

Exchange Rate Models

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The Nominal Exchange Rate

- The nominal exchange rate is the price of a currency quoted in another currency. (e.g. if a dollar currently costs 6.5 RMB, that means the RMB/dollar exchange rate is 6.5)
- The nominal exchange rate can be quoted two ways.
 - In this lecture, we express an exchange rate as the domestic currency price of a foreign currency.



- Thus an *increase* in RMB exchange rate means an RMB *depreciation*.
- We say that a currency *depreciates* when it becomes less valuable

The Nominal Exchange Rate



- Currency is an asset and the nominal exchange rate an asset price
 - It is probably the most important asset price there is
- The foreign exchange market is the largest market in the world.
 - Daily turnover in April 2016 = 5.1 trillion USD
- Changes in the nominal exchange rate frequently quoted and analyzed in news, and cited in policy debate
 - As a change in the exchange rate is supposed to alter the prices of goods in a country relative to another
- Often the discussion takes a win-lose stance
 - “If my currency appreciates, your currency depreciates.”
 - “Your exporters gain, at the expense of my exporters.”

Fair value of the exchange rate

- How does the nominal value of a currency matter?



“I heard that 11 million Chinese currency is needed to get one US dollar... Chinese don't know how to manage their country...”

川岛芳子 /
Kawashima Yoshiko (1990)

- Eleven million yuan for a dollar? How outrageous!
 - Then what is the “correct” nominal exchange rate?
 - The story happened in 1948. We don't have a sense of what the “correct” exchange rate should be.

Fair value of the exchange rate

- Consider an alternative example
- Suppose you want to travel to either Tokyo or New York, and you have a ¥6,000 budget.
 - Suppose the 6,000 yuan can be exchanged into either 1,000 dollar, or 100,000 yen
 - Does that mean you should go to Tokyo because Japanese Yen is “cheaper”?
 - In this example, one US dollar = 100 Japanese Yen
 - Is it “right” for Japanese Yen to be so much cheaper than the US dollar?
- It makes no sense to state that Japanese yen is “too cheap”, nor that Japan is a more favorable destination.
- The level of nominal exchange rate *alone* conveys essentially no information – and may be redundant

The Relative Price

- Level of nominal exchange rate is only useful conditional on the *nominal prices of goods*
 - Tokyo: a hotel room costs 32,000 yen per night
 - New York: a hotel room costs 200 dollars per night
 - To compare these prices, you found out that the dollar-yen exchange rate is 100 yen/dollar. You figured:

$$32000 \text{ yen} \times 0.01 \text{ dollar/yen} = 320 \text{ dollars}$$

- Alternatively, we could have converted the dollar price of NY room into a yen price, or both into RMB. Either way, one fact remains the same: the price of a Tokyo hotel room *relative* to a NY room is 1.6.
- What matters is *the relative price*. A change in the relative price is going to have real impact on economic decision making.
 - It makes sense to say that a stay in Tokyo is more expensive compared with New York

Law of One Price (LOOP)

- Answer to the question: “Is there a “correct” value of nominal exchange rate?”
 - *Given prices of goods* in two countries, the *efficient* relative price for goods is attained when the nominal exchange rate is at the “correct” level.
 - Then what is the efficient relative price?
- The Law of One Price (LOOP) is the statement that the same good should sell for the same price in two countries, once their prices are converted into the same currency.
 - Underlying argument: if the prices don’t equalize, there is arbitrage opportunity
 - Arbitragers buy low and sell high, until prices in two locations are equalized

- The idea behind the LOOP provides a simple method of currency valuation:
 - Say wheat is the only good being produced and traded everywhere
 - The units of wheat that ¥100 can buy in China is called the *internal purchasing power* of RMB
 - The units of wheat ¥100 can buy in a foreign country, after been converted to that country's currency, is called the *external purchasing power* of RMB
 - RMB is said to be overvalued, relative to the foreign currency, if its external purchasing power is greater than its internal purchasing power – Why?
- The arbitrage argument also makes a prediction about how the exchange rate is supposed to *change* in the future
 - How should you arbitrage if the RMB is overvalued?
 - How will the value of RMB change when arbitrage happen?



Focal point: The Big Mac Index

- Consider the price of a MacDonald Big Mac burger
 - Suppose its price in US is P^* dollars, and its price in China is P yuan
 - Let S denotes the yuan/dollar exchange rate
 - LOOP predicts that

$$P = SP^*$$

- Note that the relative price of a US Big Mac in terms of Chinese Big Mac is SP^*/P
 - LOOP is thus also the statement that the efficient relative price of any good should be one.
- Suppose however, that LOOP fails to hold, and we have instead
$$P < SP^*$$
 - We could say that Big Mac in China is too cheap compared to that in the US
 - Does that imply the exchange rate of the RMB is “wrong”?

Focal point: The Big Mac Index

- The Economist's 2011 Big Mac Index redux

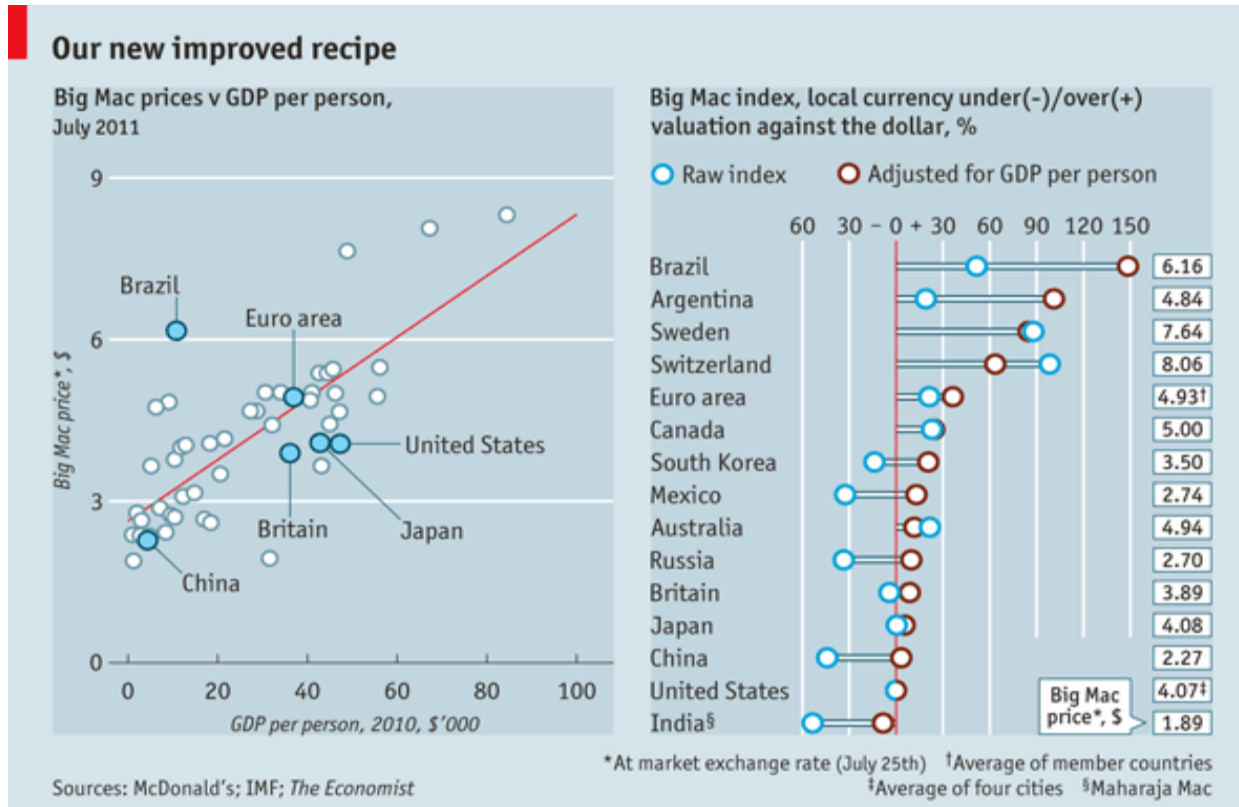
The hamburger standard

Country	Big Mac prices		Implied PPP† of the dollar	Actual dollar exchange rate July 25th	Under(-)/over(+) valuation against the dollar, %	
	in local currency	in dollars*			raw index	adjusted for GDP per person
United States‡	\$4.07	4.07	–	–	–	–
Argentina	Peso 20.0	4.84	4.92	4.13	19	101
Australia	A\$4.56	4.94	1.12	0.92	22	12
Brazil	Real 9.50	6.16	2.34	1.54	52	149
Britain	£2.39	3.89	1.70‡	1.63‡	-4	9
Canada	C\$4.73	5.00	1.16	0.95	23	24
Chile	Peso 1,850	4.00	455	463	-2	58
China	Yuan 14.7	2.27	3.60	6.45	-44	3
Colombia	Peso 8,400	4.74	2,066	1,771	17	108
Czech Republic	Koruna 69.3	4.07	17.1	17.0	nil	45
Denmark	DK 28.5	5.48	7.01	5.20	35	23
Egypt	Pound 14.1	2.36	3.47	5.96	-42	11
Euro area**	€3.44	4.93	1.18††	1.43††	21	36
Hong Kong	HK\$15.1	1.94	3.71	7.79	-52	-43
Hungary	Forint 760	4.04	187	188	-1	57
India§§	Rupee 84.0	1.89	20.7	44.4	-53	-8
Indonesia	Rupiah 22,534	2.64	5,543	8,523	-35	24
Israel	Shekel 15.9	4.67	3.91	3.40	15	43
Japan	¥320	4.08	78.7	78.4	nil	5
Malaysia	Ringgit 7.20	2.42	1.77	2.97	-40	2
Mexico	Peso 32.0	2.74	7.87	11.7	-33	13

- To make Chinese price of Big Mac identical to that in the US after conversion of currency, RMB/dollar exchange rate should be 3.6.
- Back then the actual value was 6.45. Thus the RMB is 44% undervalued.
- But countries with lower income typically have lower price level...

Focal point: The Big Mac Index

- After adjusting for the relative income, the RMB exchange rate is “about right”



- Is the “Big Mac Index” a good measure of currency misalignment?

From LOOP to PPP

- Purchasing Power Parity (PPP) is the Law of One Price stated at the aggregate level
- Suppose there are two countries: Home and Foreign. And there are N types of goods traded in the world market.
- The Home and Foreign consumer price index (CPI) are:

$$P = P_1^{\alpha_1} P_2^{\alpha_2} \dots P_N^{\alpha_N}, \quad P^* = P_1^{*\alpha_1} P_2^{*\alpha_2} \dots P_N^{*\alpha_N}$$

- The weight α_i denotes the proportion of consumption expenditure that's spent on good i . Naturally

$$\sum_{i=1}^N \alpha_i = 1$$

From LOOP to PPP



- Suppose the LOOP holds for each individual good:

$$P_i = SP_i^* \text{ for all } i$$

- Then it is easy to see that $P = SP^*$
 - Here P and P^* are the CPI of Home and Foreign, respectively.
 - The equation states that a consumption basket costs the same in Home and in Foreign, when converted to the same currency units. This is the “Purchasing Power Parity” theory.
- PPP builds on two assumptions that are *sufficient* conditions:
 1. LOOP holds at individual goods level
 2. Consumers have the same preferences (thus same weight α_i)
 - When these are true, the aggregate *price level* will equalize across countries: it is just as expensive for a typical consumer to live in China compared with Japan, or France.

PPP as a theory of exchange rate

- Note that if PPP holds, then the (log) exchange rate must equal to the difference of the nominal price levels (lower case letter denotes log):

$$s = p - p^*$$

- The PPP theory we just stated is also called the “*absolute PPP*”
- The *relative PPP*, on the other hand, requires that the price of the home consumption basket to be proportional to the price of the foreign basket

$$P_t = K S_t P_t^*$$

Question: How is *depreciation* of a currency related to the relative *inflation rates*, according to absolute and relative PPP, respectively?

- From the vast literature examining the empirical validity of PPP (use Rogoff(1996) as a hinge to explore), PPP is known to hold at most *in the long run*.

The monetary model

- If PPP does hold, the long-run nominal exchange rate is determined by the real exchange rate and the price ratio:

$$S_t = k \left(\frac{P_t}{P_t^*} \right)$$

- The monetary model asks: what determine the price level in each country then?
- It uses the theory of liquidity preference by Keynes
 - Price level is determined by money demand and money supply.
 - Money demand (in real terms) $L(r_t, Y_t)$ depends:
 - positively on output (Y)
 - negatively on interest rate (r)

Money, price, and interest rate

- Money supply is exogenously determined by the central bank
- The equilibrium of the monetary model:

i

interest
rate

$(M/P)^s$

$L(r, Y)$

M/P

real money
balances

- **In the short run:**
Price is fixed, and interest rate changes with money supply
- **In the long run:**
Change in money supply causes a proportional change in the price level, but is neutral on real variables

The monetary model: implications

- Thus, the long-run price level is determined by

$$P_t = M_t / L(i_t, Y_t)$$

- Substitute into the PPP condition:

$$S_t = k \left(\frac{M_t}{M_t^*} \frac{L(i_t^*, Y_t^*)}{L(i_t, Y_t)} \right)$$

- Implications:
 - An increase in Home money supply causes a nominal depreciation
 - An increase in Home interest rate causes a nominal depreciation
 - An increase in Home output causes a nominal appreciation



Goods market and assets market

- So far we have considered two theories of exchange rate based on goods market
- But exchange rate is most prominently an asset price
- In fact, nowadays, the volume of flow of assets across international borders is much larger than the volume of flow of goods
- Next, we will turn our attention to the assets market and ask how would the exchange rate be determined as an asset

Foreign Exchange Market



Foreign exchange: volume and growth

	1992	1995	1998	2001	2004	2007	2010
(a) Volumes							
All instruments	857	1135	1713	1480	2013	3296	3981
Spot total	434	475	637	461	657	996	1490
Forwards total	65	93	143	156	217	359	475
Growth, spot, and forwards	—	44%	54%	9%	40%	65%	32%
(b) Ratios							
Spot/Trade	31	29	30	18	21	23	36
Spot/GDP	10	10	11	7	10	12	16
Spot/Equity volume	35	25	14	4	9	7	9

Constant 2010 USD; unit: billions

Sources: BIS, IMF, WFE

Computation by King et al.(2012)

Demand for foreign currency

- What characteristics of an asset determine its desirability to investors?
- Return of an asset is a prominent consideration
- Risk and liquidity
 - Investors care about two main characteristics of an asset, other than its return:
 - Risk: The variability it contributes to the investor's wealth
 - Liquidity: The ease with which it can be sold or exchanged
- Interest rates play a central role in the determination of exchange rate
 - As currency demand is closely associated with returns of assets denominated in that currency

Uncovered Interest Parity

- To make the analysis simple, let's consider risk-free assets with no liquidity issues
- Suppose you are considering two investment options: a U.S. treasury bond that pays 3% interest rate, or a Chinese government bond that pays 4%. Suppose both bonds are absolutely safe (no chance of default). Does this mean the Chinese bond is a better option?
- To determine that, we need to consider the role of exchange rate.
 - While you hold a foreign bond, the value of the currency may change
 - Denoting the interest rate on home bond by i_t , and the interest rate on foreign bond by i_t^* . The ex-post return on the foreign bond is

$$\frac{(1 + i_t^*)S_{t+1}}{S_t}$$

Uncovered Interest Parity

- We approximate the rate of return on the foreign bond by:

$$\ln \frac{(1 + i_t^*)S_{t+1}}{S_t} \cong i_t^* + s_{t+1} - s_t$$

- The return of foreign bonds is the sum of the interest return and the currency return
- ***Uncovered interest parity*** holds if home bond and the foreign bond are expected to deliver the same rate of return

$$i_t = i_t^* + E_t s_{t+1} - s_t$$

- The term “uncovered” means that the investor is taking a risk by investing in the foreign bond
- If home currency is expected to depreciate, UIP states that the home interest rate should be higher than foreign rate – investors are compensated for the expected currency loss

UIP and the nominal exchange rate

- Note that the equation above is a theory of exchange rate determination. Everything else equal:

$$s_t \equiv i_t^* - i_t + E_t s_{t+1}$$

- An increase in home interest rate causes an appreciation
 - An increase in foreign interest rate causes a depreciation
 - An expected depreciation causes an immediate depreciation
- That is, the UIP predicts that the expected exchange rate depreciation is equal to the Home-Foreign interest differential:

$$E_t s_{t+1} - s_t \equiv i_t - i_t^*$$

- The Taylor-rule model goes one more step to ask: what determine the short-term risk free interest rate?

Exchange Rate under Taylor Rule

- Monetary policy controls real interest rate by setting nominal interest rate

$$r_t = i_t - E_t \pi_{t+1}$$

- To change the real interest rate, we need to assume that expected inflation does not change one-for-one with nominal interest rate
- This is true when prices adjust sluggishly
- E.g. with a monetary contraction – an increase in the nominal interest rate – if expected inflation doesn't change or drops, then monetary contraction induces an increase in the real interest rate
- For example, the central bank may follow an inflation targeting policy: it intends to induce a small, positive inflation rate of $\tilde{\pi}$
 - Difficult to stimulate economy when expected inflation is low, as it is difficult for the nominal interest rate to go below zero
 - Fix the problem of downward rigidity of wage over time

Real Exchange Rate under Taylor Rule

- Suppose to set a nominal interest rate, monetary policy follows a rule (sometimes called a “Taylor rule”)

$$i_t = \bar{r} + \tilde{\pi} + \sigma(E_t\pi_{t+1} - \tilde{\pi}) + v_t, \quad \sigma > 1$$

- \bar{r} is the “natural level of the real interest rate” – the real interest rate that prevails when the economy is producing at its full-employment level.
- $\sigma > 1$ is required if the central bank wants to change the real interest rate in the same direction as nominal interest rate
- v_t is an error term meant to capture other factors that might influence monetary policy aside from inflation expectations, or policy errors
- Suppose foreign central bank follows the same rule, then expected depreciation depends on *relative inflation*.
- A more general Taylor Rule assume that the central bank sets interest rates according to inflation *and* output gap.



A Brief History of RMB Exchange Rate

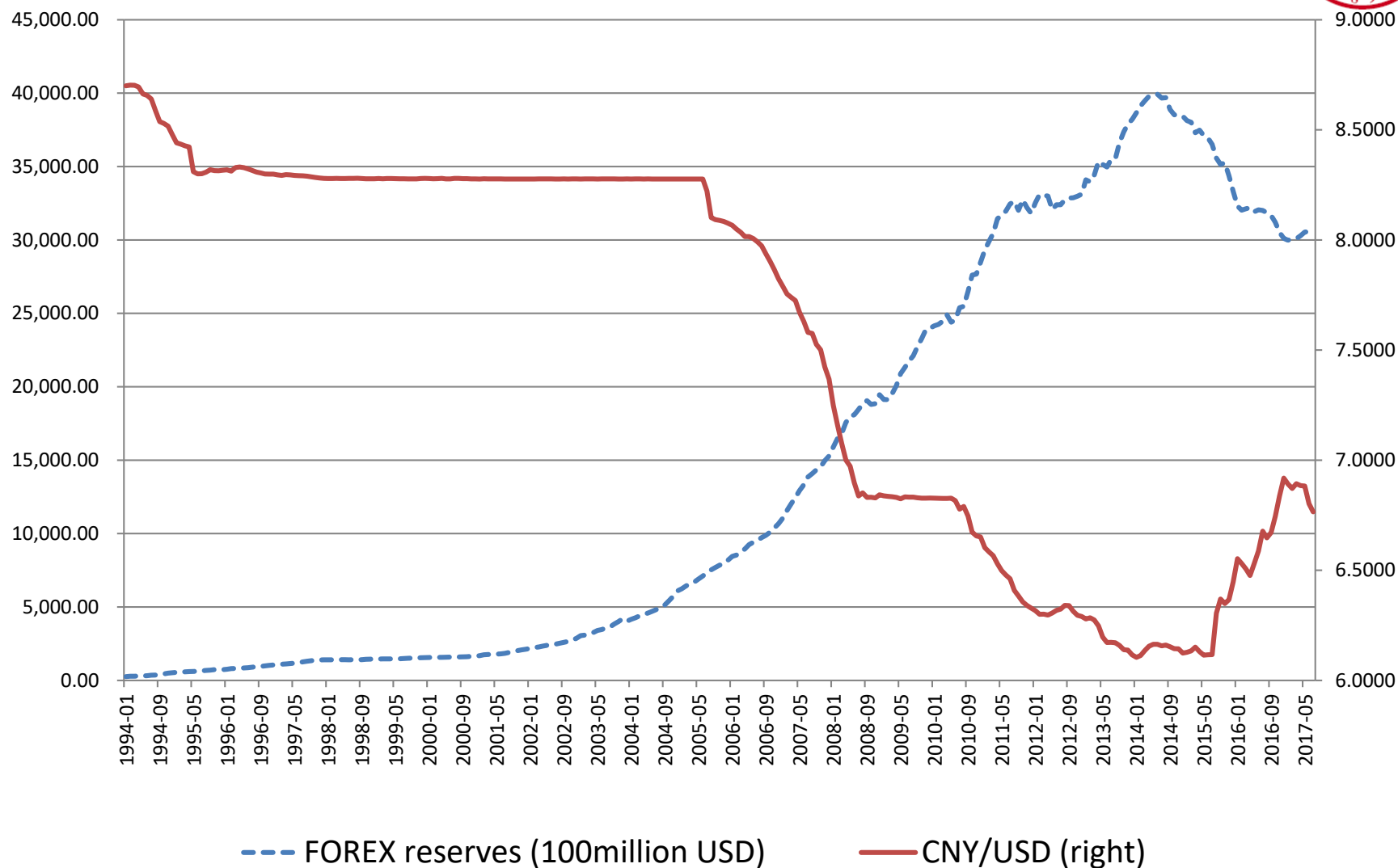
- 1949 – 1980: Exchange rate “picked” officially
 - All foreign exchange held by the nation
 - Foreign exchange quota is given to planned imports
 - Proceeds from planned exports needed to be submitted to the nation
 - From 1979: exporting sectors can partially retain foreign exchange
- 1980 – 1993: Multiple exchange rates
 - Official exchange rate remains
 - Allowed FOREX trade between qualified enterprises at an officially fixed rate
 - Then introduced the “internal exchange rate for trade” to correct export loss
 - 1986: foreign investor-specific exchange rate introduced
 - ... then there was individual exchange rate...
 - Overall trend is for RMB to depreciate when market exists



A Brief History of RMB Exchange Rate

- 1994, national interbank market established and multiple exchange rates unified. Current account convertibility initiated
- 1995-2004, fixed at about 8.3 RMB/dollar
 - Facilitates trade
 - Anchor Chinese domestic monetary policy
- Pressure from the US to appreciate RMB built overtime
 - In 2005, PBoC allowed RMB to gradually crawl up
- The appreciation continued until...

Foreign Reserve and Exchange Rate



Exchange Rate Predictability

- Engel and West (2005) show that if an asset price can be expressed as a discounted sum of expected future fundamentals in the general form

$$s_t = (1 - b) \sum_{j=0}^{\infty} b^j E_t(\mathbf{a}'_1 \mathbf{x}_{t+j}) + b \sum_{j=0}^{\infty} b^j E_t(\mathbf{a}'_2 \mathbf{x}_{t+j}), \quad 0 < b < 1$$

- And if either (1) $\mathbf{a}'_1 \mathbf{x}_t \sim I(1)$ and $\mathbf{a}_2 = 0$, or (2) $\mathbf{a}'_2 \mathbf{x}_t \sim I(1)$, then for b near one, the asset price will be well approximated by a random walk
- To be more concrete, the theorem implies that exchange rate models of the form

$$s_t = (1 - b)F_{1t} + bF_{2t} + bE_t(s_{t+1}),$$

where the F 's are linear functions of fundamentals with at least one element following $I(1)$, will imply a near-random-walk behavior of the exchange rate when b is close to one.

Exchange Rate Predictability

- To be even more concrete, note that the exchange rate implied by the class of monetary models fits into this form.
- Intuition: when the discount factor approaches unity, future fundamentals become more important. The random walk component of the fundamentals that are expected to be inherited into the future becomes more important relative to transitory ones.
- Implications:
 1. Even if the exchange rate models are correct, exchange rates may still be unpredictable
 2. But if the present-value models of exchange rates are correct, change in exchange rate can be used to predict future fundamentals
 - Engel and West (2005) verified that exchange rates indeed Granger cause fundamentals

Exchange Rate Predictability

- Engel and West (2005) also introduced Taylor rule fundamentals in a formal derivation of a model of exchange rate
 - Use UIP: $E_t s_{t+1} - s_t = i_t - i_t^* + \rho_t$
 - Assume that the interest rates i_t and i_t^* are determined by monetary policies that follow interest rate rules
 - One country uses Taylor rule that targets both inflation and output gap, and the other country also targets exchange rate
 - Resulting exchange rate follows a near random walk according to the Engel-West theorem
- Interestingly, later empirical works found that the Taylor rule fundamentals have by far the strongest predictive power
 - Molodtsova and Papell (2009) shows that Taylor rule fundamentals help predicting exchange rate over the short run