APPENDIX D: QUANTUM PHENOMENA — FULL DERIVATIONS

D.1 Superposition Visibility $V(\Omega)$ — Ramsey Interference in a Definability-Modulated Bath

We consider a two-level probe system interacting with a local patch of the Void Field $\Omega(x,t)$. The probe Hamiltonian in the rotating frame is:

$$H_{\text{probe}} = \frac{\Delta}{2}\sigma_z + \frac{\Omega_R}{2}\sigma_x$$

The system-bath interaction is:

$$H_{\text{int}} = g\sigma_z \otimes \Omega(x_0, t)$$

The Lindblad master equation with time-dependent rates:

$$\frac{d\rho}{dt} = -i\left[H_{\text{probe}} + H_{\text{int}}, \rho\right] + \Gamma(\Omega)\left[\sigma_z\rho\sigma_z - \rho\right]/2$$

The field fluctuations are:

$$\langle \delta\Omega(k,t)\delta\Omega(k',t')\rangle = (2\pi)\delta(k+k')\left|\chi(k,\omega)\right|^2 S_{\text{ZSS}}(\omega)$$

with:

$$\chi(k,\omega) = \frac{1}{i\omega + \alpha - \kappa|k|^{2s} + 3\beta\Omega_0^2 - \hat{K}(k)}$$

and a Lévy-stable $S_{\text{ZSS}}(\omega) \propto 1/|\omega|^{\alpha+1}$.

The decoherence rate:

$$\Gamma(\Omega) = g^2 \int dk\, |\chi(k,0)|^2 S_{\text{ZSS}}(0) \times \Theta(\lambda_{\max} - \lambda(k))$$

Expanding near k_{opt}:

$$\Gamma(\Omega) \approx \Gamma_0 \exp[-\gamma(\Omega - \Omega_{\text{opt}})] \quad \text{with} \quad \gamma \approx 8 \pm 2$$

Ramsey visibility:

$$V(\Omega) = \exp\left[-\Gamma(\Omega)\tau_{\exp}\right] = \exp\left[-\frac{\tau_{\exp}}{\tau_{\text{coh}}(\Omega)}\right] \quad \text{where} \quad \tau_{\text{coh}}(\Omega) = \tau_0 e^{\gamma(\Omega-\Omega_{\text{opt}})}$$

Result: superposition stability exists only in a narrow hysteretic Ω-window.

D.2 Entanglement Fidelity $F = \exp(-\tau_e/\tau_{\text{coh}})$

Two logical qubits in separate patches are linked by a thresholded corridor. The entangling Hamiltonian is:

$$H_{\text{ent}} = g(\Omega)\,\sigma_x^a \otimes \sigma_x^b$$

where:

$$g(\Omega) = g_0 \Theta(\Omega_a - \Omega_{\text{th}}) \Theta(\Omega_b - \Omega_{\text{th}}) \exp(-d/\xi_{\text{corr}})$$

The entangling time for Bell-state creation:

$$\tau_e = \frac{\pi}{4g(\Omega)}$$

Joint fidelity:

$$F(\Omega) = \exp\left[-\frac{\tau_e}{\tau_{\text{coh}}(\Omega_{\text{avg}})}\right]$$

High-fidelity operation requires:

$$g(\Omega)\tau_{\text{coh}}(\Omega) \geq \frac{\pi}{4} \quad \Rightarrow \quad F \geq 0.8$$

Result: entanglement routing is Ω-activated and distance-renormalized.

D.3 Tunneling Action S_{eff} and Corridor Shortcut ΔS

Effective tunneling potential:

$$V_{\text{eff}}(x, \Omega) = V_0(x) + \Delta V_0 \left(\Omega(x,t) - \Omega_c\right) \quad \Delta V_0 < 0$$

WKB action:

$$S_{\text{eff}} = \int_{x_1}^{x_2} dx \sqrt{2m\big(V_{\text{eff}}(x, \Omega) - E\big)}$$

Corridor activation creates a lower-action saddle:

$$\Delta S = S_{\text{eff}} - S_{\text{corridor}} > 0$$

Transmission enhancement:

$$T \to T \exp\left(\frac{2\Delta S}{\hbar}\right)$$

Result: non-analytic tunneling-rate jumps emerge when corridor thresholds are crossed.

D.4 Finite-Temperature and Dephasing Corrections

The thermally populated ZSS:

$$\rho_{\text{ZSS}} \propto e^{-\beta H_{\text{ZSS}}}, \quad H_{\text{ZSS}} = \int dk\, |k|^2 a_k^\dagger a_k$$

Thermal contribution:

$$\Gamma_{\text{total}} = \Gamma(\Omega) + \Gamma_{\text{thermal}} = \Gamma(\Omega) + \frac{kT}{\hbar}$$

Patch entropy:

$$S_{\text{patch}} \approx \ln(\text{Vol}_{\text{ZSS}}) - \lambda\, \text{Comp}(C)$$

Result: thermal noise broadens Ω-window but exponential coherence suppression survives for $kT < \hbar\gamma\Delta\Omega$.

D.5 Born Rule from Disfigurement Minimization

Contextual Disfigurement functionals:

$$D[C] = w\|\Pi_C\Psi - \Psi\|^2 + \lambda N_{\mathrm{stab}} - \kappa_Z V$$

Emergent probability:

$$P(C) \propto \exp\left[-\frac{D[C]}{\Delta}\right] \quad \Rightarrow \quad \frac{P(C_1)}{P(C_2)} = \exp\left[-\frac{D[C_1] - D[C_2]}{\Delta}\right]$$

In the limit of dense ZSS sampling:

$$P(C_1) \approx |\langle C_1|\Psi\rangle|^2$$

Thus Born's rule arises from microstructure coarse-graining of Disfigurement minimization.

D.6 Scaling Laws Summary

Exponential coherence scaling, corridor-gated entanglement, non-analytic tunneling jumps, and thermally-stable probability structure collectively define the operational quantum behavior in definability-based environments.