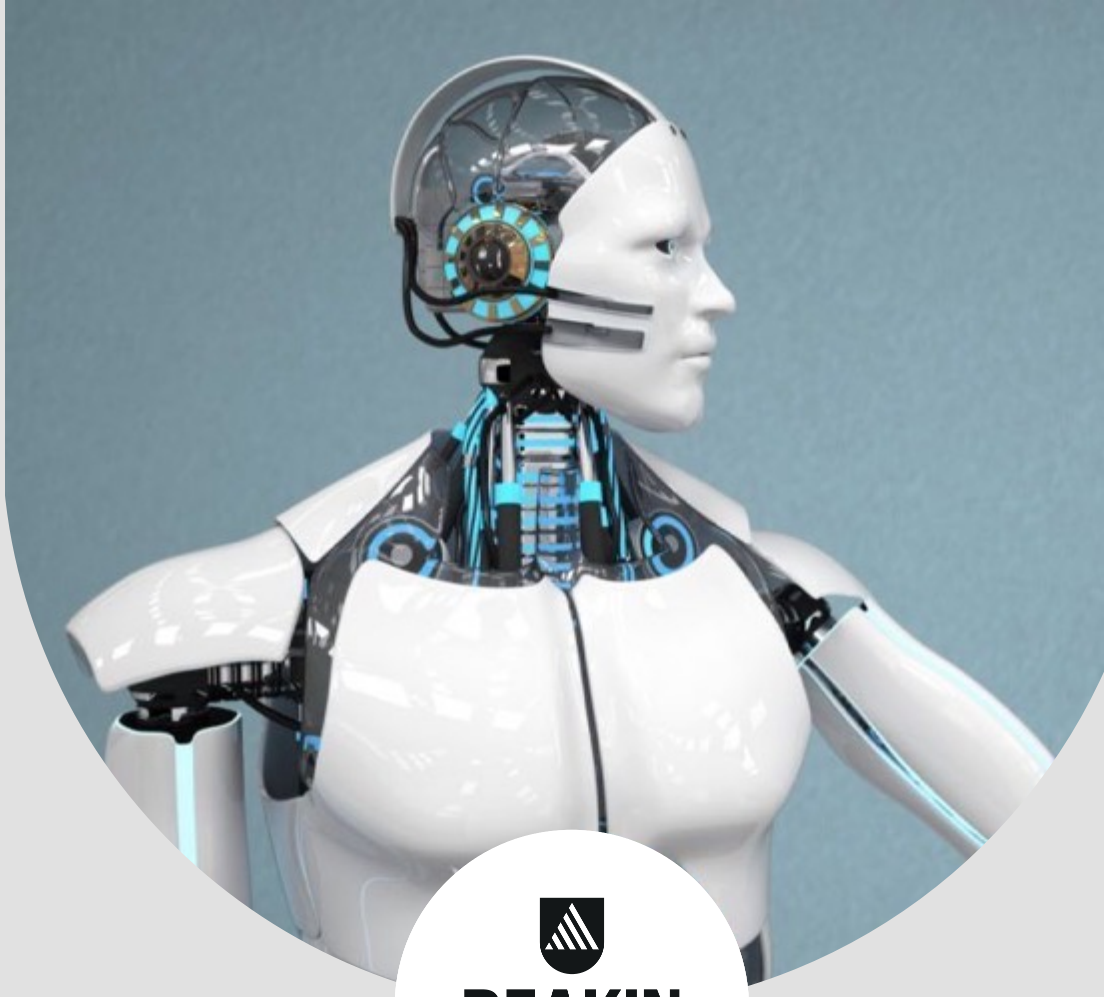
SIT330-770: Natural Language Processing

Week 8 - Sequence Labeling



Dr. Mohamed Reda Bouadjenek

School of Information Technology, Faculty of Sci Eng & Built Env

reda.bouadjenek@deakin.edu.au

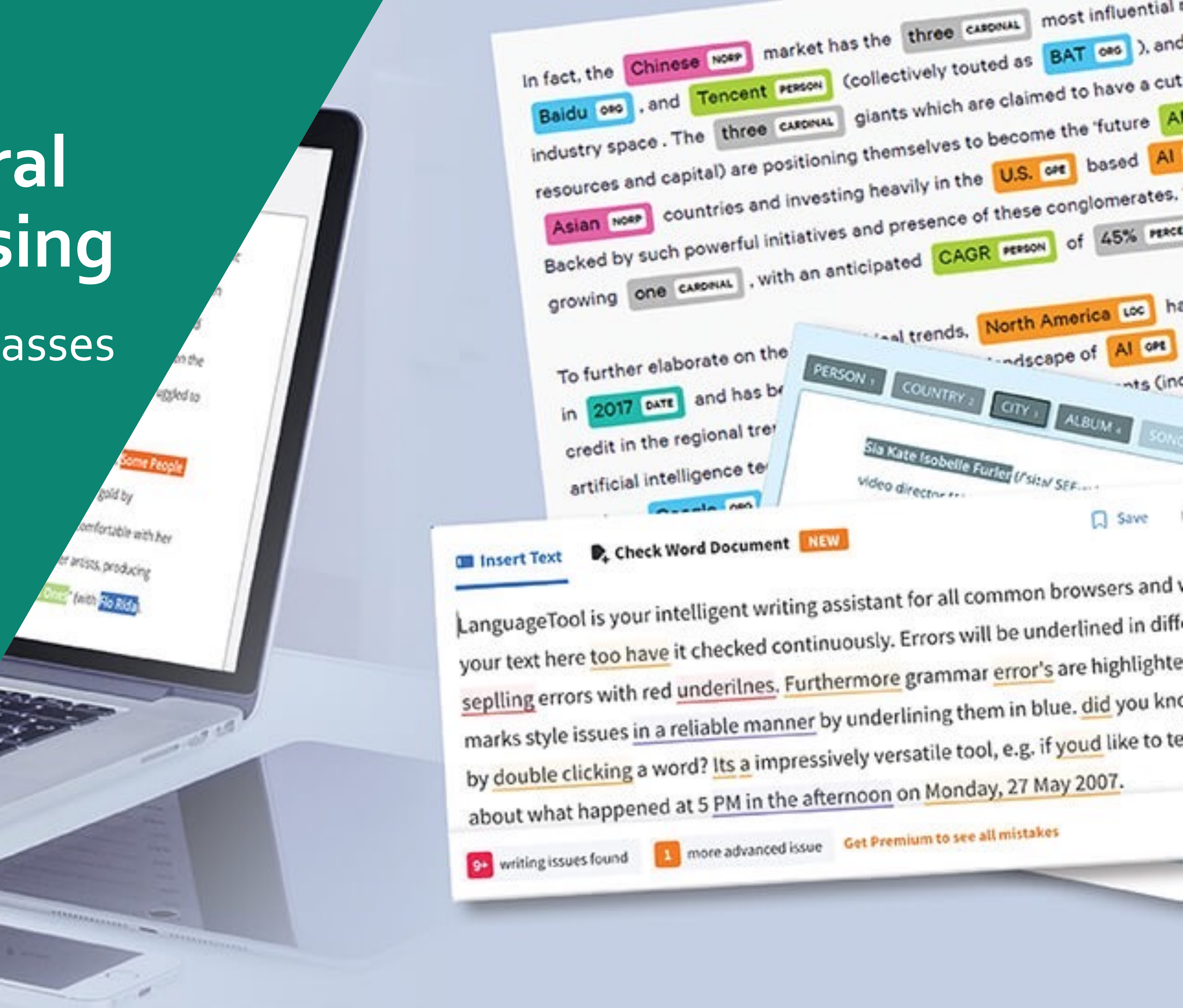
DEAKIN UNIVERSITY

SIT330-770: Natural Language Processing

Week 8.1 - English Word Classes

Dr. Mohamed Reda Bouadjenek

School of Information Technology,
Faculty of Sci Eng & Built Env



Parts of Speech

- From the earliest linguistic traditions (Yaska and Panini 5^{th} C. BCE, Aristotle 4^{th} C. BCE), the idea that words can be classified into grammatical categories
  - part of speech, word classes, POS, POS tags
- 8 parts of speech attributed to Dionysius Thrax of Alexandria (c. 1^{st} C. BCE):
  - noun, verb, pronoun, preposition, adverb, conjunction, participle, article
  - These categories are relevant for NLP today.

Deakin University CRICOS Provider Code: 00113B

Two classes of words: Open vs. Closed

- Closed class words
  - Relatively fixed membership
  - Usually **function** words: short, frequent words with grammatical function
    - determiners: ***a, an, the***
    - pronouns: ***she, he, I***
    - prepositions: ***on, under, over, near, by, …***
- Open class words
  - Usually **content** words: Nouns, Verbs, Adjectives, Adverbs
    - Plus interjections: **oh, ouch, uh-huh, yes, hello**
  - New nouns and verbs like *iPhone* or *to fax*

Deakin University CRICOS Provider Code: 00113B

Open class ("content") words

Nouns

- Proper: *Janet*, *Italy*
- Common: *cat, cats*, *mango*

Verbs

- Main: *eat*, *went*
- Auxiliary: *can*, *had*

Adjectives

old green tasty

Adverbs

slowly yesterday

Interjections

Ow hello

Numbers

122,312
one

… more

Closed class ("function")

- Determiners *the some*
- Conjunctions *and or*
- Pronouns *they its*
- Prepositions *to with*
- Particles *off up*

… more

Deakin University CRICOS Provider Code: 00113B

Part-of-Speech Tagging

- Assigning a part-of-speech to each word in a text.
- Words often have more than one POS.
- **book**:
  - VERB: (***Book*** *that flight*)
  - NOUN: (*Hand me that* ***book***).

Deakin University CRICOS Provider Code: 00113B

"Universal Dependencies" Tagset

	Tag	Description	Example
Open Class	**ADJ**	Adjective: noun modifiers describing properties	*red*, *young*, *awesome*
	ADV	Adverb: verb modifiers of time, place, manner	*very*, *slowly*, *home*, *yesterday*
	NOUN	words for persons, places, things, etc.	*algorithm*, *cat*, *mango*, *beauty*
	VERB	words for actions and processes	*draw*, *provide*, *go*
	PROPN	Proper noun: name of a person, organization, place, etc..	*Regina*, *IBM*, *Colorado*
	INTJ	Interjection: exclamation, greeting, yes/no response, etc.	*oh*, *um*, *yes*, *hello*
Closed Class Words	**ADP**	Adposition (Preposition/Postposition): marks a noun's spacial, temporal, or other relation	*in, on, by under*
	AUX	Auxiliary: helping verb marking tense, aspect, mood, etc.,	*can, may, should, are*
	CCONJ	Coordinating Conjunction: joins two phrases/clauses	*and*, *or*, *but*
	DET	Determiner: marks noun phrase properties	*a, an, the, this*
	NUM	Numeral	*one, two, first, second*
	PART	Particle: a preposition-like form used together with a verb	*up, down, on, off, in, out, at, by*
	PRON	Pronoun: a shorthand for referring to an entity or event	*she, who, I, others*
	SCONJ	Subordinating Conjunction: joins a main clause with a subordinate clause such as a sentential complement	*that*, *which*
Other	**PUNCT**	Punctuation	; , ()
	SYM	Symbols like $ or emoji	$, %
	X	Other	asdf, qwfg

Deakin University CRICOS Provider Code: 00113B

Nivre et al. 2016

Sample "Tagged" English sentences

- There/PRO were/VERB 70/NUM children/NOUN there/ADV ./PUNC
- Preliminary/ADJ findings/NOUN were/AUX reported/VERB in/ADP today/NOUN ’s/PART New/PROPN England/PROPN Journal/PROPN of/ADP Medicine/PROPN

Deakin University CRICOS Provider Code: 00113B

SIT330-770: Natural Language Processing

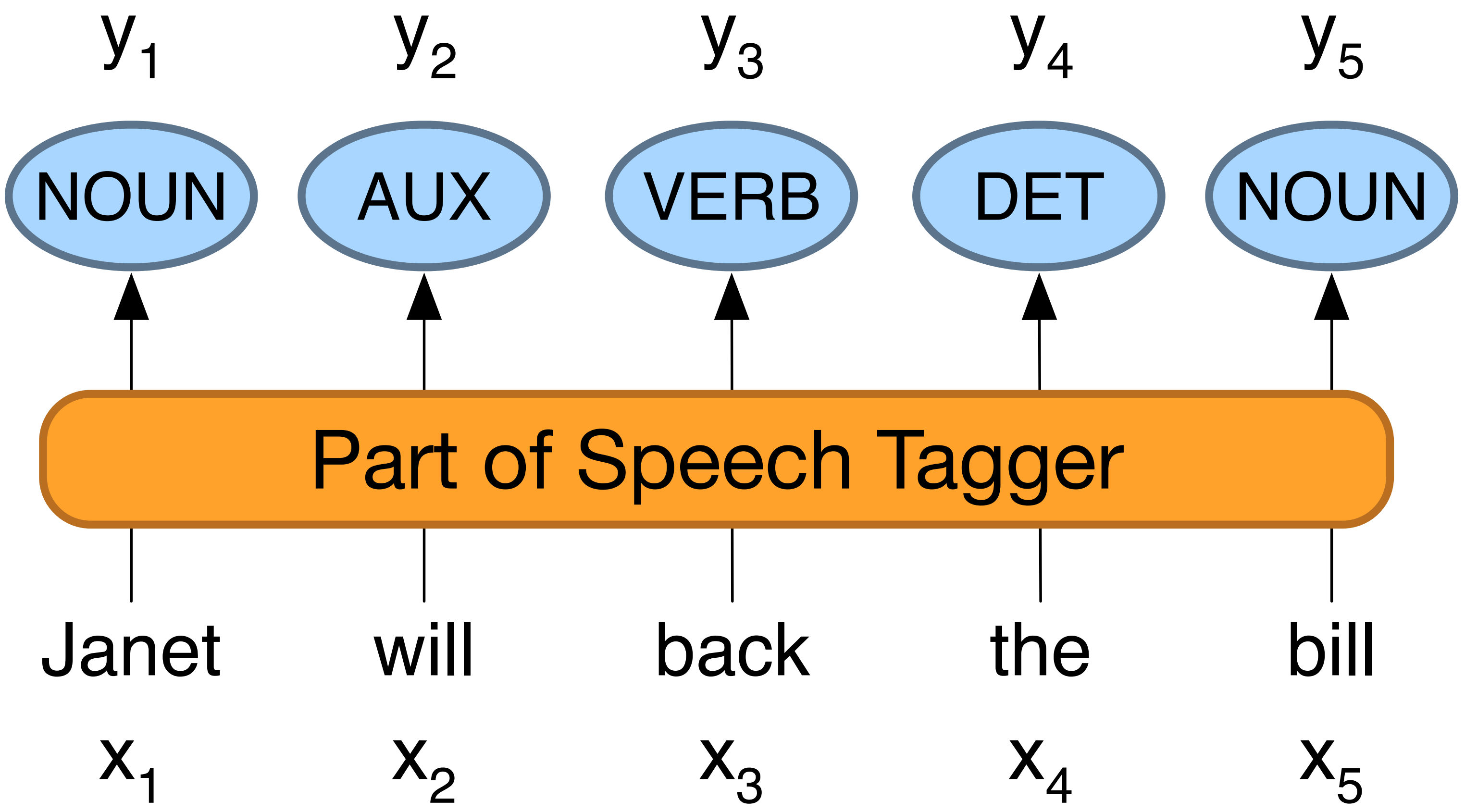
Week 8.2 - Part of Speech Tagging

Dr. Mohamed Reda Bouadjenek

School of Information Technology,
Faculty of Sci Eng & Built Env

Part-of-Speech Tagging

- Map from sequence $x_1,\ldots,x_n$ of words to $y_1,\ldots,y_n$ of POS tags



Deakin University CRICOS Provider Code: 00113B

Why Part of Speech Tagging?

- Can be useful for other NLP tasks
  - Parsing: POS tagging can improve syntactic parsing
  - MT: reordering of adjectives and nouns (say from Spanish to English)
  - Sentiment or affective tasks: may want to distinguish adjectives or other POS
  - Text-to-speech (how do we pronounce “lead” or "object"?)
- Or linguistic or language-analytic computational tasks
  - Need to control for POS when studying linguistic change like creation of new words, or meaning shift
  - Or control for POS in measuring meaning similarity or difference

Deakin University CRICOS Provider Code: 00113B

How difficult is POS tagging in English?

- Roughly 15% of word types are ambiguous
  - Hence 85% of word types are unambiguous
  - *Janet* is always PROPN, *hesitantly* is always ADV
- But those 15% tend to be very common.
- So ~60% of word tokens are ambiguous
- E.g., *back*
  - earnings growth took a *back*/ADJ seat
  - a small building in the *back*/NOUN
  - a clear majority of senators *back*/VERB the bill
  - enable the country to buy *back*/PART debt
  - I was twenty-one *back*/ADV then

Deakin University CRICOS Provider Code: 00113B

POS tagging performance in English

- How many tags are correct? (Tag accuracy)
  - About 97%
    - Hasn't changed in the last 10+ years
    - HMMs, CRFs, BERT perform similarly .
    - Human accuracy about the same
- But baseline is 92%!
  - Baseline is performance of stupidest possible method
    - "Most frequent class baseline" is an important baseline for many tasks
      - Tag every word with its most frequent tag
      - (and tag unknown words as nouns)
  - Partly easy because
    - Many words are unambiguous

Deakin University CRICOS Provider Code: 00113B

Sources of information for POS tagging

Janet will back the bill

AUX/NOUN/VERB? NOUN/VERB?

- Prior probabilities of word/tag
  - "will" is usually an AUX
- Identity of neighboring words
  - "the" means the next word is probably not a verb
- Morphology and wordshape:
  - Prefixes unable: un- → ADJ
  - Suffixes importantly: -ly → ADV
  - Capitalization Janet: CAP → PROPN

Deakin University CRICOS Provider Code: 00113B

Standard algorithms for POS tagging

- Supervised Machine Learning Algorithms:
  - Hidden Markov Models
  - Conditional Random Fields (CRF)/ Maximum Entropy Markov Models (MEMM)
  - Neural sequence models (RNNs or Transformers)
  - Large Language Models (like BERT), finetuned
- All required a hand-labeled training set, all about equal performance (97% on English)
- All make use of information sources we discussed
  - Via human created features: HMMs and CRFs
  - Via representation learning: Neural LMs

Deakin University CRICOS Provider Code: 00113B

SIT330-770: Natural Language Processing

Week 8.3 - Named Entity Recognition (NER)

Dr. Mohamed Reda Bouadjenek

School of Information Technology,
Faculty of Sci Eng & Built Env

Named Entities

- **Named entity**, in its core usage, means anything that can be referred to with a proper name. Most common 4 tags:
  - PER (Person): “Marie Curie”
  - LOC (Location): “New York City”
  - ORG (Organization): “Stanford University”
  - GPE (Geo-Political Entity): "Boulder, Colorado”
- Often multi-word phrases
- But the term is also extended to things that aren't entities:
  - dates, times, prices

Deakin University CRICOS Provider Code: 00113B

Named Entity tagging

- The task of named entity recognition (NER):
  - find spans of text that constitute proper names
  - tag the type of the entity.

Deakin University CRICOS Provider Code: 00113B

NER output

Citing high fuel prices, [$_{\text{ORG}}$ **United Airlines**] said [$_{\text{TIME}}$ **Friday**] it has increased fares by [$_{\text{MONEY}}$ **$6**] per round trip on flights to some cities also served by lower-cost carriers. [$_{\text{ORG}}$ **American Airlines**], a unit of [$_{\text{ORG}}$ **AMR Corp.**], immediately matched the move, spokesman [$_{\text{PER}}$ **Tim Wagner**] said. [$_{\text{ORG}}$ **United**], a unit of [$_{\text{ORG}}$ **UAL Corp.**], said the increase took effect [$_{\text{TIME}}$ **Thursday**] and applies to most routes where it competes against discount carriers, such as [$_{\text{LOC}}$ **Chicago**] to [$_{\text{LOC}}$ **Dallas**] and [$_{\text{LOC}}$ **Denver**] to [$_{\text{LOC}}$ **San Francisco**].

Deakin University CRICOS Provider Code: 00113B

Why NER?

- Sentiment analysis: consumer's sentiment toward a particular company or person?
- Question Answering: answer questions about an entity?
- Information Extraction: Extracting facts about entities from text.

Deakin University CRICOS Provider Code: 00113B

Why NER is hard

1) Segmentation

- In POS tagging, no segmentation problem since each word gets one tag.
- In NER we have to find and segment the entities!

2) Type ambiguity

[$_{PER}$ Washington] was born into slavery on the farm of James Burroughs.
[$_{ORG}$ Washington] went up 2 games to 1 in the four-game series.
Blair arrived in [$_{LOC}$ Washington] for what may well be his last state visit.
In June, [$_{GPE}$ Washington] passed a primary seatbelt law.

Deakin University CRICOS Provider Code: 00113B

BIO Tagging

- How can we turn this structured problem into a sequence problem like POS tagging, with one label per word?

- [PER Jane Villanueva] of [ORG United] , a unit of [ORG United Airlines Holding] , said the fare applies to the [LOC Chicago] route.

Deakin University CRICOS Provider Code: 00113B

BIO Tagging

- [PER Jane Villanueva] of [ORG United] , a unit of [ORG United Airlines Holding] , said the fare applies to the [LOC Chicago] route.

Words	BIO Label
Jane	B-PER
Villanueva	I-PER
of	O
United	B-ORG
Airlines	I-ORG
Holding	I-ORG
discussed	O
the	O
Chicago	B-LOC
route	O
.	O

Now we have one tag per token!!!

Deakin University CRICOS Provider Code: 00113B

BIO Tagging

B: token that *begins* a span

I: tokens *inside* a span

O: tokens outside of any span

# of tags (where n is #entity types):

1 O tag,

n B tags,

n I tags

total of 2*n*+1

Words	BIO Label
Jane	B-PER
Villanueva	I-PER
of	O
United	B-ORG
Airlines	I-ORG
Holding	I-ORG
discussed	O
the	O
Chicago	B-LOC
route	O
.	O

Deakin University CRICOS Provider Code: 00113B

BIO Tagging variants: IO and BIOES

- [PER Jane Villanueva] of [ORG United] , a unit of [ORG United Airlines Holding] , said the fare applies to the [LOC Chicago] route.

Words	IO Label	BIO Label	BIOES Label
Jane	I-PER	B-PER	B-PER
Villanueva	I-PER	I-PER	E-PER
of	O	O	O
United	I-ORG	B-ORG	B-ORG
Airlines	I-ORG	I-ORG	I-ORG
Holding	I-ORG	I-ORG	E-ORG
discussed	O	O	O
the	O	O	O
Chicago	I-LOC	B-LOC	S-LOC
route	O	O	O
.	O	O	O

Deakin University CRICOS Pr

Standard algorithms for NER

- Supervised Machine Learning given a human-labeled training set of text annotated with tags
  - Hidden Markov Models
  - Conditional Random Fields (CRF)/ Maximum Entropy Markov Models (MEMM)
  - Neural sequence models (RNNs or Transformers)
  - Large Language Models (like BERT), finetuned

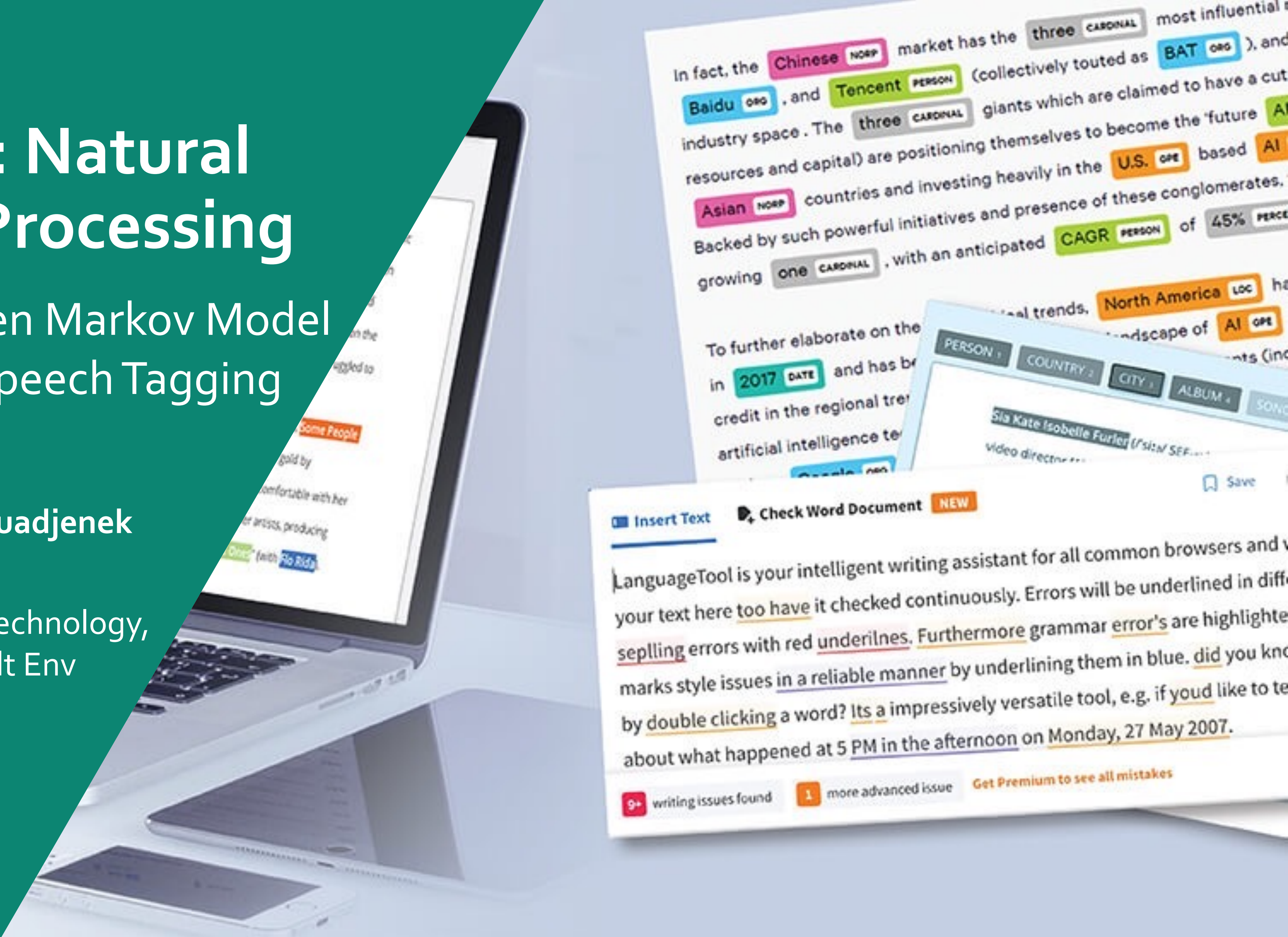
Deakin University CRICOS Provider Code: 00113B

SIT330-770: Natural Language Processing

Week 8.4 – Hidden Markov Model (HMM) Part-of-Speech Tagging

Dr. Mohamed Reda Bouadjenek

School of Information Technology,
Faculty of Sci Eng & Built Env



Introduction to Markov Chains

- A Markov chain models the probabilities of state sequences, each drawn from a specific set.
- It assumes the future state depends only on the current state, not any prior ones.
- Markov chains are used to predict various phenomena
  - E.g., modeling weather patterns or word sequences.

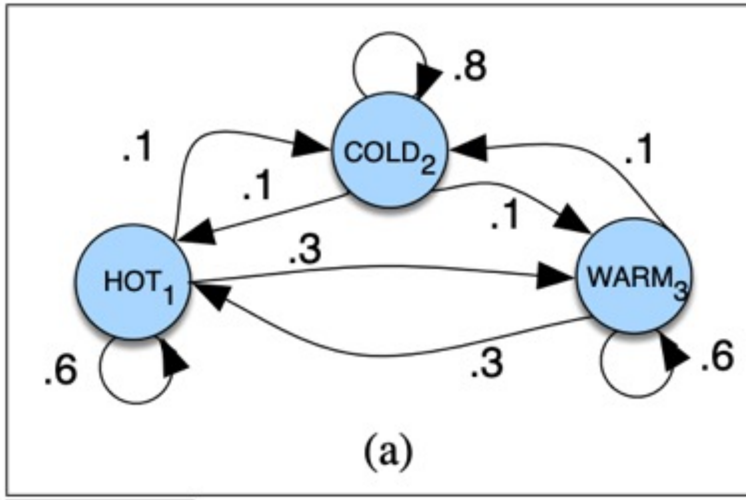


Figure 8.8 A Markov chain for weather

Deakin University CRICOS Provider Code: 00113B

Markov Chain Representation

$Q = q_1 q_2 \ldots q_N$	a set of N **states**
$A = a_{11} a_{12} \ldots a_{N1} \ldots a_{NN}$	a **transition probability matrix** A, each a_{ij} representing the probability of moving from state i to state j, s.t. $\sum_{j=1}^{n} a_{ij} = 1 \quad \forall i$
$\pi = \pi_1, \pi_2, \ldots, \pi_N$	an **initial probability distribution** over states. π_i is the probability that the Markov chain will start in state i. Some states j may have $\pi_j = 0$, meaning that they cannot be initial states. Also, $\sum_{i=1}^{n} \pi_i = 1$

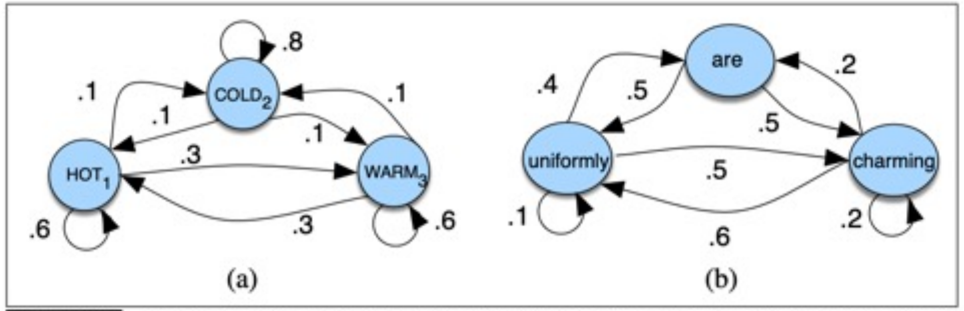


Figure 8.8 A Markov chain for weather (a) and one for words (b), showing states and transitions. A start distribution π is required; setting $\pi = [0.1, 0.7, 0.2]$ for (a) would mean a probability 0.7 of starting in state 2 (cold), probability 0.1 of starting in state 1 (hot), etc.

- Markov Assumption:
  - Formally stated as: $P(q_i{=}a|q_1 \ldots q_{i-1}) = P(q_i{=}a|q_{i-1})$ implying that when predicting the future, only the present state matters

Deakin University CRICOS Provider Code: 00113B

The Hidden Markov Model

- A Markov chain computes probabilities for sequences of observable events.
- But often, the events of interest are hidden.
  - **Example:** Part-of-speech tags in text—hidden because we don't observe them directly.
- **Solution:** Hidden Markov Model (HMM) handles both observed and hidden events.
  - HMMs augment Markov chains

Deakin University CRICOS Provider Code: 00113B

Probabilistic Sequence Modeling with HMMs

- A Hidden Markov Models (HMM) is a probabilistic sequence model that, given a sequence of units (words, letters, morphemes, sentences, etc.), computes a probability distribution over possible sequences of labels.
    - HMMs determine the likelihood of different label sequences and select the most probable sequence based on the observed data.
    - HMM is based on augmenting the **Markov chain**

Deakin University CRICOS Provider Code: 00113B

Input and Assumptions

- Input (O): Sequence of observations (o_1, o_2, ..., o_T) drawn from vocabulary V.
- Assumptions of first-order HMM:
  - Markov Assumption:
    - Probability of state q_i depends only on the previous state (q_{i-1}).
      - $P(q_i|q_1 \dots q_{i-1}) = P(q_i|q_{i-1})$
  - Output Independence:
    - Probability of observation o_i depends only on the state that produced it q_i
      - $P(o_i|q_1, \dots q_i, \dots, q_T, o_1, \dots, o_i, \dots, o_T) = P(o_i|q_i)$

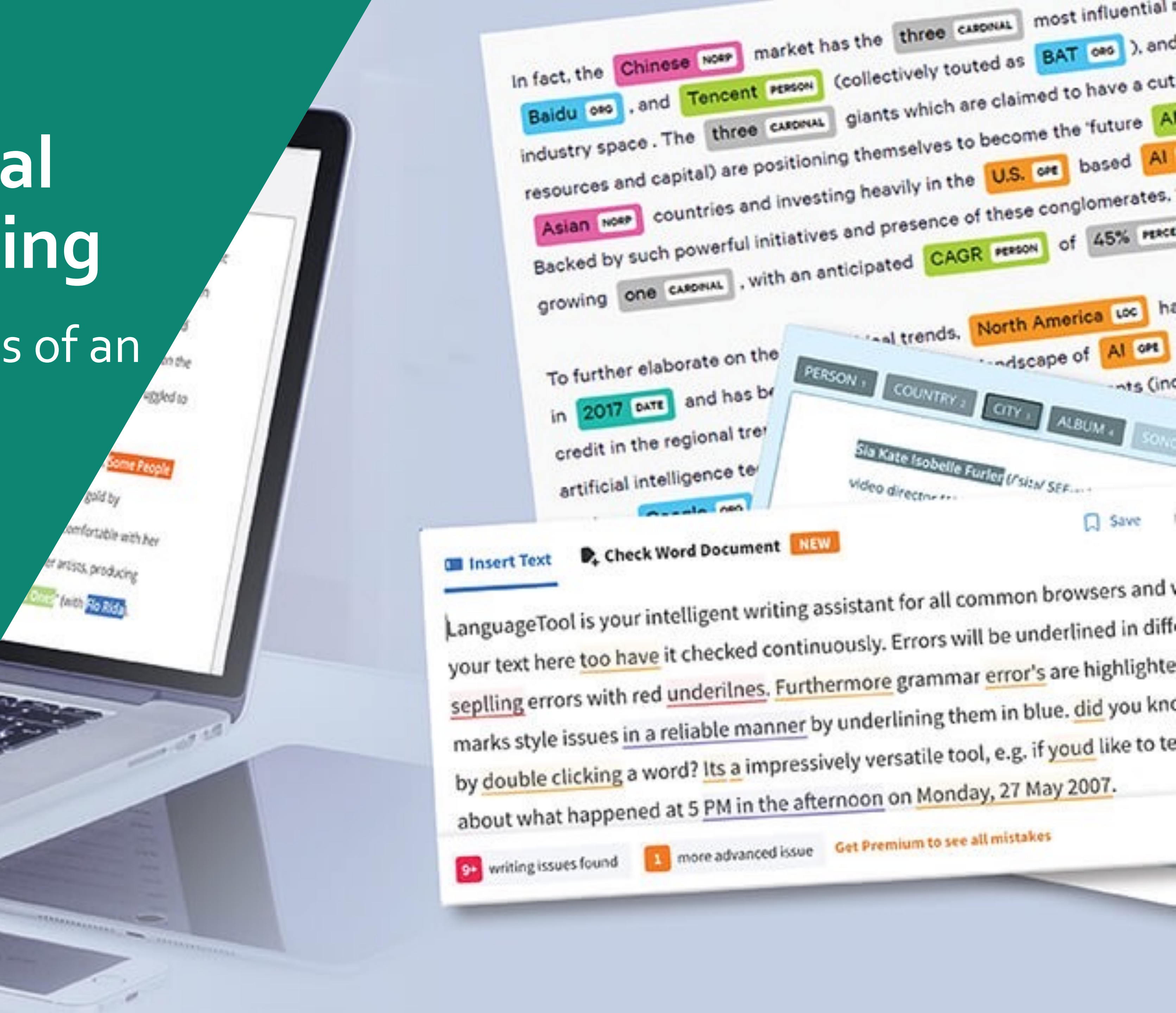
Deakin University CRICOS Provider Code: 00113B

SIT330-770: Natural Language Processing

Week 8.5 – The components of an HMM tagger

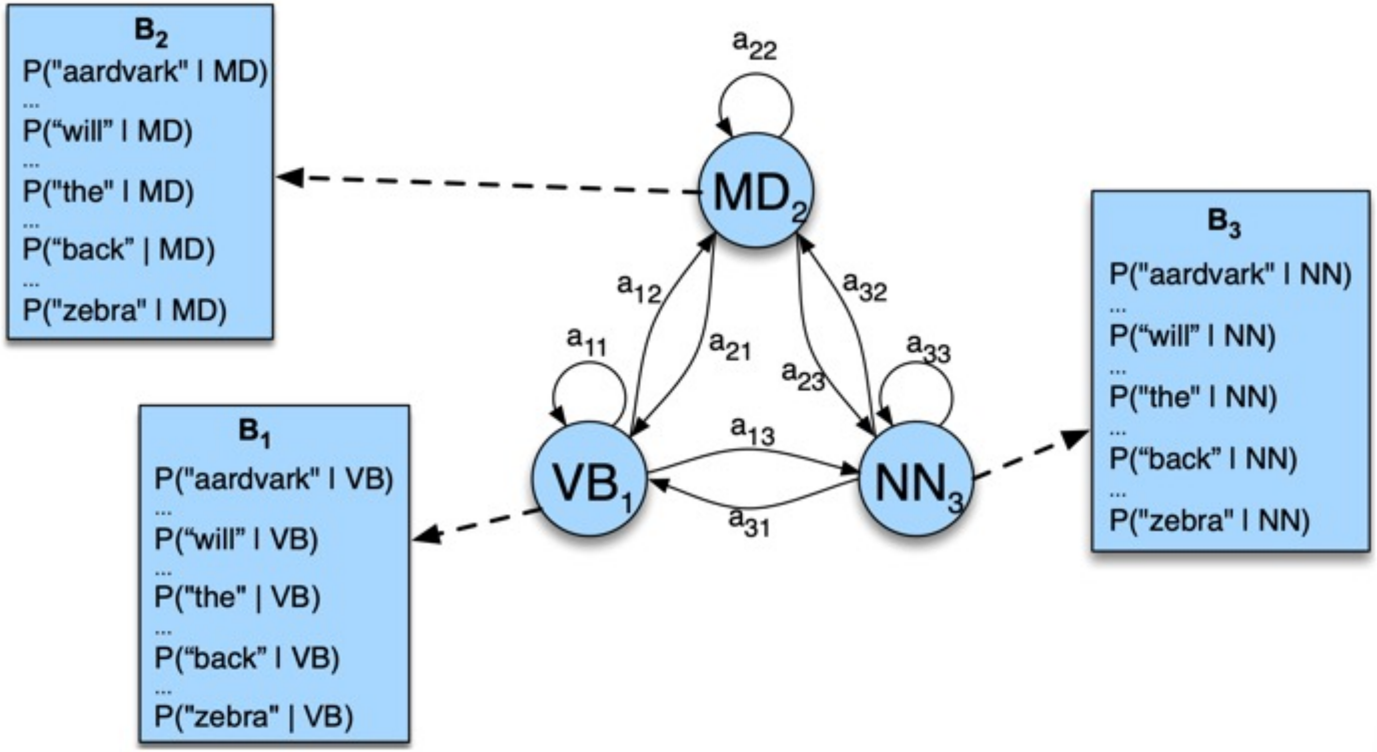
Dr. Mohamed Reda Bouadjenek

School of Information Technology,
Faculty of Sci Eng & Built Env



Components of an HMM Tagger

- An HMM tagger consists of two main components:
  - Matrix A which represents the tag transition.
  - Matrix B which represents emission probabilities.



Deakin University CRICOS Provider Code: 00113B

The A Matrix - Transition Probabilities

- The A matrix encapsulates the tag transition probabilities, $P(t_i|t_{i-1})$, which express how likely a tag follows its predecessor.
  - Example:
    - The modal verb "***will***" commonly precedes the base form of a verb (VB), as in "***will race***", leading to a high transition probability.
  - These probabilities are derived using maximum MLE by counting tag occurrences in a labeled corpus.
- Calculating Transition Probabilities:
  - In the WSJ corpus example, the modal verb tag (MD) is observed 13,124 times.
  - Out of these, MD transitions to a base verb (VB) 10,471 times.
  - Using MLE, we estimate $P(VB|MD) = C(MD, VB) / C(MD) = 10{,}471 / 13{,}124 \approx 0.80$.

Deakin University CRICOS Provider Code: 00113B

The B Matrix - Emission Probabilities

- The B matrix contains emission probabilities, $P(w_i|t_i)$, which quantify the likelihood of a word being tagged with a specific tag.
- Emission Probability Calculation
  - To calculate emission probabilities, we count how often a word occurs with a particular tag in a corpus.
  - For instance, the MD tag associated with the word 'will' occurs 4,046 times in the WSJ corpus.
  - Hence, P(will|MD) is calculated as C(MD, will) / C(MD) = 4,046 / 13,124 ≈ 0.31.

Deakin University CRICOS Provider Code: 00113B

Components of HMM

$Q = q_1 q_2 \ldots q_N$	a set of N **states**
$A = a_{11} \ldots a_{ij} \ldots a_{NN}$	a **transition probability matrix** A, each a_{ij} representing the probability of moving from state i to state j, s.t. $\sum_{j=1}^{N} a_{ij} = 1 \quad \forall i$
$B = b_i(o_t)$	a sequence of **observation likelihoods**, also called **emission probabilities**, each expressing the probability of an observation o_t (drawn from a vocabulary $V = v_1, v_2, \ldots, v_V$) being generated from a state q_i
$\pi = \pi_1, \pi_2, \ldots, \pi_N$	an **initial probability distribution** over states. π_i is the probability that the Markov chain will start in state i. Some states j may have $\pi_j = 0$, meaning that they cannot be initial states. Also, $\sum_{i=1}^{n} \pi_i = 1$

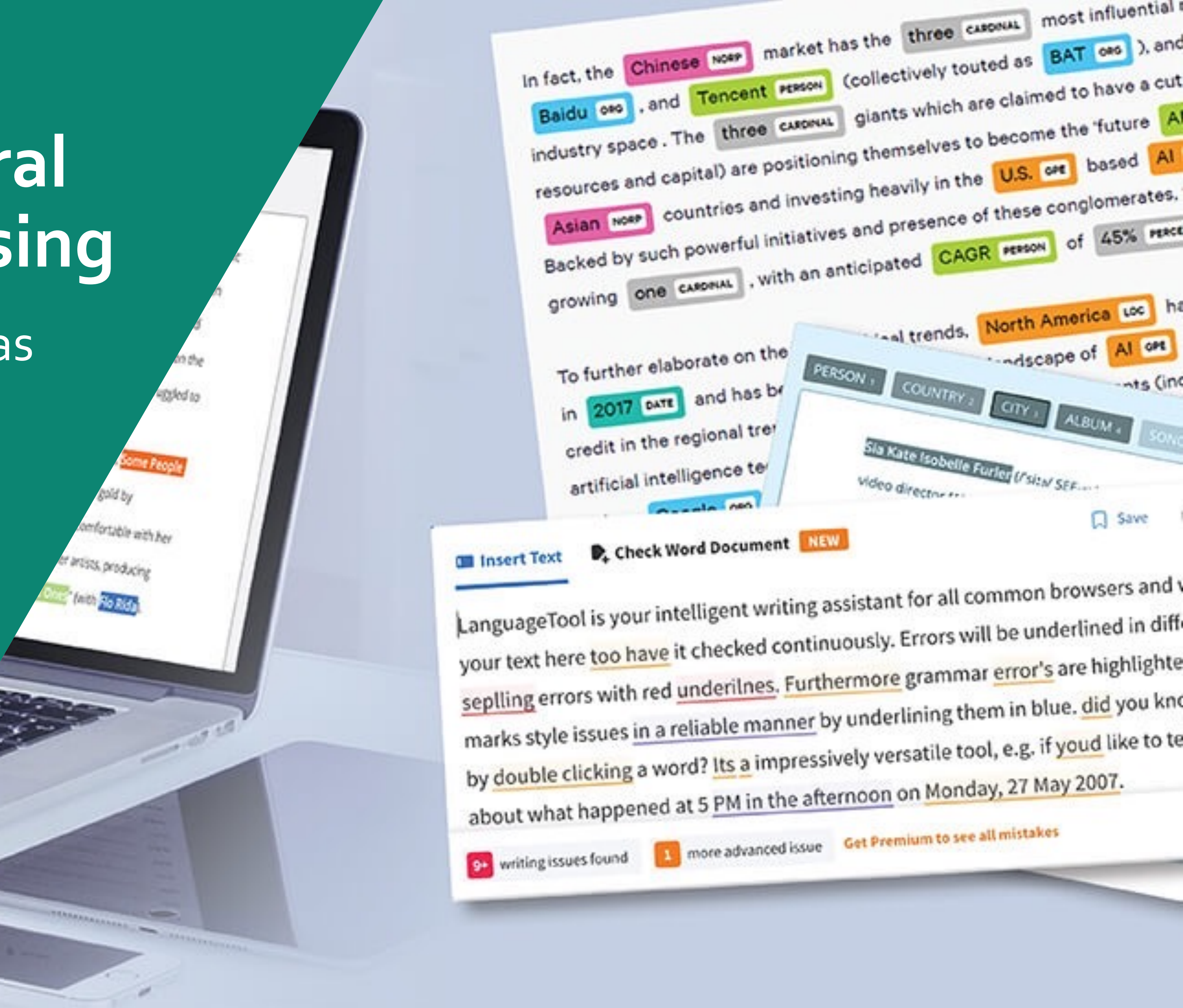
Deakin University CRICOS Provider Code: 00113B

SIT330-770: Natural Language Processing

Week 8.6 – HMM tagging as decoding

Dr. Mohamed Reda Bouadjenek

School of Information Technology,
Faculty of Sci Eng & Built Env



Decoding with Hidden Markov Models

- Decoding is the process of determining the most probable sequence of hidden states (tags) based on observed data.
  - Given a sequence of observations $O = o_1, o_2, \ldots, o_T$, decoding aims to find the most probable sequence of states $Q = q_1 q_2 \ldots q_T$.
  - The input is an HMM $\lambda = (A, B)$, with **A** being the transition probabilities and **B** the emission probabilities.

$$\hat{t}_{1:n} = \underset{t_1 \ldots t_n}{\operatorname{argmax}} P(t_1 \ldots t_n | w_1 \ldots w_n)$$

Deakin University CRICOS Provider Code: 00113B

Decoding with Hidden Markov Models (i)

$$\hat{t}_{1:n} = \underset{t_1 \ldots t_n}{\operatorname{argmax}} P(t_1 \ldots t_n | w_1 \ldots w_n)$$

MAP is "maximum a posteriori" = most likely sequence

$$\hat{t}_{1:n} = \underset{t_1 \ldots t_n}{\operatorname{argmax}} \frac{P(w_1 \ldots w_n | t_1 \ldots t_n) P(t_1 \ldots t_n)}{P(w_1 \ldots w_n)}$$

Bayes Rule

$$\hat{t}_{1:n} = \underset{t_1 \ldots t_n}{\operatorname{argmax}} P(w_1 \ldots w_n | t_1 \ldots t_n) P(t_1 \ldots t_n)$$

Dropping the denominator

Deakin University CRICOS Provider Code: 00113B

Decoding with Hidden Markov Models (ii)

DEAKIN UNIVERSITY

"Likelihood" "Prior"

$$\hat{t}_{1:n} = \underset{t_1 \ldots t_n}{\operatorname{argmax}} P(w_1 \ldots w_n | t_1 \ldots t_n) P(t_1 \ldots t_n)$$

- HMM taggers make two further simplifying assumptions.
  - The probability of a word appearing depends only on its own tag and is independent of neighboring words and tags:

$$P(w_1 \ldots w_n | t_1 \ldots t_n) \approx \prod_{i=1}^{n} P(w_i | t_i)$$

  - The second assumption, the bigram assumption, is that the probability of a tag is dependent only on the previous tag, rather than the entire tag sequence;

$$P(t_1 \ldots t_n) \approx \prod_{i=1}^{n} P(t_i | t_{i-1})$$

Deakin University CRICOS Provider Code: 00113B

Decoding with Hidden Markov Models (iii)

- Plugging the simplifying assumptions results in the following equation for the most probable tag sequence from a bigram tagger:

$$\hat{t}_{1:n} = \underset{t_1 \ldots t_n}{\operatorname{argmax}} P(t_1 \ldots t_n | w_1 \ldots w_n) \approx \underset{t_1 \ldots t_n}{\operatorname{argmax}} \prod_{i=1}^{n} \overbrace{P(w_i | t_i)}^{\text{emission}} \overbrace{P(t_i | t_{i-1})}^{\text{transition}}$$

- The two parts correspond neatly to the **B** emission probability and **A** transition probability that we defined previously!

Deakin University CRICOS Provider Code: 00113B

SIT330-770: Natural Language Processing

Week 8.7 – The Viterbi Algorithm

Dr. Mohamed Reda Bouadjenek

School of Information Technology,
Faculty of Sci Eng & Built Env

Computing the most probable sequence of tags

- A brute force approach to identify the most probable sequence of tags faces exponential complexity
  - This method is impractical for large datasets or real-time applications.
- Solution: The Viterbi algorithm **1967**
  - Leverages dynamic programming, streamlining the process by breaking the problem into manageable sub-problems
  - This approach significantly reduces computational demands and enhances processing speed, making it viable for complex tasks in real-world scenarios

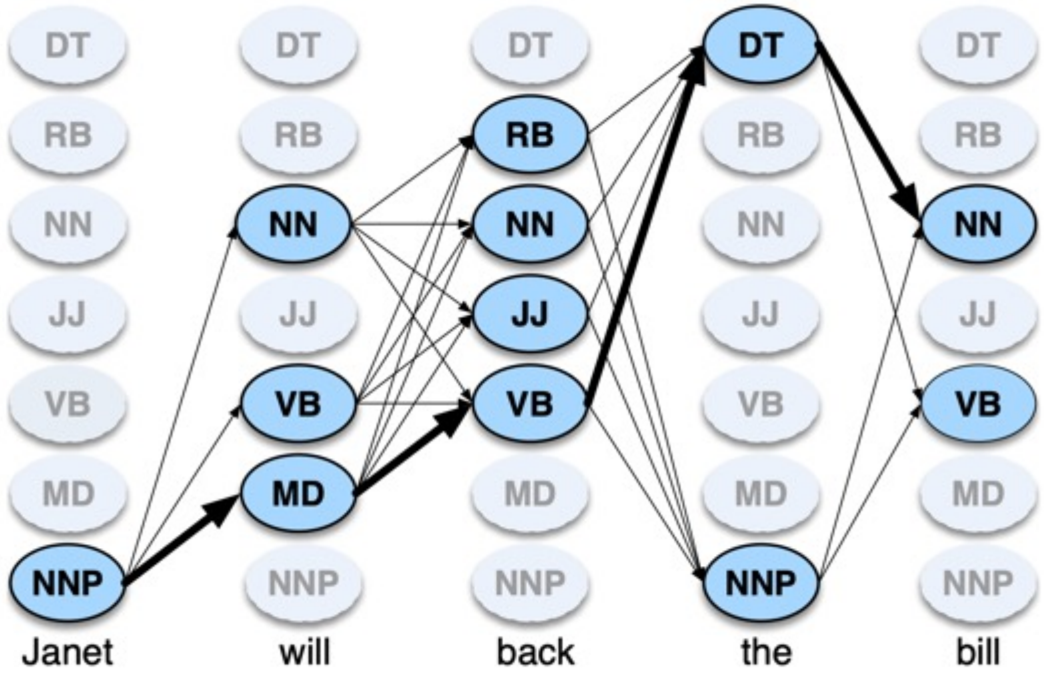
Andrew Viterbi

Deakin University CRICOS Provider Code: 00113B

The Viterbi Algorithm (i)

- The decoding algorithm for HMMs is the **Viterbi algorithm**
  - As an instance of **dynamic programming**, Viterbi resembles the dynamic programming minimum edit distance algorithm
- The Viterbi algorithm first sets up a probability matrix or lattice:
  - **Columns as observables** (words of a sentence in the same sequence as in sentence)
  - Rows as hidden states (all possible POS Tags are known)

tag the sentence
Janet will back the bill



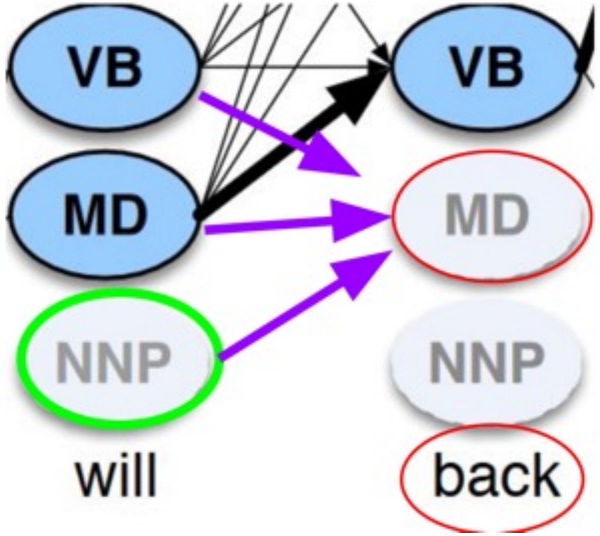
Deakin University CRICOS Provider Code: 00113B

The Viterbi Algorithm (ii)

- Each cell of the matrix is represented by **$V_t(j)$** (Viterbi value for t: column, j: row) having the probability that the HMM is in **state j** (present POS Tag) after seeing the **first t observations** (past words for which matrix (cell) values has been calculated) and passing through the most **probable state sequence (previous POS Tag)** $q_1 \ldots q_{t-1}$
- Computed by recursively taking the most probable path that could lead us to this cell

$$v_t(j) = \max_{i=1}^{N} v_{t-1}(i) \, a_{ij} \, b_j(o_t)$$

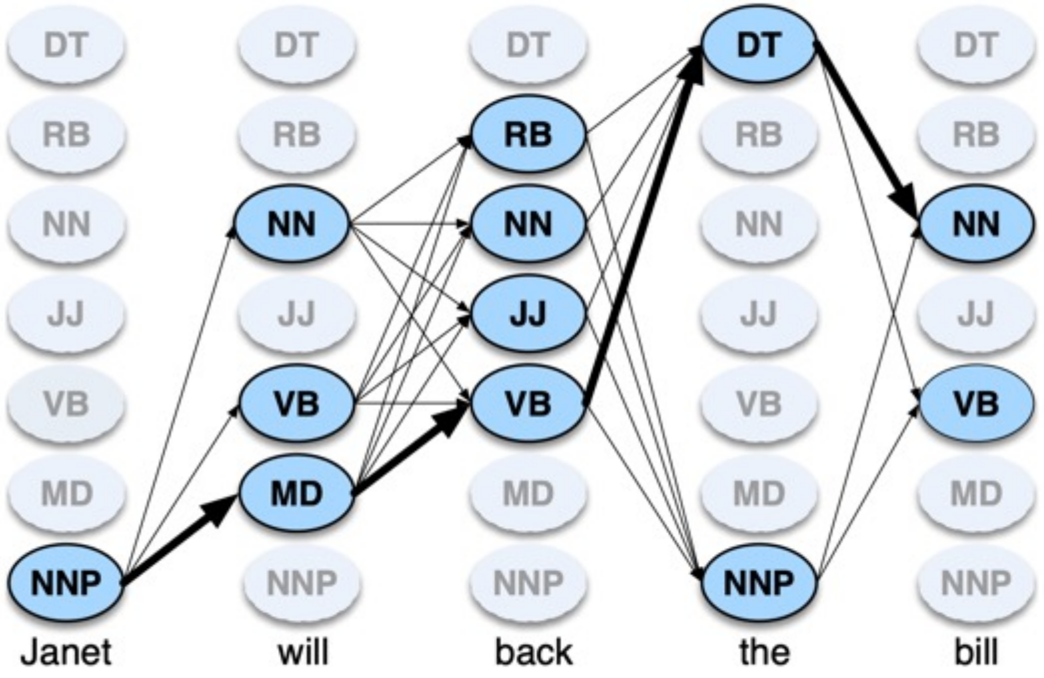
$v_{t-1}(i)$	the **previous Viterbi path probability** from the previous time step
a_{ij}	the **transition probability** from previous state q_i to current state q_j
$b_j(o_t)$	the **state observation likelihood** of the observation symbol o_t given the current state j



Deakin University CRICOS Provider Code: 00113B

The Viterbi Algorithm (iii)

- Each cell of the matrix is represented by **$V_t(j)$** (Viterbi value for t: column, j: row) having the probability that the HMM is in **state j** (present POS Tag) after seeing the **first t observations** (past words for which matrix (cell) values has been calculated) and passing through the most **probable state sequence (previous POS Tag)** $q_1.....q_{t-1}$



A sketch of the matrix for Janet will back the bill, showing the possible tags (q_i) for each word and highlighting the path corresponding to the correct tag sequence through the hidden states

States (parts of speech) which have a zero probability of generating a particular word according to the B matrix (such as the probability that a determiner DT will be realized as Janet) are greyed out

Deakin University CRICOS Provider Code: 00113B

Working Example (i)

- Janet will back the bill → Janet/NNP will/MD back/VB the/DT bill/NN

The A transition probabilities $P(t_i|t_{i-1})$ computed from the WSJ corpus without smoothing

	NNP	MD	VB	JJ	NN	RB	DT
$<s>$	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

Observation likelihoods B computed from the WSJ corpus without smoothing, simplified slightly

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

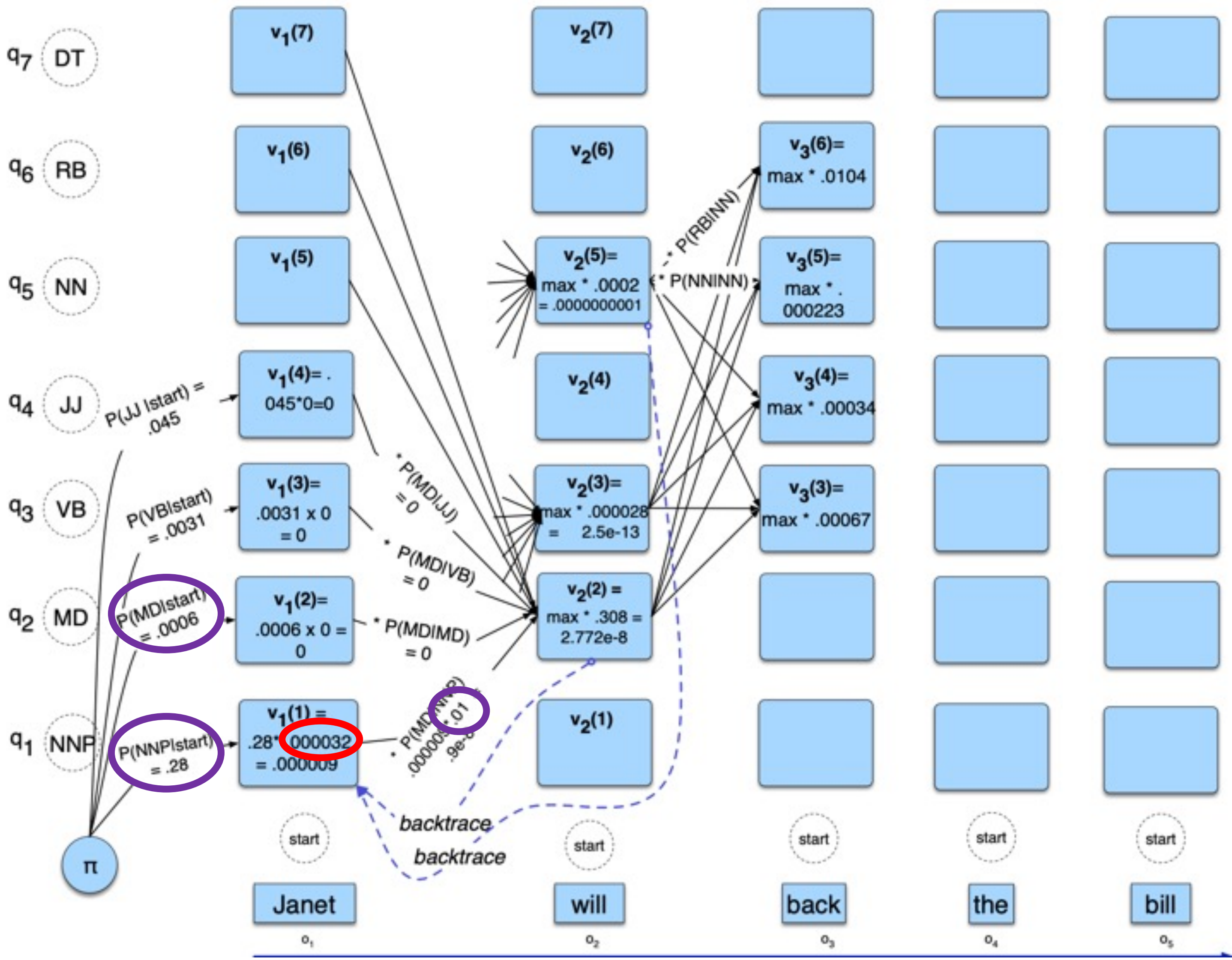
Deakin University CRICOS Provider Code: 00113B

Working Example (ii)

$$v_t(j) = \max_{i=1}^{N} v_{t-1}(i)\, a_{ij}\, b_j(o_t)$$

	NNP	MD	VB	JJ	NN	RB	DT
$<s>$	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0



Deakin University CRICOS Provider Code: 00113B