

# Computer-generated hologram watermarking resilient to rotation and scaling

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**Abstract.** We propose a new scheme of computer-generated hologram (CGH) watermarking to resist rotation and scaling. To embed the inverse log-polar mapping of a mark pattern's CGH into a cover image, the twin image of the mark pattern can be directly reconstructed by fast Fourier transformation from the log-polar mapping of the watermarked image after rotation and scaling, not requiring a registration step in the extracting procedure. In an experiment, the information position of the twin image is located in the high-frequency domain and the redundant information of the low-frequency component is properly eliminated, so the contrast of the twin image is appropriately enhanced and the basic information of the mark pattern is effectively preserved to be recognized. The experimental results show that the mark-pattern's information can be effectively reconstructed when the watermarked image is scaled by 0.5 to 2 or rotated by any angle, so this watermarking scheme is effectively verified by experiment. © 2007 Society of Photo-Optical Instrumentation Engineers.

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## 1 Introduction

In this paper, we propose a scheme of computer-generated hologram (CGH) watermarking resilient to rotation and scaling, based on the CGH, log-polar mapping (LPM), inverse LPM (ILPM), and the fast Fourier transform (FFT), as shown in Fig. 1.

We have investigated some documents about holographic watermarking such as Refs. 1–3. They mainly emphasize the robustness of common signal-processing operations such as image compression, and they all have difficulty in dealing with geometrical attacks, especially rotation and scaling. Because computation of a rotation angle or scaling factor is very fragile to errors, a little imprecision will make the watermark hard to extract.

In order to solve those problems, LPM is usually adapted to integrate it with the FFT, because the LPM algorithm can transform rotation and scaling into translation, and the amplitude of the FFT is translation-invariant.<sup>4</sup> If we embed a CGH of the mark pattern into the LPM domain of a cover image, the geometrical attacks of rotation and scaling

will be transformed into translation, and will not affect the twin image's reconstruction from the image rotated and scaled. So the reconstruction of the twin image from the watermarked image can resist rotation and scaling. But when the lossy LPM algorithm is applied to the cover image, it will degrade the cover image's quality. So we embed the ILPM of the CGH into the cover image instead of directly embedding the CGH into the LPM domain of the cover image.

Therefore, our scheme can directly reconstruct the embedded information from the watermarked image after rotation and scaling, and the reconstructed twin image can be easily recognized. It avoids the registration step in the extracting procedure, without involving the calculation complexity of the rotation angle or scaling factor. It is a blind watermarking scheme resilient to rotation and scaling.

## 2 Computer-Generated Hologram and Reconstruction

According to the off-axis hologram principle of Leith and Upatnieks,<sup>5</sup> we can get the transmittance as

$$h(x, y) = |O(x, y) + R(x, y)|^2 = |A(x, y) \exp[j\varphi(x, y)] + R \exp[j2\pi(\alpha x + \beta y)]|^2 = A(x, y)^2 + R^2 + 2R \cdot A(x, y) \times \cos[2\pi(\alpha x + \beta y) - \varphi(x, y)], \quad (1)$$

where  $A(x, y) \exp[j\varphi(x, y)]$  represents the object wave  $O(x, y)$ , and  $R \exp[j2\pi(\alpha x + \beta y)]$  represents the reference beam  $R(x, y)$ . According to Eq. (1), we made a CGH [Fig. 2(b)] from the mark pattern  $W(p, q)$ , the Chinese character "guang" [Fig. 2(a)].

In order to simulate the diffusion effect,  $W(p, q)$  should be multiplied by a random phase  $\text{rand}(p, q) \in (0, 1)$ , i.e.,

$$W'(p, q) = W(p, q) \exp[j2\pi \text{rand}(p, q)]; \quad (2)$$

thus  $O(x, y) = A(x, y) \exp[j\varphi(x, y)] = F\langle W'(p, q) \rangle$ , where  $F\langle \square \rangle$  indicates the discrete Fourier transform (DFT).

According to the properties of the DFT,

$$F\langle A(x, y) \exp[j\varphi(x, y)] \rangle = F\langle F\langle W'(p, q) \rangle \rangle = \frac{1}{MN} W'(M - p, N - q),$$

$$F\langle A(x, y) \exp[-j\varphi(x, y)] \rangle = F\left\langle \frac{1}{MN} F^{-1}\langle W'(p, q) \rangle \right\rangle = \frac{1}{MN} W'(p, q), \quad (3)$$

where  $M, N$  are the dimensions of the mark pattern, and  $F^{-1}\langle \square \rangle$  indicates the inverse DFT (IDFT).

The reconstruction of Fourier hologram can be expressed as

$$F\langle h(x, y) \rangle = F\langle A(x, y)^2 + R^2 \rangle + r(p, q) + v(p, q), \quad (4)$$

where  $r(p, q)$  indicates the real image  $(R/MN)W(p - \alpha M, q - \beta N)$ , and  $v(p, q)$  indicates the virtual image  $(R/MN)W(M - p - \alpha M, N - q - \beta N)$ . Figure 2(c) shows the

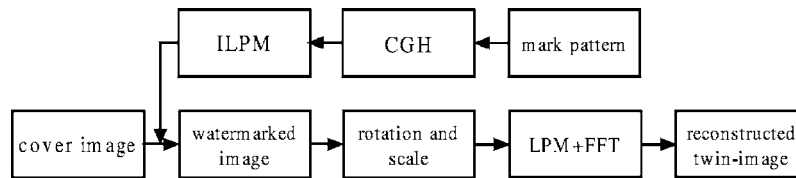


Fig. 1 A scheme of CGH watermarking resilient to rotation and scaling.

twin image reconstructed from the CGH by the FFT;  $r(p, q)$  and  $v(p, q)$  are locates at symmetrical positions, determined by  $\alpha$  and  $\beta$ .<sup>6</sup>

### 3 Log-Polar Mapping

The log-polar mapping is defined as

$$x = e^u \cos \theta,$$

$$y = e^u \sin \theta, \quad (5)$$

here  $(\mu, \theta)$  is the corresponding point in the log-polar coordinates of the point  $(x, y)$  in the Cartesian coordinates. LPM converts rotation and scaling in the Cartesian domain to translations in the log-polar domain, which can be expressed as

$$(\rho, \theta) \leftrightarrow (\mu + \log \rho, \theta),$$

$$(x \cos(\theta + \delta) - y \sin(\theta + \delta), x \sin(\theta + \delta) + y \cos(\theta + \delta))$$

$$\leftrightarrow (\mu, \theta + \delta). \quad (6)$$

### 4 Watermarking Scheme

In order to avoid the cover image's quality loss caused by LPM, we embed the CGH's ILPM into the cover image, so the embedding procedure can be expressed as

$$w(p, q) = \text{ILPM}[h(\mu, \theta)],$$

$$I'(p, q) = I(p, q) + \gamma \cdot w(p, q), \quad (7)$$

where  $I(p, q)$  indicates the intensity of the cover image,  $I'(p, q)$  indicates the intensity of the watermarked image,  $\gamma$  is the weighting factor of the watermark,  $h(\mu, \theta)$  is the function of the CGH made from the mark pattern  $W(p, q)$

[Fig. 2(a)],  $\text{LPM}[\square]$  indicates log-polar mapping, and  $\text{ILPM}[\square]$  indicates inverse log-polar mapping.

If  $w(p, q)$  is rotated by  $\delta$  and scaled by  $\rho$ , resulting in  $w'(p, q)$ , then the reconstruction of the twin image from  $w'(p, q)$  can be expressed as

$$\begin{aligned} F[\text{LPM}[w'(p, q)]] &= F[\text{LPM}\{\text{ILPM}[h(\mu + \log \rho, \theta + \delta)]\}] \\ &= F[h(\mu + \log \rho, \theta + \delta)] = F[h(\mu, \theta)] \\ &= r(p, q) + v(p, q). \end{aligned} \quad (8)$$

We can see that reconstructed twin image is resilient to rotation and scaling.

### 5 Experimental Analysis

In an experiment, the ILPM of the mark pattern's CGH ( $512 \times 512$  samples) was embedded into a cover image [Fig. 3(a)], and yielded the watermarked image [Fig. 3(b),  $\gamma=0.3$ , PSNR=37.8612]. When the watermarked image [Fig. 3(b)] is rotated or scaled, the mark pattern's twin image can be directly reconstructed by FFT from the LPM of the image rotated or scaled [Fig. 4(a)], not needing a registration step.

When we adopt the method of Eq. (8) to reconstruct the information of the mark pattern, the cover image  $I(p, q)$  will go through the same procedure to produce redundant information; therefore we have to properly eliminate some noise and effectively enhance the contrast of the reconstructed twin image. It is well known that the ordinary image's spectrum is concentrated in its low-frequency component  $l(p, q)$ , so we can select appropriate  $\alpha$  and  $\beta$  to locate the information position of the twin image in the high-frequency domain, as shown in Fig. 4(b), and eliminate the redundant information  $l(p, q)$  to enhance the contrast of the reconstructed twin image. As shown in Fig. 4(c), we eliminated the redundant information and put the reconstructed real image and virtual image together in the reconstruction plane.

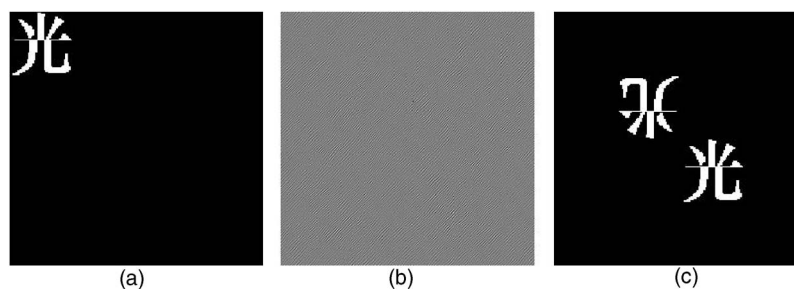


Fig. 2 (a) Mark pattern; (b) CGH of mark pattern; (c) twin image reconstructed from CGH. The image size is  $256 \times 256$  pixels.



**Fig. 3** (a) Cover image; (b) watermarked image,  $\gamma=0.2$ , PSNR=37.8612. The image size is  $512 \times 512$  pixels.

We adopted the peak signal-to-noise ratio (PSNR) to evaluate the quality of the reconstructed twin image, and found that when the PSNR is larger than 12.00, the reconstructed twin image can be effectively recognized. To view the reconstructed twin image (PSNR=16.1378) in Fig. 4(c), we found that when the image is scaled by 1.5 and rotated by  $45^\circ$  [Fig. 4(a)], the information of “guang” is basically preserved and the shape of the reconstructed twin image can be effectively recognized. We also did other, similar experiments, and found that this holographic watermarking method is robust to rotation through any angle and scaling by any factor from 0.5 to 2.0.

## 6 Conclusion

In this paper, we have established a scheme of CGH watermarking to resist the geometrical attacks of rotation and scaling, based on CGH, ILPM, LPM, and FFT. Through embedding the ILPM of a mark pattern's CGH into a cover image, the twin image of the mark pattern can be effectively reconstructed from the watermarked image rotated and scaled, and the registration step in the extracting procedure can be avoided. The information position of the twin image is located in the high-frequency domain, and the redundant information of the low-frequency component is properly eliminated; thus the contrast of the twin image is

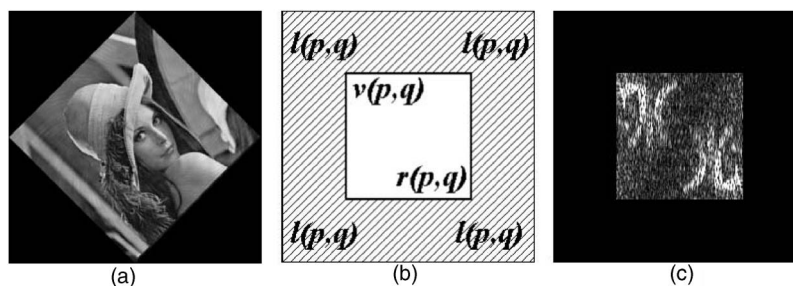
effectively enhanced, and the information of the mark pattern is preserved well enough to be recognized. Although the LPM and ILPM algorithms caused some loss in image quality, the experimental result is acceptable. If the precision of LPM and ILPM algorithms is sufficiently improved, the experimental results will become better. This processing scheme has potential application in rotation- and scaling-resilient watermarking.

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**Fig. 4** (a) The geometrically attacked watermarked image (scaled by 1.5 and rotated by  $45^\circ$ ); (b) the positions of the low-frequency component  $l(p,q)$  and twin image  $[r(p,q)$  and  $v(p,q)]$ ; (c) the reconstructed twin image, PSNR=16.1378.