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62.Consider a birth-death process on the state space $\{0, 1, 2, 3\}$. The birth rates are given by $\lambda_0 = 1$, $\lambda_1 = 1$, $\lambda_2 = 2$, and $\lambda_3 = 0$. The death rates are given by $\mu_0 = 0$, $\mu_1 = 1$, $\mu_2 = 1$, and $\mu_3 = 1$. If $[\pi_0, \pi_1, \pi_2, \pi_3]$ is the unique stationary distribution, then $\pi_0 + 2\pi_1 + 3\pi_2 + 4\pi_3$ (rounded off to two decimal places) equals

Solution: : Given, a birth-death process on the state space $\{0, 1, 2, 3\}$ with The birth rates $\lambda_0 = 1$, $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 0$ and the death rates $\mu_0 = 0$, $\mu_1 = 1$, $\mu_2 = 1$, and $\mu_3 = 1$ defined over a stationary distribution $[\pi_0, \pi_1, \pi_2, \pi_3]$. So,

$$\Sigma_{i=0}^{3} \pi_i = 1 \tag{1}$$

The steady-state balanced solution of the birth and death process is given by

$$\begin{aligned} \lambda_0\pi_0 &= \mu_1\pi_1 \quad (\text{for } i = 0) \\ (\lambda_i + \mu_i)\pi_i &= \mu_{i+1}\pi_{i+1} + \lambda_{i-1}\pi_{i-1} (i = 1, 2, \ldots) \end{aligned} \tag{2}$$

Using (2) for each state continuously and solving will yield us with

$$\begin{aligned} \lambda_i\pi_i &= \mu_{i+1}\pi_{i+1} \quad \text{for } i = 0, 1, 2, \ldots \\ \implies \pi_n &= \prod_{i=1}^{n} \frac{\lambda_{i-1}}{\mu_i}\pi_0 \end{aligned} \tag{3}$$

Applying the (3) for the given birth and death process, we get

$$\begin{aligned} \pi_1 &= \pi_0 \\ \pi_2 &= \pi_0 \\ \pi_3 &= 2\pi_0 \end{aligned} \tag{4}$$

Using the results obtained in equations from (4) and (1), we get

$$\begin{aligned} \pi_0 &= \frac{1}{5} \\ \therefore \pi_0 + 2\pi_1 + 3\pi_2 + 4\pi_3 &= (1 + 2 + 3 + 8)\pi_0 \\ &= 2.80 \end{aligned} \tag{5}$$