QUESTION : 12.13.3.7

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9.3.27.The probability of a man hitting the target is 0.25. He shoots 7 times.What is the probability of his hitting the target atleast twice?

Solution: :

TABLE 0
VARIABLES

Variable	Value	Description
n	7	Number of trails
p	0.25	The probability of man hitting the target
q	0.75	The probability of man not hitting the target
$\mu = np$	1.75	mean of distribution
$\sigma = \sqrt{npq}$	1.145	variance of distribution
X	$X \geq 2$	Number of times man hits the target

From gaussian,

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

CDF of Y is defined as:

$$F_Y(X) = \Pr(Y < X) \tag{2}$$
$$= \Pr\left(\frac{Y-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right) \tag{3}$$
$$\implies \frac{Y-\mu}{\sigma} \sim N(0,1) \tag{4}$$
$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{X-\mu}{\sigma}\right) \tag{5}$$
$$= \begin{cases} 1 - Q\left(\frac{X-\mu}{\sigma}\right) & X \geq \mu \\ Q\left(\frac{\mu-X}{\sigma}\right) & X < \mu \end{cases} \tag{6}$$

Hence, the probability of hitting target atleast twice using gaussian distribution is:
without correction:

$$\Pr(Y \geq 2) = 1 - \Pr(Y < 2) \tag{7}$$
$$= 1 - F_Y(2) \tag{8}$$
$$\implies \Pr(Y \geq 2) = Q\left(\frac{X-\mu}{\sigma}\right) \tag{9}$$
$$= Q(0.218) \tag{10}$$
$$\Pr(Y \geq 2) = 0.4137 \tag{11}$$

with correction:

$$\Pr(Y \geq 2) = Q\left(\frac{X-0.5-\mu}{\sigma}\right) \tag{12}$$
$$= Q(-0.218) \tag{13}$$
$$\Pr(Y \geq 2) = 0.5862 \tag{14}$$

Hence, the probability of hitting target atleast twice using binomial distribution is:

$$\Pr(X \geq 2) = 1 - \Pr(X < 2) \tag{15}$$
$$= 1 - \sum_{k=0}^{1} {}^{n}C_k p^k (1-p)^{n-k} \tag{16}$$
$$= 0.55 \tag{17}$$