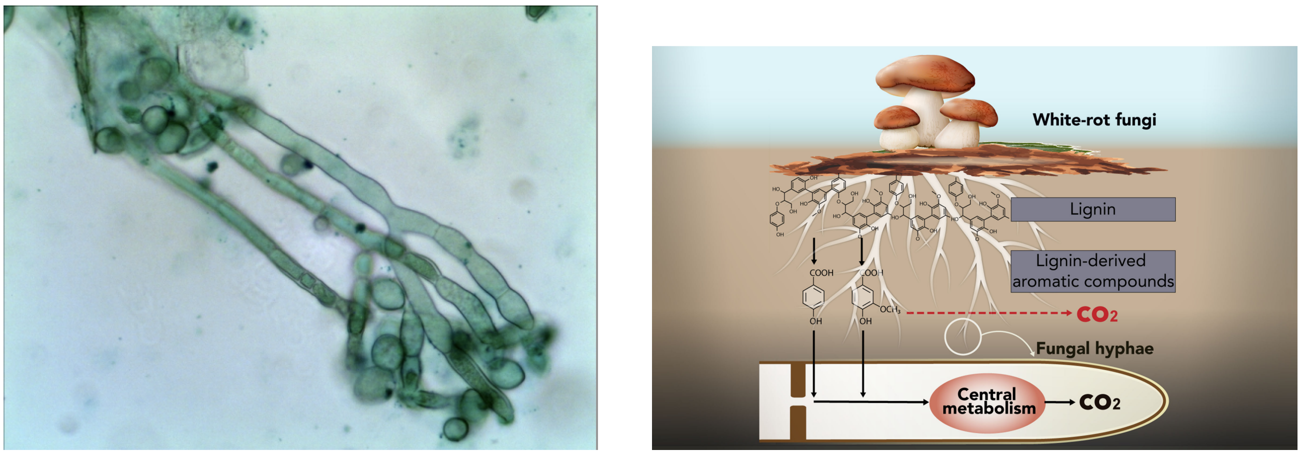

Figure 1: Left: Brown-rot fungi at 400X; Right: The decay process of white-rot fungi

What's more, the interaction between different kinds of fungus also will change the speed of decay rate.With the climate change, the environment where fungus live will also change, which will impact the overall efficiency of the carbon cycle system. According to the type of decay they cause, wood decay fungi can be classified into white-rot fungi, brown-rot fungi, and soft-rot fungi. The detailed of interaction must be examined.

1.2 Problem Restatement

- Build a mathematical model to find out decomposition speed in the situation where several species of fungi are exist at a given temperature and district in certain district.
- Find out what will happen when fungus are interacted in short-term and long-term.
- Examine the different kinds of combinations of species and its advantages and disadvantages in different environment(arid, semi-arid, temperate, arboreal, and tropical rain forests).
- Analyzing the sensitivity of our model in rapid fluctuations and changing atmospherical trends.

1.3 Our approach

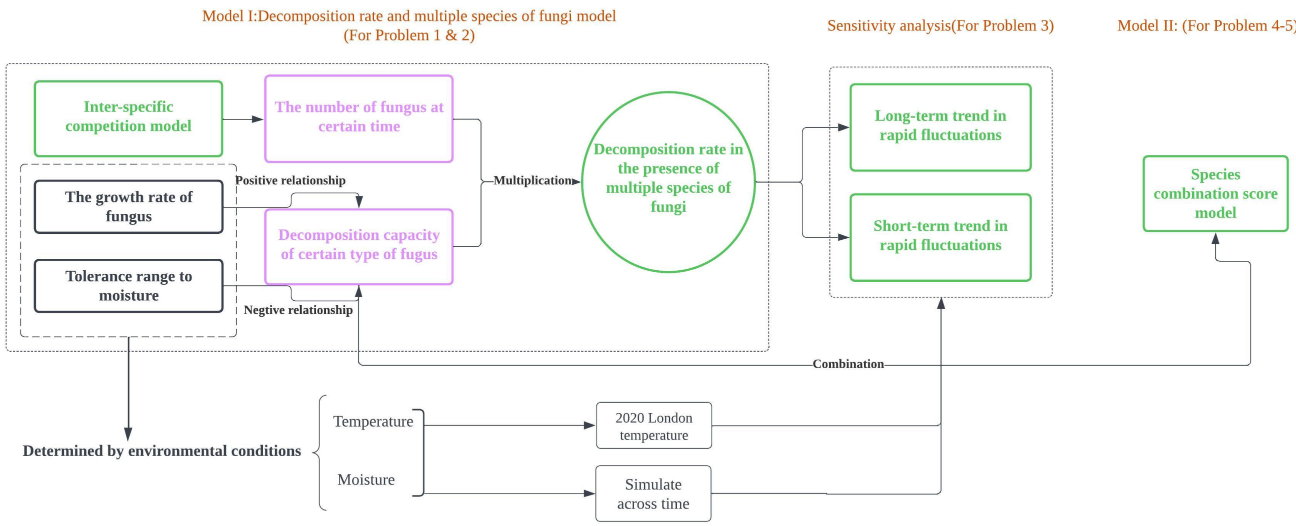

Figure 2: The framework of our article

- In order to discover the decomposition rate when there is various kinds of fungus, we will firstly use textbfCompetitive Lotka–Volterra equations to find out the density of fungi at certain time. After that, we will consider the decomposition capacity of one certain type of fungi and multiply it to the density of this type of fungus to get the total decomposition rate at certain time period.
- To examine the interaction of fungi in different environment, we heritage our previous model and develop it into Fungi trend model after considering the fluctuations of daily temperature and moisture.
- We will extract the temperature data of corresponding cities and simulate the moisture conditions with Gamma distribution to predict the relative advantages for different combinations of species under different environments.
- Finally, we try to find the decomposition efficiency considering all elements we mentioned before to examine the importance of biodiversity in the presence of different degrees of variability.

2 Assumptions

Assumption 1: We assume the maximum capacity for one kind of fungi is 3 g/m^3.

Justification: The capacity for certain kind of fungi is limited in a given area. So there is a maximum density. To simplify the question, we use 3 g/m^3 as this capacity.

Assumption 2: The temperature in the same day is constant.

Justification: To simplify the question, we will assume the temperature in the same day is a constant. Actually, the fluctuations of temperature in certain day is really small compared to the long time period fluctuation.

3 Notations

Table 1: Notation of this article
In this article, we will use the following notations to solve problems.

Symbols	Definition	Unit
T	Temperature of the environment	°C
r_p, r_t, r_x	The hyphal extension rate of different fungi	%
n_p, n_t, n_x	The maximum capacity of different fungi	g/m^3
p,t,x	The density of different fungi	g/m^3
Matrix D	The density of each fungi at certain time period	g/m^3
Matrix V	The speed of growth	mm/time
FR	Fibre remain	g

4 The Data

4.1 Data collection

We collect relevant data for our model building from

Table 2: Data source

Data source	Data websites	Data description
Artical	https://github.com/dsmaynard/fungal_biogeography	2020 London Temperature in °C, The relevant data of fungi

5 Model I: Decomposition rate and multiple species of fungi model

The decomposition rate is determined by several environmental factors and characteristics of fungi. Among these, the most factors are temperature and moisture. The majority of fungi are mesophiles, and grow at temperatures in the range of 5-35 °C, which optimum temperatures for growth between 20 and 30 °C. In contrast, the sensitivity of fungi to moisture has huge differences. Different fungal species have different tolerances to moisture and some will grow at lower moisture levels. Moreover, the rate will not only be affected by the behaviour of a single fungi, but also be determined by the interactions of different species of fungi. Due to the limited resource, the density of different kinds of fungus will also change and affect the total decomposition model.

To simplify this problem, in our model, we choose three typical fungus to simulate the procession, which are Tyromyces chioneus(denote in T), Pycnoporus sanguineus(denote in P) and Xylobolus subpileatus(denote in X). The reasons that we choose these three kinds of fungus are that they are all typical and common fungus in the nature and have similar suitable temperature and moisture. In that case, we can better simulate the process of the competition and find the result of what will happen after a long period.

5.1 Competitive Lotka–Volterra equations

5.1.1 Introduction of model

The competitive Lotka–Volterra equations are a model of the population dynamics of species competing for some common resource. It shows the interactive relationship between different species under limited resource. There are several elements in the equations.

In our problem, the competition is within the scope of different kinds of fungi. The basic model is shown in below:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{\sum_{j=1}^{N} \alpha_{ij} x_j}{K_i}\right) \tag{1}$$

In this equation, x_i denote the amount of i species, r_i is the inherent per-capita growth rate, K_i is the carrying capacity of i species, α_{ij} represents the effect species i has on the population of species j. To simplify this problem, in our model, we choose three typical fungus to simulate the procession, which are Tyromyces chioneus(denote in T), Pycnoporus sanguineus(denote in P) and Xylobolus subpileatus(denote in X).

5.1.2 Conditions of our model

In the case that temperature and moisture will affect the growth rate of fungus. We choose certain environment for our simulation. To better simulate the situation, it's important to choose a suitable environment for these three kinds of fungus. We collect the data of these three kinds of fungus and plot the relevant data into one graph. Below is the result:

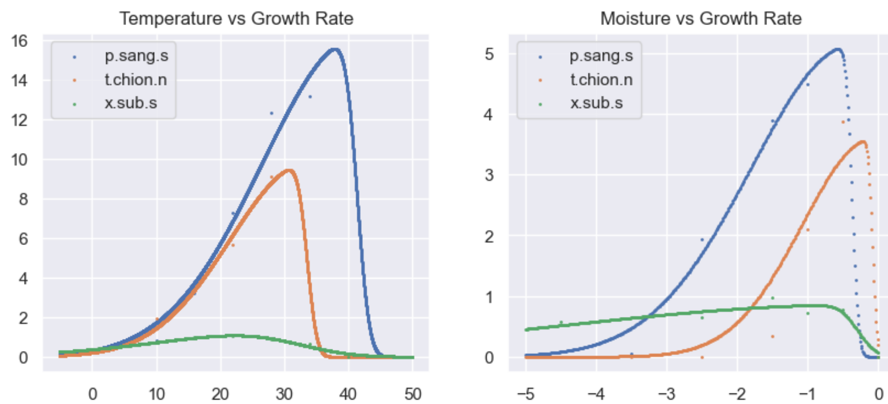

Figure 3: The relationship between growth rate and moisture and temperature

From the graph, we found that the suitable temperature interval is between 20-30 °C for all fungus. In that case, we will choose 22 °C, which Xylobolus subpileatus has the largest growth rate while others are also remain high.

The measurement of moisture for soil is matrix potential. Matrix potential is forces between the water molecules and the solid particles in combination with attraction among water molecules promote surface tension and the formation of menisci within the solid matrix. The conversion equation is Van Genuchten model which show above:

$$\theta(\psi) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|\psi|)^n]^{1-1/n}} \tag{2}$$

We will choose the moisture of matrix potential -0.5kPa for our model.

5.1.3 Fungi Competitive Lotka–Volterra equations

Applying the basic ideas of Competitive Lotka–Volterra equations, we build the following equations for this problem.

$$\frac{dp}{d\ time} = r_p \ * p \left(1 - \frac{p}{n_p} - s_{pt}(\frac{t}{n_t}) - s_{px}(\frac{x}{n_x})\right) \tag{3}$$

$$\frac{dt}{d\ time} = r_t * t \left(1 - \frac{t}{n_t} - s_{tp}(\frac{p}{n_p}) - s_{tx}(\frac{x}{n_x})\right) \tag{4}$$

$$\frac{dx}{d\ time} = r_x * x \left(1 - \frac{x}{n_x} - s_{xp}(\frac{p}{n_p}) - s_{xz}(\frac{z}{n_z})\right) \tag{5}$$

In these equations:

- r_p, r_t, r_x to denote The hyphal extension rate of different fungi
- n_p, n_t, n_x to denote the maximum capacity of different fungi (in g/m^3)
- p,t,x denote the density of different fungi (in g/m^3)

Table 3: The density of fungi

Name	t1	t2	t3	t4	t5	t6
p.sang.s	0.02	1.20	2.51	2.90	2.98	3.00
t.chion.n	0.06	1.17	0.46	0.09	0.02	0.00
x.sub.s	1.74	0.78	0.03	0.00	0.00	0.00

In our model, we choose the initial data $init_p, init_t, init_x$ = 0.02, 0.06, 1.74 g/m^3 and found the dynamic movement of the density of fungi.

We will use matrix D to denote the density of each fungi at certain time period:

$$D = \begin{bmatrix} p_{t1} & p_{t2} & p_{t3} & p_{t4} \\ t_{t1} & t_{t2} & t_{t3} & t_{t4} \\ x_{t1} & x_{t2} & x_{t3} & x_{t4} \end{bmatrix} \tag{6}$$

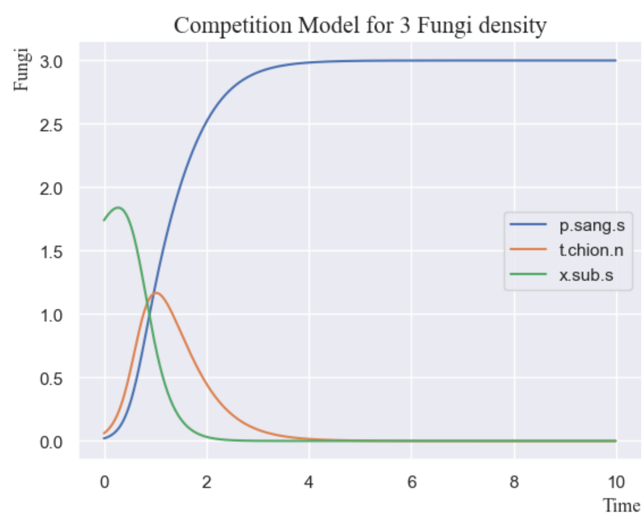

Figure 4: Competition model

In the short run, all fungi does increase for a short time. However, after a long competitive time, we found Pycnoporus sanguineus dominates the environment.

5.2 Decomposition rate with various fungi model

The decomposition speed of fungi will change over time due to X. The speed of each fungi is shown below:

Table 4: The density of fungi

Name	t1	t2	t3	t4	t5	t6
p.sang.s	0.0031	0.1842	0.3848	0.4455	0.4577	0.4598
t.chion.n	0.0024	0.0463	0.0182	0.0036	0.0006	0.0001
x.sub.s	0.0007	0.0003	0.0001	0.0000	0.0000	0.0000

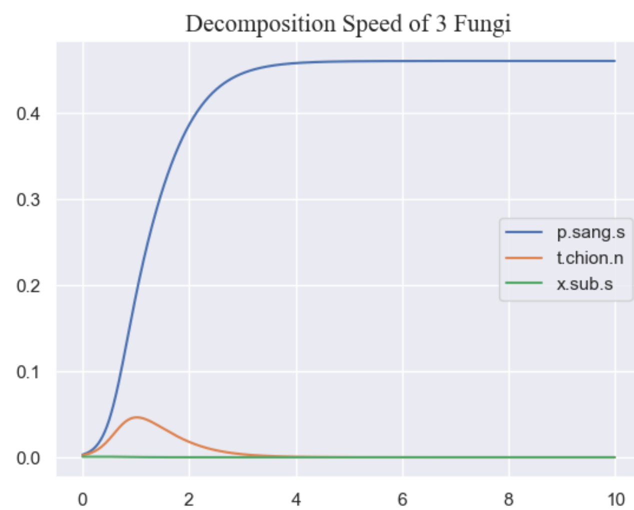

Figure 5: Decomposition speed of 3 fungi

Here, matrix V is used to denote the density of each fungi at certain time period:

$$V = \begin{bmatrix} Speed_p \\ Speed_t \\ Speed_x \end{bmatrix} \tag{7}$$

The decomposition of wooden fibre depends on the density of each fungi and their corresponding speed. In that case, total decomposition speed can be calculated by the product of speed and density. We will use $\frac{df}{d\ time}$ to denote the decomposition

$$\frac{df}{d\ time} = \left[Speed_p, Speed_t, Speed_x\right] * \begin{bmatrix} p_{t1} & p_{t2} & p_{t3} & p_{t4} \\ t_{t1} & t_{t2} & t_{t3} & t_{t4} \\ x_{t1} & x_{t2} & x_{t3} & x_{t4} \end{bmatrix} \tag{8}$$

5.3 Conclusions

We substitute the notations into real data we extract from the database. We found the although x.sub.s is the most initially, it has little effect on the decomposition of fibre. By contrast, p.sang.s decomposes most of the wooden fibre by itself.

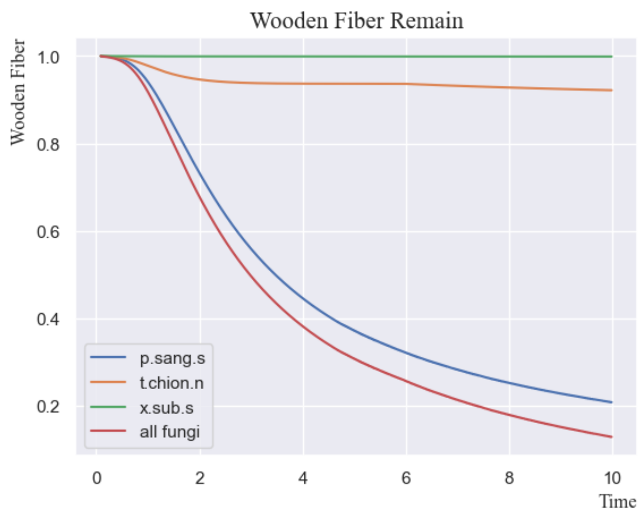

Figure 6: Wooden fiber remain

6 Model II: Fungi trend model under reality

In Model I, we analyze the relationship between decomposition rate and multiple fungi. However, the behaviour and interaction will be different against time due to the limited resource and external environment. In short-term, because there is sufficient nutrient in the environment , there is no obvious interest conflict between groups of fungus. However, as the time goes by, the nutrient will decrease to an extent that it can't support all fungus to survive. In that case, there will be competition among species and reach a balance in the long run. We will use our previous model to solve this problem while consider several elements.

To better illustrate this problem, we choose another five kinds of fungi, which are:

- Armillaria_gallica_FP102531(a.gal1.s)
- Armillaria_gallica_FP102535(a.gal3.s)
- Hyphodontia_crustosa_HHB13392(h.crust.n)
- Lentinus_crinitus_PR2058(l.crin.s)
- Phellinus_hartigii_DMR94(p.har.n)

6.1 Trend of decomposition under real temperature situation

6.1.1 The growth rate under real temperature

Different fungi has different sensitivity to the temperature. The temperature will also affect the growth rate. With the fluctuations of the temperature in reality, it's critical to analyze the actual competition model under real situation. In order to analyze this problem, we choose temperature of London in 2020 as an example.

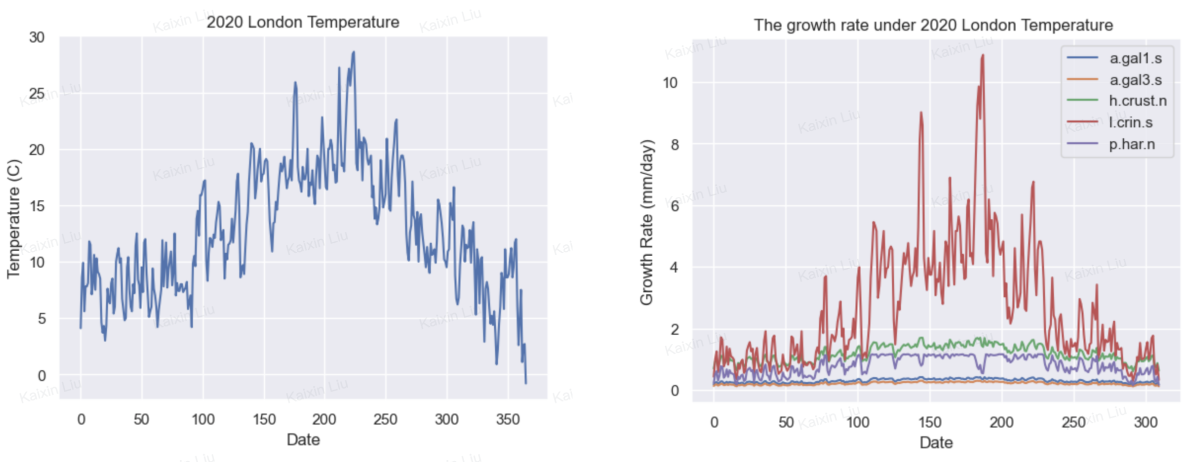

Figure 7: Left: The temperature of London in 2020; Right: The growth rate under 2020 temperature

We firstly extract the temperature data in 2020 which is fluctuate across the time. And then we calculate the growth rate of five fungus under each temperature basing on the data. From the graph below, we found that the curve is volatile for Lentinus_crinitus_PR2058 while others are remain stable. After applying the model we previously build, we try to discover the short-term behaviour of decomposition and long-term trend.

6.2 Short-term behaviour of competition

First, we consider the short-term behaviour of our model. From the graph, we find that Hyphodontia crustosa grows faster than any other four kinds of fungi. Two kinds of Armillaria_gallica grows slowly during the first five days.

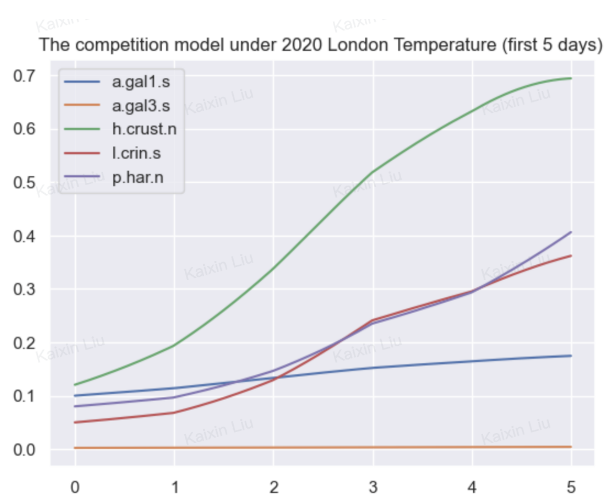

Figure 8: Short-term behaviour of five fungi

6.3 Long-term behaviour of competition

However, the long-term behaviour seems different. After approximate ten days, Phellinus_hartigii almost dominates the environment. And twenty days later, the other four kinds of fungi seems almost disappear while Phellinus_hartigii occupy the environment

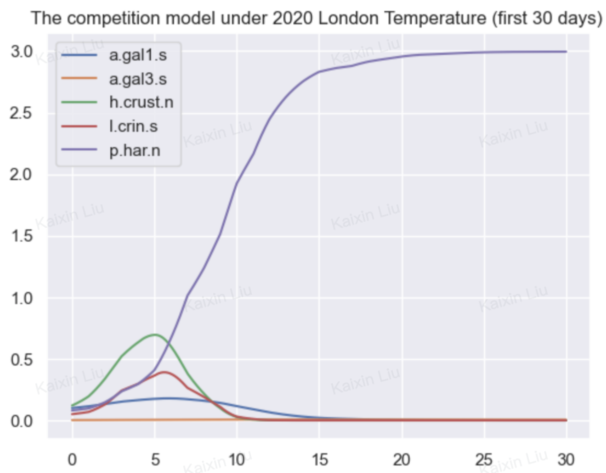

Figure 9: Long-term behaviour of five fungi

7 Model III: Advantage of fungi and environment model

Competition does exist among different kinds of fungus. However, the suitable combination of them may result in different situations. There are variable kinds of fungi which has different method of decaying. For example, white-rot fungi mainly secrete cell oxidases for delignification while brown-rot fungi can degrade cellulose and hemicellulose.

In that section, we will analyze the relative advantages and disadvantages for each species and combinations of species likely to persist. Also we will consider different kinds of environment including arid, semi-arid, temperate, arboreal, and tropical rain forests.

7.1 Introduction of model III

To find out the relevant advantages and disadvantages of fungi in different environment, we need to consider what is the behaviour of different fungi in different environment. In this model, we still choose the same kind of fungi in model II to make sure the consistency of our model variable. Then, we heritage our previous model to predict the relative growth of fungi in different environment.

7.2 Advantage of fungi across temperature

To discover the relevant advantage and disadvantage under each environment, we choose five corresponding cities as our temperature standard.

Table 5: The environment and cities

City name	Environment
Birmingham	tropical rain forests
Brainerd	Arboreal
Washington	Temperate
San Angelo	Semi-arid
Phonix	Arid

After applying our model before, we draw the following graph and find several phenomena:

1.Pycnoporus sanguineus will finally dominate the environment in the long-run

In graph 10, we note that although there is some fluctuation in the first ten days, Pycnoporus sanguineus will dominate the environment in the long term.

2.The peak of fungi density will happen in a later time during the transition of arid to aboreal, while decreasing from aboreal to rain forest environment

From the graph, we find that the peak of fungi density will happen in different time. After examining some article, we found that the reason is the suitable temperature for fungi is 20-30 °C. In that case, the environment which has a suitable temperature interval will boost the fungi to grow for a ling time.

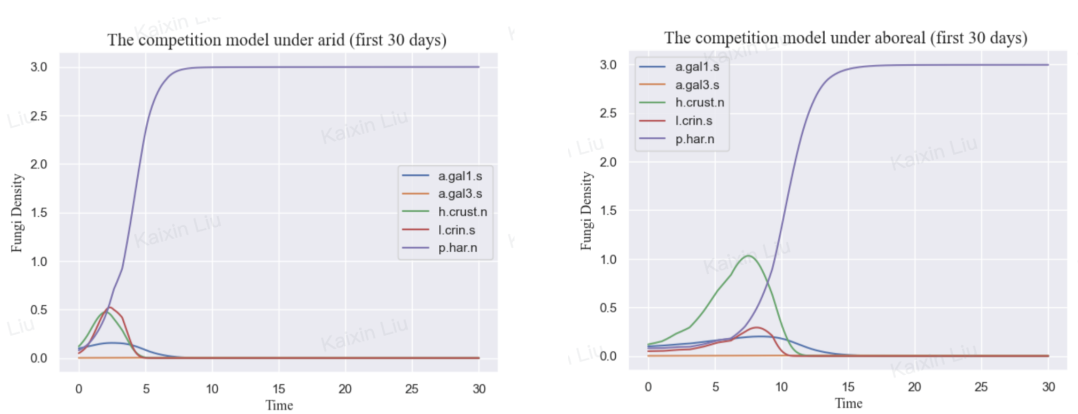

Figure 10: The fungi density under aboreal and arid during first 30 days

3.Lentinus_crinitus will dominate the environment in arid and semi-arid environment within the short-term, while Hyphodontia_crustosa will dominate in other three environments.

In our result, we find that within a short period, the temperature environment will affect the dominance of fungi. Lentinus_crinitus will dominate the environment in arid and semi-arid environment within the short-term, while Hyphodontia_crustosa will dominate in other three environments. It may because that Lentinus_crinitus is more suitable in arid area while Hyphodontia_crustosa is more likely to survive in moist districts.

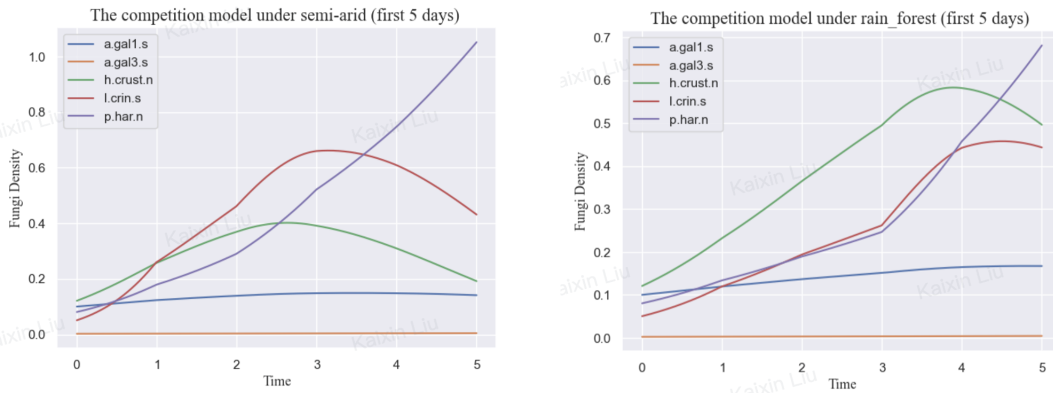

Figure 11: The fungi density under semi-arid and rain forest during first 5 days

7.3 Advantage of fungi across moisture

To discover the relevant advantage and disadvantage under each environment, we choose to use gamma distribution to simulate the moisture environment in five areas and discover the relative relationship. After applying our model before, we draw the following graph and find several phenomena:

1. Pycnoporus sanguineus will finally dominate the environment in the long-run

In graph 12, we note that although there is some fluctuation in the first ten days, Pycnoporus sanguineus will dominate the environment in the long term.

2.The advantage of Pycnoporus sanguineus will happen later when environment become drier.

From the graph, we find that the peak of fungi density will happen in different time. After examining some article, we found that the reason is the suitable temperature for fungi is 20-30 °C. In that case, the environment which has a suitable temperature interval will boost the fungi to grow for a ling time.

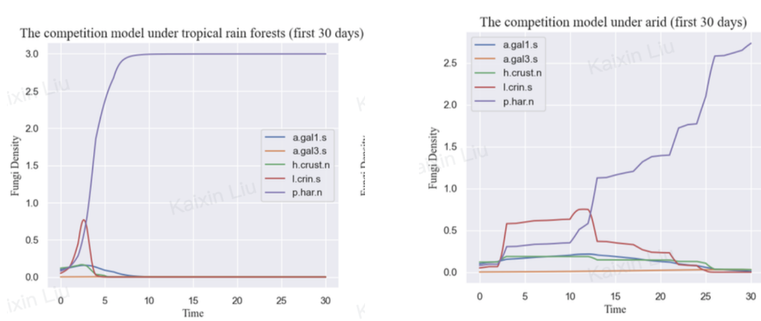

Figure 12: The fungi density under aboreal and arid during first 30 days

8 Model IV: Fungi decomposition efficiency with biodiversity model

Biodiversity is critical in the nature, so as for fungi. From our previous model, we found that decomposition of wood fibre depends on the fungi density and corresponding speed of each fungi.

Also, temperature and moisture does play an important role in the interaction between various species of fungi. In this model, we try to find the decomposition efficiency considering all elements we mention before to examine the importance of biodiversity in nature.

8.1 Introduction of model IV

9 Conclusion

9.1 Result of Problem I & II

In order to discover the decomposition rate when there are various kinds of fungus, we build **Decomposition rate with multiple species of fungi model** to solve this problem. We will first build a dynamic system to calculate fungus density at a specific time. To simplify this problem, we choose three typical fungi to simulate the procession, which are Tyromyces chioneus(T), Pycnoporus sanguineus(P), and Xylobolus subpileatus(X). We will use textbfCompetitive Lotka–Volterra equations to simulate the competitive process and reach the final result. After that, we will take the decomposition capacity of each type of fungi into consideration so that we can get the total decomposition rate within a certain time period.

9.2 Result of Problem III

9.3 Result of Problem IV

We found three phenomenon in Problem IV. First, Pycnoporus sanguineus will finally dominate the environment in the long-run. Second, the peak of fungi density will happen in a later time during the transition of arid to aboreal, while decreasing from aboreal to rain forest environment.Third, Lentinus_crinitus will dominate the environment in arid and semi-arid environment within the short-term, while Hyphodontia_crustosa will dominate in other three environments.

9.4 Result of Problem V

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