

62.2023

ROLL NO:EE22BTECH11027
NAME: KATARI SIRI VARSHINI

62.Consider a birth-death process on the state space $\{0, 1, 2, 3\}$. The birth rates are given by $\lambda_0 = 1$, $\lambda_1 = 1$, $\lambda_2 = 2$, and $\lambda_3 = 0$. The death rates are given by $\mu_0 = 0$, $\mu_1 = 1$, $\mu_2 = 1$, and $\mu_3 = 1$. If $[\pi_0, \pi_1, \pi_2, \pi_3]$ is the unique stationary distribution, then $\pi_0 + 2\pi_1 + 3\pi_2 + 4\pi_3$ (rounded off to two decimal places) equals

Solution: : Given, a birth-death process on the state space $\{0, 1, 2, 3\}$ with The birth rates $\lambda_0 = 1$, $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 0$ and the death rates $\mu_0 = 0$, $\mu_1 = 1$, $\mu_2 = 1$, and $\mu_3 = 1$ defined over a stationary distribution $[\pi_0, \pi_1, \pi_2, \pi_3]$.

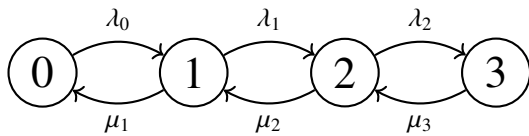


Fig. 0. Markov Chain for Birth-Death Process

$$\text{Let, } \pi = \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} \quad (1)$$

The steady-state balanced solution of the birth and death process is given by

$$\begin{aligned} \lambda_0 \pi_0 &= \mu_1 \pi_1 \quad (\text{for } i = 0) \\ (\lambda_i + \mu_i) \pi_i &= \mu_{i+1} \pi_{i+1} + \lambda_{i-1} \pi_{i-1} \quad (i = 1, 2, \dots) \end{aligned} \quad (2)$$

\therefore Transition Matrix P for the given states is given by

$$P = \begin{bmatrix} -\lambda_0 & \mu_1 & 0 & 0 \\ \lambda_0 & -(\lambda_1 + \mu_1) & \mu_2 & 0 \\ 0 & \lambda_1 & -(\lambda_2 + \mu_2) & \mu_3 \\ 0 & 0 & \lambda_2 & -\mu_3 \end{bmatrix} \quad (3)$$

$$\Rightarrow P = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 2 & -1 \end{bmatrix}$$

Stationary distribution π satisfies $P\pi = 0$ and $\sum_{i=0}^3 \pi_i = 1$. Converting P to echelon form we get,

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

From the above we get

$$\pi = \begin{bmatrix} \frac{1}{5} \\ \frac{1}{5} \\ \frac{1}{5} \\ \frac{2}{5} \end{bmatrix} \quad (5)$$

Using the results obtained in equations from (5), we get

$$\therefore \pi_0 + 2\pi_1 + 3\pi_2 + 4\pi_3 = 2.80 \quad (6)$$