SIT330-770: Natural Language Processing

Week 1 - Information Retrieval Part 1

Inverted Indices Scoring, term weighting and the vector space model

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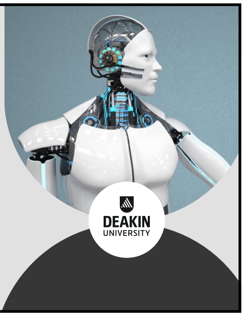

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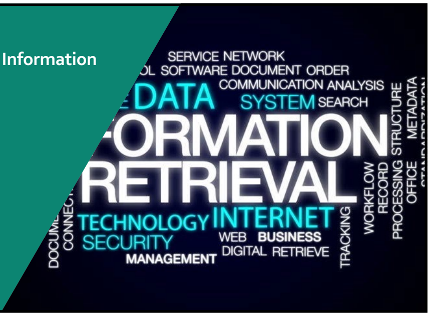
1

Introducing Information Retrieval



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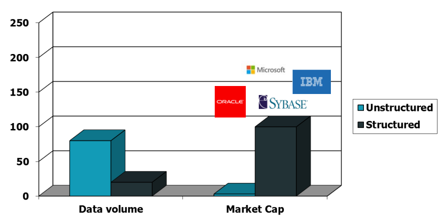
Information Retrieval

- Information Retrieval (IR) is finding material (usually documents) of an unstructured nature (usually text) that satisfies an information need from within large collections (usually stored on computers).
  - These days we frequently think first of web search, but there are many other cases:
    - E-mail search
    - Searching your laptop
    - Corporate knowledge bases
    - Legal information retrieval

3

Unstructured (text) vs. structured (database) data in 1996



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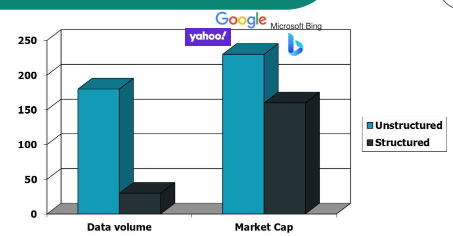
Unstructured (text) vs. structured (database) data today



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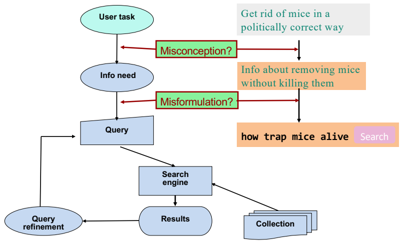
Basic assumptions of Information Retrieval

- Collection: A set of documents
  - Assume it is a static collection for the moment
- Goal: Retrieve documents with information that is relevant to the user's information need and helps the user complete a task

6

The classic search model



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How good are the retrieved docs?

- *Precision* : Fraction of retrieved docs that are relevant to the user's information need
- *Recall* : Fraction of relevant docs in collection that are retrieved
  - More precise definitions and measurements to follow later

8

8

Term-document incidence matrices

9

9

Unstructured data in 1620

- Which plays of Shakespeare contain the words ***Brutus*** AND ***Caesar*** but NOT ***Calpurnia***?
- One could grep all of Shakespeare's plays for ***Brutus*** and ***Caesar***, then strip out lines containing ***Calpurnia***?
- Why is that not the answer?
  - Slow (for large corpora)
  - NOT ***Calpurnia*** is non-trivial
  - Other operations (e.g., find the word ***Romans*** near ***countrymen***) not feasible
  - Ranked retrieval (best documents to return)
    - Later lectures

10

10

Term-document incidence matrices

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

1 if play contains word, 0 otherwise

Brutus *AND* ***Caesar*** *BUT NOT* ***Calpurnia***

11

11

Incidence vectors

- So, we have a 0/1 vector for each term.
- To answer query: take the vectors for ***Brutus, Caesar*** and ***Calpurnia*** (complemented) ➔ bitwise *AND*.
  - 110100 *AND*
  - 110111 *AND*
  - 101111 =
  - **100100**

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
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Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

12

12

Answers to query

- Antony and Cleopatra, Act III, Scene ii

Agrippa [Aside to DOMITIUS ENOBARBUS]: Why, Enobarbus,
When Antony found Julius ***Caesar*** dead,
He cried almost to roaring; and he wept
When at Philippi he found ***Brutus*** slain.

- Hamlet, Act III, Scene ii

Lord Polonius: I did enact Julius ***Caesar*** I was killed i' the
Capitol; ***Brutus*** killed me.


13

Bigger collections

- Consider N = 1 million documents, each with about 1000 words.
- Avg 6 bytes/word including spaces/punctuation
  - 6GB of data in the documents.
- Say there are M = 500K *distinct* terms among these.

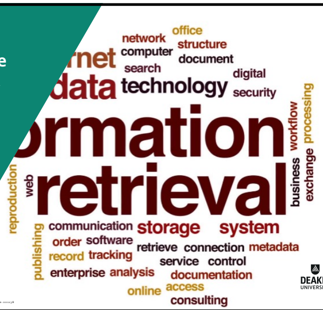
14

Can't build the matrix

- 500K x 1M matrix has half-a-trillion 0's and 1's.
- But it has no more than one billion 1's. Why?
  - matrix is extremely sparse.
- What's a better representation?
  - We only record the 1 positions.

15

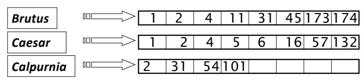
The Inverted Index
The key data structure underlying modern IR



16

Inverted index

- For each term *t*, we must store a list of all documents that contain *t*.
  - Identify each doc by a **docID** (doc serial number)
- Can we use fixed-size arrays for this?

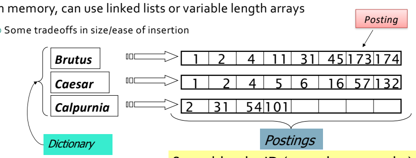


What happens if the word ***Caesar*** is added to document 14?

17

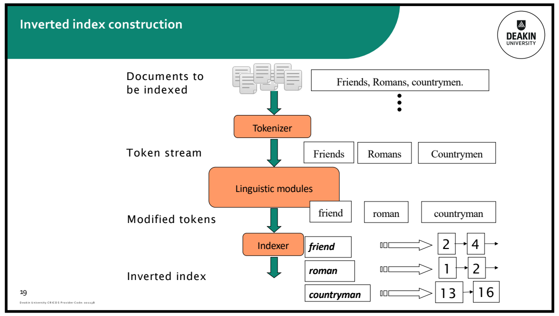
Inverted index

- We need variable-size postings lists
  - On disk, a continuous run of postings is normal and best
  - In memory, can use linked lists or variable length arrays
    - Some tradeoffs in size/ease of insertion



Sorted by docID (more later on why).

18



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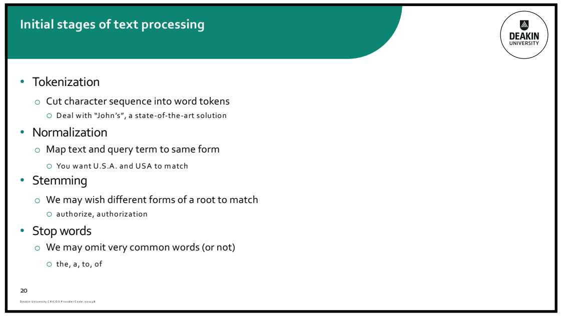

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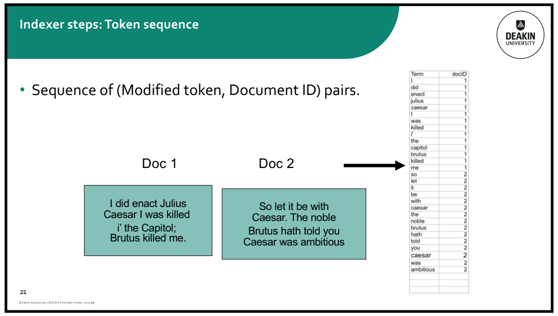

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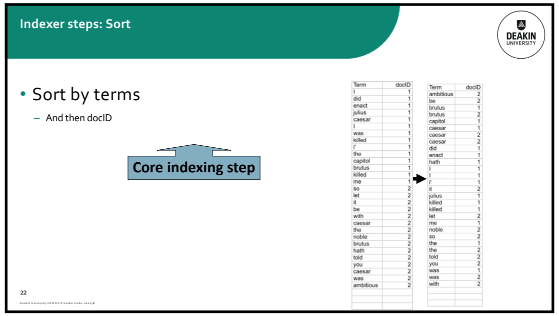

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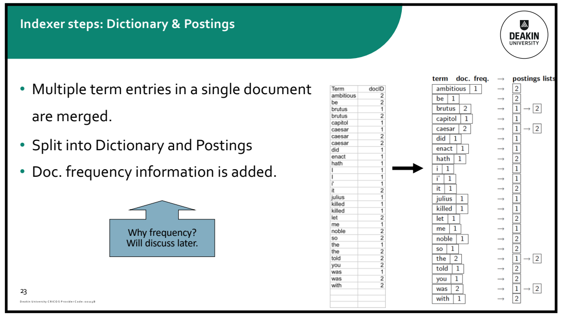

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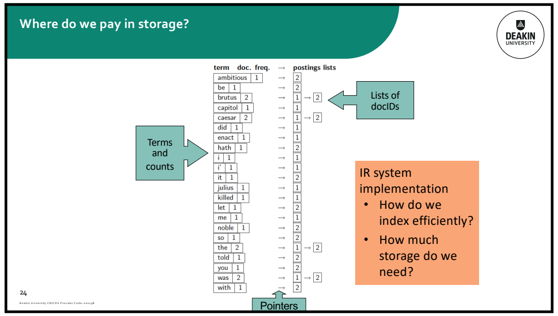
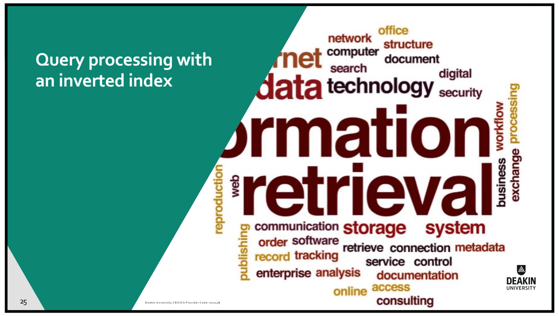


24



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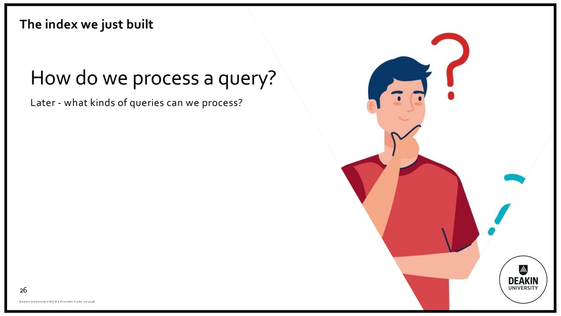

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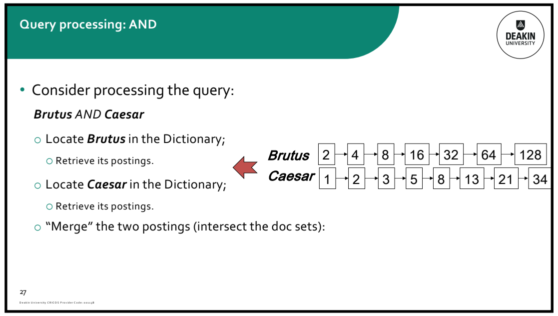

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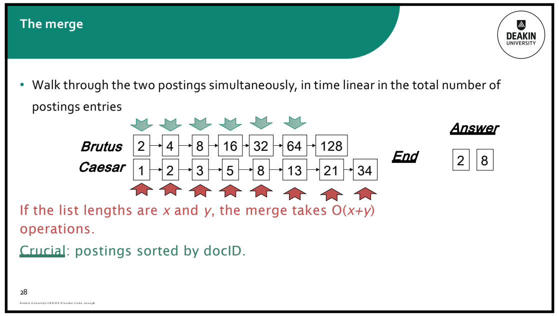

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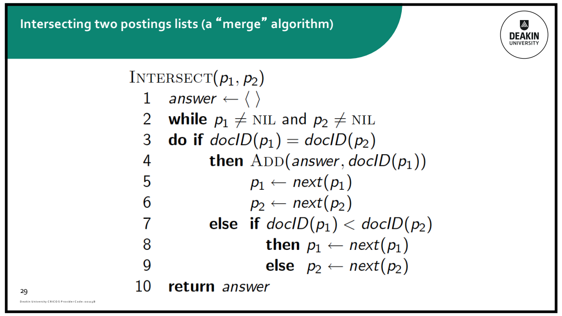

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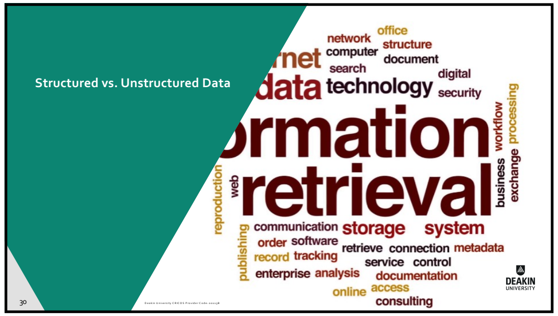


30

IR vs. databases: Structured vs unstructured data

- Structured data tends to refer to information in "tables"

Employee	Manager	Salary
Smith	Jones	50000
Chang	Smith	60000
Ivy	Smith	50000

Typically allows numerical range and exact match (for text) queries, e.g.,
Salary < 60000 AND Manager = Smith.

31

Unstructured data

- Typically refers to free text
- Allows
  - Keyword queries including operators
  - More sophisticated "concept" queries e.g.,
    - find all web pages dealing with *drug abuse*
- Classic model for searching text documents

32

Semi-structured data: Fielded Indices

- In fact almost no data is "unstructured"
- E.g., this slide has distinctly identified zones such as the *Title* and *Bullets*
  - … to say nothing of linguistic structure
- Facilitates "semi-structured" search such as
  - *Title* contains data AND *Bullets* contain search
- Or even
  - *Title* is about Object Oriented Programming AND *Author* something like stro*rup
  - where * is the wild-card operator

33

Modeling in Information Retrieval

34

IR Models

- **Modeling** in IR is a complex process aimed at producing a ranking function
  - **Ranking function:** a function that assigns scores to documents with regard to a given query
- This process consists of two main tasks:
  - The conception of a logical framework for representing documents and queries
  - The definition of a ranking function that allows quantifying the similarities among documents and queries

35

Modeling and Ranking

- IR systems usually adopt **index terms** to index and retrieve documents
- Index term:
  - In a restricted sense: it is a keyword that has some meaning on its own; usually plays the role of a noun
  - In a more general form: it is any word that appears in a document
- Retrieval based on index terms can be implemented efficiently
- Also, index terms are simple to refer to in a query
- Simplicity is important because it reduces the effort of query formulation

36

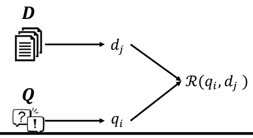
Ranking

- A **ranking** is an ordering of the documents that (hopefully) reflects their **relevance** to a user query
- Thus, any IR system has to deal with the problem of predicting which documents the users will find relevant
- This problem naturally embodies a degree of uncertainty, or vagueness

37

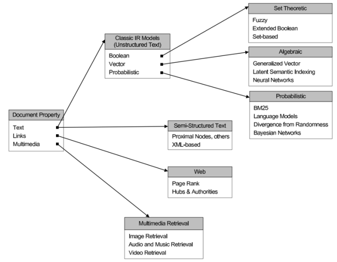
IR Models

- An IR model is a quadruple $[\boldsymbol{D}, \boldsymbol{Q}, \mathcal{F}, \mathcal{R}(q_i, d_j)]$ where:
  1. $\boldsymbol{D}$ is a set of logical views for the documents in the collection
  2. $\boldsymbol{Q}$ is a set of logical views for the user queries
  3. $\mathcal{F}$ is a framework for modeling documents and queries
  4. $\mathcal{R}(q_i, d_j)$ is a ranking function



38

A Taxonomy of IR Models



39

Modeling in Information Retrieval

- In this lecture, we will discuss the following models:
  - The Boolean Model
  - The Vector Model
  - Probabilistic Model

40

The Boolean Model

41

Boolean queries: Exact match

- The Boolean retrieval model is being able to ask a query that is a Boolean expression:
  - Boolean Queries are queries using *AND, OR* and *NOT* to join query terms
    - Views each document as a set of words
    - Is precise: document matches condition or not.
  - Perhaps the simplest model to build an IR system on
- Primary commercial retrieval tool for 3 decades.
- Many search systems you still use are Boolean:
  - Email, library catalog, Mac OS X Spotlight

42

Example: WestLaw

- Largest commercial (paying subscribers) legal search service (started 1975; ranking added 1992; new federated search added 2010)
- Tens of terabytes of data; ~700,000 users
- Majority of users *still* use boolean queries
- Example query:
  – What is the statute of limitations in cases involving the federal tort claims act?
  – LIMIT! /3 STATUTE ACTION /S FEDERAL /2 TORT /3 CLAIM
    - /3 = within 3 words, /S = in same sentence

43

Example: WestLaw

- Another example query:
  - Requirements for disabled people to be able to access a workplace
  - disabl! /p access! /s work-site work-place (employment /3 place
- Note that SPACE is disjunction, not conjunction!
- Long, precise queries; proximity operators; incrementally developed; not like web search
- Many professional searchers still like Boolean search
  - You know exactly what you are getting
- But that doesn't mean it actually works better….

44

Boolean queries: More general merges

- Exercise: Adapt the merge for the queries:

  Brutus *AND NOT* ***Caesar***

  Brutus *OR NOT* ***Caesar***

- Can we still run through the merge in time O(*x*+*y*)? What can we achieve?

45

Merging

- What about an arbitrary Boolean formula?

  (Brutus *OR* ***Caesar)*** *AND NOT* ***(Antony*** *OR* ***Cleopatra)***

- Can we always merge in "linear" time?
  - Linear in what?
- Can we do better?

46

Query optimization

- What is the best order for query processing?
- Consider a query that is an *AND* of *n* terms.
- For each of the *n* terms, get its postings, then *AND* them together.

Brutus	2, 4, 8, 16, 32, 64, 128
Caesar	1, 2, 3, 5, 8, 16, 21, 34
Calpurnia	13, 16

Query: ***Brutus*** *AND* ***Calpurnia*** *AND* ***Caesar***

47

Query optimization example

- Process in order of increasing freq:
  - *start with smallest set, then keep cutting further.*

This is why we kept document freq. in dictionary

Brutus	2, 4, 8, 16, 32, 64, 128
Caesar	1, 2, 3, 5, 8, 16, 21, 34
Calpurnia	13, 16

Execute the query as (***Calpurnia*** *AND* ***Brutus)*** *AND* ***Caesar***.

48

More general optimization

- e.g., *(**madding** OR **crowd**) AND (**ignoble** OR **strife**)*
- Get doc. freq.'s for all terms.
- Estimate the size of each *OR* by the sum of its doc. freq.'s (conservative).
- Process in increasing order of *OR* sizes.

49

Exercise

- Recommend a query processing order for

***(tangerine** OR **trees)** AND*
***(marmalade** OR **skies)** AND*
(kaleidoscope** OR **eyes)

Term	Freq
eyes	213312
kaleidoscope	87009
marmalade	107913
skies	271658
tangerine	46653
trees	316812

- Which two terms should we process first?

50

Phrase queries and positional indexes

51

Phrase queries

- We want to be able to answer queries such as **"stanford university"** – as a phrase
- Thus the sentence *"I went to university at Stanford"* is not a match.
  - The concept of phrase queries has proven easily understood by users; one of the few "advanced search" ideas that works
  - Many more queries are implicit phrase queries
- For this, it no longer suffices to store only
  <term : docs> entries

52

Standard Solution: Positional indexes

- In the postings, store, for each ***term*** the position(s) in which tokens of it appear:

<***term***, number of docs containing ***term***;
doc1: position1, position2 … ;
doc2: position1, position2 … ;
etc.>

53

Positional index example

<***be***: 993427;
1: 7, 18, 33, 72, 86, 231;
2: 3, 149;
4: 17, 191, 291, 430, 434;
5: 363, 367, …>

Which of docs 1,2,4,5 could contain "***to be or not to be***"?

- For phrase queries, we use a merge algorithm recursively at the document level
- But we now need to deal with more than just equality

54

Processing a phrase query

- Extract inverted index entries for each distinct term: **to, be, or, not**
- Merge their doc:position lists to enumerate all positions with "**to be or not to be**"

➢**to**:
  - 2:1,17,74,222,551; 4:8,16,190,429,433; 7:13,23,191; ...

➢**be**:
  - 1:17,19; 4:17,191,291,430,434; 5:14,19,101; ...

- Same general method for proximity searches

55

55

Proximity queries

- LIMIT! /3 STATUTE /3 FEDERAL /2 TORT
  - Again, here, /k means "within k words of".
- Clearly, positional indexes can be used for such queries.
- Exercise: Adapt the linear merge of postings to handle proximity queries. Can you make it work for any value of k?
  - This is a little tricky to do correctly and efficiently

56

56

Positional index size

- A positional index expands postings storage *substantially*
  - Even though indices can be compressed
- Nevertheless, a positional index is now standardly used because of the power and usefulness of phrase and proximity queries ... whether used explicitly or implicitly in a ranking retrieval system.

57

57

Positional index size

- Need an entry for each occurrence, not just once per document
- Index size depends on average document size
  - Average web page has <1000 terms
  - SEC filings, books, even some epic poems ... easily 100,000 terms
- Consider a term with frequency 0.1%

Document size	Postings	Positional postings
1000	1	1
100,000	1	100

58

58

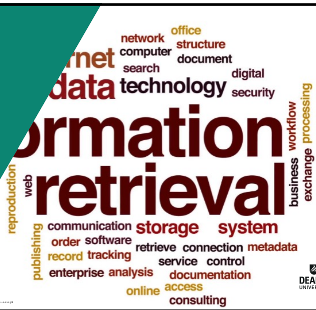
Rules of thumb

- A positional index is 2–4 as large as a non-positional index
- Positional index size 35–50% of volume of original text
  - Caveat: all of this holds for "English-like" languages

59

59

The Vector Model



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60

Outline

- Ranked retrieval
- Scoring documents
- Term frequency
- Collection statistics
- Weighting schemes
- Vector space scoring

61

Ranked retrieval

62

Ranked retrieval

- So far, our queries have all been Boolean
  - Documents either match or don't
- Good for expert users with precise understanding of their needs and the collection
  - Also good for applications: Applications can easily consume 1000s of results
- Not good for the majority of users
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work)
  - Most users don't want to wade through 1000s of results
    - This is particularly true of web search

63

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query 1: "*standard user dlink 650*" → 200,000 hits
- Query 2: "*standard user dlink 650 no card found*": 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

64

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval, the system returns an ordering over the (top) documents in the collection for a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval has normally been associated with free text queries and vice versa

65

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
  - Indeed, the size of the result set is not an issue
  - We just show the top k (≈ 10) results
  - We don't overwhelm the user
  - Premise: the ranking algorithm works

66

67

Scoring documents

68

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query "match".

69

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this

70

Take 1: Jaccard coefficient

- $jaccard(A,B) = |A \cap B| / |A \cup B|$
- $jaccard(A,A) = 1$
- $jaccard(A,B) = 0$ if $A \cap B = 0$
- A and B don't have to be the same size
- Always assigns a number between 0 and 1

71

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

72

Issues with Jaccard for scoring

- It doesn't consider *term frequency* (how many times a term occurs in a document)
- Rare terms in a collection are more informative than frequent terms. Jaccard doesn't consider this information
- We need a more sophisticated way of normalizing for length
- Later in this lecture, we'll use $|A \cap B|/\sqrt{|A \cup B|}$
- . . . instead of $|A \cap B|/|A \cup B|$ (Jaccard) for length normalization.

73

Term frequency

74

Recall: Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

75

Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a count vector in $\mathbb{N}^{v}$: a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

76

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary* and *Mary is quicker than John* have the same vectors
- This is called the bag of words model
- In a sense, this is a step back: The positional index was able to distinguish these two documents
- The IIR book considers "recovering" positional information
- For now: bag of words model

77

Term frequency tf

- The term frequency $tf_{t,d}$ of term t in document d is defined as the number of times that t occurs in d
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term
  - But not 10 times more relevant
- Relevance does not increase proportionally with term frequency

NB: frequency = count in IR

78

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10}(tf_{t,d}), & \text{if } tf_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

  - 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.
- Score for a document-query pair: sum over terms t in both q and d:

$$score(q,d) = \sum_{t \in q \cap d} [1 + \log_{10}(tf_{t,d})]$$

- The score is 0 if none of the query terms is present in the document

79

Collection statistics

80

Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- → We want a high weight for rare terms like *arachnocentric*

81

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance-
- → For frequent terms, we want high positive weights for words like *high, increase, and line*
- But lower weights than for rare terms
- We will use document frequency (df) to capture this

82

idf weight

- df_t is the document frequency of t: the number of documents that contain t
  - df_t is an inverse measure of the informativeness of t
  - $df_t \le N$
- We define the idf (inverse document frequency) of t by

$$idf_t = \log_{10}\left({}^{N}/_{df_t}\right)$$

  - We use $\log_{10}({}^{N}/_{df_t})$ instead of ${}^{N}/_{df_t}$ to "dampen" the effect of idf

Will turn out the base of the log is immaterial.

83

idf example, suppose N = 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$idf_t = \log_{10}\left({}^{N}/_{df_t}\right)$$

There is one idf value for each term t in a collection

84

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like
  - iPhone
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

85

Collection vs. Document frequency

- The collection frequency of *t* is the number of occurrences of *t* in the collection, counting multiple occurrences
- Example:

Word	Collection frequency	Document frequency
insurance	10,440	3,997
try	10,422	8,760

- Which word is a better search term (and should get a higher weight)?

86

TF-IDF Properties

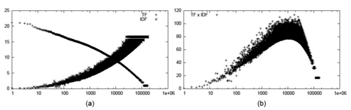
- Consider the tf, idf, and $tf-idf$ weights for the Wall Street Journal reference collection
- To study their behavior, we would like to plot them together
- While idf is computed over all the collection, tf is computed on a per document basis. Thus, we need a representation of tf based on all the collection, which is provided by the term collection frequency
- This reasoning leads to the following tf and idf term weights:

$$w_t = 1 + log_{10}\sum_{j=1}^{N} tf_{i,j}, \quad idf_t = log_{10}\left({}^{N}/_{df_t}\right)$$

87

TF-IDF Properties

- Plotting tf and idf in logarithmic scale yields
- We observe that tf and idf weights present power-law behaviors that balance each other



- The terms of intermediate idf values display maximum $tf-idf$ weights and are most interesting for ranking

88

Weighting schemes

89

tf-idf weighting

- The $tf-idf$ weight of a term is the product of its tf weight and its idf weight

$$tf-idf_{t,d} = \left(1 + log_{10}(tf_{t,d})\right) \times log_{10}\left({}^{N}/_{df_t}\right)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in $tf-idf$ is a hyphen, not a minus sign!
  - Alternative names: $tf.idf$, $tf \times idf$
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

90

Score for a document given a query

$$score(q,d) = \sum_{t \in q \cap d} tf-idf_{t,d}$$

- There are many variants
  - How "tf" is computed (with/without logs)
  - Whether the terms in the query are also weighted
  - …

91

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91

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

92

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92

Vector space scoring

93

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93

Documents as vectors

- So we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero

94

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94

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents

95

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95

Formalizing vector space proximity

- First cut: distance between two points
  - (= distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is large for vectors of different lengths

96

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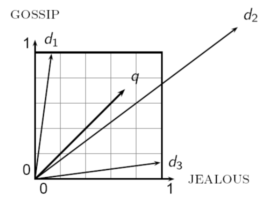
96

Why distance is a bad idea

- The Euclidean distance between q and d_2 is large even though the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar.



97

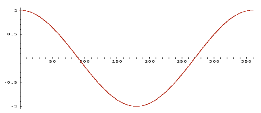
Use angle instead of distance

- Thought experiment: take a document *d* and append it to itself. Call this document *d′*
- "Semantically" d and d′ have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity
- Key idea: Rank documents according to angle with query

98

From angles to cosines

- The following two notions are equivalent
  - Rank documents in decreasing order of the angle between query and document
  - Rank documents in increasing order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0°, 180°]



But how – *and why* – should we be computing cosines?

99

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
  - Long and short documents now have comparable weights

100

cosine(query,document)

Dot product | Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term *i* in the query
d_i is the tf-idf weight of term *i* in the document

cos(*q*,*d*) is the cosine similarity of *q* and *d* … or, equivalently, the cosine of the angle between *q* and *d*.

101

Cosine for length-normalized vectors

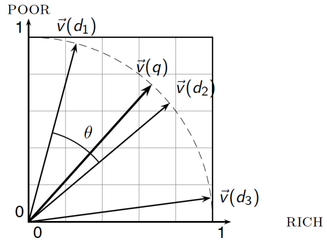
- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q}\cdot\vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

  - for q, d length-normalized.

102

Cosine similarity illustrated



103

Cosine similarity amongst 3 documents

- How similar are the novels:
  - SaS: *Sense and Sensibility*
  - PaP: *Pride and Prejudice*, and
  - WH: *Wuthering Heights?*

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting

104

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈ 0.789 × 0.832 + 0.515 × 0.555 + 0.335 × 0.0 + 0.0 × 0.0
≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69

Why do we have cos(SaS,PaP) > cos(SaS,WH)?

105

Computing cosine scores for ranking

```
CosineScore(q)
float Scores[N] = 0
float Length[N]
for each query term t
do calculate w_{t,q} and fetch postings list for t
    for each pair(d, tf_{t,d}) in postings list
    do Scores[d] += w_{t,d} × w_{t,q}
Read the array Length
for each d
do Scores[d] = Scores[d]/Length[d]
return Top K components of Scores[]
```

106

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1+\log(\mathrm{tf}_{t,d})$	t (idf)	$\log\frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5+\frac{0.5\times\mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0,\log\frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases}1 & \text{if } \mathrm{tf}_{t,d}>0\\ 0 & \text{otherwise}\end{cases}$			b (byte size)	$1/\mathit{CharLength}^{\alpha}$, $\alpha<1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t\in d}(\mathrm{tf}_{t,d}))}$				

Columns headed 'n' are acronyms for weight schemes

Why is the base of the log in idf immaterial?

107

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation *ddd.qqq*, using the acronyms from the previous table
- A very standard weighting scheme is: lnc.ltc A bad idea?
- Document: logarithmic tf (l as first character), no idf and cosine normalization
- Query: logarithmic tf (l in leftmost column), idf (t in second column), no normalization …

108

tf-idf example: lnc.ltc

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query						Document				Prod
	tf-raw	tf-wt	df	idf	wt	n'lize	tf-raw	tf-wt	wt	n'lize	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Exercise: what is N, the number of docs?

Doc length $= \sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

109

109

Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., K = 10) to the user

110

110