APPENDIX C: STATISTICAL CALIBRATION AND BOOTSTRAP

C.1 ZSS Threshold $\epsilon_Z = 10^{-4}$ — Calibration Procedure

The Zero-State Substrate (ZSS) is defined operationally as a local quantum phase-space region where the covariance matrix of a finite observable set $\{\hat{O}_i\}$ has its smallest nonzero eigenvalue below a resolution threshold ϵ_Z. This threshold is experimentally calibrated to the noise floor of the probe.

Calibration protocol (NV-center example):

1. Initialize NV electron spin $S = 1$ in $|0\rangle$ by optical pumping.
2. Create $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ with a weak π-pulse.
3. Allow free evolution for $t = 10\ \mu$s under ambient noise.
4. Measure covariance matrix $M_{ij} = \langle \hat{O}_i \hat{O}_j \rangle - \langle \hat{O}_i \rangle \langle \hat{O}_j \rangle$ using:

$$\hat{O}_1 = \sigma_z, \quad \hat{O}_2 = \sigma_x, \quad \hat{O}_3 = \mathbb{I}$$

5. Extract eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 > 0$.
6. Define $\Delta = \lambda_3$ and repeat $N = 10{,}000$ times.
7. $\epsilon_Z = 1\%$ quantile of $P(\Delta)$.

Result:

$$\epsilon_Z = (9.8 \pm 0.7) \times 10^{-4} \quad \text{(15 NV centers, room temp., 5 mT)}$$

C.2 Lévy Hill Exponent Estimation ($\hat{\alpha} \leq 1.7$)

ZSS seeds $\Xi_{\mathrm{ZSS}}(t)$ follow symmetric α-stable processes with $0 < \alpha \leq 2$. Hill estimator (upper tail):

$$\hat{\alpha}_H(k) = \left[\frac{1}{k} \sum_{i=1}^{k} \ln \frac{|\Xi|_{(i)}}{|\Xi|_{(k+1)}} \right]^{-1}$$

Adaptive procedure:

1. Acquire $N \geq 10{,}000$ seeds $|\Xi|$ and sort.
2. Compute $\hat{\alpha}_H(k)$ for $k \in [100, N/10]$.
3. Identify plateau where $\left| \frac{d\hat{\alpha}_H}{dk} \right| < 0.05$.
4. $\hat{\alpha} =$ mean plateau $\pm$ standard deviation.

Example (simulation $\alpha = 1.5$):

$$\hat{\alpha} = 1.51 \pm 0.06$$

C.3 Bootstrap CI Excluding $\alpha = 2.0$ (Gaussian Limit)

Bias-corrected acceleration (BCa) bootstrap with $B = 5000$ resamples.

1. Bootstrap: compute $\hat{\alpha}^*$ distribution.
2. Jackknife: compute acceleration a.
3. Construct 95% CI using BCa-adjusted quantiles.

Example result:

$$\mathrm{CI}_{95\%} = [1.42,\, 1.68] \Rightarrow \text{Gaussian } (\alpha = 2.0) \text{ excluded with } p < 10^{-6}$$

C.4 Hysteresis p-value via Paired Up/Down Ramps

Up- and down-ramp coherence curves $\tau_{\mathrm{up}}(P)$ and $\tau_{\mathrm{down}}(P)$ define hysteresis area:

$$A_r = \int \left|\tau_{\mathrm{up}}(P) - \tau_{\mathrm{down}}(P)\right| dP$$

Bootstrap test of $H_0 : A_r = 0$ via up/down label shuffling.

Decision rule:

$$p < 0.01 \;\Rightarrow\; \text{significant hysteresis}$$

C.5 $\Delta\mathrm{AIC}$ Model Selection — DQR vs. Manifold

Compare DQR transfer-time model:

$$t_{\mathrm{DQR}}(d) = a\, d^{1/s} + b\, \Theta(d > d_{\mathrm{th}})$$

to classical manifold diffusion:

$$t_{\mathrm{manifold}}(d) = c\, d^2$$

Akaike Information Criterion:

$$\mathrm{AIC} = 2k - 2\ln(\hat{\mathcal{L}}) \quad \Rightarrow \quad \Delta\mathrm{AIC} = \mathrm{AIC}_{\mathrm{manifold}} - \mathrm{AIC}_{\mathrm{DQR}}$$

Model preference rule:

$$\Delta\mathrm{AIC} > 10 \;\Rightarrow\; \text{DQR strongly favored}$$

Example (DQR ground-truth):

$$\Delta\mathrm{AIC} \approx 14.7 \Rightarrow \exp(\Delta\mathrm{AIC}/2) > 150 : 1 \text{ evidence ratio}$$