QUESTION : 12.13.6.9

ROLL NO:EE22BTECH11027
NAME: KATARI SIRI VARSHINI

12.13.6.9.An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

Solution: : The repetition in conduction of experiment constitute the bernoulli trails. Let X denote the number of times the experiment succeeds in all the 6 attempts.
Let p be the probability for the experiment to succeed and q for the failure.
Here, it is given that probability of success is twice that of the failure, so

$$\begin{aligned} p &= 2q \\ p + q &= 1 \\ 3q &= 1 \\ q &= \frac{1}{3} \\ p &= \frac{2}{3} \end{aligned} \tag{1}$$

Clearly X is the benoulli distribution with

$$n = 6, p = \frac{2}{3}$$

bernoulli distribution is given by:

$$P_X(k) = \binom{n}{k} q^{n-k} p^k \tag{2}$$

Here,

$$n = 6, p = \frac{2}{3}, q = \frac{1}{3}, k = 0, 1, 2...6$$

Therefore,

$$P_X(k) = \binom{6}{k} \left(\frac{1}{3}\right)^{6-k} \left(\frac{2}{3}\right)^k \tag{3}$$

We need to find the probability for the experiment to succeed to atleast 4 times, which means that we want probability for experiment to succeed for 4, 5, or 6 times i.e. $P_X(X \geq 4)$

$$P_X(X \geq 4) = P_X(4) + P_X(5) + P_X(6) \tag{4}$$

Using equation 3 we get,

$$\begin{aligned} P_X(X \geq 4) &= \binom{6}{4}\left(\frac{1}{3}\right)^{6-4}\left(\frac{2}{3}\right)^4 + \binom{6}{5}\left(\frac{1}{3}\right)^{6-5}\left(\frac{2}{3}\right)^5 + \binom{6}{6}\left(\frac{1}{3}\right)^{6-6}\left(\frac{2}{3}\right)^6 \\ &= \binom{6}{4}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4 + \binom{6}{5}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5 + \binom{6}{6}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6 \\ &= \frac{6!}{2! \times 4!}\left(\frac{16}{3^6}\right) + \frac{6!}{1! \times 5!}\left(\frac{32}{3^6}\right) + \frac{6!}{0! \times 6!}\left(\frac{64}{3^6}\right) \\ &= \frac{240}{3^6} + \frac{192}{3^6} + \frac{64}{3^6} = \frac{496}{3^6} = \frac{496}{729} \end{aligned} \tag{5}$$

Therefore the probability that in the next six trials, there will be atleast 4 successes is $\frac{496}{729}$.