

# 62.2023

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62.Consider a birth-death process on the state space  $\{0, 1, 2, 3\}$ . The birth rates are given by  $\lambda_0 = 1$ ,  $\lambda_1 = 1$ ,  $\lambda_2 = 2$ , and  $\lambda_3 = 0$ . The death rates are given by  $\mu_0 = 0$ ,  $\mu_1 = 1$ ,  $\mu_2 = 1$ , and  $\mu_3 = 1$ . If  $[\pi_0, \pi_1, \pi_2, \pi_3]$  is the unique stationary distribution, then  $\pi_0 + 2\pi_1 + 3\pi_2 + 4\pi_3$  (rounded off to two decimal places) equals

**Solution:** : Given, a birth-death process on the state space  $\{0, 1, 2, 3\}$  with The birth rates  $\lambda_0 = 1$ ,  $\lambda_1 = 1$ ,  $\lambda_2 = 2$ ,  $\lambda_3 = 0$  and the death rates  $\mu_0 = 0$ ,  $\mu_1 = 1$ ,  $\mu_2 = 1$ , and  $\mu_3 = 1$  defined over a stationary distribution  $[\pi_0, \pi_1, \pi_2, \pi_3]$ .

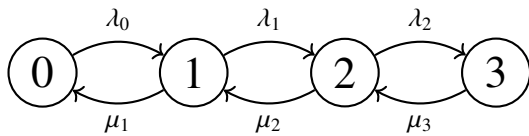


Fig. 0. Markov Chain for Birth-Death Process

$$\text{Let, } \pi = \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} \quad (1)$$

The steady-state balanced solution of the birth and death process is given by

$$\begin{aligned} \lambda_0 \pi_0 &= \mu_1 \pi_1 \quad (\text{for } i = 0) \\ (\lambda_i + \mu_i) \pi_i &= \mu_{i+1} \pi_{i+1} + \lambda_{i-1} \pi_{i-1} \quad (i = 1, 2, \dots) \end{aligned} \quad (2)$$

$\therefore$  Transition Matrix  $P$  for the given states is given by

$$\begin{aligned} P &= \begin{bmatrix} -\lambda_0 & \mu_1 & 0 & 0 \\ \lambda_0 & -(\lambda_1 + \mu_1) & \mu_2 & 0 \\ 0 & \lambda_1 & -(\lambda_2 + \mu_2) & \mu_3 \\ 0 & 0 & \lambda_2 & -\mu_3 \end{bmatrix} \\ \Rightarrow P &= \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 2 & -1 \end{bmatrix} \end{aligned} \quad (3)$$

Stationary distribution  $\pi$  satisfies,

$$\begin{aligned} P\pi &= 0 \\ \sum_{i=0}^3 \pi_i &= 1 \end{aligned} \quad (4)$$

final matrix equation can be deduced as

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (5)$$

From the above we get

$$\pi = \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \\ \frac{1}{5} \\ \frac{2}{5} \end{bmatrix} \quad (6)$$

Using the results obtained in equations from (5), we get

$$\therefore \pi_0 + 2\pi_1 + 3\pi_2 + 4\pi_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \cdot \pi = 2.80 \quad (7)$$