Problem Chosen	2023 MCM/ICM Summary Sheet	Team Control Number
A		2302480

Small fungi plays a big role in carbon cycle

summary

Carbon cycle draws more people's attention recent years due to the climate change and greenhouse effect. One of the major component in this cycle is the decomposition of plant material and woody fibers. Among several factors, fungi plays a key role in decomposition process. In order to find out what determines the decomposition speed of fungi and what is the interaction among different kinds of fungi, we establish several models to examine.

In order to discover the decomposition rate when there is various kinds of fungus, we build **Decomposition rate with multiple species of fungi model** to solve this problem. We will firstly build a dynamic system to calculate the number of fungus at certain time. To simplify this problem, we choose three typical fungus to simulate the procession, which are Tyromyces chioneus(T), Pycnoporus sanguineus(P) and Xylobolus subpileatus(X). We will use textbfCompetitive Lotka–Volterra equations to reach the final result. After that, we will consider the decomposition capacity of one certain type of fungi and multiply it to the density of this type of fungus to get the total decomposition rate at certain time period.

To find out the relationship between decomposition rate and the key two factor(the growth rate of fungus and the range of fungus' tolerance to moisture), We found out that there is a positive relationship between the growth rate and the decomposition. By contrast, a negative correlation was found between the range of fungus's tolerance to moisture. After incorporating the interactions between different types of fungi, we found out XXXX.

The interaction between various kinds of fungus will cause different results. In short-term, because there is sufficient nutrient in the environment , there is no obvious interest conflict between groups of fungus. However, as the time goes by, the nutrient will decrease to an extent that it can't support all fungus to survive. In this case, we will use **XXXXX model** to solve this problem. We will also examine its sensitivity to rapid fluctuations in the environment and its overall impact.

Different fungi has different functions to degrade the wood. In that case, the combination of fungus will have an impact on the speed of decaying given certain temperature and moisture. We will apply our model built before to analyze the relative the advantages and disadvantages for each species and combinations of species likely to persist. Also we will consider different kinds of environment including arid, semi-arid, temperate, arboreal, and tropical rain forests.

Key words: Competitive Lotka–Volterra equations, Sensitivity analysis

Contents

1 Introduction

1.1 Problem Background

Carbon cycle is the process in which carbon atoms continually travel from the atmosphere to the Earth and then back into the atmosphere. The balance of carbon cycle is critical for the live beings on the earth. There are several tunnels for carbon to back into the atmosphere, among which the decomposition of wood plays an important part.

Wood decay is caused by any species of fungus that digests moist wood, causing it to rot. The decomposition speed of different fungus is determined by both intrinsic (e.g., tree species properties) and environmental (e.g., temperature, moisture) factors. Among the associated organisms, wood decay fungi are essential and extremely important, being the only forms of life capable of degrading wood to its initial constituents.

Figure 1: Brown-rot fungi at 400X

What's more, the interaction between different kinds of fungus also will change the speed of decay rate.With the climate change, the environment where fungus live will also change, which will impact the overall efficiency of the carbon cycle system. According to the type of decay they cause, wood decay fungi can be classified into white-rot fungi, brown-rot fungi, and soft-rot fungi. The detailed of interaction must be examined.

1.2 Problem Restatement

- Build a mathematical model to find out decomposition speed in the situation where several species of fungi are exist at a given temperature and district in certain district.
- Find out what will happen when fungus are interacted in short-term and long-term.
- Examine the different kinds of combinations of species and its advantages and disadvantages in different environment(arid, semi-arid, temperate, arboreal, and tropical rain forests).
- Analyzing the sensitivity of our model in rapid fluctuations and changing atmospherical trends.

1.3 Our approach

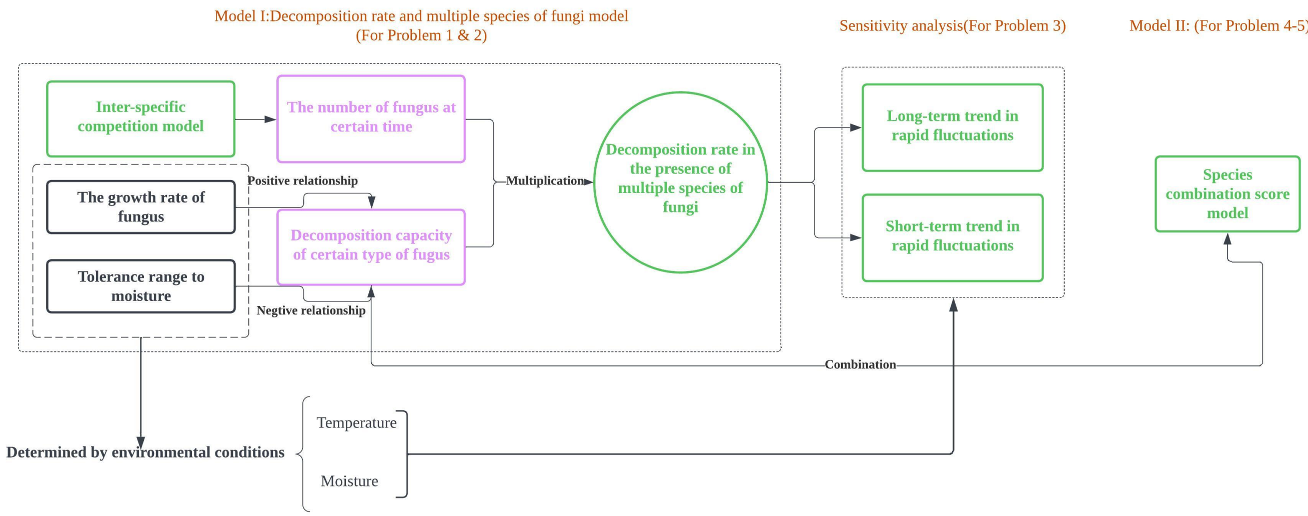

Figure 2: The framework of our article

- In order to discover the decomposition rate when there is various kinds of fungus, we build **Decomposition rate with multiple species of fungi model** to solve this problem. We will use textbfCompetitive Lotka–Volterra equations to reach the final result. After that, we will consider the decomposition capacity of one certain type of fungi and multiply it to the density of this type of fungus to get the total decomposition rate at certain time period.
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2 Assumptions

Assumption 1: We assume the maximum capacity for one kind of fungi is 3 g/m^3.

Justification: The capacity for certain kind of fungi is limited in a given area. So there is a maximum density. To simplify the question, we use 3 g/m^3 as this capacity.

Assumption 2:

Justification:

Assumption 3:

Justification:

3 Notations

Table 1: Notation

Symbols	Definition	Unit
T	Temperature of the environment	°C
r_p, r_t, r_x	The hyphal extension rate of different fungi	Unit
n_p, n_t, n_x	The maximum capacity of different fungi	g/m^3
p,t,x	The density of different fungi	g/m^3

4 The Data

4.1 Data collection

We collect relevant data for our model building from

Table 2: Data source

Data source	Database websites	Data description

4.2 Data Processing

5 Model I: Decomposition rate and multiple species of fungi model

The decomposition rate is determined by several environmental factors and characteristics of fungi. Among these, the most factors are temperature and moisture. The majority of fungi are mesophiles, and grow at temperatures in the range of 5-35 °C, which optimum temperatures for growth between 20 and 30 °C. In contrast, the sensitivity of fungi to moisture has huge differences. Different fungal species have different tolerances to moisture and some will grow at lower moisture levels. Moreover, the rate will not only be affected by the behaviour of a single fungi, but also be determined by the interactions of different species of fungi. Due to the limited resource, the density of different kinds of fungus will also change and affect the total decomposition model.

To simplify this problem, in our model, we choose three typical fungus to simulate the procession, which are Tyromyces chioneus(denote in T), Pycnoporus sanguineus(denote in P) and Xylobolus subpileatus(denote in X). The reasons that we choose these three kinds of fungus are that they are all typical and common fungus in the nature and have similar suitable temperature and moisture. In that case, we can better simulate the process of the competition and find the result of what will happen after a long period.

5.1 Competitive Lotka–Volterra equations

5.1.1 Introduction of model

The competitive Lotka–Volterra equations are a model of the population dynamics of species competing for some common resource. It shows the interactive relationship between different species under limited resource. There are several elements in the equations.

In our problem, the competition is within the scope of different kinds of fungi. The basic model is shown in below:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{\sum_{j=1}^{N} \alpha_{ij} x_j}{K_i}\right) \tag{1}$$

In this equation, x_i denote the amount of i species, r_i is the inherent per-capita growth rate, K_i is the carrying capacity of i species, α_{ij} represents the effect species i has on the population of species j. To simplify this problem, in our model, we choose three typical fungus to simulate the procession, which are Tyromyces chioneus(denote in T), Pycnoporus sanguineus(denote in P) and Xylobolus subpileatus(denote in X).

5.1.2 Conditions of our model

In the case that temperature and moisture will affect the growth rate of fungus. We choose certain environment for our simulation. To better simulate the situation, it's important to choose a suitable environment for these three kinds of fungus. We collect the data of these three kinds of fungus and plot the relevant data into one graph. Below is the result:

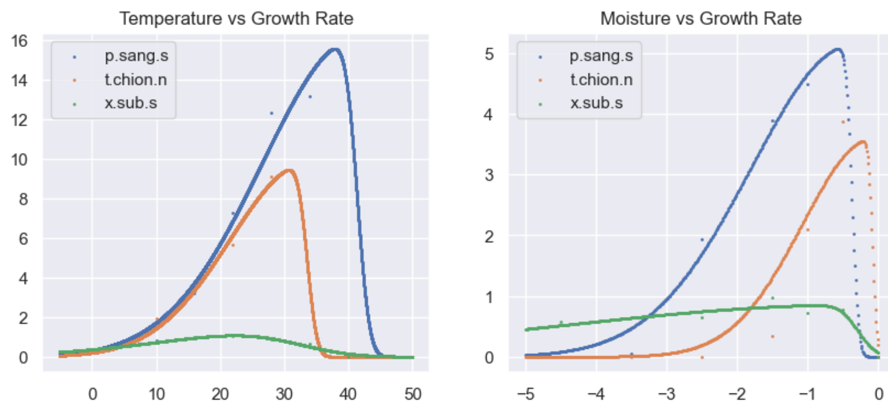

Figure 3: The relationship between growth rate and moisture and temperature

From the graph, we found that the suitable temperature interval is between 20-30 °C for all fungus. In that case, we will choose 22 °C, which Xylobolus subpileatus has the largest growth rate while others are also remain high.

The measurement of moisture for soil is matrix potential. Matrix potential is forces between the water molecules and the solid particles in combination with attraction among water molecules promote surface tension and the formation of menisci within the solid matrix. The conversion equation is Van Genuchten model which show above:

$$\theta(\psi) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|\psi|)^n]^{1-1/n}} \tag{2}$$

We will choose the moisture of matrix potential -0.5kPa for our model.

5.1.3 Fungus Competitive Lotka–Volterra equations

Applying the basic ideas of Competitive Lotka–Volterra equations, we build the following equations for this problem.

$$\frac{dp}{d\ time} = r_p \ * p \left(1 - \frac{p}{n_p} - s_{pt}(\frac{t}{n_t}) - s_{px}(\frac{x}{n_x})\right) \tag{3}$$

$$\frac{dt}{d\ time} = r_t * t \left(1 - \frac{t}{n_t} - s_{tp}(\frac{p}{n_p}) - s_{tx}(\frac{x}{n_x})\right) \tag{4}$$

$$\frac{dx}{d\ time} = r_x * x \left(1 - \frac{x}{n_x} - s_{xp}(\frac{p}{n_p}) - s_{xz}(\frac{z}{n_z})\right) \tag{5}$$

In these equations, we use r_p, r_t, r_x to denote The hyphal extension rate of different fungi, n_p, n_t, n_x to denote the maximum capacity of different fungi (in g/m^3) and p,t,x denote the density of different fungi (in g/m^3).

In our model, we choose the initial data $init_p, init_t, init_x$ = 0.02, 0.06, 1.74 g/m^3 and found the dynamic movement of the density of fungi. We will use matrix D to denote the density of each fungi at certain time period:

$$D = \begin{bmatrix} p_{t1} & p_{t2} & p_{t3} & p_{t4} \\ t_{t1} & t_{t2} & t_{t3} & t_{t4} \\ x_{t1} & x_{t2} & x_{t3} & x_{t4} \end{bmatrix} \tag{6}$$

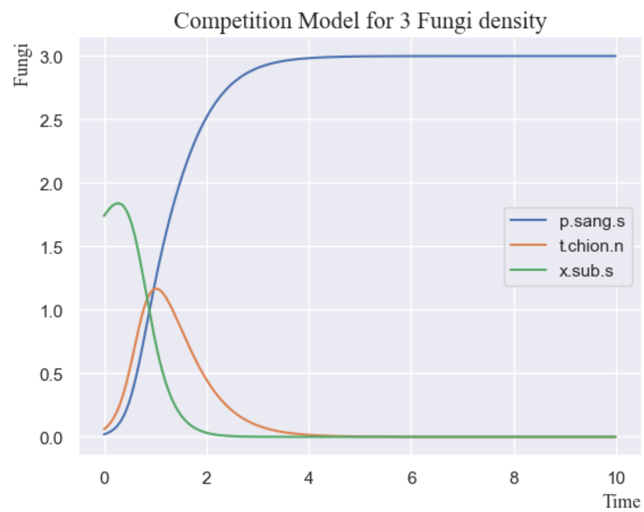

Figure 4: Competition model

In the short run, all fungi does increase for a short time. However, after a long competitive time, we found Pycnoporus sanguineus dominates the environment.

5.2 Decomposition rate with various fungi model

The decomposition speed of fungi will change over time due to X. The speed of each fungi is shown below:

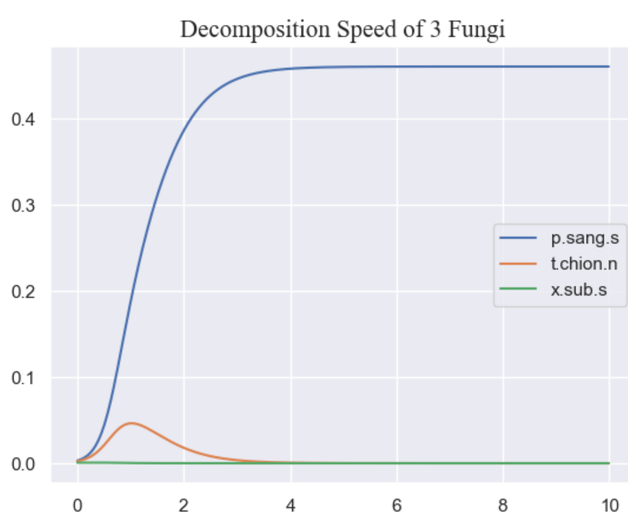

Figure 5: Decomposition speed of 3 fungi

We will use matrix D to denote the density of each fungi at certain time period:

$$V = \begin{bmatrix} Speed_p \\ Speed_t \\ Speed_x \end{bmatrix} \tag{7}$$

$$\frac{df}{d\ time} = \left[Speed_p, Speed_t, Speed_x\right] * \begin{bmatrix} p_{t1} & p_{t2} & p_{t3} & p_{t4} \\ t_{t1} & t_{t2} & t_{t3} & t_{t4} \\ x_{t1} & x_{t2} & x_{t3} & x_{t4} \end{bmatrix} \tag{8}$$

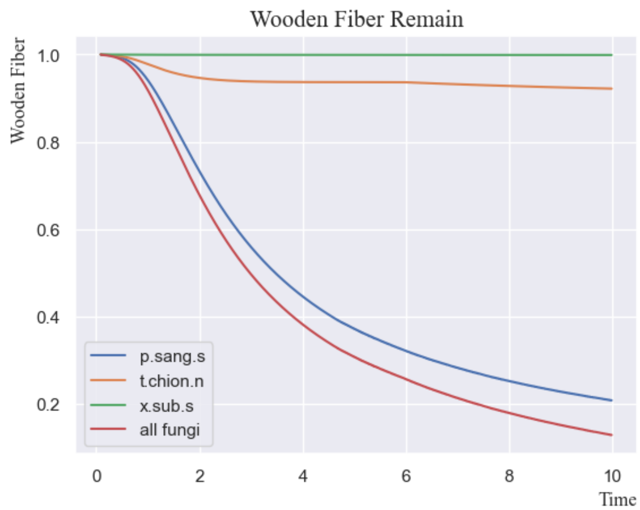

Figure 6: Wooden fiber remain

5.3 Conclusions

6 Model II:

In Model I, we analyze the relationship between decomposition rate and multiple fungi. However, the behaviour and interaction will be different against time due to the limited resource and external environment. In short-term, because there is sufficient nutrient in the environment , there is no obvious interest conflict between groups of fungus. However, as the time goes by, the nutrient will decrease to an extent that it can't support all fungus to survive. In that case, there will be competition among species and reach a balance in the long run. We will use X model to solve this problem.

6.1 model

6.2 Sensitivity analysis

6.3 Conclusion

7 Model III: Fungus combination model

Competition does exist among different kinds of fungus. However, the suitable combination of them may result in different situations. There are variable kinds of fungi which has different method of decaying. For example, white-rot fungi mainly secrete cell oxidases for delignification while brown-rot fungi can degrade cellulose and hemicellulose.

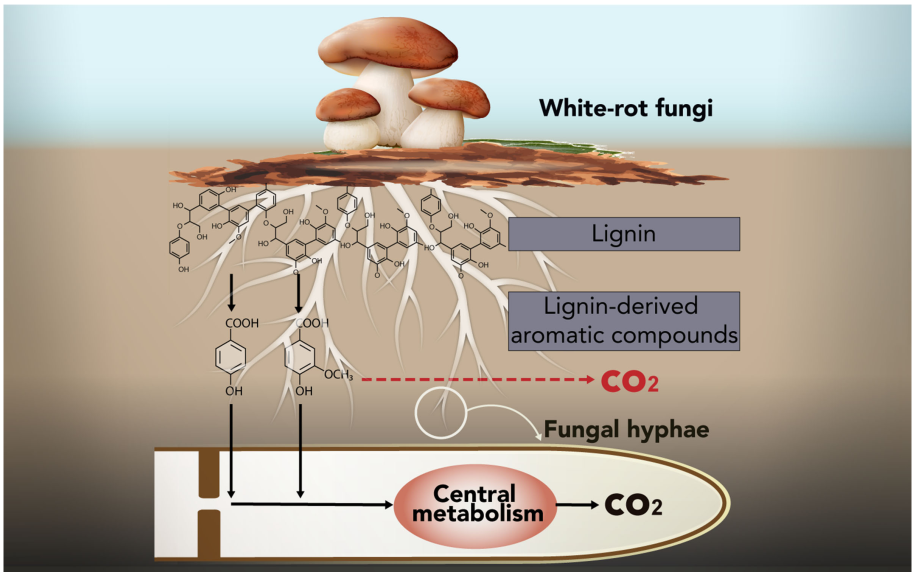

Figure 7: White-rot fungi

In that section, we will analyze the relative the advantages and disadvantages for each species and combinations of species likely to persist. Also we will consider different kinds of environment including arid, semi-arid, temperate, arboreal, and tropical rain forests.

7.1 Introduction of model III

8 Conclusion

8.1 Result of Problem I & II

8.2 Result of Problem III

8.3 Result of Problem IV

8.4 Result of Problem V

References

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