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A		2302480

A Simulation-Based Modelling of Plant Surviving Analysis

Summary

In recent years, climate change has led to more frequent and intense extreme weather events worldwide. The irregularity of extreme weather, particularly drought, threatens the growth of plants, changing the behaviors of ecology. However, the biodiversity of plant communities may mitigate the effect and have a positive impact on adaptation and resilience against drought. This paper will analyze and predict the long-term interactions between different species in irregular drought environments. We will also accommodate the suitable actions that could be done in the future.

To begin with, in model I, we establish **Cooperation-Competition model** based on Lotka–Volterra equations to simulate the dynamic systems among different species. The **Multi-Factor Regression Model** is applied to simulate the irregular cycle after generating the weather conditions with sine function with random parameters following normal distributions. The cycle includes the unexpected drought period, which largely determines the growth rate of the plants. After applying the cooperation-competition model, we find that Blue Grama can obtain the highest density in a long-term trend, and all drought-intolerant plants will soon be nudged towards extinction in the irregular weather cycle.

Second, we build a **Benefit Score Function** in model II to quantatively reflect the benefit degree of the co-existence conditions of different plants' combinations. We choose three criteria in a stable system to evaluate the system. Then, we simulate the long-term behaviors of various combinations of plants and use the medium score of one-plant-only living conditions. After calculating the score of all combinations of species, we find that the score will grow with respect to the increase of the number of species co-existing in the planting community. The selection of the plant type matters as well, and we conclude that among all types we choose, Blue Grama plays the most vital role in the sustainability of the overall system, and all drought-intolerant species have similar beneficial effects.

Third, the community's sensitivity to the drought cycle will be analyzed by **Drought Cycle Sensitivity Model**. We first simulate a more volatile weather cycle inherited from Model I and add the traits of life cycle of plants into our interaction model to increase the authenticity. We find the frequency of the drought cycle will not apparently affect the total population when the cycle exceeds 12 months. However, the wider variation will apparently impede the population's speed toward stability. Further, the frequency of drought has nearly no effect on the relationship between the number of species and the overall population, unless the variation exceeds a certain limit.

Fourth, in model IV, we use **External Environment Sensitivity Model** to examine the influence of external environments, such as pollution and habitat reduction on long-term trends. We introduce the **Pollution Rate** into our interaction model to reflect the effect of pollution. It is determined by the concentration of the pollution and the percentage of area polluted, Then we build a **Pollution Diffusion Model** to estimate the effect of pollution and habitat reduction. We found that there exists an effect threshold of pollution concentration of roughly $5\times\frac{S_1}{S_2}$, where S_1 is the habitat area and S_2 is the pollutant discharge area, and the impact of habitat reduction is linear.

Further, we consider feasible actions such as natural reserve construction considering the model we built before to ensure long-term viability and examine the impact on the large environment.

Finally, we analyze the strengths including the species chosen with representatives and progressive model building with precise interpretability. Meanwhile, the rough estimation of the maximum capacity and uniform distribution assumption may be our weaknesses.

Key words: Multi-factors regression, Cooperation-competition model, Analytic hierarchy model

Contents

1 Introduction

1.1 Problem Background

Plants are quite sensitive to the external environment such as temperature and moisture. In the past, the rainfall and temperature are seasonal which provide a regular cycle for these plants to survive. In the wet season, plants will reproduce the next generation and conserve nutrients for the dry season. However, climate change shifts the weather cycle worldwide. Drought happens more frequently which influences the interactions among different species.

Figure 1: Left: Drought area in North America; Right: The competition among species

Moreover, human activities expand rapidly in recent years and occupy more habitats of species, producing more and more pollution which damages the living environment of plants. The damage to the plant community from both environmental conditions and humans may result in the imbalance of the system. In our article, we will probe into the actual result of these factors and predict the future, providing some suggestions.

1.2 Problem Restatement

- Build a model to predict the interaction among various plants when the plant community confronts irregular weather cycles with unexpected droughts when precipitation should be abundant.
- Examine the long-term interactions behaviors and the benefit when there are different numbers of species and various types of plants
- Find the impact of drought frequency and the species numbers on the long-term population of the system
- Discover the influence of external environmental factors such as habitat reduction and pollution on the long-term population of plants

1.3 Our Approach

- We build Cooperation-Competitive model for the dynamic system of the plant community. Then we apply a multi-factor regression model to simulate the irregular cycle in which there are several expected droughts that happen when there should be abundant precipitation.

- The benefit score function is built to quantify the benefit of the species and numbers. We calculate the percentage of types and combination numbers which above the baseline to find the relationship.
- The volatile irregular weather cycle will be simulated by changing the parameters inheriting from Model I to examine the long-term behavior of the whole system.
- We introduce the pollution reduction ratio(RR) into our cooperation-competition model considering the reduction of the habitat environment to examine the long-term trends in various extent situations.

1.4 Data Collection & Visualization

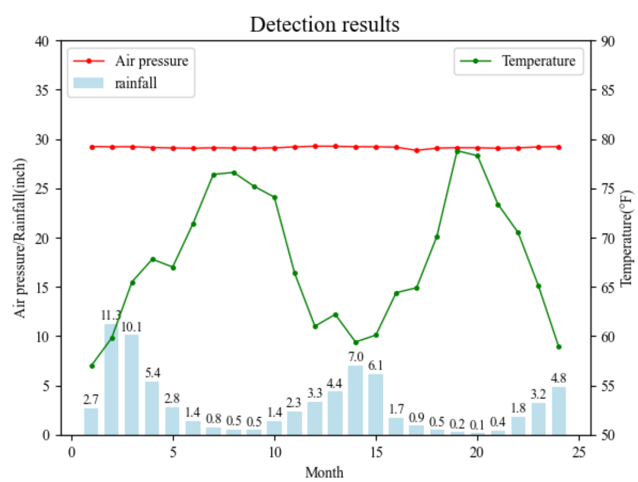
In our model, we collect various kinds of data from different data sources. The climatic data we collected is California's monthly temperature and rainfall statistics. The reason we choose California is that it is a Mediterranean climate city[2] with warm and dry weather in summer while cool and wet in winter. The data source is listed in Table 1:

Table 1: Data source

Data source	Data websites	Data description
CEDA Archive	https://data.ceda.ac.uk	The Volumetric Soil Moisture worldwide(m^3/m^3)
NCEI	https://www.ncei.noaa.gov	The temperature of California in 2017-2018 (°F)
San Bernardino country	http://www.sbcounty.gov	The information of plants
Arbor Day Foundation	https://www.arborday.org	The information of plants

From CEDA Archive, we collected the monthly weather data in Los Angeles, California between 2017-2018. The actual data is shown in the graph below:

(a) The map of Los Angelo



(b) The monthly weather in California in 2017-2018

In this climate condition, there are various kinds of plants with varying degrees of drought tolerance, which are suitable for our model. We choose several plants which live in California with different characteristics: Coastal Sage Scrub, Chaparral, Rhododendron, Blue Grama, Perennial ryegrass, Desert Willow, and Redwood. The general information about these seven different types is listed in table 2 below sorted by name in alphabetical order. The unit of growth rate is $n/year$, the unit of optimal rainfall is $inch/year$, and the unit of the life cycle is $year$.

Table 2: Information of plants

Plant name	Types	Growth rate	Optimal rainfall interval	Life cycle
Blue Grama	Drought-tolerance grass	0.023	8-15	5
Coastal Sage Scrub	Drought-tolerance cactus	0.03	10-20	100
Chaparral	Drought-tolerance bushes	0.1	10-17	10
Desert Willow	Drought-tolerance trees	0.08	12-20	40
Redwood	Drought-intolerance trees	0.01	50-100	2000
Rhododendron	Drought-intolerance bushes	0.105	40-60	5
Perennial ryegrass	Drought-intolerance grass	0.04	32-40	1

2 Assumptions & Notations

2.1 Assumption

Assumption 1: We assume plants are uniformly distributed in the environment discussed.

Justification: This is an assumption without loss of generality. Although this assumption may not always be true in reality, the deviation is somehow meaningless, and this assumption can ease our modeling and calculation processes.

Assumption 2: We assume all of the combinations of plants can co-exist at the initial state.

Justification: Among these combinations, some may not exist in nature. But most of them can coexist actually. Also, it provides us an opportunity to examine why there are not able to exist together in our model

Assumption 3: We assume the habitat area is a circle.

Justification: To simplify our model and calculation process, we will assume the environment is in a circle shape. It can provide us convenience when calculating the diffusion areas. It is also realistic for us to choose a circle environment in reality.

Assumption 4: We assume the diffusion speed is a constant v.

Justification: In reality, the speed can hardly be a constant rate due to other factors such as soil density. However, in a broader view, the speed is approximately constant and will not significantly affect our model.

2.2 Notation

Table 3: Notation of this article

Symbols	Definition	Unit
n	The number of the plant	N/A
N_i	Carrying capacity of i species	n/m^2
x_i	The density of the plant i	n/m^2
x_{steady}	The density of the plant i in stable system	n/m^2
$O_{i,j}$	The net effect of plant j on plant i	N/A
r_i	The growth rate of the plant i	n/(m^2· mon
r_{opt}	The optimal growth rate	n/(m^2· mon
l	Life cycle of plants	year
yT	Temperature	°F
y_{Rf}	Average monthly rainfall	inch/year
R_{opt}	Optimal monthly rainfall	inch/year
M	Moisture	m^3/m^3
M_{opt}	Optimal moisture	m^3/m^3
M_s	Simulated soil moisture	m^3/m^3
G_i	The related growth rate parameter with respect to the deviation of real moisture data	
ω	angular frequency	N/A
POP	Average population of plants alive	n/m^2
$OVER(S)$	Average overflow rate	N/A
$T_{min}(S)$	The earliest time system reaches stable	month
$c(t)$	Density of pollution at time t	$0.1g/(kg \cdot n$
H_r	Habitat reduction ratio	N/A
h	Concentration of pollution in the environment	$0.1g/(kg \cdot n$
P_r	Reduction ratio of pollution	N/A
R_p	The radius of pollution area	m
R_h	The radius of habitat area	m
d	The distance between the center and the pollution center	m
S_{ph}	The intersection area of polluted and habitat	m^2

3 Model I: Plant Community Interaction Model

Regular weather cycle is critical to the growth of plants. For plants that are adapted to certain weather patterns, the growth will follow some regulations. For example, in the wet season, there is suitable moisture for growth and reproduction while the dry season allows them to conserve resources and survive until the next wet season. Moreover, it ensures the species in the plant community interact with each other so that a long-term balance can be sustained. However, due to the influence of irregular weather such as drought and flood, the plants will be affected in varying degrees, which results in the changing of the dynamic system. In this model, we will predict the long-term interaction behavior among various kinds of species under the irregular weather cycle, including the unexpected drought period.

3.1 Introduction of Model I

In this model, we focus on establishing **Cooperation-Competition Model** to simulate the dynamic systems among different species considering the irregular weather cycle[5], which includes temperature, atmosphere pressure, and rainfall. These factors will influence the soil moisture in different weights, which then determine the inherited growth rate of each plant. To find the relationship between weather conditions and soil moisture, We build the **Multi-Factor Regression Model** based on data in California. The temperature and rainfall data will then be generated by the sine function with a period of one year and simulated in the irregular cycles and corresponding interactions result. Find our flow chart for Model I in Figure 2:

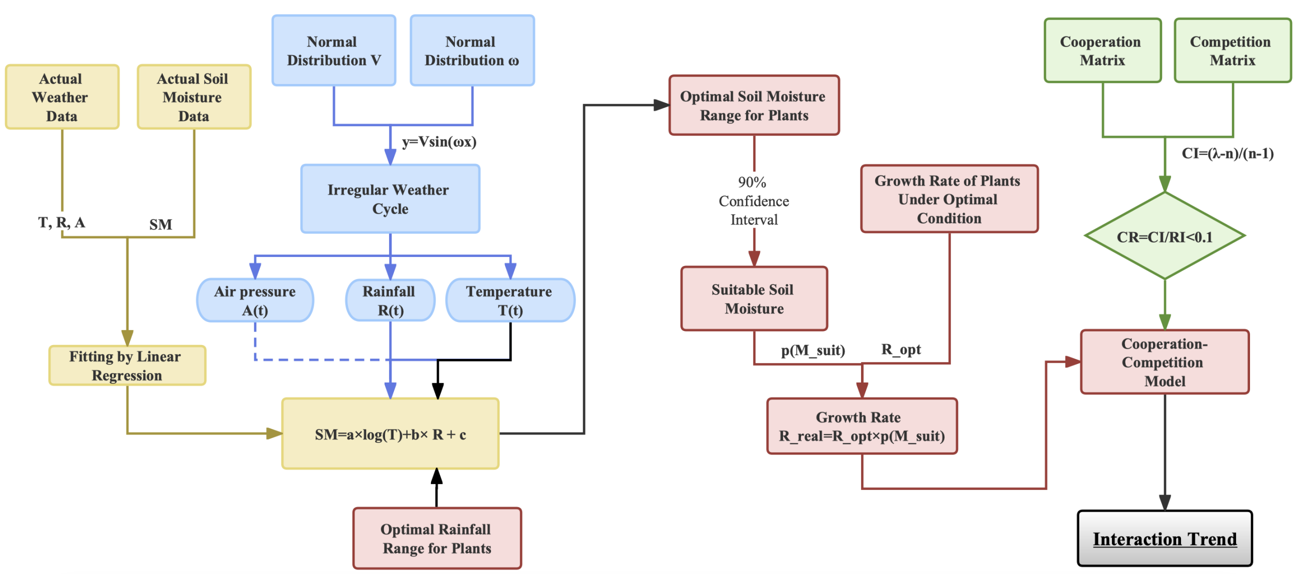


Figure 2: The flow chart of Model I

The subjective selection of cooperation and competition ratios are primarily determined by factors like environment adaptions and growth rates of different species. The consistency of these ratios will be further examined by **The Analytic Hierarchy Process**.[6]

3.2 Irregular Cycle Simulation Model

Plant inherited growth rate is determined by several factors, among which soil moisture is the most important one. Meanwhile, soil moisture is determined by the weather cycle. In order to predict the soil moisture and its corresponding growth rate in irregular weather, we need to first simulate the cycle behaviors.

3.2.1 Multi-Factor Regression Model

Soil moisture is determined by the weather[5], whose traits can be generalized as three factors — temperature, rainfall, and atmospheric pressure. In this section, we will build a multi-factor regression model to discover the connection between these factors mentioned before and soil moisture.

We collect the weather data for California in 2017-2018 from National Centers for Environmental Information(NCEI) website which is shown in Figure 2. We find the monthly atmospheric pressure is relevantly stable in California compared to the other two factors and this feature should not have much impact on soil moisture. The effect of rainfall on soil moisture should be much larger than the effect of temperature. So in our regression model, we will just focus on temperature and rainfall factors. Hence, we will use the log function to mitigate the weight of temperature and convert the monthly temperature y_T into y'_T, which

$$y'_T = ln\ y_T \tag{1}$$

We substitute the temperature and rainfall data into our previous equation, and we get the following regression model:

$$M = a \times y'_T + b \times\ y_{Rf}\ +\ c \tag{2}$$

where y'_T denotes the converted monthly temperature y_T and y_{Rf} denotes the monthly rainfall which we get from data procession, M denotes the soil moisture.

The result of the weather data fitting is shown in Figure 3 below:

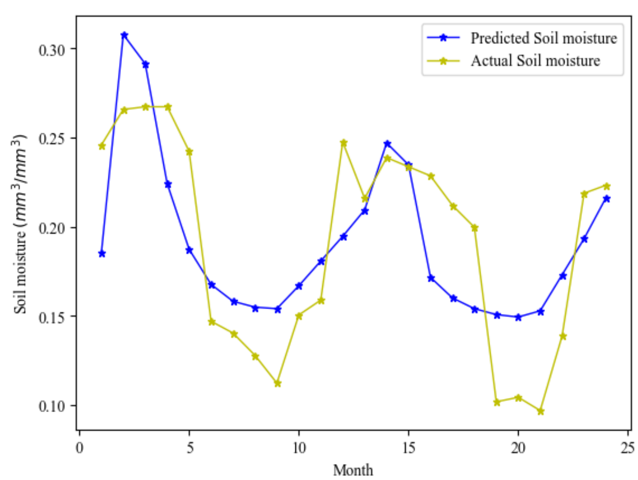


Figure 3: The comparison of the fitting curve and the original curve

3.2.2 From Weather Cycle to Soil Moisture

In a normal year, the regular temperature and rainfall show a regular pattern in California. The rainfall decreases in the first half of the year and then increases through the second half of the year. In contrast, the monthly temperature increases in the first of the year while decreasing in the second half. We draw a graph for the monthly temperature and rainfall data of California in 2017-2018. We discover the cycle from Figure(2) is like a sine function approximately which we can use to simulate the temperature and rainfall.

$$y = A \times sin(\omega x + \theta) + B \tag{3}$$

where A means the peak of the function value, ω denotes the cycle interval, B is the mean of the maximum and minimum data throughout one year. The generation of A and w should follow the normal distribution $\mathcal{N}(\mu, \sigma^2)$ which is defined as:

$$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(M-\mu)^2}{2\sigma^2}}$$

By using the method mentioned above, we are able to generate irregular rainfall and temperature cycle. The simulated temperature cycle graph is shown in Figure 4 below:

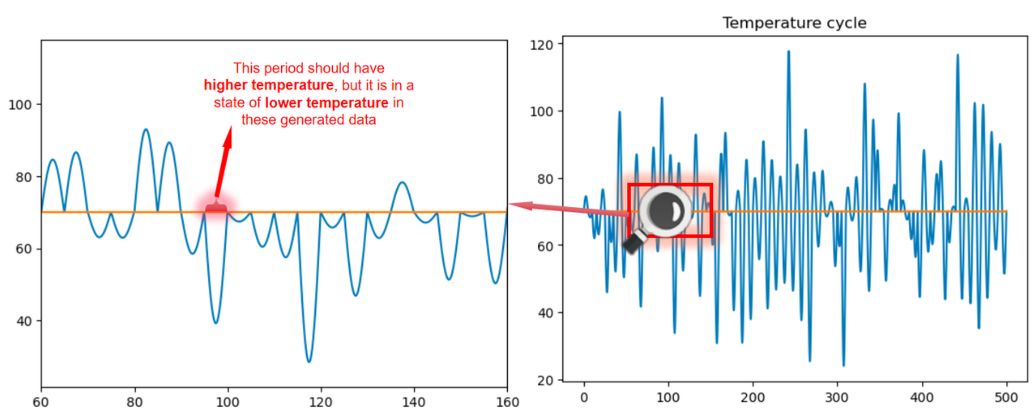


Figure 4: The simulation of the **irregular** temperature cycle.

From the graph, we find that there are several times in this cycle when there should be abundant precipitation, but it is actually drought. The red segment is one of these times.

Combining the fitting we built before, we can simulate the soil moisture in regular years from the temperature cycle and rainfall cycle as:

$$M = a \times log(y_T) + b \times \ y_{Rf} \ + \ c \tag{4}$$

3.3 Cooperation-Competition model

The increasing population of certain species follows the logistic model in isolation areas with limited nature.

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{x_i}{N_i}\right) \tag{5}$$

where N_i denotes the maximum environment capacity and r_i denotes the intrinsic growth rate. We will assume the species in our model also follow this kind of growth.

However, in nature, there are interactions among various kinds of species, which maintain a balance of ecosystems in the long-term period. We consider the interaction as competitive and cooperation where competitive means the content for some common limited resource among species while cooperation means the mutual growth beneficial. We will denote the competitive ratio as p while the cooperation ratio will be denoted as n. So equation (2) can be expressed as:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{x_i}{N_i}\right) + p_{i,j} x_i x_j \ - \ n_{i,j} x_i x_j \tag{6}$$

In this equation,

- x_i, denotes the density of the plant i at time t
- N_i denotes the carrying capacity of i species which is a pre-assumed constant
- r_i denotes the growth rate of plant i, which will be affected by soil moisture
- p_i represents the positive effect of species j has on the population of species i
- n_i represents the negative effect of species j has on the population of species i

To simplify the equation, we define the combination of these two factors p and n as the net cooperation ratio which we denote as O, where

$$O_{ij} = p_{ij} - n_{ij}, \ \forall i, j \in [1, m] \tag{7}$$

m is the number of plants in the plant community. Then we can get the Cooperation-Competition model which is shown below:

$$\frac{dx_i}{dt} = r_i x_i (1 - \frac{x_i}{N_i} + \sum_{j=1}^{m} \frac{O_{i,j}}{N_j} x_j) \tag{8}$$

where O_{ij} denotes the net effect species j has on the population of species i considering the cooperation and competition effect.

In order to apply this Cooperation-Competition model, we still need several parameters. While the environment capacity N_i is a constant for each species i, we still need to examine the net effect ratio O_{ij} and the inherited growth rate r_i. We build relevant models to estimate these parameters in the following sections and finally reach our conclusion.

3.3.1 Inherited Growth Rate and Moisture

The growth rate of certain species depends on several factors, among which soil moisture plays a vital role. During the irregular weather cycle, the growth of plants will be affected by varying degrees. In this subsection, we will first probe into the relationship between moisture and growth rate. Then, we will apply our irregular cycle simulation model before to get the inherited growth rate of each plant in an irregular weather cycle.

First, We will use our regression model built before to calculate the simulated soil moisture:

$$M_s = a \times ln\ y_T + b \times\ y_{Rf}\ +\ c \tag{9}$$

After getting the simulated soil moisture, we need to know the concrete relation between the growth rate and moisture. It is apparent that every plant should grow the fastest when the soil moisture lies in its own optimal range. Further, the growth rate should vary with the change of real soil moisture when the real moisture leaves the optimal range, and finally reach 0 when the real moisture deviates from the preference too much. Here our model indicates that the relationship between growth rate and soil moisture should follow a normal distribution $\mathcal{N}(\mu, \sigma^2)$. The general formula is shown below:

$$G_i = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(M-\mu)^2}{2\sigma^2}}, & \text{if } M \in [a_i, b_i] \\ 0 & \text{if } M \notin\ [a_i, b_i] \end{cases} \tag{10}$$

$$p(G_i) = \frac{G_i - min(G)}{max(G) - min(G)} \tag{11}$$

where G_i means the related growth rate parameter with respect to the deviation of real moisture data from the plant's preference, and $p(G_i)$ denotes the percentage of maintenance of actual growth rate, obtained as the normalization of the related growth rate parameter. The parameters μ can be defined as the middle point of the optimal soil moisture range. σ is manually determined. Note that the oσ is generally closely related to the lifetime, as the species with faster metabolism speed should usually be more sensitive to external change. After fixing the normal distribution, we are required to determine the valid moisture interval. As suggested by Literature [1], the proper confidential interval to represent the range where most data lie in should be among $90\% - 95\%$. The border of this interval should be interpreted as the least tolerated soil moisture and the most tolerated soil moisture. Hence we can deduce the general features of seven plants. The comparison is shown in Figure 5 below.

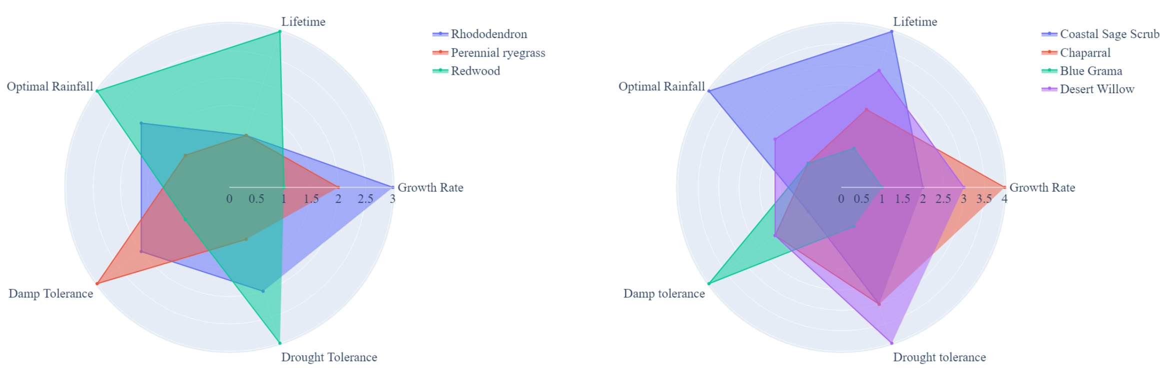


Figure 5: The characteristics of seven plants

Finally, we can calculate the real growth rate under specific soil moisture. The function can be expressed as:

$$r = r_{opt} \times p(G_i) \tag{12}$$

where r_{opt} is the inherited growth rate under the soil moisture of M_i

3.3.2 Cooperation-Competition Ratio

In order to get the cooperation ratio p_{ij} and competition ratio n_{ij}, we will use the analytic hierarchy process. To apply the analytic hierarchy model, we need to first build the judgment matrix for the cooperation model and competition to denote the relevant importance between element i and element j.

$$\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mm} \end{pmatrix} \tag{13}$$

where p_{ij} denotes the ratio of mean suitable rainfall of i to that of j.

For example, from the data we collect, we find the mean suitable of Costal Sage Scrub is 15 inches and that of Chaparral is 13.5. Then p_{12} is calculated as:

$$p_{12} = \frac{15}{13.5} \tag{14}$$

We collect the optimal rainfall interval for our seven plants and calculate the mean of the interval.

By the same method, we get the judgment matrix for the competition ratio:

$$\begin{pmatrix} n_{11} & n_{12} & \cdots & n_{1m} \\ n_{21} & n_{22} & \cdots & n_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ n_{m1} & n_{m2} & \cdots & n_{mm} \end{pmatrix} \tag{15}$$

After building two judgment matrices, we should examine the consistency of a paired comparison matrix. The consistency denoted by CI can be expressed as:

$$CI = \frac{\lambda_{\max}(A) - n}{n - 1} \tag{16}$$

where $\lambda_{\max}(A)$ represents the largest eigenvector of matrix A and n is the dimension of the matrix. Then we calculate the consistency ratio CR which is:

$$CR = \frac{CI}{RI} \tag{17}$$

where RI is the Random consistency Index which will only be affected by the dimensions of the matrix. The CR we get here is less than 0.1, which means it has good consistency.

3.4 Conclusion

We calculate the soil moisture in each period and the corresponding growth rate of seven plants. We substitute all data into our cooperation-competition model and get the following result. We choose the initial density of all species to be $10/m^2$. The interaction result is shown in Figure 6 below.

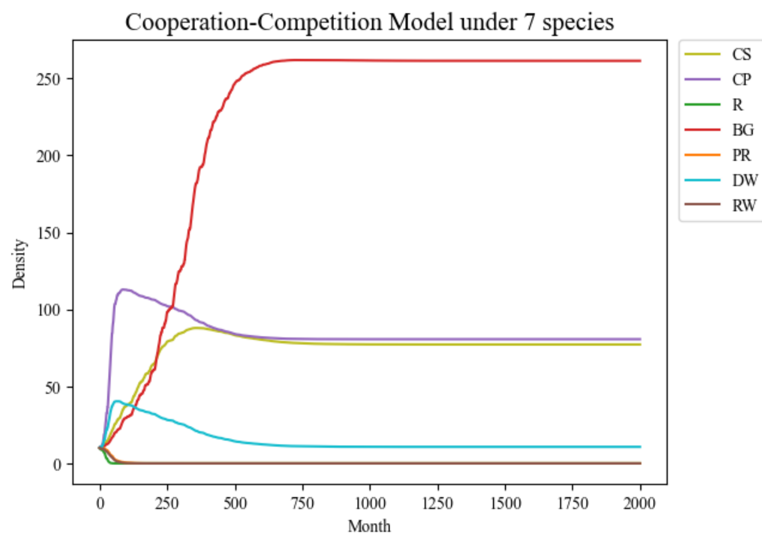


Figure 6: The long-term trend of seven plants in irregular weather cycles

In this graph, CS represents Coastal Sage Scrub, CP represents Chaparral, R represents Rhododendron, BG represents Blue Grama, PR represents Perennial ryegrass, DW represents Desert Willow, and RW represents Redwood.

From this result, we can generate several conclusions:

- Blue Grama shows a strong growth rate and dominates the environment after approximately thirty years
- The other three drought-tolerant species can survive in stable at a certain rate, while all four drought-intolerant species will go extinct in a very short time.
- Blue Grama stabilizes at the density over the pre-assumed maximum environmental capacity. This is a reasonable result since we introduce the cooperation factor in our model, hence Blue Grama should "benefit" from the plant community and be promoted by other species.

4 Model II: Biodiversity Beneficial Model

Biodiversity is critical to the ecosystem. It can not only create a plentiful nature on the earth but also play an important role in the sustainability of the system. In nature, each specie of plant has its own function while the types of the plant will also affect the interaction among plants, especially in extreme or irregular weather.

In this model, we will analyze the benefits of biodiversity in drought areas in two dimensions. First, we will discover the influence of the increasing **number** of plant species in the environment. Next, we will examine whether the **types** of species can impact the result. Below is the flow chart for this model:

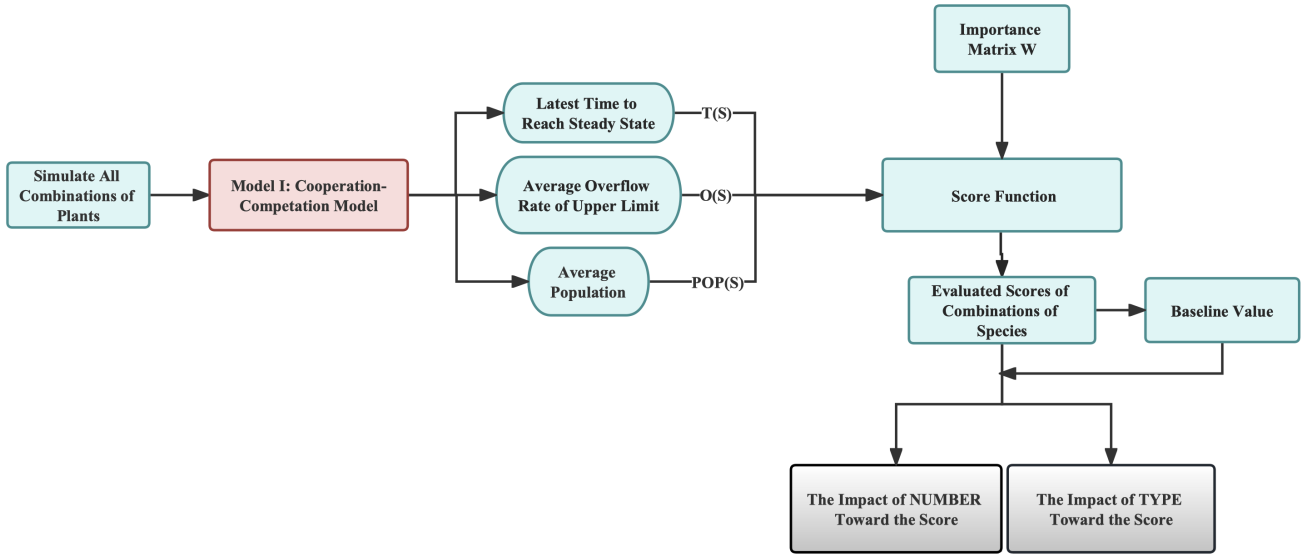


Figure 7: The flow chart of model II

4.1 Introduction of Model II

We divide our seven plants into two categories: Drought tolerant plants whose optimal rainfall is under 20 inches per year and drought-intolerant plants whose optimal rainfall is higher than 30 inches per year. In order to explicitly quantify the benefits, we will use three criteria to build our benefit score function:

- The average population of the plants which still alive in the stable system is denoted as POP. The larger the average population the system has, the larger the benefit of bio-diversity. The reason is that in a system in which exists cooperation and competition, the average population suggests cooperation extent among various species. If there is a higher average population in a stable environment, plants can benefit each other and build a better environment.

- The average overflow ratio of each plant's density in a stable system to the maximum carry capacity for this plant. The overflow ratio for each plant is expressed below:

$$OVER(S) = \begin{cases} 0, & x_{\text{steady}} < N \\ \frac{x_{\text{steady}}}{N} - 1, & x_{\text{steady}} > N \end{cases} \tag{18}$$

  where x_{steady} means the density of one plant in a stable system, N means the maximum environment capacity.

  The larger the average ratio, the higher benefits for the community. The reason is that a larger ratio means the cooperation among various species is helping plants to live exceed the environmental limit. Most species can benefit from this plant system.

- Finally, the earliest time when the system reaches stability which is denoted as $T_{min}(S)$, where S is a particular combination of species in a plant community. The earlier the system reaches, the higher benefits for the community. The reason is that the earliest stability time reveals the capacity of adaptation of the environment. If the system can become stable in a short time, the input species will adapt to the environment quicker and better.

The earliest stable time is defined as the time when the changing rate of every plant in the environment is less than 0.01, compared with the previous year.

4.2 Benefit Score Model

In order to explicitly express the capacity of adaptation to the environment by the three criteria stated in Section 4.1, we will introduce an overall score model as the weighted sum of the three criteria. We will still apply the analytic hierarchy process to get the overall score model which is expressed below:

$$Score = [w_1, w_2, w_3] \times \begin{bmatrix} \mathrm{POP}(S) \\ \mathrm{OVER}(S) \\ \mathrm{T}_{\mathrm{min}}(S) \end{bmatrix}, \tag{19}$$

$$S \in P(\{CS, CP, R, BG, PR, DW, RW\})$$

Here S represents a certain combination in the power set of P, $POP(S)$ denotes the average population, $OVER(S)$ denotes the average overflow ratio of each plant's density in a stable system to the maximum carry capacity, T_{min} denotes the time period when the system reaches stable. CS, CP, R, BG, PR, DW, RW are the seven chosen plants. W is the judgment matrix.

Judgment Matrix

The judgment matrix is the matrix that denotes the relative weight of different factors. In our model, we have three weights for each element: Extinction time period(w_1), Average density ratio(w_2), and Stable time period(w_3). The matrix we build is as below

$$W = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \frac{w_1}{w_3} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \frac{w_2}{w_3} \\ \frac{w_3}{w_1} & \frac{w_3}{w_2} & \frac{w_3}{w_3} \end{pmatrix} \tag{20}$$

Since the ratios of weights are transitive, the analytic hierarchy process result $CI = 0$, which means the judgment- matrix is entirely consistent.

Average Ratio

The average ratio is defined as the ratio of each plant's density in a stable system to the maximum carry capacity for this plant which is expressed below:

$$OVER(x_i) = \begin{cases} 0, & x_{\mathrm{steady}} < N \\ \frac{x_{\mathrm{steady}}}{N} - 1, & x_{\mathrm{steady}} > N \end{cases} \tag{21}$$

which x_{steady} denotes the density of plant i when the system becomes stable.

Combination Simulation In order to get the data of these three criteria, we will apply the Monte Carlo simulation for each combination. The overall set of the combination includes seven plants which are Coastal Sage Scrub, Chaparral, Rhododendron, Blue Grama, Perennial ryegrass, Desert Willow, Redwood, which are the representative of each type of plant. We will use Monte Carlo simulation to test each combination of the community

$$C_7^i,\ i \in \{1, 2, \ldots, 7\} \tag{22}$$

There will be 127 combinations with different numbers of species and we will examine types and examine the corresponding benefits. We first examine the benefit of different numbers of species, then the influence of the types of plants.

We will use a medium score number in the situation where there is only one plant living in the environment as our baseline to lead the result.

4.3 Simulation result

4.3.1 The Benefit Score of Different Numbers of Species

Among our 127 combinations, we divide them into six categories which have different scales. We calculate the score in each category and compare it with the baseline. Since the result is highly entangled with the type selection, the impact of number in this process is not linear. Therefore, we calculate the percentage of combinations with the score higher than the baseline in each group and get the table below:

The percentage of combinations above the baseline

Number of plants in combination	Numbers of combination	Percentage of combinations above the baseline
2	21	0.6190
3	28	0.6286
4	28	0.6857
5	21	0 .7619
6	7	0.8571
7	1	1

From the table, we can draw two conclusions:

- If the standard benefit result is the benefit base, then the plant community begin to have beneficial effects with the number of species = 2
- When the number of species grows, the benefit effect is enlarged simultaneously. The relationship between the number of species and overall benefit is consistent

4.3.2 The Benefit of Different Types of Species

To examine the benefit of types of species, we calculate the number of times each species appears in the combination above the baseline. The following graphs simulate the comparison between the the number of times each species appears in the combination. All the seven species is selected randomly to the circle at random position, and the each species is represented by a specific type of circle, where the radius of cycle represents the occurrence of a particular species in the plant communities that go beyond the beneficial base.

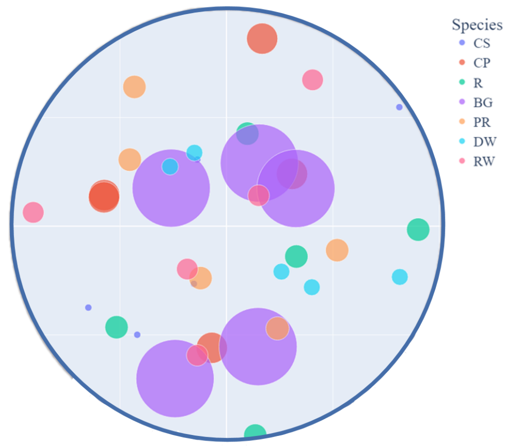


Figure 8: The number of times each species appear above the baseline

We find among all types we choose, Blue Grama plays a vital role in the sustainability of the overall system. The ranking of the role of drought-tolerant plants, from high to low, are Blue Grama, Chaparral, Desert Willow, Coastal Sage Scrub, while the role of drought-intolerant plants are nearly the same.

5 Model III: Drought Cycle Sensitivity Model

In this model, we focus on predicting the long-term trends of the dynamic systems if the weather irregular cycle fluctuates with different variances. We will first simulate the volatile weather cycle inheriting from the irregular cycle simulation model. Then, we will consider the different extent effects of the drought cycle on plants with different life cycles. See our flow chart for this model in Figure 9 below:

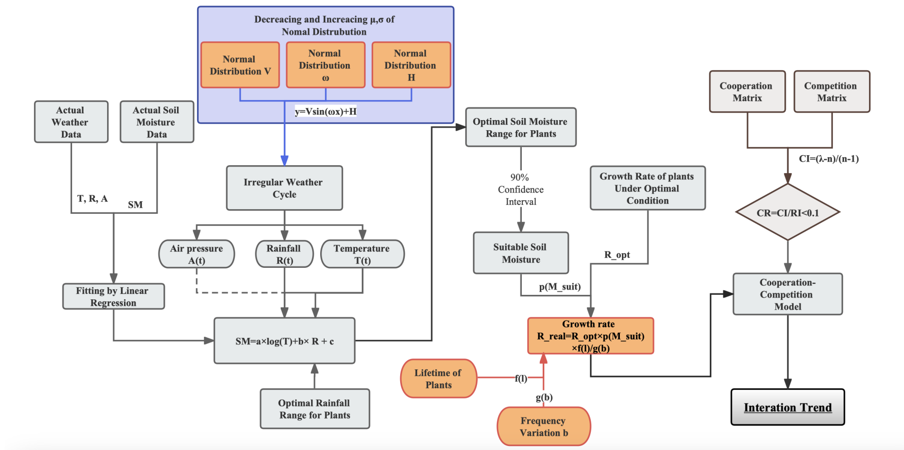


Figure 9: The number of times each species appear above baseline

In this model, the frequency can be expressed as the different value of ω in model II. The high variance can be denoted by the rainfall interval. We will change the coefficient every half of the cycle. Hence the weather cycle generation function can be expressed as:

$y = V sin(\omega x) + B$

Here the bias value B is added to the function mainly to simulate the effect of less frequency of occurrence of drought — when the sin function curve moving towards the moist side, the frequency and intensity of droughts will drop. Hence, we can probe the effect of less frequency of occurrence of drought, by increasing the value of B.

Further, to expose the impact of frequency more explicitly and accurately, we introduce the feature of plants' lifetime into our model, as the lifetime of a specific plant is closely related to its tolerance capability of the frequent weather variations.

5.1 Drought Cycle and Growth Rate

Plants' sensitivity to the irregular drought cycle is varying across different types of species. There are several factors that result in this difference, among which the lifetime of the plant plays a vital role. For the plant which has a longer life cycle, it can survive better than the plant whose life is shorter than one year, which can be shown in the growth rate.

For this reason, we will build a drought cycle and growth convert function which is expressed below:

$$r' = r \times \frac{ln(l + e - 1)}{1 + (\frac{2\pi}{\omega_t} - \frac{2\pi}{12})^2} \tag{23}$$

where l denotes the life cycle of plant, ω_t denotes the cycle of drought happen, r is the growth rate we simulated in Model I.

Element Analysis

- In this model, we use $ln(l + e - 1)$ to represent the life cycle's effect on the total growth rate. The $ln(x)$ function is used to minimize the impact of large lifetime. The addition operation of the constant $e - 1$ ensures that the numerator is greater than 1.

- We will use the square of the difference of two periods of half cycles to indicate the variation of weather cycle frequency. The use of square assumes that the smaller or the larger frequency of the weather cycle should have similar effect.

5.2 Conclusion

We first choose different weather peaks (V) to discover what will happen when drought happens in greater frequency. The result is shown below:

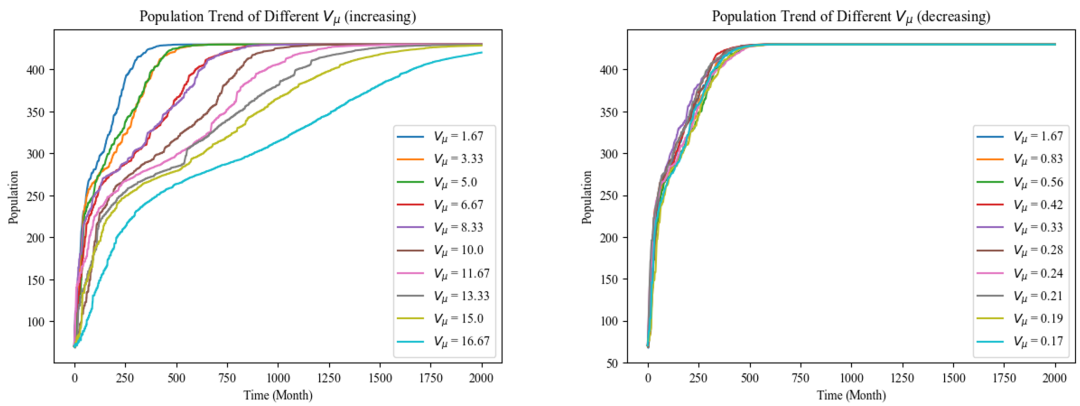


Figure 10: The long-term trend of seven plants in weather cycles with irregular peaks

We find from the graph that the variation of the peak of drought cycle will not affect the total population when the peak become smaller than before, but when the peak value goes up, the adaption capability of plant communities to the environment goes down, making the stabilization process slower. The left figure in Figure 11 reveal the concrete changing trend of the stable time, revealing a growing trend of shortest stable time when the mean value of the peak value of the circle increases.

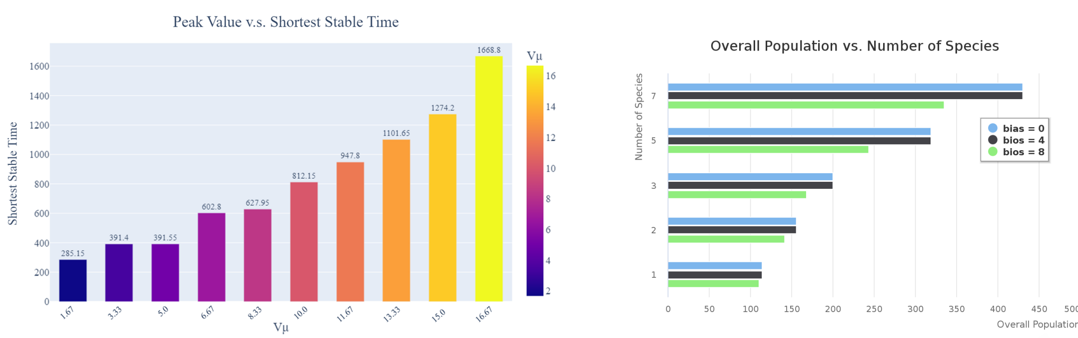


Figure 11: Left: Long-term trend of seven plants in volatile irregular weather cycles; Right: The long-term trend with different numbers of species in less frequent drought situation

The right figure in Figure 11 shows how the bias value impact the relationship of the number of species in the plant community and overall population. The graph shows that the impact is **negligible** when the bias is 0 and 4, but when the adding goes to 8, the overall population has an apparent reduction. This reduction can be attributed to the role of over biasing. When the curve is moving too much, there will be nearly no drought conditions, and it contradicts the original assumption (irregular weather cycle) and all the previous results can no longer held. In other words, when the frequency of drought is reduced to a certain degree (nearly no occurrence of drought for example), the overall population may decrease.

6 Model IV: External Environment Sensitivity Model

The external environment will significantly influence the long-term interaction among different species. Pollution such as heavy metals in water will damage the inherited growth rate of plants and influence the cooperation among various species. Moreover, human activity occupies more and more space that originally belong to the plant community which restrains the growth of species. We will examine three external environment factors' effect on the interaction in the long-term trend in this model — pollution concentration, the distance between the environment center and pollution discharge center, the habitat reduction rate.

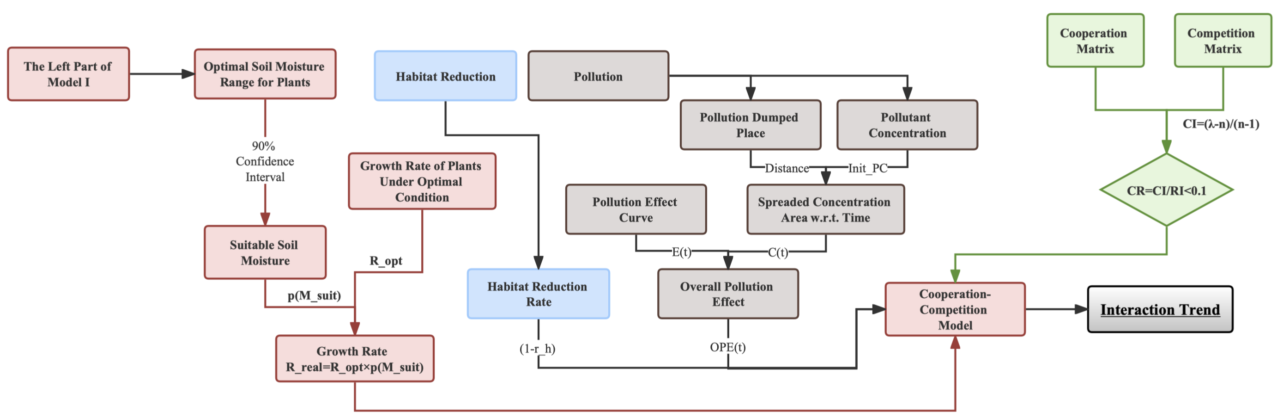


Figure 12: The flow chart of Model IV

6.1 The Effect of Habitat Reduction

Human activity such as cutting down trees will reduce the habitat area of the plant community. The reduction of living space will significantly affect the maximum environment-carrying capacity of each plant by a certain degree. We define this habitat reduction ratio as H_r. So the actual habitat area S_a for the plant community can be expressed as:

$$S_a = (1 - H_r) \times S_o \tag{24}$$

where S_o is the original habitat area.

6.2 The Effect of Pollution Diffusion

In this model, we examine the effect of pollution on plants. We define the negative effect of pollution as the reduction ratio denoted as P_r, which is determined by the pollution ability and percentage of pollution area among total actual habitat area. We will apply the reduction ratio to scrutinize the long-term interaction among species.

6.2.1 Pollution Ability

Pollution ability is determined by the concentration, which we denote as h (in 0.1mg/kg). After referring to relative articles[3], we find the extent of harm follows the 'S' curve when the concentration increases from 0. We define the extent of harm as E which can be expressed as:

$$E(h) = \frac{1}{1 + e^{-(h-M_\beta)}} \tag{25}$$

which M_β is the mean of pollution interval and h is the concentration of the pollutants.

6.2.2 Pollution Diffusion

To simplify our model, we assume the total amount of pollutants will not evaporate and remain constant. We will assume that both the habitat area and the pollution area are circular with radios of R_h and R_p. We assume the pollution source is in the environment.

As pollutants diffusing, the schematic diagram of changes in the pollution area and habitat is shown below:

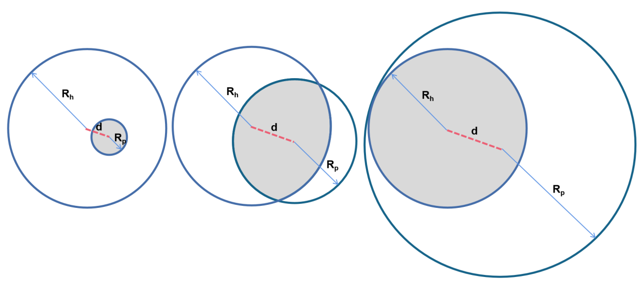


Figure 13: Three Conditions of Diffusion states

The area of the habitat contaminated by pollutants can be expressed as:

$$S_{ph} = \begin{cases} \pi R_p^2, & R_p \le R_h - d \\ R_p^2 \cdot arccos(\frac{d^2+R_p^2-R_h^2}{2\cdot d\cdot R_p}) + R_h^2 \cdot arccos(\frac{d^2+R_h^2-R_p^2}{2\cdot d\cdot R_h}) - \frac{1}{2}\sqrt{\Pi}, & R_h - d < R_p \le R_h + d \\ \pi R_h^2, & R_p > R_h + d \end{cases} \tag{26}$$

in which:

$$\Pi = (-d + R_p + R_h) \cdot (d + R_p - R_h) \cdot (d - R_p + R_h) \cdot (d + R_p + R_h) \tag{27}$$

The percentage of pollution areas(P_p) can be expressed as:

$$P_p = \frac{S_{ph}}{\pi R^2} \tag{28}$$

Finally, it will pollute all areas in the environment.

6.2.3 Reduction Ratio of Pollution

In this model, we will assume the pollution is distributed uniformly among the polluted area. In that case, the reduction ratio for one unit of area can be defined as:

$$P_r = E(h(t)) \times P_p \tag{29}$$

The reduction ratio will affect both the cooperation ratio and the inherited growth rate which we will examine in the next section.

6.3 Competitive Model in Polluted Environment

In a polluted environment, the growth of plants will be affected by varying aspects, both in growth rate(r) and cooperation ratio(p). After considering the pollution factor in our previous Competition-Cooperation model, we get the dynamic system in the polluted environment:

$$\frac{dx_i}{dt} = r_i x_i (1 - P_r)\left(1 - \frac{x_1}{N_1} + \sum_{j=1}^{n}\left(\frac{p_{i,j}}{N_j} x_j (1 - P_r) - \frac{n_{i,j}}{N_j} x_j\right)\right) \tag{30}$$

6.4 Model Result

We choose different pollution concentrations from 1000(0.1g/$kg \cdot m^2$) to 100000(0.1g/$kg \cdot m^2$) to examine the long-term population change when the habitat reduction rate is 0.2. See our result in figure 14 below:

Figure 14: The external environment's influence on the population

We find that when the initial pollution is within 10000, the plant can still grow and reach sustainability. However, when pollution exceeds 30000, the population will be affected significantly. When it reaches 50000, the plants will decrease to an extremely small number. Actually the result can be generalized. Assume the initial pollution area is S_1, the total area is S_2, the pollution with concentration over $3 \times \frac{S_1}{S_2}$ $(0.1g/kg \cdot m^2)$ will have a respectable effect on overall population, while the pollution with concentration over $5 \times \frac{S_1}{S_2}(0.1g/kg \cdot m^2)$ will almost kill every plant in the communities within several years. As for pollution distance, this factor only has a small effect on overall population, compared with the effect of concentration.

The impact of habitat reduction on overall population is linear. Following the statement in our assumption 1, the rate of reduction on habitat corresponds to the one on maximum holding capacity of each plants. The overall population will decrease in proportional to the habitat.

7 Actions for Long-Term Viability and Corresponding Impact

From model II, we discover the importance of biodiversity from the aspects of numbers and types of species. When the number of species in the environment reaches more than two, the sustainability of the environment will greatly increase. Moreover, the types of plants are also critical in the environment. Blue Grama has the best ability to maintain long-term stability and increase the overall population of the plant community. In model III, we find that the variation of rainfall only has a slight impact on the whole system if the irregular weather cycle is maintained. When the drought happens less frequently, the increasing number of species will also benefit the overall system. Pollution and habitat reduction are also vital factors for long-term viability. If there exists pollution over a particular concentration or a huge reduction in plant habitat, the overall population in plant communities will be greatly reduced, even going to extinctions.

According to our discovery, we put forward the following recommended actions:

- In irregular weather cycles, if there exists **drought-tolerant species** in the plant communities, do not introduce too much **drought-intolerant species**. Model 1 indicates that all drought-intolerant species co-existed with drought-tolerant species will all go into extinctions and the larger environment will remain almost the same as the one before introducing drought-intolerant species, after a period of time.

- Maintain the biodiversity in plant communities as much as possible. From model 2, we can conclude that the plant community will benefit more with the number of species increases, and this trend will hold for all numbers examined.

- Monitor the atmosphere condition and the variance of rainfall, ensuring the occurrence of weather cycles. Otherwise, from Model III, the overall population in plant community might be reduced, hence the benefit score of the community will decrease as well.

- Restrict the pollution under a specific value. In model IV, the pollution will have nearly no impact on the larger environment if the concentration is lower than $3 \times \frac{S_1}{S_2}$ $(0.1g/kg \cdot m^2)$, but will draw all plants to death with the concentration higher than $5 \times \frac{S_1}{S_2}$ $(0.1g/kg \cdot m^2)$. Hence, if the pollution can't be avoided, then confine the pollution concentration and radius at discharge.

- Do more detailed and comprehensive pollution supervision. As suggested in model IV, the pollution distance has little impact on the overall population trend. Hence whether the pollution with the same concentration is discharged in the center of environment and at the border of environment, the final result will be nearly the same.

- Try stop the habitat reduction actions like assarting land. The impact of habitat reduction has much more linear effect on plant community. Reduce the habitat is equal to reduce the benefit score of the plant community.

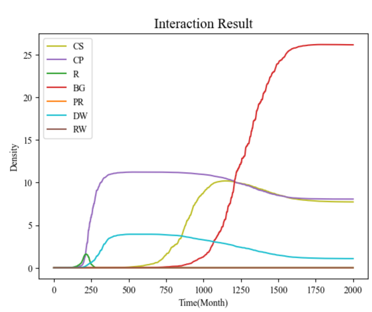
After implementing the necessary strategy, the larger environment
the larger environment will also benefit such as preventing land degradation.

8 Robustness Analysis

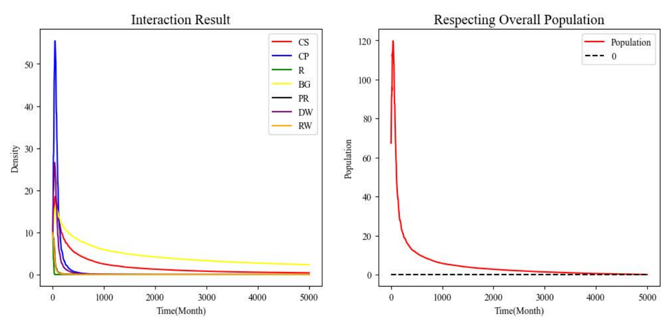
In this section, we will analyze the robustness of our model.

First, we change the initial density data from 10 to 10^{-4}. We find although the growth rate for each plant is really slow in the initial time, the system will finally reach a stable state after approximately 1500 months which shows the robustness of the model.

Then we analyze the robustness of our external environment sensitivity model. When we choose an extremely large pollution density in our model, we find all plants will extinct after 5000 months. It is realistic because when the pollution reaches a certain threshold, the environment will not be suitable for plants to survive.



(a) The long-term trend when the initial density is in $10^{-4}/m^2$



(b) The long-term trend when the density of pollution is $10^{10} * 0.1g/kg(m^2)$

9 Strengths and Weaknesses

9.1 Strength

- **Representative Selection of Data Samples**. In our model, we select 7 types of plants as our data sample, comprised of 4 drought tolerant plants and 3 drought-intolerance plants. Further, these plants can be categorized into 4 types, which include a potential part of plant species in reality, so the result is representative to lead to more general plant interaction rules.

- **Progressive Model**. In each small problem, we are trying to put some additional traits and factors into our model, pushing the model's capability forward concerning the complicity of problems. Hence, our model is readable, understandable, and reasonable. In other words, we try to adjust our model to fit the individual need of different problems.

- **Precise Interpretability**. The result of our model reveals high reliability since it follows our common sense and ecological knowledge. Moreover, some of our conclusions can also be evidenced by some academic papers related to the ecologic field.

9.2 Weakness

- **Rough Estimation of the Maximum Capacity**. In our cooperation-competition model, we assign the ratio maximum capacity of four types (cacti, bushes, grasses, trees) to be 1:1:2:0.5, since the real ratio of densities is nearly impossible to be measured and calculated.

- **Uniform Distribution of Contaminant**. In Model IV, we consider the distribution of contaminants in the polluted cycle to be uniform, but in reality, there should be differences in concentration between the center and the border. We neglect this difference for easier and smoother calculation and simulation processes.

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