QUESTION : 12.13.3.7

ROLL NO:EE22BTECH11027
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12.13.3.7.A and B are two events such that $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{3}$ and $\Pr(AB) = \frac{1}{4}$.
Find:

i $\Pr(A|B)$
ii $\Pr(B|A)$
iii $\Pr(A'|B)$
iv $\Pr(A'|B')$

Solution: : Given, $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{3}$ and $\Pr(AB) = \frac{1}{4}$. Then,

$$\begin{aligned}\Pr(A') &= 1 - \Pr(A) = \frac{1}{2}\\ \Pr(B') &= 1 - \Pr(B) = \frac{2}{3}\\ \Pr(A+B) &= \Pr(A) + \Pr(B) - \Pr(AB)\\ &= \frac{7}{12}\end{aligned} \tag{1}$$

1)

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} = \frac{3}{4} \tag{2}$$

2)

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} = \frac{1}{2} \tag{3}$$

3)

$$\Pr(A'|B) = \frac{\Pr(A'B)}{\Pr(B)}$$

We have,

$$B = AB + A'B$$

Applying probabilities on both sides,

$$\begin{aligned}\Pr(B) &= \Pr(AB) + \Pr(A'B)\\ \Pr(A'B) &= \Pr(B) - \Pr(AB)\\ &= \frac{1}{12}\\ \therefore \Pr(A'|B) &= \frac{1}{4}\end{aligned} \tag{4}$$

4)

$$\Pr(A'|B') = \frac{\Pr(A'B')}{\Pr(B')}$$

$$\because \Pr(A'B') = \Pr(A+B)' = 1 - \Pr(A+B)$$

using the conclusions from equation 1

$$\begin{aligned}\Pr(A'B') &= \frac{5}{12}\\ \therefore \Pr(A'|B') &= \frac{5}{8}\end{aligned} \tag{5}$$