QUESTION : 12.13.6.9

ROLL NO:EE22BTECH11027
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12.13.6.9.An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

Solution: : The repetition in conduction of experiment constitute the bernoulli trails. Let X denote the number of times the experiment succeeds in all the 6 attempts.
Let p be the probability for the experiment to succeed and q for the failure.
Here, it is given that probability of success is twice that of the failure, so

$$\begin{aligned} p &= 2q \\ p+q &= 1 \\ 3q &= 1 \\ q &= \frac{1}{3} \\ p &= \frac{2}{3} \end{aligned} \tag{1}$$

Now, let's consider a single trial as a bernoulli random variable $X_i = 1$ represents success and $X_i = 0$ represents failure.

Outcome	X_i
success(p)	1
failure(q)	0

Therefore we have,

$$P(X_i = 1) = p = \frac{2}{3} \tag{2}$$

$$P(X_i = 0) = q = \frac{1}{3} \tag{3}$$

Since we have n=6 trials, the random variable X representing the number of successes in 6 trials follows a binomial distribution. The cumulative distribution function (CDF) of X is given by

$$F_X(k) = P_X(X \le k) = \Sigma_{k=0}^{n} {}^{n}C_k q^{n-k} p^k \tag{4}$$

Here,

$$n = 6, p = \frac{2}{3}, q = \frac{1}{3}, k = 0, 1, 2...6$$

We need to find the probability for the experiment to succeed to atleast 4 times, which means that we want probability for experiment to succeed for 4, 5, or 6 times i.e. $P_X(X \ge 4)$ Using equation 4 we get,

$$\begin{aligned} P_X(X \ge 4) &= 1 - P_X(X \le 3) \\ &= 1 - F_X(3) \\ &= 1 - {}^{6}C_0 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^0 + {}^{6}C_1 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1 \\ &+ {}^{6}C_2 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + {}^{6}C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 \\ &= 1 - \frac{1}{3^6} + \frac{12}{3^6} + \frac{60}{3^6} + \frac{160}{3^6} \\ &= 1 - \frac{233}{3^6} = \frac{496}{729} \approxeq 0.680 \end{aligned} \tag{5}$$

Therefore the probability that in the next six trials, there will be atleast 4 successes is 0.680.