QUESTION : 12.13.6.9

ROLL NO:EE22BTECH11027
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12.13.6.9.An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

Solution: : The repetition in conduction of experiment constitute the bernoulli trails. Let X denote the number of times the experiment succeeds in all the 6 attempts.
Let p be the probability for the experiment to succeed and q for the failure.

Here, it is given that probability of success is twice that of the failure, so

$$\begin{aligned} p &= 2q \\ p+q &= 1 \\ 3q &= 1 \\ q &= \frac{1}{3} \\ p &= \frac{2}{3} \end{aligned} \tag{1}$$

Clearly X is the benoulli distribution with n=6 and p=$\frac{2}{3}$

Therefore,

$$\Pr(X=k) = \binom{n}{k} q^{n-k} p^k \tag{2}$$

Here, n=6,p= $\frac{2}{3}$,q= $\frac{1}{3}$

where, k= 0,1,2...6

$$\Pr(X=k) = \binom{6}{k}\left(\frac{1}{3}\right)^{6-k}\left(\frac{2}{3}\right)^k \tag{3}$$

We need to find the probability for the experiment to succeed to atleast 4 times, which means that we want probability for experiment to succeed for 4, 5, or 6 times i.e. $\Pr(X \geq 4)$

$$\Pr(X \geq 4) = \Pr(X=4) + \Pr(X=5) + \Pr(X=6) \tag{4}$$

Using equation 3 we get,

$$\begin{aligned} \Pr(X \geq 4) &= \binom{6}{4}\left(\frac{1}{3}\right)^{6-4}\left(\frac{2}{3}\right)^4 + \binom{6}{5}\left(\frac{1}{3}\right)^{6-5}\left(\frac{2}{3}\right)^5 + \binom{6}{6}\left(\frac{1}{3}\right)^{6-6}\left(\frac{2}{3}\right)^6 \\ &= \binom{6}{4}\left(\frac{1}{3}\right)^{2}\left(\frac{2}{3}\right)^4 + \binom{6}{5}\left(\frac{1}{3}\right)^{1}\left(\frac{2}{3}\right)^5 + \binom{6}{6}\left(\frac{1}{3}\right)^{0}\left(\frac{2}{3}\right)^6 \\ &= \frac{6!}{2!\times 4!}\left(\frac{16}{3^6}\right) + \frac{6!}{1!\times 5!}\left(\frac{32}{3^6}\right) + \frac{6!}{0!\times 6!}\left(\frac{64}{3^6}\right) \\ &= \frac{240}{3^6} + \frac{192}{3^6} + \frac{64}{3^6} = \frac{496}{3^6} = \frac{496}{729} \end{aligned} \tag{5}$$

Therefore the probability that in the next six trials, there will be atleast 4 successes is $\frac{496}{729}$.