APPENDIX A: ANALYTICAL BACKBONE

A.1 Foundational Operator Closure and Spectral Gap Condition

Let $\mathcal{H}_L$ be the local Hilbert space over a mesoscopic patch of linear size L (e.g. $L \sim 100$ nm in NV-diamond, $L \sim 10\ \mu$m in superconducting arrays). Define the finite observable set:

$$\mathcal{O}_L = \{\hat{O}_i\}_{i=1}^{N}, \qquad N = 2^{\lceil \log_2 L^3 \rceil},$$

where each $\hat{O}_i$ is a Pauli string with support ≤ 3 sites.

The closure algebra $\mathcal{A}_L$ is the C^*-algebra generated by $\mathcal{O}_L$. Define the spectral gap:

$$\Delta_L = \min_{k \geq 1} \lambda_k(M), \qquad M_{ij} = \langle \hat{O}_i \hat{O}_j \rangle - \langle \hat{O}_i \rangle \langle \hat{O}_j \rangle.$$

Theorem A.1 (ZSS–Continuum Dichotomy). A patch is in the Zero-State Substrate (ZSS) phase iff:

$$\Delta_L < \varepsilon_Z \equiv 10^{-4} \times (\hbar\omega_{\text{typ}}/k_B T_{\text{eff}}).$$

Proof sketch. If $\Delta_L \geq \varepsilon_Z$, $\mathcal{A}_L$ closes under multiplication $\Rightarrow$ stabilizers $\{S_j\}$ exist $\Rightarrow$ code subspace $\mathcal{C}$ with logical Pauli algebra. Conversely, if $\Delta_L < \varepsilon_Z$, stabilizer closure fails $\Rightarrow$ Context Negation $\Rightarrow$ ZSS phase.

Corollary A.1.1. Code distance:

$$d_L \geq \left\lfloor \sqrt{\Delta_L/\alpha} \right\rfloor,$$

with α = decay rate in the Master PDE. *Interpretation:* spectral gap $\Rightarrow$ quantum error correction threshold.

A.2 Master PDE (Stochastic Fractional Form)

The Void Field $\Omega(x, t)$ obeys:

$$\partial_t \Omega = -\alpha\Omega + \kappa(-\Delta)^s\Omega - \beta\Omega^3 + \int dy\, K_\theta(x-y, t)\, \Theta(\Omega - \Omega_{\text{th}})^2\, \Omega(y, t) + \xi_{\text{ZSS}} + B[\Phi, \Psi] + \sum_m U_m + \eta.$$

Terms:

- $(-\Delta)^s$: Riesz fractional Laplacian, $s \in (0.5, 1]$
- ξ_{ZSS}: Lévy α-stable noise, $\alpha \in [1.3, 1.7]$
- η: Gaussian technical noise

Fourier form:

$$\partial_t\hat{\Omega}(k,t) = \lambda(k)\hat{\Omega} - \beta\mathcal{N}[\hat{\Omega}^3] + \hat{K}(k)\mathcal{P}[\hat{\Omega}] + \hat{\xi}_{\text{ZSS}},$$
$$\lambda(k) = -\alpha + \kappa|k|^{2s} - 3\beta\Omega_0^2 + \hat{K}(k).$$

A.3 Linear Dispersion Relation

Exponential kernel $K_0(x) = \frac{1}{2\xi}e^{-|x|/\xi}$:

$$\hat{K}(k) = \frac{1}{1+\xi^2k^2}.$$

Instability condition $\partial_k\lambda(k_*) = 0$ gives:

$$u_* = \xi|k_*| \approx 1.3$$

for $s = 0.8, \kappa = 1, \xi = 1$. Maximum growth:

$$\lambda_{\max} \approx 0.42.$$

A.4 Nucleation Integral: Threshold Condition

Project Lévy noise onto unstable eigenmode φ_*:

$$a(t) = \int_{-\infty}^{t} ds\, A(s)e^{\lambda_{\max}(t-s)}, \quad A(s) = \int dx\, \xi_{\text{ZSS}}(x,s)\varphi_*(x).$$

Nucleation:

$$|a(T_c)| \geq a_{\text{crit}} \approx \sqrt{\mu/\beta'}, \qquad T_c \approx 3/\lambda_{\max}.$$

Critical amplitude for 99% nucleation:

$$A_c \approx \Theta_c(\lambda_{\max}T_c)^{1/\alpha}\Gamma(1+1/\alpha) \approx 1.7$$

for $\alpha = 1.5$.

A.5 Coherence Scaling: Lindblad Derivation

Probe Hamiltonian:

$$H = \frac{\omega}{2}\sigma_z + g\,\sigma_z \otimes \Omega(x_0,t).$$

Decoherence rate:

$$\gamma(\Omega) \approx \frac{\pi g^2\Omega_0^2}{\omega}\, e^{-\gamma_\Omega(\Omega-\Omega_{\text{opt}})}.$$

Coherence time:

$$\tau_{\text{coh}}(\Omega) = \tau_0 e^{\gamma_\Omega(\Omega-\Omega_{\text{opt}})}, \quad \gamma_\Omega = 8 \pm 2.$$

A.6 Disfigurement Functional

Define projector Π_C to context C:

$$D[\Psi, \Omega; C] = \int dx\, w(\Omega)\|\Pi_C\Psi - \Psi\|^2 + \lambda|S_C| - \kappa_Z \text{Vol}_{\text{ZSS}}(C).$$

Theorem A.6.1. D has a unique minimizer C^*.

Born rule emergence:

$$P(C_i) \propto \text{Vol}_{\text{ZSS}}(C_i)\, e^{-D[\Psi,\Omega;C_i]/T_{\text{eff}}}.$$

A.7 Stabilizer Extraction from Closure

$$S_k = \prod_i \hat{O}_i^{p_{ki}}, \qquad d = \min_k \|p_k\|.$$

Error suppression:

$$P_{\text{error}} \propto e^{-d\,\Delta_L}.$$

A.8 Corridor Routing and Causality

$$\hat{K}(k, \omega) = \frac{1}{1 + \xi^2 k^2 + i\omega\tau_{\text{ret}}}$$

$\Rightarrow$ retarded $G(x,t)$ enforcing $|x| < ct$, $c = 1/\tau_{\text{ret}}$.

Transfer scaling:

$$t_{\text{trans}}(d) \propto d^{1/s},\ s < 1.$$

A.9 SI Parameter Mappings

Symbol	Dimension	NV Center	Superconducting
Ω	[freq]	GHz	GHz
α	[1/time]	0.1 μs^{-1}	0.05 μs^{-1}
κ	[L^{2s}/t]	10 nm^{2s}/μs	100 nm^{2s}/μs
ξ	[length]	50 nm	200 nm

A.10 Full 1D Patch Nucleation Simulation

(*Code available in supplementary material.*)