Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} \qquad (1)$$

1 Vectors

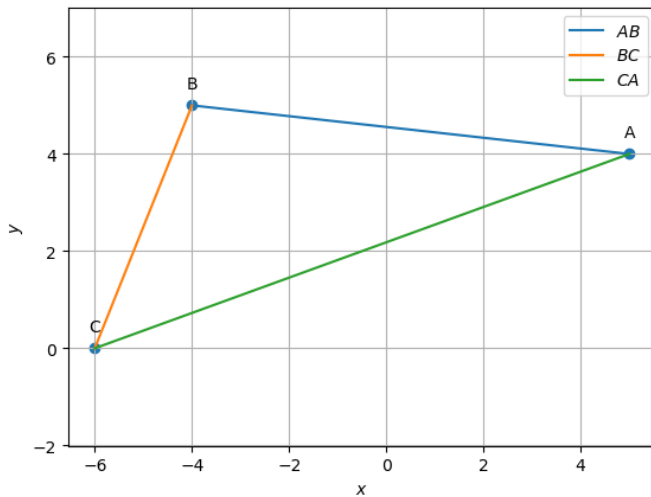| parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} -9 \\ 1 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -2 \\ 5 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} 11 \\ 4 \end{pmatrix}$	CA
$\|A - B\|$	5.38	length of AB
$\|B - C\|$	9.06	length of BC
$\|C - A\|$	11.70	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} 1 \\ 9 \end{pmatrix}$	AB
c_1	10	
$\mathbf{n_2}$	$\begin{pmatrix} -5 \\ 2 \end{pmatrix}$	BC
c_2	-2	
$\mathbf{n_3}$	$\begin{pmatrix} 4 \\ -11 \end{pmatrix}$	CA
c_3	-4	
Area	23.5	Area of Triangle
$\angle A$	26.32°	
$\angle B$	105.46°	Angles
$\angle C$	48.22°	

TABLE 1: Vectors.

Fig. 1: triangle plotted using python

2 Median

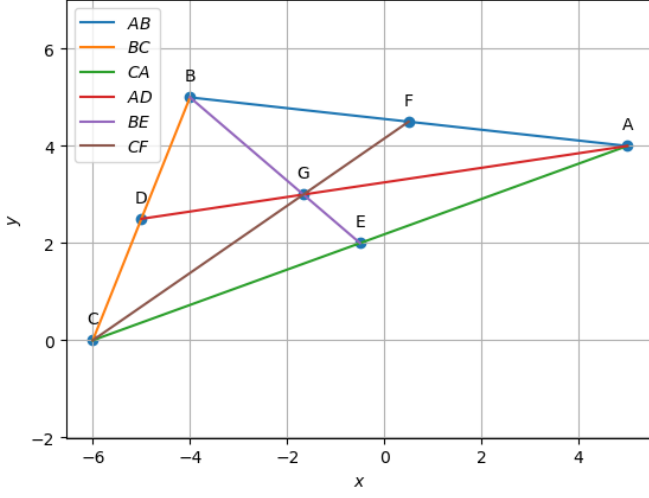| parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} -5 \\ 2.5 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} -0.5 \\ 2 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} 0.5 \\ 4.5 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} -10 \\ -1.5 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -1.5 \\ 10 \end{pmatrix}$	
c_4	32.5	
$\mathbf{m_5}$	$\begin{pmatrix} 3.5 \\ -3 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} -3 \\ 3.5 \end{pmatrix}$	
c_5	-5.5	
$\mathbf{m_6}$	$\begin{pmatrix} 6.5 \\ 4.5 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 4.5 \\ -6.5 \end{pmatrix}$	
c_6	27	
$\mathbf{G}$	$\begin{pmatrix} -1.67 \\ 3 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$ $\frac{CG}{GF}$ $\frac{AG}{GD}$	2	Division ratio by $\mathbf{G}$
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.
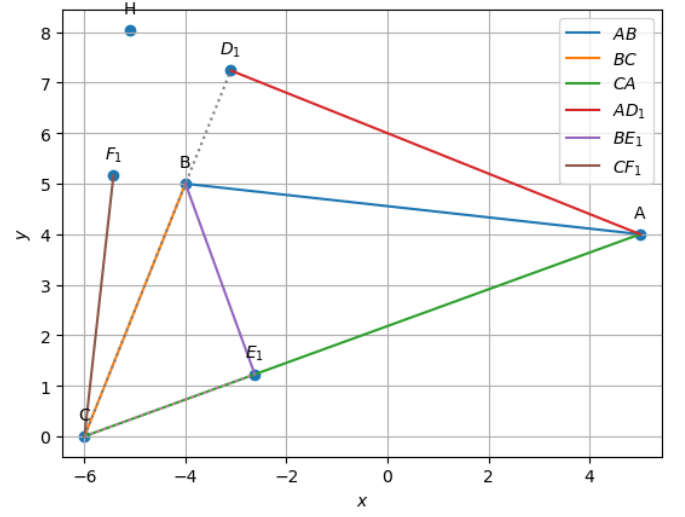
Fig. 2: medians plotted using python

Fig. 3: altitudes plotted using python

4 PERPENDICULAR BISECTOR

3 ALTITUDE

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix} -3.10 \\ 7.24 \end{pmatrix}$	Foot of altitude from $\mathbf{A}$
$\mathbf{E_1}$	$(-2.63, 1.23)$	Foot of altitude from $\mathbf{B}$
$\mathbf{F_1}$	$\begin{pmatrix} -5.43 \\ 5.16 \end{pmatrix}$	Foot of altitude from $\mathbf{C}$
$\mathbf{m_7}$	$\begin{pmatrix} -8.10 \\ 3.24 \end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix} 3.24 \\ 8.10 \end{pmatrix}$	
c_7	48.62	
$\mathbf{m_8}$	$\begin{pmatrix} 1.37 \\ -3.77 \end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix} -3.77 \\ -1.37 \end{pmatrix}$	
c_8	8.23	
$\mathbf{m_9}$	$\begin{pmatrix} 0.57 \\ 5.16 \end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix} 5.16 \\ -0.57 \end{pmatrix}$	
c_9	-30.95	
$\mathbf{H}$	$\begin{pmatrix} -5.11 \\ 8.04 \end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix} -5 \\ 2 \end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix} -2 \\ -5 \end{pmatrix}$	
c_{10}	-2.5	
$\mathbf{m_{11}}$	$\begin{pmatrix} -4 \\ 11 \end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix} -11 \\ -4 \end{pmatrix}$	
c_{11}	-2.5	
$\mathbf{m_{12}}$	$\begin{pmatrix} -1 \\ -9 \end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix} 9 \\ -1 \end{pmatrix}$	
c_{12}	0	
$\mathbf{O}$	$\begin{pmatrix} 0.05 \\ 0.48 \end{pmatrix}$	Circumcentre
$\|\mathbf{O - A}\|$		$OA = OB = OC = R$
$\|\mathbf{O - B}\|$	6.07	
$\|\mathbf{O - C}\|$		
R		
$\angle BOC$	52.65°	$\angle BOC = 2\angle BAC$
$\angle BAC$	26.32°	
$\angle AOC$	149.08°	$\angle AOC = 2\angle ABC$
$\angle ABC$	105.46°	
$\angle AOB$	263.57°	$\angle AOB = 2\angle BCA$
$\angle BCA$	48.22°	

TABLE 4: Perpendicular Bisector.

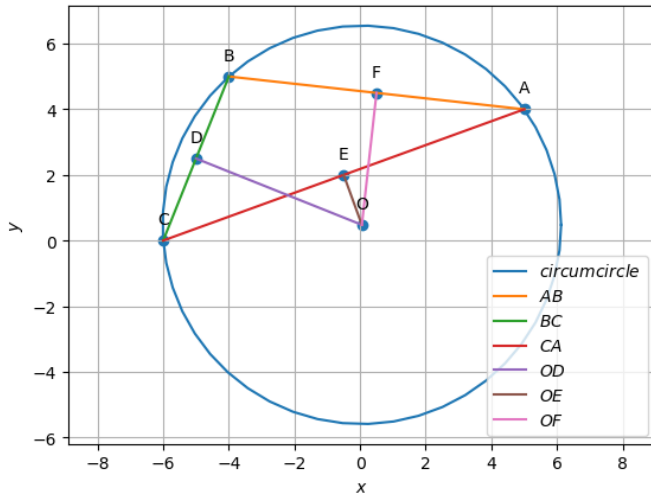Fig. 4: perpendicular bisectors plotted using python

| parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix} 1.93 \\ 0.23 \end{pmatrix}$	
$\mathbf{n_{13}}$	$\begin{pmatrix} -0.23 \\ -1.93 \end{pmatrix}$	AI
c_{13}	-6.58	
$\mathbf{m_{14}}$	$\begin{pmatrix} 0.62 \\ -1.04 \end{pmatrix}$	
$\mathbf{m_{14}}$	$\begin{pmatrix} 1.04 \\ 0.62 \end{pmatrix}$	BI
c_{14}	-1.04	
$\mathbf{m_{15}}$	$\begin{pmatrix} -1.31 \\ -1.27 \end{pmatrix}$	
$\mathbf{n_{15}}$	$\begin{pmatrix} 1.27 \\ -1.31 \end{pmatrix}$	CI
c_{15}	-7.62	
$\mathbf{I}$	$\begin{pmatrix} -2.84 \\ 3.06 \end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix} -4.50 \\ 3.73 \end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix} -2.22 \\ 1.37 \end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix} -2.64 \\ 4.85 \end{pmatrix}$	Point of contact with AB
$\|\mathbf{I} - \mathbf{D_3}\|$		
$\|\mathbf{I} - \mathbf{E_3}\|$		
$\|\mathbf{I} - \mathbf{F_3}\|$	1.80	$ID_3 = IE_3 = IF_3 = r$
r		
$\angle BAI$		
$\angle CAI$	13.16°	$\angle BAI = \angle CAI$
$\angle ABI$		
$\angle CBI$	52.73°	$\angle ABI = \angle CBI$
$\angle ACI$		
$\angle BCI$	24.11°	$\angle ACI = \angle BCI$

TABLE 5: Angle Bisectors.
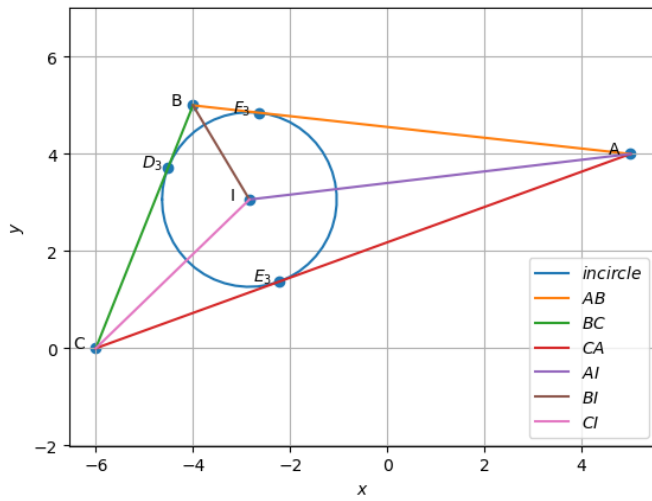
Fig. 5: Angle bisectors plotted using python