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Optimization scheme for synthesizing kinoform with genetic algorithm

Ruifeng Zheng
Guanglin Yang
Haiyan Xie

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Ruifeng Zheng
Guanglin Yang
Peking University
State Key Laboratory on Advanced Optical, Communication System and Network
School of Electronic Engineering and Computer Science
Beijing, 100871, China
E-mail: ygl@pku.edu.cn

Haiyan Xie
China Science Patent Trademark Agents Ltd.
Beijing, 100083, China

Abstract. An optimization scheme based on a genetic algorithm (GA) is proposed for kinoform synthesis. Unlike conventional optimization schemes, the initial kinoforms here are obtained by Fourier transform of the original image with random phase masks. The phase masks are then optimized by GA in order to reduce the reconstruction noise caused by amplitude negligence and phase quantization. Compared with the conventional methods of the genetic algorithm, in which optimization is directly performed to the kinoforms, the scheme can significantly improve the convergence and reduce the computation cost. [DOI: 10.1117/1.3621516]

Subject terms: kinoform; optimization; genetic algorithm; random phase mask.

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1 Introduction

The kinoform is a phase-only optical element generated by a computer. The amplitude of its Fourier transfer function is assumed to be unity. Because of its flexibility of design and high diffraction efficiency near 100%, it has application potential.[1,2] As shown in Fig. 1, the kinoform $K(\mu, \nu)$ is illuminated by a plane wave and the reconstructed image $U(x, y)$ is derived in the focal plane of the Fourier lens. The wave front $K(\mu, \nu)$ from the kinoform is given by $U(\mu, \nu) = \exp[i\theta(\mu, \nu)]$, where $\theta(\mu, \nu)$ is the phase distribution of kinoform designed in the range $[0,2\pi)$. The $\theta(\mu, \nu)$ can be expressed with a quantization level of L, that is, with L equally spaced phase levels, the transfer function of the kinoform takes the value of $\exp[2\pi i(m\text{-}1)/L]$, $(m = 1,\ldots L)$. Assume that the kinoform has a size of $N\times N$, the reconstructed image $U(x, y)$ in the image plane from the kinoform is given by

$$U(x, y) = \frac{1}{N^2}\sum_{\mu=1}^{N}\sum_{\nu=1}^{N} K(\mu, \nu)\exp\left[2\pi i\,\frac{\mu x + \nu y}{N}\right] = \frac{1}{N^2}\sum_{\mu=1}^{N}\sum_{\nu=1}^{N}\exp\left[-2\pi\,\theta(\mu, \nu)\,\frac{\mu x + \nu y}{N}\right]. \tag{1}$$

However, the reconstructed images of the kinoforms include inevitable noise caused by amplitude negligence and phase quantization. In this letter, the cost function E is the normalized mean square error between the reconstructed image and the input image, which is used in most optimization algorithms.[2,3] The E can be expressed as

$$E = \frac{\sum_{x=1}^{N}\sum_{y=1}^{N} |\, I_o(x, y) - A\, I(x, y)\,|^2}{\sum_{x=1}^{N}\sum_{y=1}^{N} |\, I_o(x, y)\,|^2}, \tag{2}$$

where A is a scale factor determined by the diffraction efficiency to coordinate the total powers, and $I_o(x, y)$ and $I(x, y)$ are the intensity of the original image and the reconstructed image, respectively. $I_o(x, y)$, $I(x, y)$, and A are also given by

$$I_o(x, y) = |U_o(x, y)|^2, \quad I(x, y) = |U(x, y)|^2, \quad A = \sum_{x=1}^{N}\sum_{y=1}^{N} I_o(x, y) \Big/ \sum_{x=1}^{N}\sum_{y=1}^{N} I(x, y). \tag{3}$$

The reconstruction noise poses a serious obstacle to practical applications. Therefore, it is necessary that the cost function E be minimized to improve reconstruction image quality.

At present, there exists several optimization schemes of a genetic algorithm (GA) for the reconstruction improvement.[3,4] However, in those schemes, their computation cost is high, especially in gray image processing. In this letter, we present an optimization scheme of GA for kinoform synthesis. In this scheme, the initial kinoforms are obtained by the Fourier transform of the original image with uniform random phase masks which are directly optimized by GA to reduce the reconstruction noise. Compared with the other two schemes, we find that our scheme has better performance.

2 Proposed Scheme

In our scheme, because the initial kinoforms are obtained by Fourier transform of the input image with random phase masks $\varphi(x, y)$, the amplitude of the spectrum $|\mathrm{FFT}\{U_o(x, y)\exp[i\varphi(x, y)]\}|$ gets close to a constant. Its details are described in Ref. 5. The phase masks $\varphi(x, y)$ are directly optimized by GA to flatten the amplitude. If the amplitude absolutely equals a constant, the reconstructed image is exactly the input image and no reconstruction noise exists.

As shown in Fig. 2, the proposed scheme consists of the following steps:

Step 1: Initialization. A phase mask is encoded as an individual chromosome in GA, which is represented by a real-valued $N\times N$ matrix. An initial random phase mask is obtained by a sequence of random numbers uniformly distributed in the interval $[0, 2\pi)$. The population of the genetic algorithm consists of M chromosomes. With the M random phase masks attached to the input image, M complex matrices are obtained.

Step 2: Kinoform Generation, Quantization, Reconstruction and Cost Function Computation. After fast Fourier

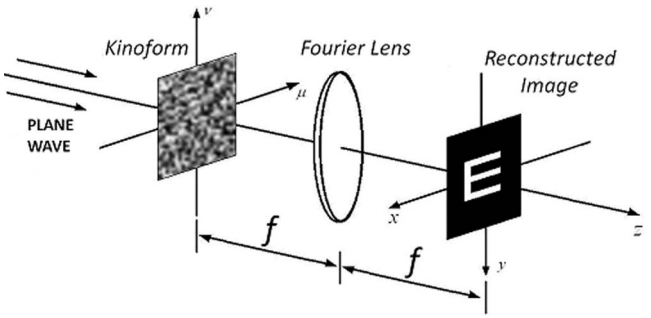

Fig. 1 Optical reconstruction setup used for Fourier kinoform.

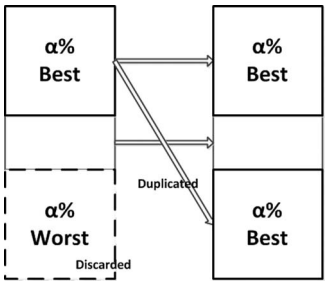

Fig. 3 Selection among population.

transform and amplitude part discard, M real-valued kinoforms are generated from these complex matrices. The quantization level L must be greater than 2 to avoid the conjugate-image. The higher the quantization level, the better the reconstruction quality. After quantization, the reconstructed images are obtained by inverse Fourier transform,[6] and the cost functions of the whole population can be obtained according to Eq. (2). The best chromosomes have the lowest cost functions.

Step 3: Selection. The elitism principle is adopted as the selection method in order to avoid discarding the optimal chromosomes and to save the computation time. The excellent chromosomes are treated more fairly and the population diversity can be maintained better. As shown in Fig. 3, for each generation, the $\alpha\%$ best chromosomes are selected twice and the $\alpha\%$ worst chromosomes are replaced.

Step 4: Cross-over. The cross-over in this step generates new chromosomes by combining genes from their parents. Two chromosomes are selected randomly in population. The two parent chromosomes are arbitrarily divided into two parts and the parts are mutually exchanged. Finally, $\beta\%$ of the M chromosomes is crossed over.

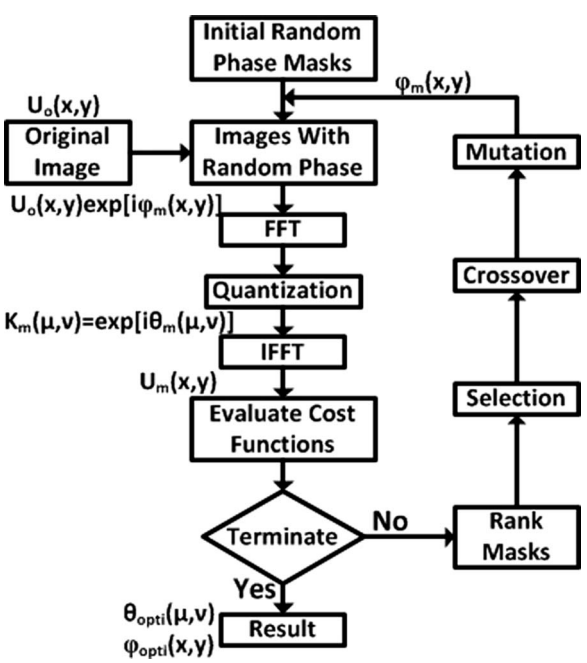

Fig. 2 Flowchart of proposed optimization scheme.

As shown in Fig. 4(a), some cross-over manners[4] use four random variables to represent this rectangle: coordinates of two pixels on a diagonal line (x_{start}, y_{start}, x_{end} and y_{end}). Both two pixels are uniformly randomly chosen in the chromosome. This manner is widely used, but it has one serious disadvantage: the probabilities of choosing a pixel clearly vary with its position, i.e., the closer the pixel is to the center, the more probable it is chosen, and vice versa. As shown in Fig. 4(b), we use width and height instead of the end pixel. The width and height are uniformly randomly chosen in the interval [1, N]. Then another problem arises: the end pixel [x_{start} + width, y_{start} + height] sometimes protrudes beyond the boundary. To prevent the protrusion, we manipulate the cross-over rectangle with periodicity[3] and every gene is treated absolutely fairly.

Step 5: Mutation. In order to maintain a good genetic diversity in a population, the genes are mutated with the mutation probability $\gamma\%$. And the $\gamma\%$ of all genes is arbitrarily chosen to be added on a uniform noise with amplitude equaling 0.1π.

Step 6: Output. The evaluation of the kinoform is defined as the minimum value of the cost function in population. If the evaluation decreases to small enough or other circulation conditions are satisfied, the optimization is finished, otherwise, the steps 2 to 5 are repeated.

3 Experimental Analysis

As shown in Fig. 5(a), a gray "Lena" image (128×128) is adopted as the input image. The parameters have to be chosen carefully, [4] and they are listed in Table 1. Other two schemes (i.e., the classical GA and the step-quantization GA[3]) are performed. The reconstructed images by these optimization

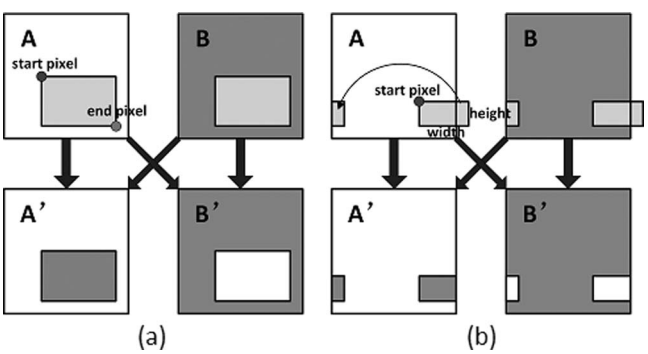

Fig. 4 Cross-over between two chromosomes. (a) Conventional cross-over and (b) cross-over with periodicity.

Table 1 Simulation configurations.

Parameters	Value
Population (M)	100
Quantization level (L)	4
Selection rate (α%)	40%
Cross-over rate (β%)	80%
Mutation rate (γ%)	0.08%

Table 2 Cost functions of the reconstructions.

Optimization algorithm	Cost function
Proposed GA	0.070
Classical GA	0.229
Step-quantization GA	0.131
Classical GA (initialized)	0.215
Step-quantization GA (initialized)	0.103

Fig. 5 Input image and reconstructions by different GAs. (a) Input image, (b) image reconstructed by the classical GA, (c) image reconstructed by the step-quantization GA, and (d) image reconstructed by the proposed GA.

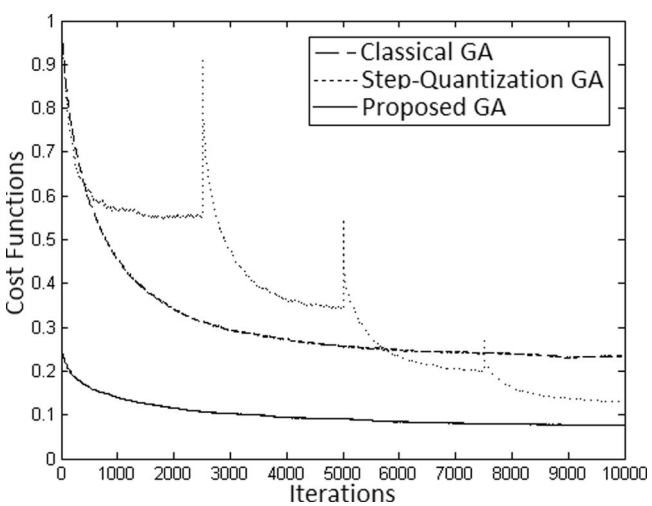

Fig. 6 Convergences of cost functions.

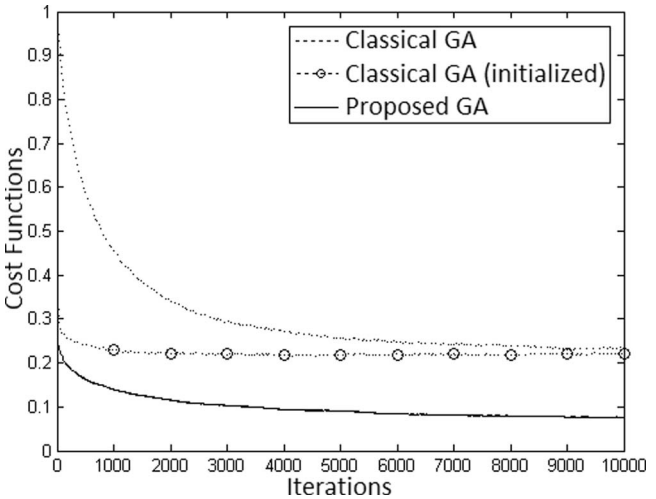

Fig. 7 Cost functions of the classical GA, the classical GA (initialized), and the proposed GA.

algorithms at 10,000 iterations are shown in Figs. 5(b)–5(d), and the cost functions of these reconstructions are listed in Table 2.

The convergence procedures of the cost functions are shown in Fig. 6. In the beginning, the cost function in our scheme is much better due to the initial random phase masks. When the iteration number reaches 6000, the cost function of the step-quantization GA just gets close to the initial value in our scheme. The cost function in our scheme is sharply reduced to 0.115 at 2000 iterations, and then slowly reduced to 0.070 at 10,000 iterations. Our scheme can promptly synthesize a kinoform with a cost function less than 10%, which can be accepted in many applications. It can be obviously found in Fig. 4 that the convergence procedure of the proposed GA is much better than other two GAs.

To demonstrate the importance of optimization on the phase mask, two additional simulations are carried out: with the same initial condition as the proposed GA, the classical

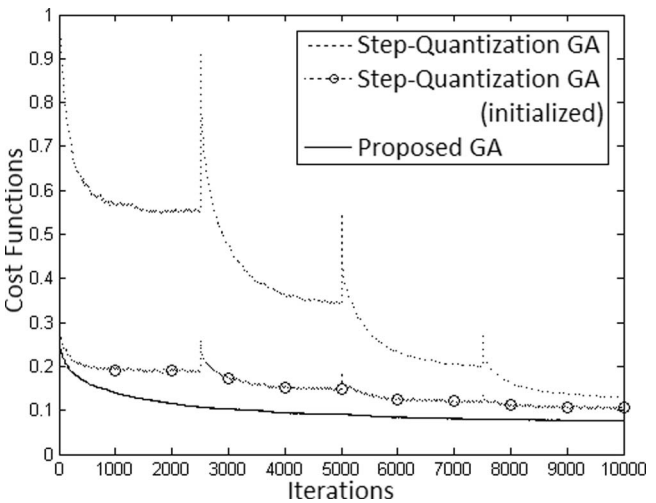

Fig. 8 Cost functions of the step-quantization GA, the step-quantization GA (initialized), and the proposed GA.

GA, and the step-quantization GA are performed again. The cost functions of the simulations are also listed in Table 2. As shown in Figs. 7 and 8, although the initial conditions are better, the convergences of cost functions of classical GA and step-quantization GA are not significantly improved. Therefore, the change of optimization object from the kinoform to phase mask play a much more important role than the better initial condition in the faster convergence of our scheme.

4 Conclusion

A new optimization scheme based on GA for synthesizing Fourier kinoform is proposed. The initial kinoforms are obtained by Fourier transform of the input image with random phase masks. The phase masks are then optimized by GA in order to synthesize the kinoform with minimum reconstruction noise. This scheme is demonstrated to evidently accelerate the convergence procedure and significantly reduce the computation cost.

Acknowledgments

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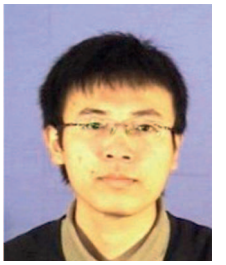

Ruifeng Zheng received his BEng degree from the department of instrument science and opto-electronics engineering, Hefei University of Technology in July 2004. His MEng degree from the department of electronics, school of electronics engineering and computer science, Peking University in July 2011. His research interests include image processing, digital holography.

Guanglin Yang is an associate professor in the image processing laboratory, department of electronics, school of electronics engineering and computer science, Peking University. He received his DEng degree from Faculty of Engineering, Osaka City University of Japan in September 2001, and his MEng degree from the department of opto-electronics engineering, Huazhong University of science and technology of China in July 1993. He became a lecturer in faculty of automatic control engineering, Beijing University of Aeronautics and Astronautics of China in September 1994. His research interests mainly include the optical information processing, digital holography, image processing, artificial neural network, intelligent infrared detecting, and digital control technique. He respectively is a member of IEEE and OSA. E-mail: ygl@pku.edu.cn

Haiyan Xie is a patent engineer in China Science Patent Trademark Agents Ltd., she respectively received her MS and PhD degrees from the school of ccience, Osaka City University of Japan in March 2002 and April 2005. Moreover, she respectively received her BEng and MEng degrees from the department of opto-electronic and scientific instrumentation engineering, Zhejiang University of China in July 1990 and December 1992. She became a lecturer in the faculty of automatic control engineering, Beijing University of Aeronautics and Astronautics of China in 1995. Her interests mainly include the thin film optics and the semiconductor material.