QUESTION : 12.13.3.7

ROLL NO:EE22BTECH11027
NAME: KATARI SIRI VARSHINI

9.3.27.The probability of a man hitting the target is 0.25. He shoots 7 times.What is the probability of his hitting the target atleast twice?

Solution: : Let the probability of hitting the target be p and not hitting the target be q. Then,

$$\begin{aligned} p &= 0.25 \\ q &= 1 - p \\ &= 0.75 \end{aligned} \tag{1}$$

For gaussian approximation we will start with finding μ(mean) and σ(standard deviation) of the binomial distribution with $n=7$:

$$\begin{aligned} \mu &= np = 7(0.25) = 1.75 \\ \sigma &= \sqrt{npq} = \sqrt{7(0.25)(0.75)} = 1.145 \end{aligned} \tag{2}$$

We know that Q-function is given as

$$\begin{aligned} Q(x) &= \Pr(X > x) \\ &= \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \end{aligned} \tag{3}$$

Using equations (1),(2) and (**??**) we need to find the probability of hitting target atleast twice i.e. $\Pr(X > 1)$,

$$\begin{aligned} \Pr(X > 1) &= \Pr\left(\frac{X - \mu}{\sigma} > \frac{1 - 1.75}{1.145}\right) \\ &= \Pr\left(Z > \frac{-0.75}{1.145}\right) \\ &= \Pr(Z > -0.655) \\ &= Q(-0.655) \\ &= \int_{-0.655}^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} dx \\ &= 0.7437 \end{aligned} \tag{4}$$

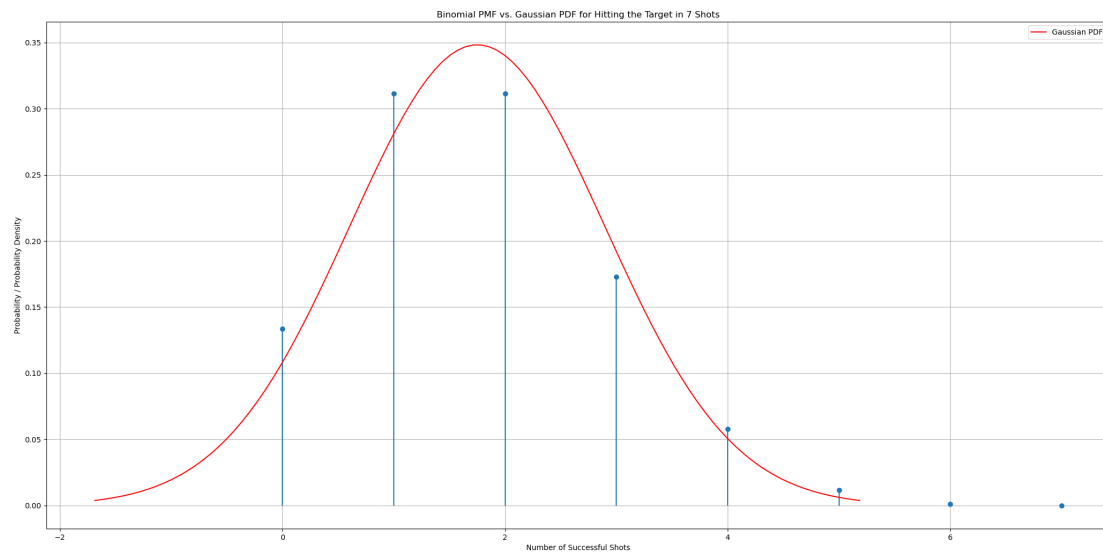$\therefore$ the probability of hitting target atleast twice is 0.7437.

Fig. 0. pmf and pdf