

# INTRODUCTION

Fuzzy matrix theory was first introduced by Michael G. Thomson in 1977 as a branch of fuzzy set theory, which was developed by L.A. Zadeh twelve years prior. The motivation behind Zadeh's exploration of fuzzy sets was the fact that in physical reality, there exist objects that cannot be placed under clearly defined criteria of membership. For instance, Zadeh points to the "Class of all real numbers which are much greater than 1". It would be impossible to precisely define such a set of real numbers, and therefore we would consider this to be a fuzzy set.

Fuzzy Set Theory also plays vital role in the field of decision making. Decision making is a most important scientific, social and economic endeavour. For decision making in fuzzy environment, one may refer Bellman and Zadeh. Most probably the fuzzy decision model in which over all ranking or ordering of different fuzzy sets are determined by using comparison matrix introduced and developed by Shimura. In 2010 Cagman et.al defined fuzzy soft matrix theory and its application in decision making.

Fuzzy matrices (FMs) are currently very rich modelling issues in science, automata theory, binary connection logic, medical diagnosis, etc... Thomason introduced fuzzy matrices for the first time by talking about the convergence of their powers. Several authors have presented a number of results on the convergence of the power sequence of fuzzy matrices. Ragab at presented some properties on determinant and adjoint of square fuzzy matrix. Hashimoto studied the canonical form of transitive matrix.

Fuzzy Set Theory permits membership function valued in the interval  $[0,1]$ . Fuzzy matrices have applications in a broad spectrum of fields. For instance, fuzzy matrices have proven very useful within the medical field. Since there is often uncertainty in information about Cases, Manifestation and Diagnoses, fuzzy matrices assist in more accurately representing such uncertainty while also pointing to the most likely candidate for diagnosis. Meenakshi and Kaliraja, in their work on interval valued fuzzy matrices for medical diagnosis, state that by using fuzzy matrices with sets of Manifestations, Illness, and Cases, we can calculate diagnosis scores both for an against respective diseases.

They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of medical diagnosis on it. Fuzzy set theory also

plays a vital role in the field of decision making. Decision Making is a most important scientific, Social and economic endeavour. Fuzzy matrices (FMs) are currently very rich modelling issues in Science, automata theory, binary connection logic, medical diagnosis, etc. The field of medicine is one of the most fruitful and interesting areas of applications for fuzzy set theory. Sanchez formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between symptoms and diseases

Fuzziness can be represented in different ways. One of the most useful representations is membership function. Also, depending the nature or shape of membership function a fuzzy number can be classified in different ways, Such as triangular fuzzy number (TFN), trapezoidal fuzzy number etc. Triangular fuzzy number (TFN) are frequently used in applications. It is well known that the matrix formulation of a mathematical formula gives extra facility to handle\study the problem. Due to the presence of uncertainty in many mathematical formulations in different branches of science and technology, we introduce triangular fuzzy matrices (TFMs).

The field of medicine is one of the most fruitful and interesting areas of applications for fuzzy set theory. Sanchez formulated the diagnostic models involving fuzzy matrices representing the medical knowledge between manifestation and illness. Esogbue and Eder utilized fuzzy cluster analysis to model medical diagnostic. Meenakshi and Kali raja have extended Sanchez's approach for medical diagnosis using representation of a interval valued fuzzy matrix. They have also introduced the arithmetic mean matrix of an interval valued fuzzy matrix and directly applied Sanchez's method of medical diagnosis on it.

# **CHAPTER 1**

## **BASIC DEFINITIONS FOR FUZZY SETS**

### **1.1.1.SET**

A set is a term, which is a collection of unordered or ordered elements. Following are various example of a set:

1. A set of all whole numbers.
2. A set of all students in ground.
3. A set of all states in India.
4. A set of all real numbers.

### **1.1.2. TYPES OF SET**

There are the following various categories of set:

1. Subset
2. Singleton
3. Equivalent set
4. Disjoint set
5. Universal
6. Proper
7. Infinite
8. Empty
9. Finite

### **1.1.3. CLASSICAL SET**

It is a type of set which collects the distinct in a group. The sets with the crisp

boundaries are classical sets. In any sets, each single entity is called an element or member of that set.

$$\mu_A = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

#### 1.1.4. OPERATIONS ON CLASSICAL SET:

1. Union operation
2. Intersection operation
3. Difference operation
4. Complement operation

#### 1.1.5. UNION

This operation is defined by  $(R \cup S)$ . If  $R \cup S$  is the set of those elements which exists in two different sets R and S. This operation combines all the elements from both the set and makes a new set. It is called a logical OR operation.

It can be described as:

$$R \cup S = \{x \mid x \in A \text{ (OR) } x \in B\}$$

Example: 1.1.1

Set R = {20, 21, 22, 23, 24} and Set S = {21, 22, 23, 24, 25}

$$R \cup S = \{20, 21, 22, 23, 24, 25\}$$

#### 1.1.6. INTERSECTION

This operation is defined by  $(R \cap S)$ . If  $R \cap S$  is the set of those elements which are common in both set R and S. It is also called a logical AND operation.

It can be described as:

$$R \cap S = \{x \mid x \in R \text{ and } x \in S\}.$$

### **Example :1.1.2**

Set  $R = \{20, 21, 22, 23, 24\}$ , Set  $S = \{21, 24, 25\}$

$$R \cap S = \{21, 24\}$$

### **1.1.7. DIFFERENCE OPERATION**

This operation is denoted by  $(R-S)$ ,  $R-S$  is a set of only those elements which exist only in set  $R$  but not in set  $S$ .

It can be described as:

$$R-S = \{x \mid x \in R \text{ and } x \notin S\}.$$

### **1.1.8. COMPLEMENT OPERATION**

This operation is denoted by  $(\bar{R})$ . It is applied on a single set.  $\bar{R}$  is the set of elements which does not exist in set  $R$ .

It can be described as:

$$\bar{R} = \{x \mid x \notin R\}.$$

## **1.2. PROPERTIES OF CLASSICAL SET**

### **1.2.1. COMMUTATIVE PROPERTY**

$R$  and  $S$  are two finite sets,

$$R \cup S = S \cup R$$

$$R \cap S = S \cap R$$

### **1.2.2. ASSOCIATIVE PROPERTY**

$R$ ,  $S$  and  $T$  are three different finite sets,

$$R \cup (S \cup T) = (R \cup S) \cup T$$

$$R \cap (S \cap T) = (R \cap S) \cap T$$

### 1.2.3. IDEMPOTENCY PROPERTY

R is a single finite set,

$$R \cup R = R$$

$$R \cap R = R$$

### 1.2.4. ABSORPTION PROPERTY

R and S are two finite sets,

$$R \cup (R \cap S) = R$$

$$R \cap (R \cup S) = R$$

### 1.2.5. DISTRIBUTIVE PROPERTY

R, S and T are three finite sets,

$$R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$$

$$R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$$

### 1.2.6. IDENTITY PROPERTY

R is a finite set and X is a universal set,

$$R \cup \varnothing = R$$

$$R \cap X = R$$

$$R \cap \varnothing = \varnothing$$

$$R \cup X = X$$

### 1.2.7. TRANSITIVE PROPERTY

R, S and T are three different finite sets,

If  $R \subseteq S \subseteq T$ , then  $R \subseteq T$

### 1.2.8. INVOLUTION PROPERTY

R is a finite set,

$$\bar{\bar{R}} = R$$

### 1.2.9. DE MORGAN'S LAW

De Morgan's law gives the following rules for proving the contradiction and tautologies,

$$\overline{R \cap S} = \bar{R} \cup \bar{S}$$

$$\overline{R \cup S} = \bar{R} \cap \bar{S}$$

### 1.3. FUZZY SETS

A fuzzy set is a set with a boundary. Fuzzy logic is based on the theory of fuzzy set, which is a generalization of classical set theory.

- “The classical set theory is a set of the theory of fuzzy sets”
- Fuzzy sets are denoted or represented by the tilde ( $\sim$ ) character.
- This theory is denoted mathematically. A Fuzzy set  $(\tilde{A})(\tilde{R})$  is a pair of U and M.
- U is the universe of discourse and M is the membership function which takes on Value in the interval [0,1]. The universe of discourse (U) is also denoted by  $\Omega$  Or X.

$$\tilde{R} = \{(x, \mu_{\tilde{A}}(\mu_{\tilde{R}}(x))) \mid x \in X\}$$

EXAMPLE:

$$R = \{(x, 0.5), (x, 0.7), (x, 0)\}$$

$$\mu_{1(x1)} = 1 - \mu_{1(x1)}$$

$$= 1 - 0.5$$

$$= 0.3$$

$$\mu_{1(x1)} = 0.3 \text{ and } \mu_{1(x1)} = 1$$

#### 1.3.1. TYPES OF FUZZY SETS

There are two types of Fuzzy set

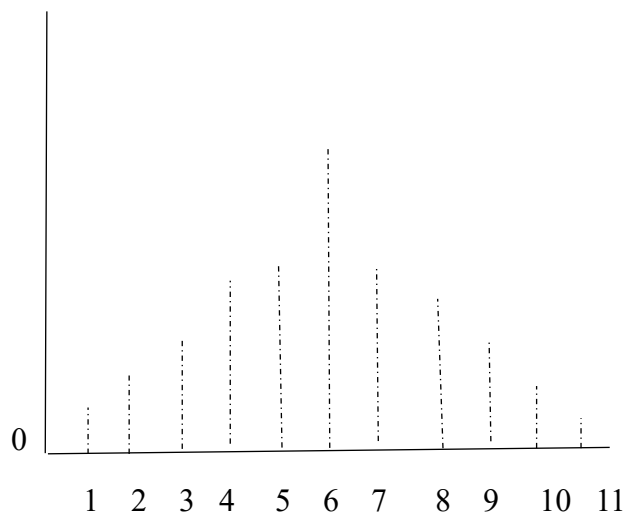
1. Discrete Fuzzy set
2. Continuous Fuzzy set

## 1. Discrete Fuzzy Set

$$A(x) = \sum_{i=1}^n \mu_A(x_i) \backslash x_i,$$

n: number of elements present in the sets.

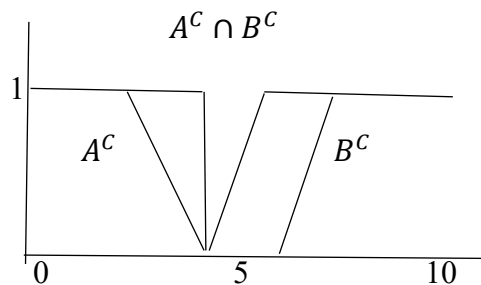
Example:



## 2. Continuous Fuzzy set

$$A(x) = \int \mu_A(x) \backslash x.$$

Example:





### 1.3.2. OPERATION OF FUZZY SET

Given R and S are the two fuzzy sets and X be the universe of discourse with the following respective member functions:

$$\mu_R(x) \text{ and } \mu_S(x)$$

The operation of fuzzy set are as follows:

### 1.3.3. UNION OPERATION

$$\mu_{R \cup S}(x) = \max(\mu_R(x), \mu_S(x))$$

### 1.3.4. INTERSECTION OPERATION

$$\text{Let } \mu_{R \cap S}(x) = \min(\mu_R(x), \mu_S(x))$$

### 1.3.5. COMPLEMENT OPERATION

$$\mu_{\bar{R}}(x) = 1 - \mu_R(x)$$

### 1.3.6. CRISP SET

Crisp set defines the value is either 0 or 1.

S is called as classical set.

It shows full membership.

### 1.3.7. FUZZY MATRIX

A fuzzy matrix is a matrix with elements having values in the fuzzy interval. R of order  $m \times n$  is defined as where  $\mu_{ij}$  is the membership value of the element  $ij$  in R.

Example:

$R = \{a, b, c\}$ . Then a fuzzy relation  $R$  on  $X$  may be:

$$R = \frac{0.2}{(a,a)} + \frac{1}{(a,b)} + \frac{0.4}{(a,c)} + \frac{0.6}{(b,b)} + \frac{0.3}{(b,c)} + \frac{1}{(c,b)} + \frac{0.8}{(c,c)}$$

(or)

$$R = \begin{pmatrix} 0.2 & 1 & 0.4 \\ 0 & 0.6 & 0.3 \\ 0 & 1 & 0.8 \end{pmatrix}$$

### 1.3.8. FUZZY ROW MATRIX

Let,  $R \in [0,1]$ ;  $j = 1, 2, \dots, n$ .

Then  $R$  is called  $1 \times n$  fuzzy row matrix (or) fuzzy row vector.

Example:

$$\text{Let } R = [1, 0.4, 0.6, 0, 1, 0.7, 0.1]$$

$R$  is a row fuzzy matrix

$R$  is a  $1 \times 7$  row fuzzy matrix.

### 1.3.9. FUZZY DIAGONAL MATRIX

A fuzzy square matrix  $A = [a_{ij}]$   $n \times n$  is said to be a fuzzy diagonal matrix if

$a_{ij} = 0$  when  $i \neq j$ , where  $a_{ij} \in [0,1]$ ,  $1 \leq i, j \leq n$ .

Example:

$$\begin{pmatrix} 0.4 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.9 \end{pmatrix}$$

Is a fuzzy diagonal matrix of order 3.

## 1.4. FUZZY SCALAR MATRIX

A fuzzy square matrix is claimed to be fuzzy diagonal matrix, if all its diagonal entries are equal thus a fuzzy square matrix  $A = [a_{ij}] \times n$  is said to be fuzzy scalar matrix if operation of ‘Maximum \ most’ > ‘most \ Minimum’.

$$\text{Matrices} = \begin{cases} a_{ij} = 0 \text{ when } i \neq j \\ a_{ij} = a \text{ when } i = j \end{cases} \text{ where } a \in [0,1], 1 \leq j, i \leq n.$$

EXAMPLE:

$$[0.3] \text{ and } \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix} \text{ are fuzzy scalar matrices of order 1 and 2.}$$

### 1.4.1. FUZZY SQUARE MATRIX

Let a fuzzy square matrix is a matrix that has an equal number of rows and columns, where  $a \in [0,1]$ .  $1 \leq i, j \leq n$ .

EXAMPLE:

$$\begin{pmatrix} 1 & 4 & 0 \\ 8 & 15 & 3 \\ 1 & 9 & 2 \end{pmatrix}$$

### 1.4.2. FUZZY ZERO MATRIX

Let a matrix that has all its elements equal to zero. Since a zero matrix contains only zeros as its elements, therefore it is also called as a null matrix.

A zero matrix can be a square matrix. A zero matrix is denoted by ‘0’.

EXAMPLE:

$$[0 \ 0 \ 0] \text{ is a null (or) zero matrix .}$$

## CHAPTER 2

### SOME EXAMPLES OF FUZZY SETS AND MEMBERSHIP FUNCTIONS

#### 1.2. Properties of fuzzy subset of a set:

Let  $R, S, T$  are three fuzzy subset of a set  $X$  and  $0, 1$  are fuzzy null is fuzzy universal subsets then we've

##### 1.2.1 Commutativity:

$$\text{i) } R \cap S = R \cap S$$

$$\text{ii) } R \cup S = R \cup S$$

To prove: commutativity

First, we've to prove  $R \cap S = R \cap S$

We know that,  $R(x) \leq S(x) \forall x \in X$

Now consider,  $(R \cap S)(x) = \min \{R(x), S(x)\}$

$$= S(x) \longrightarrow (1)$$

Now consider,  $(S \cap R)(x) = \min \{S(x), R(x)\}$

$$= S(x) \longrightarrow (2)$$

From (1) and (2) are equal

$$(R \cap S)(x) = (S \cap R)(x)$$

$$R \cap S = S \cap R$$

Similarly,

$$R \cup S = S \cup R$$

Hence proved.

### 1.2.2. ASSOCIATIVE PROPERTY

First, we've to pt.  $(R \cup S) \cup T = R \cup (S \cup T)$

W.K.T

$$R(x) \leq S(x) \leq T(x)$$

Now consider,

$$\begin{aligned}(R \cup S) \cup T(x) &= \max \{(R \cup S)(x), T(x)\} \\ &= \max \{\max \{R(x), S(x)\}, T(x)\} \\ &= \max \{S(x), T(x)\}\end{aligned}$$

$$[(R(x) \cup S(x)) \cup T](x) = T(x) \longrightarrow (1)$$

Now consider

$$\begin{aligned}[R \cup (S \cup T)](x) &= \max \{R(x), (S \cup T)(x)\} \\ &= \max \{R(x), \max \{S(x), T(x)\}\} \\ &= \max \{R(x), T(x)\} \\ &= T(x) \longrightarrow (2)\end{aligned}$$

From (1) and (2)

$$[(R \cup S) \cup T](x) = [R \cup (S \cup T)](x)$$

$$R \cup (S \cup T) = (R \cup S) \cup T$$

II<sup>ly</sup>

$$(R \cap S) \cap T = R \cap [S \cap T]$$

### 1.2.5. DISTRIBUTIVE PROPERTY:

- (i)  $(\mu_1 \cup \mu_2) \cap \mu_3 = (\mu_1 \cap \mu_3) \cup (\mu_2 \cap \mu_3)$
- (ii)  $(\mu_1 \cap \mu_2) \cup \mu_3 = (\mu_1 \cup \mu_3) \cap (\mu_2 \cup \mu_3)$

To prove Distributivity:

First we've to prove that  $(\mu_1 \cup \mu_2) \cap \mu_3 = (\mu_1 \cap \mu_3) \cup (\mu_2 \cap \mu_3)$

We know that  $\mu_1(x) \leq \mu_2(x) \leq \mu_3(x)$

Now consider,

$$\begin{aligned}
 (R \cup S) \cap T(x) &= \min \{ (R \cup S)(x), T(x) \} \\
 &= \min \{ \max \{ R(x), S(x) \}, T(x) \} \\
 &= \min \{ S(x), T(x) \} \\
 &= S(x) \longrightarrow (1)
 \end{aligned}$$

Now consider,

$$\begin{aligned}
 (R \cap S) \cup (S \cap T)(x) &= \max \{ (R \cap S)(x), (S \cap T)(x) \} \\
 &= \max \{ \min \{ R(x), S(x) \}, \\
 &\quad \min \{ S(x), T(x) \} \} \\
 &= \max \{ R(x), S(x) \} \\
 &= S(x) \longrightarrow (2)
 \end{aligned}$$

From (1) and (2)

$$\begin{aligned}
 [(R \cup S) \cap T](x) &= [(R \cap S) \cup (S \cap T)](x) \\
 (R \cup S) \cap T &= (R \cap S) \cup (S \cap T)
 \end{aligned}$$

*Illy*

$$(R \cap S) \cap T = (R \cup S) \cap (S \cup T)$$

### 1.3.3. UNION OPERATION

Some examples for union operations

example:

Let's suppose R is a set which contains following elements:

$$R = \{(x_1, 0.5), (x_2, 0.2), (x_3, 1)\}$$

And S is a set which contains following elements:

$$S = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\}$$

Then

$$R \cup S = \{(x_1, 0.1), (x_2, 0.7), (x_3, 1)\}$$

Because, according to this operation

For  $x_1$ ,

$$\mu_{R \cup S}(x_1) = \max(\mu_R(x_1), \mu_S(x_1))$$

$$\mu_{R \cup S}(x_1) = \max(0.5, 0.1)$$

$$\mu_{R \cup S}(x_1) = 0.5$$

For  $x_2$ ,

$$\mu_{R \cup S}(x_2) = \max(\mu_R(x_2), \mu_S(x_2))$$

$$\mu_{R \cup S}(x_2) = \max(0.2, 0.7)$$

$$\mu_{R \cup S}(x_2) = 0.7$$

For  $x_3$ ,

$$\mu_{R \cup S}(x_3) = \max(\mu_R(x_3), \mu_S(x_3))$$

$$\mu_{R \cup S}(x_3) = (1, 0)$$

$$\mu_{R \cup S}(x_3) = 1$$

### 1.3.4. INTERSECTION OPERATION

Example:

Suppose R is a set which contains following elements.

$$R = \{(x_1, 0.2), (x_2, 0.6), (x_3, 0.4)\}$$

$$S = \{(x_1, 0.7), (x_2, 0.1), (x_3, 0.5)\}$$

Then

$$R \cap S = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.4)\}$$

Because, according to the operation

For  $x_1$ ,

$$\mu_{R \cap S}(x_1) = \min(\mu_R(x_1), \mu_S(x_1))$$

$$\mu_{R \cap S}(x_1) = (0.2, 0.7)$$

$$\mu_{R \cap S}(x_1) = 0.2$$

For  $x_2$ ,

$$\mu_{R \cap S}(x_2) = \min(\mu_R(x_2), \mu_S(x_2))$$

$$\mu_{R \cap S}(x_2) = (0.6, 0.1)$$

$$\mu_{R \cap S}(x_2) = 0.1$$

For  $x_3$ ,

$$\mu_{R \cap S}(x_3) = \min(\mu_R(x_3), \mu_S(x_3))$$

$$\mu_{R \cap S}(x_3) = (0.4, 0.5)$$

$$\mu_{R \cap S}(x_3) = 0.4$$

### 1.3.5. COMPLEMENT OPERATION

Example:

$$R = \{(x_1, 0.2), (x_2, 0.7), (x_3, 0.4)\}$$

Then

$$\bar{R} = \{(x_1, 0.8), (x_2, 0.3), (x_3, 0.6)\}$$

For  $x_1$ ,



$$\mu_{\bar{R}}(x_1) = 1 - \mu_R(x_1)$$

$$\mu_{\bar{R}}(x_1) = 1 - 0.2$$

$$\mu_{\bar{R}}(x_1) = 0.8$$

For  $x_2$ ,

$$\mu_{\bar{R}}(x_2) = 1 - \mu_R(x_2)$$

$$\mu_{\bar{R}}(x_2) = 1 - 0.7$$

$$\mu_{\bar{R}}(x_2) = 0.3$$

For  $x_3$ ,

$$\mu_{\bar{R}}(x_3) = 1 - \mu_R(x_3)$$

$$\mu_{\bar{R}}(x_3) = 1 - 0.4$$

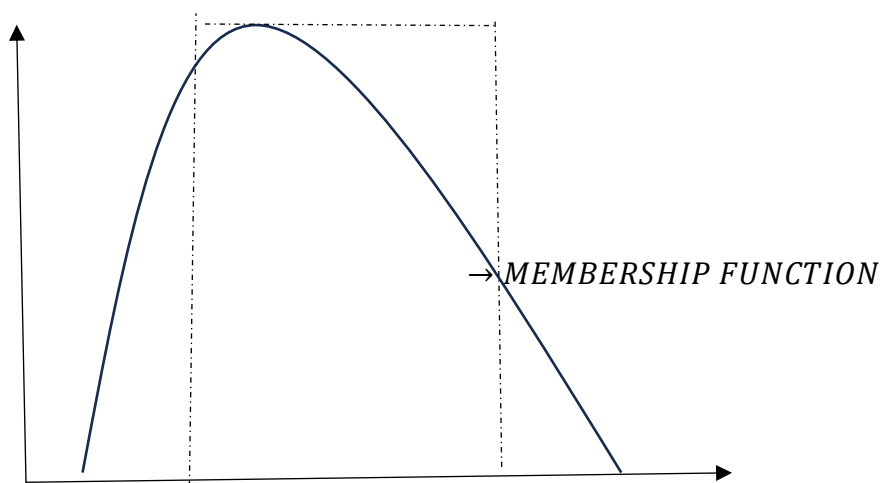
$$\mu_{\bar{R}}(x_3) = 0.6$$

### 1.4.3. MEMBERSHIP FUNCTION

A membership function is curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.

There can be multiple membership functions applicable to fuzzify a numerical value. Simple membership functions are used as use of complex functions does not add more Precision in output.

All membership function for **LP, MP, S, MN, LN**

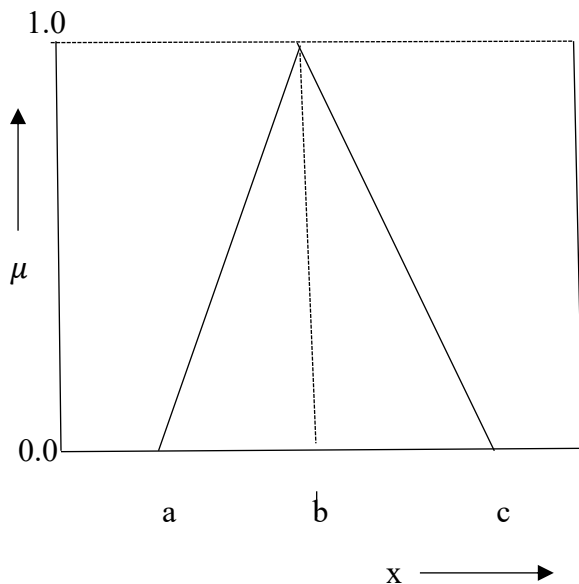


The triangular membership function shapes are most common among various other membership function shapes such as Trapezoidal, Bell-shaped, Gaussian.

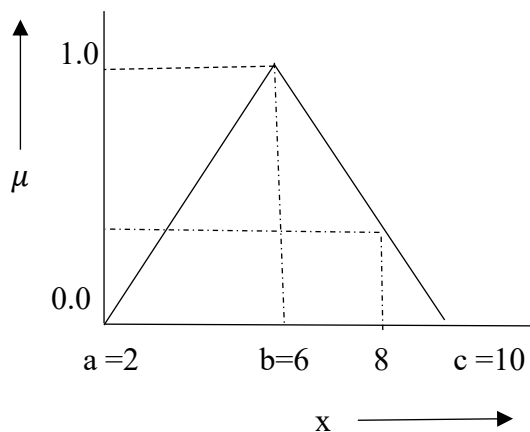
#### 1.4.4. TRIANGULAR MEMBERSHIP FUNCTION

It can be defined by three parameters a, b and c where a and c are the left and right based points of the triangle and b is the peak point. The function is zero outside the interval [a, c] and linearly increases (or) decreases within the interval.

$$\mu_{triangle} = \max \left[ \min \left[ \frac{x-a}{b-a}, \frac{c-x}{c-b} \right], 0 \right]$$



Example:



$$\begin{aligned}
\mu_{triangle} &= \max \left[ \min \left[ \frac{x-a}{b-a}, \frac{c-x}{c-b} \right], 0 \right] \\
&= \max \left[ \min \left[ \frac{x-2}{6-2}, \frac{10-x}{10-6} \right], 0 \right] \\
&= \max \left[ \min \left[ \frac{x-2}{4}, \frac{10-x}{4} \right], 0 \right]
\end{aligned}$$

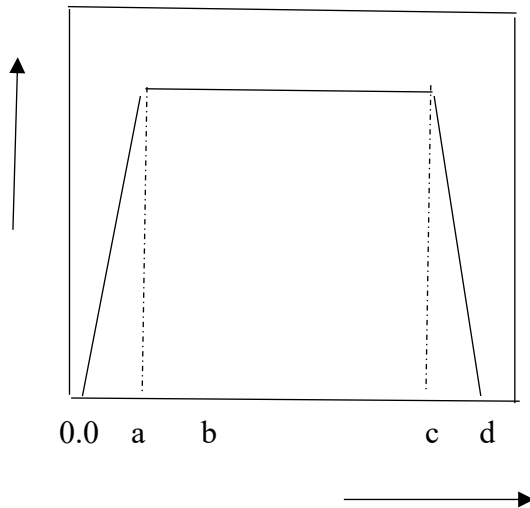
We put,  $x=0.8$

$$\mu_{triangle} = \max \left[ \min \left( \frac{3}{2}, \frac{1}{2} \right), 0 \right] = \frac{1}{2} = 0.5$$

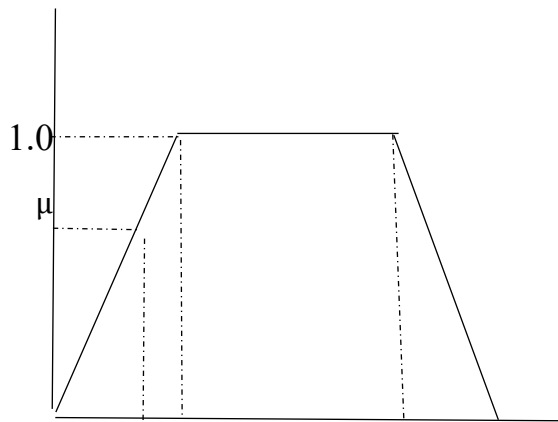
### 1.4.5. TRAPEZOIDAL MEMBERSHIP

When the shape of the membership function depends on the relative values of  $b$  and  $c$ . when  $c$  is greater than  $b$ , the resulting membership function is trapezoidal.

$$\mu_{trapezoidal} = \max \left( \min \left( \frac{x-a}{b-a}, 1, \frac{d-x}{d-c} \right), 0 \right)$$



Example:



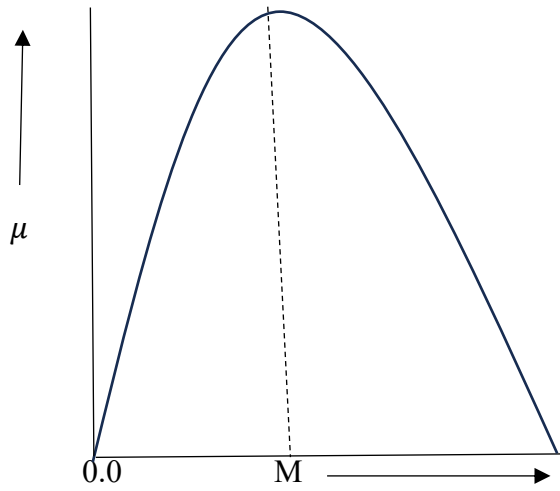
$$\begin{aligned}\mu_{Trapezoidal} &= \max \left[ \min \left( \frac{x-a}{b-a}, 1, \frac{d-x}{d-c} \right), 0 \right] \\ &= \max \left[ \min \left( \frac{x-2}{4-2}, 1, \frac{10-x}{10-8} \right), 0 \right] \\ &= \max \left[ \min \left( \frac{x-2}{2}, 1, \frac{10-x}{2} \right), 0 \right]\end{aligned}$$

We put  $x=3.5$

$$\begin{aligned}&= \max \left[ \min \left( \frac{1.5}{2}, 1, \frac{6.5}{2} \right), 0 \right] \\ &= \max [0.75, 0] \\ &= 0.75\end{aligned}$$

#### 1.4.6. GAUSSIAN MEMBERSHIP

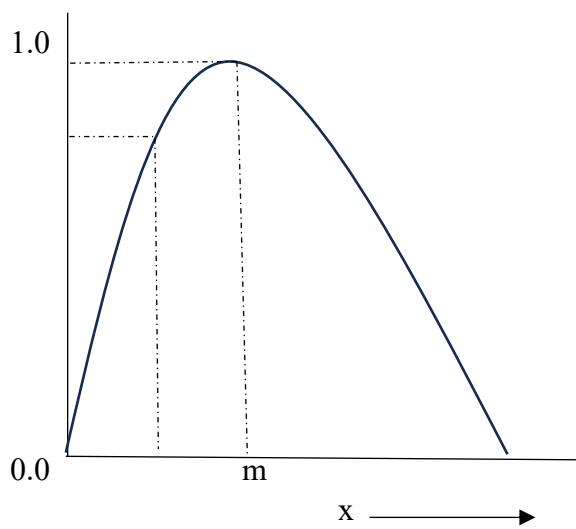
A gaussian membership function is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1.



$$\mu_{\text{gaussian}} = \frac{1}{e^{\frac{1}{2}(\frac{x-m}{\sigma})^2}}$$

EXAMPLE:

$$\mu_{\text{gaussian}} = \frac{1}{e^{\frac{1}{2}(\frac{x-m}{\sigma})^2}}$$



Take  $m=10.0$  and  $\sigma=3.0$

$$\mu_{gaussian} = \frac{1}{e^{\frac{1}{2}(\frac{x-10.0}{3.0})^2}}$$

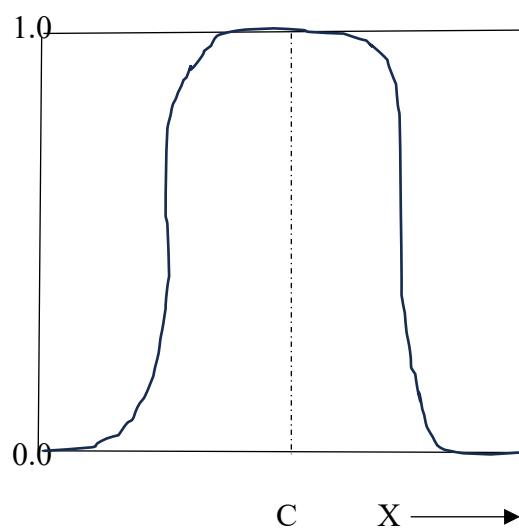
We put x=9.0

$$\begin{aligned}\mu_{gaussian} &= \frac{1}{e^{\frac{1}{2}(\frac{9.0-10.0}{3.0})^2}} \\ &= 0.9459\end{aligned}$$

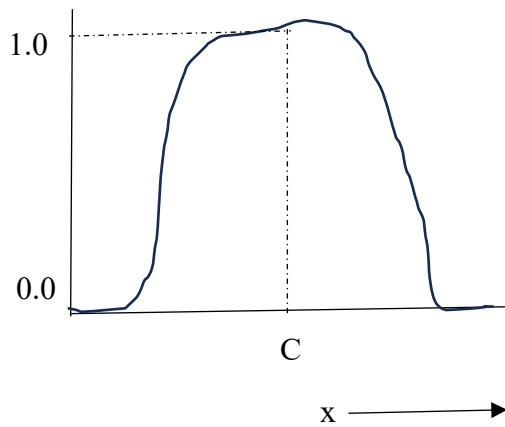
### 1.4.7. BELL SHAPED MEMBERSHIP

It is symmetrical shape similar to a bell. This function employs three parameters: a determines the width of the bell like curve, b is a positive integer, while c sets the center of the curve in universe of discourse.

$$\mu_{Bell-shaped} = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$



Example:



Take  $c=10.0$ ,  $a=2.0$ ,  $b=3.0$

$$\mu_{Bell-shaped} = \frac{1}{1 + \left| \frac{x-c}{a} \right|^{2b}}$$

Take  $c = 10.0$ ,  $a = 2.0$ ,  $b = 3.0$

$$\mu_{Bell-shaped} = \frac{1}{1 + \left| \frac{x-10}{2} \right|^6}$$

We put  $x = 8.0$

$$\mu_{Bell-shaped} = \frac{1}{1 + \left| \frac{8-10}{2} \right|^6} = 0.5$$

### 1.4.7. $\alpha$ – LEVEL SET

The set of elements that belonging to the fuzzy set  $R$  at least to the degree

$\alpha$  is called the  $\alpha$  -level set (or)  $\alpha$  – cut set  $R$ .

$$\alpha = \{u \in U \mid \mu_R(u) \geq \alpha\}$$

Fuzzy set  $R$  can be considered as the union of all its level sets:

$$R = \bigcup_{\alpha \in [0,1]} R_\alpha$$

### 1.4.8. CROSSOVER POINT

The element of the universal set, for which the membership function has the value of 0.5, is called a cross over point. The crossover element marks the point where the possibility of belonging becomes lower than the possibility of not belonging. Although an exact position of a cross over point is usually not very important, it characterizes a shape of the membership function.

### 1.4.9. ARITHMETIC MEAN (TFMF)

Let  $R = (R_1, R_2, R_3)$  be a triangular fuzzy number then arithmetic

$$\text{Mean}(R) = \frac{R_1 + R_2 + R_3}{3}$$

## 1.5. ADDITION OPERATION ON TFMF

Assume  $\bar{R} = (r_{ij})$  and  $\bar{S} = (s_{ij})$  be a triangular fuzzy number of same order matrices. Then  $\bar{R} (+) \bar{S} = (r_{ij} + s_{ij})$  where  $r_{ij} + s_{ij} = (r_{ijL} + s_{ijU}, r_{ijM} +$

$s_{ijL}, r_{ijU} + s_{ijM})$  is the  $ij^{th}$  of  $\bar{R} (+) \bar{S}$ .

### 1.5.1. SUBTRACTION OPERATION ON TFMF

Assume  $\bar{R} = (r_{ij})$  and  $\bar{S} = (s_{ij})$  be a triangular fuzzy number of same Order matrices. Then  $\bar{R} (-) \bar{S} = (r_{ij} - s_{ij})$  where  $r_{ij} - s_{ij} = r_{ijL} - s_{ijU}, r_{ijM} - s_{ijL}, r_{ijU} - s_{ijM})$  is the  $ij^{th}$  of  $\bar{R} (-) \bar{S}$ .



### 1.5.2. MULTIPLICATION OPERATION ON TFMF

Assume  $\bar{R} = r_{ij}$  and  $\bar{S} = s_{ij}$  be two triangular fuzzy number matrix.

Then the multiplication operation  $R(.)S = (c_{ij})$  where  $(c_{ij}) = \sum_{k=1}^p R_{ik} S_{jk}$  for  $i= 1,2,3,\dots,m$  and  $j=1,2,3,\dots,n$ .

### 1.5.3. MAX-MIN COMPOSITION ON FMVM

Let  $F_{mn}$  denote the set of all  $m \times n$  matrices over F. Element of  $F_{mn}$

Are called as fuzzy membership value matrices.

For  $R = (r_{ij}) \in F$  and  $S = (s_{ij}) \in F$  the max -min product

$$R(.)S = (\sup [\{\inf \{r_{ik} : s_{kj}\}\}]) \in F$$

### 1.5.4. MAXIMUM OPERATION ON TFM

Let  $R = (r_{ij})$  where  $r_{ij} = (r_{ijL}, r_{ijM}, r_{ijU})$  and  $S = (s_{ij})$

Where  $s_{ij} = (s_{ijL}, s_{ijM}, s_{ijU})$  be two triangular fuzzy number matrices of same order . Then

The maximum operation on it is given by  $L_{max} = \max (R, S) = (\sup \{r_{ij} ; s_{ij}\})$  where

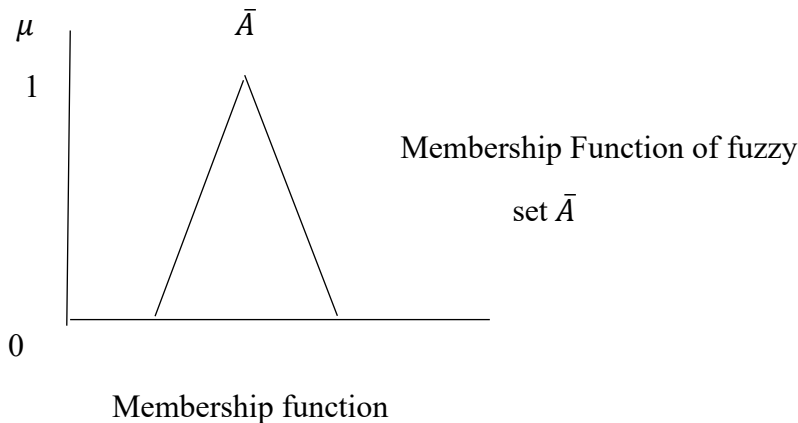
$\sup \{r_{ij} , s_{ij}\} = (\sup (r_{ijL}; s_{ijL}), \sup(r_{ijM}, s_{ijM}) \sup (r_{ijU}, s_{ijU}))$  is the  $ij^{th}$  element of  $\max (R, S)$ .

## CHAPTER 3

### FEATURES OF MEMBERSHIP FUNCTIONS AND ALGORITHM

#### 3.1. FEATURES OF MEMBERSHIP FUNCTIONS

The fuzzy logic is not logic that is fuzzy but logic that is used to describe fuzziness. This fuzziness is best characterized by its membership function. In other words, we can say that membership function represents the degree of truth in fuzzy logic.



Following are a few important points relating to the membership function –

- Membership functions were first introduced in 1965 by Lofti A. Zadeh in his first research paper “fuzzy sets”.
- Membership functions characterize fuzziness (i.e., all the information in fuzzy set), whether the elements in fuzzy sets are discrete or continuous.
- Membership functions can be defined as a technique to solve practical problems by experience rather than knowledge.
- Membership functions are represented by graphical forms.
- Rules for defining fuzziness are fuzzy too.

### 3.1.1. MATHEMATICAL NOTATION

We have already studied that a fuzzy set  $\tilde{A}$  in the universe of information  $U$  can be defined as a set of ordered pairs and it can be represented mathematically as –

$$\tilde{A} = \{(y, \mu_{\tilde{A}}(y)) \mid y \in U\}$$

- Here  $\mu_{\tilde{A}}(\bullet)$  = membership function of  $\tilde{A}$ ; this assumes values in the range from 0 to 1, i.e.,  $\mu_{\tilde{A}}(\bullet) \in [0, 1]$ . The membership function  $\mu_{\tilde{A}}(\bullet)$  maps  $U$  to the membership space  $M$ .
- The dot ( $\bullet$ ) in the membership function described above, represents the element in a fuzzy set; whether it is discrete or continuous.
- Advantages of features of membership function:

We will now discuss the different features of Membership Functions are:

### 3.1.2. CORE

For any fuzzy set  $\tilde{A}$ , the core of a membership function is that region of universe that is characterized by full membership in the set. Hence, core consists of all those elements  $Y$  of the universe of information such that,

$$\mu_{\tilde{A}}(y) = 1$$

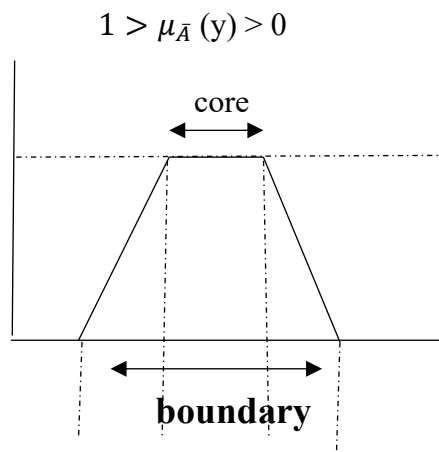
### 3.1.3. SUPPORT

For any fuzzy set  $\tilde{A}$ , the support of a membership function is the region of universe that is characterized by a nonzero membership in the set. Hence core consists of all those elements  $y$  of the universe of information such that,

$$\mu_{\tilde{A}}(y) > 0$$

### 3.1.4. BOUNDARY

For any fuzzy set  $\tilde{A}$ , the boundary of a membership function is the region of universe that is characterized by a nonzero but incomplete membership in the set. Hence, core consists of all those elements  $y$  of the universe of information such that,



### 3.1.5. FUZZIFICATION

It may be defined as the process of transforming a crisp set to a fuzzy set or a fuzzy set to fuzzier set. Basically, his operation translates accurate crisp input values into linguistic variables.

### 3.1.6. DEFUZZIFICATION

It may be defined as the process of reducing a fuzzy set into a crisp set or to convert a fuzzy member into a crisp member. Mathematically, the process of defuzzification is also called as ‘**rounding it off**’.

### 3.1.7. MAX-MEMBERSHIP METHOD

This method is limited to peak output functions and also known as height method. Mathematically it can be represented as follows

$$\mu_{\tilde{A}}(x^*) > \mu_{\tilde{A}}(x) \text{ for all } x \in X. \text{ Here, } x^* \text{ is the defuzzied output}$$

### 3.1.8. CENTROID METHOD

This method is also known as the centre of area or the centre of gravity method. Mathematically, the defuzzied output  $x^*$  will be represented as

$$x^* = \frac{\int \mu_{\tilde{A}}(x) \cdot x dx}{\int \mu_{\tilde{A}}(x) \cdot dx}$$

### 3.1.9. WEIGHTED AVERAGE METHOD

In this method, each membership function is weighted by its maximum membership value. Mathematically, the defuzzied output  $x^*$  will be represented as

$$X^* = \frac{\sum \mu_{\tilde{A}}(\bar{x}_i) \cdot \bar{x}_i}{\sum \mu_{\tilde{A}}(\bar{x}_i)}$$

## 3.2. MEAN-MAX MEMBERSHIP

This method is also known as the middle of the maxima. Mathematically, the defuzzied output  $x^*$  will be represented as

$$X^* = \frac{\sum_{i=1}^n x_i}{n}$$

## ALGORITHM

### STEP 1:

Construct triangle fuzzy number matrix  $(F, I)$  over  $M$ , where  $F$  is a mapping given by  $F: I \longrightarrow \check{F}(s)$ ,  $\check{F}(s)$  is a set of all triangular fuzzy sets of  $M$ . This matrix is denoted by  $G_o$  which is the fuzzy occurrence matrix or manifestation -illness triangular fuzzy number matrix.

### STEP 2:

Construct another triangular fuzzy number matrix  $(F_1, M)$  over  $C$ , where  $F_1$  is a mapping given by  $F: M \longrightarrow F(C)$ . This matrix is denoted by  $G_s$  which is the cases - Manifestation triangular fuzzy number matrix.

### STEP 3:

Convert the elements of triangular fuzzy number matrix into its membership function as follows:

Membership function of  $(a_{ij}) = (a_{ijL}, a_{ijM}, a_{ijU})$  is defined as  $\mu_{a_{ijL}}$   
 $= (\frac{a_{ijL}}{10}, \frac{a_{ijM}}{10}, \frac{a_{ijU}}{10})$ , if  $0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \leq 10$ , where  $0 \leq a_{ijL} \leq a_{ijM} \leq a_{ijU} \leq 1$ .

Now the matrix  $G_o$  and  $G_s$  are converted into triangular fuzzy membership matrices namely  $(G_o)_{mem}$  and  $(G_s)_{mem}$ .

### STEP 4:

Compute the following relation matrices:

$G_1 = (G_s)_{mem} \bullet (G_o)_{mem}$  it is calculated using definition

$G_2 = (G_s)_{mem} \bullet J(-)(G_o)_{mem}$ , where  $J$  is the triangular fuzzy membership matrix in which all entries are  $(1,1,1)$   $\bullet J(-)(G_o)_{mem}$  is the complement of  $(G_o)_{mem}$  and it is called as non- manifestation -Illness triangular fuzzy membership matrix.

$G_3 = (J(-)(G_s)mem)(\bullet)(G_o)mem$  where  $(J(-)(G_s)mem)$  is the complement of  $G_s$  and it is called as non-Illness – manifestation triangular fuzzy membership matrix.  $G_2$  and  $G_3$  are calculated using subtraction operation and definition

$G_4 = \max \{G_2 ; G_3\}$  . It is calculated using definition

The element of  $G_1, G_2, G_3, G_4$  is of the form  $Y_{ij} = (Y_{ijL}, Y_{ijM}, Y_{ijU})$  where  $0 \leq Y_{ijL} \leq Y_{ijM} \leq Y_{ijU} \leq 1$ .

$G_5 = G_1(-)G_2$ . It is calculated using subtraction operation .The elements of  $G_5$  is of the form  $z_{ij} = (z_{ijL}, z_{ijM}, z_{ijU}) \in [-1,1]$  where  $z_{ijL} \leq z_{ijM} \leq z_{ijU}$  .

### STEP 5:

Calculate  $G_6 = AM(Z_{ij})$  and Row I = Maximum of ith row which helps the decision maker to strongly confirm the Illness for the cases.

## CHAPTER 4

### CASE STUDY ABOUT DIABETES AND HYPERTENSION CASES BY USING FUZZY MATRIX MODEL

#### CASE STUDY

Consider three cases  $C_1, C_2$  and  $C_3$  admitted in a hospital with Manifestation of Headache, Blurred vision, Chest pain, Lack of energy. Suppose possible Illness with these Manifestation be Diabetes and Hypertension.

We assume that  $M_1, M_2, M_3$  and  $M_4$  represents the Manifestation Headache, Blurred vision, Chest pain, Lack of energy respectively. Assume that  $I_1$  and  $I_2$  represents the Illness Diabetes and Hypertension respectively. Let  $M = M_1, M_2, M_3, M_4$  and  $I = I_1, I_2$  be the parameter set representing the manifestation and Illness respectively. Also let  $C = C_1, C_2, C_3$  be the set of cases.

#### Step:1

Let us consider the set  $M = \{m_1, m_2, m_3, m_4\}$  as a universal set. There are represent the manifestation of Headache, Blurred vision, Chest pain, Lack of energy.

The set  $C = \{c_1, c_2\}$  represents the parameters diabetes and hypertension respectively.

$$F(I_1) = [<e_1(7,8.5,10)>, <e_2(3,4.5,6)>, <e_3(7,7.5,8)>, <e_4(4,5,6)>]$$

$$F(I_2) = [<e_1(7,7.5,8)>, <e_2(6,7,8)>, <e_4(5,6.5,8)>, <e_5(7,8,9)>]$$

Triangular fuzzy number matrix (F, I) is parameterized family  $(F(I_1), F(I_2))$  of all triangular fuzzy number matrix over the set M are determined from expert medical documentation.

Triangular fuzzy number matrix (F, I) represent a relation matrix  $G_0$  gives approximation description of the triangular fuzzy number matrix medical knowledge of two diseases are



$$\text{their symptoms is } G_0 = \begin{matrix} & \begin{matrix} M_1 & M_2 & M_3 & M_4 \end{matrix} \\ \begin{matrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{matrix} & \begin{pmatrix} (7,8.5,10) & (7,7.5,8) \\ (3,4.5,6) & (6,7,8) \\ (7,7.5,8) & (5,6.5,8) \\ (4,5,6) & (7,8,9) \end{pmatrix} \end{matrix}$$

## Step 2:

We take  $C = \{c_1, c_2, c_3\}$  as the universal set where  $c_1, c_2$  and  $c_3$  represent cases respectively and  $M = \{m_1, m_2, m_3, m_4\}$  as the set of parameters.

$$F_1(m_1) = [<C_1, (7,7.5,8)> ; <C_2, (4,5,6)> ; <C_3, (7,8,9)>]$$

$$F_1(m_2) = [<C_1, (5,6,7) > ; <C_2, (5,7,9)> ; <C_3, (4,6,8)>]$$

$$F_1(m_3) = [<C_1, (7,8,9)> ; <C_2, (4,5,6)> ; <C_3, (7,8,9)>]$$

$$F_1(m_4) = [<C_1, (6,7.5,9)> ; <C_2, (5,6,7)> ; <C_3, (4,5.5,7)>]$$

Triangular fuzzy number matrix  $(F_1, M)$  is another parameterized family of triangular fuzzy number matrix and gives a collection of approximate description of the cases-manifestations in the hospital. The triangular fuzzy number matrix  $(F_1, M)$  represent a relation matrix  $G_s$  called cases-manifestation matrix given by

$$G_s = \begin{matrix} & \begin{matrix} M_1 & M_2 & M_3 & M_4 \end{matrix} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (7,7.5,8) & (5,6,7) & (7,8,9) & (6,7.5,9) \\ (4,5,6) & (5,7,9) & (4,5,6) & (5,6,7) \\ (7,8,9) & (4,6,8) & (7,8,9) & (4,5.5,7) \end{pmatrix} \end{matrix}$$

### Step 3:

$$(G_0)_{mem} = \begin{matrix} & I_1 & I_2 \\ \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} & \begin{pmatrix} (0.7, 0.85, 1) \\ (0.3, 0.45, 0.6) \\ (0.7, 0.75, 0.8) \\ (0.4, 0.5, 0.6) \end{pmatrix} & \begin{pmatrix} (0.7, 0.75, 0.8) \\ (0.6, 0.7, 0.8) \\ (0.5, 0.65, 0.8) \\ (0.7, 0.8, 0.9) \end{pmatrix} \end{matrix}$$

$$(G_s)_{mem} = \begin{matrix} & M_1 & M_2 & M_3 & M_4 \\ \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} & \begin{pmatrix} (0.7, 0.75, 0.8) \\ (0.4, 0.5, 0.6) \\ (0.7, 0.8, 0.9) \end{pmatrix} & \begin{pmatrix} (0.5, 0.6, 0.7) \\ (0.5, 0.7, 0.9) \\ (0.4, 0.6, 0.8) \end{pmatrix} & \begin{pmatrix} (0.7, 0.8, 0.9) \\ (0.4, 0.5, 0.6) \\ (0.7, 0.8, 0.9) \end{pmatrix} & \begin{pmatrix} (0.6, 0.75, 0.9) \\ (0.5, 0.6, 0.7) \\ (0.4, 0.55, 0.7) \end{pmatrix} \end{matrix}$$

### Step 4:

Computing the following relation matrices. Now we find the Triangular membership function. So, we use the mini max formula.

$$A = [a_{ij}] = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$$

$$B = [b_{ij}] = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}$$

$$C_{11} = \max [\min (a_{11}, b_{11}), \min (a_{12}, b_{21})]$$

$$C_{12} = \max [\min (a_{11}, b_{12}), \min (a_{12}, b_{22})]$$

$$C_{13} = \max [\min (a_{11}, b_{13}), \min (a_{12}, b_{23})]$$

$$C_{21} = \max [\min (a_{21}, b_{11}), \min (a_{22}, b_{21})]$$

$$C_{22} = \max [\min (a_{21}, b_{12}), \min (a_{22}, b_{22})]$$

$$c_{23} = \max [\min (a_{21}, a_{13}), \min (a_{22}, b_{23})]$$

$$C_{31} = \max [\min (a_{31}, b_{11}), \min (a_{32}, b_{21})]$$

$$C_{32} = \max [\min (a_{31}, b_{12}), \min (a_{32}, b_{22})]$$

$$c_{33} = \max [\min (a_{31}, b_{13}), \min (a_{32}, b_{23})]$$

Using this formula to find the value of  $G_0$  and  $G_S$  respectively.

$$\begin{aligned} C_{11} &= \max [\min(0.7, 0.85, 1) (0.7, 0.75, 0.8), (0.3, 0.45, 0.6) (0.5, 0.6, 0.7), \\ &\quad (0.7, 0.75, 0.8) (0.7, 0.8, 0.9), (0.4, 0.5, 0.6) (0.6, 0.75, 0.9)] \\ &= \max [(0.7, 0.75, 0.8) (0.3, 0.45, 0.6) (0.7, 0.75, 0.8) (0.4, 0.5, 0.6)] \\ C_{11} &= (0.7, 0.75, 0.8) \end{aligned}$$

$$\begin{aligned} C_{12} &= \max [\min (0.7, 0.75, 0.8) (0.7, 0.75, 0.8), (0.6, 0.7, 0.8) (0.5, 0.6, 0.7), \\ &\quad (0.5, 0.65, 0.8) (0.7, 0.8, 0.9), (0.7, 0.8, 0.9) (0.6, 0.75, 0.9)] \\ &= \max [(0.7, 0.75, 0.8), (0.5, 0.6, 0.7), (0.5, 0.65, 0.8), (0.6, 0.75, 0.9)] \\ C_{12} &= (0.7, 0.75, 0.9) \end{aligned}$$

$$\begin{aligned} C_{21} &= \max [\min (0.7, 0.85, 1) (0.4, 0.5, 0.6), (0.3, 0.45, 0.6) (0.5, 0.7, 0.9), \\ &\quad (0.7, 0.75, 0.8) (0.4, 0.5, 0.6), (0.4, 0.5, 0.6) (0.5, 0.6, 0.7)] \\ &= \max [(0.4, 0.5, 0.6), (0.3, 0.45, 0.6), (0.4, 0.5, 0.6), (0.4, 0.5, 0.6)] \\ C_{21} &= (0.4, 0.5, 0.6) \end{aligned}$$

$$\begin{aligned} C_{22} &= \max [\min (0.7, 0.75, 0.8) (0.4, 0.5, 0.6), (0.6, 0.7, 0.8) (0.5, 0.7, 0.9), \\ &\quad (0.5, 0.65, 0.8) (0.4, 0.5, 0.6), (0.7, 0.8, 0.9) (0.5, 0.6, 0.7)] \end{aligned}$$

$$= \max [(0.4,0.5,0.6), (0.5,0.7,0.8), (0.4,0.5,0.6), (0.5,0.6,0.7)]$$

$$C_{22} = (0.5,0.7,0.8)$$

$$C_{31} = \max [\min (0.7,0.85,1) (0.7,0.8,0.9), (0.3,0.45,0.6) (0.4,0.6,0.8), \\ (0.7,0.75,0.8) (0.7,0.8,0.9), (0.4,0.5,0.6) (0.4,0.55,0.7)]$$

$$= \max [(0.7,0.8,0.9), (0.3,0.45,0.6), (0.7,0.75,0.8), (0.4,0.5,0.6)]$$

$$C_{31} = (0.7,0.8,0.9)$$

$$C_{32} = \max [\min (0.7,0.75,0.8) (0.7,0.8,0.9), (0.6,0.7,0.8) (0.4,0.6,0.8) \\ (0.5,0.65,0.8) (0.7,0.8,0.9), (0.7,0.8,0.9) (0.4,0.55,0.7)]$$

$$= \max [(0.7,0.75,0.8), (0.4,0.6,0.8), (0.5,0.65,0.8), (0.4,0.55,0.7)]$$

$$C_{32} = (0.7,0.75,0.8)$$

$$G_1 = (G_s)_{mem} (.) (G_0)_{mem} = \begin{matrix} & I_1 & I_2 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (0.7,0.75,0.8) & (0.7,0.75,0.9) \\ (0.4,0.5,0.6) & (0.5,0.7,0.9) \\ (0.7,0.8,0.9) & (0.7,0.75,0.8) \end{pmatrix} \end{matrix}$$

$$[(J)(-)(G_0)_{mem}] = \begin{matrix} & I_1 & I_2 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{matrix} & \begin{pmatrix} (0.3,0.15,0) & (0.3,0.25,0.2) \\ (0.7,0.55,0.4) & (0.4,0.3,0.2) \\ (0.3,0.25,0.2) & (0.5,0.35,0.2) \\ (0.6,0.5,0.4) & (0.3,0.2,0.1) \end{pmatrix} \end{matrix}$$

$$C_{11} = \max [\min (0.7,0.75,0.8) (0.3,0.15,0), (0.5,0.6,0.7) (0.7,0.55,0.4), \\ (0.7,0.8,0.9) (0.3,0.25,0.2), (0.6,0.75,0.9) (0.6,0.5,0.4)]$$

$$= \max [(0.3,0.15,0), (0.5,0.55,0.4), (0.3,0.25,0.2), (0.6,0.5,0.4)]$$

$$= (0.6, 0.55, 0.4)$$

$$\begin{aligned} C_{12} &= \max [ \min (0.7, 0.75, 0.8) (0.3, 0.25, 0.2), (0.5, 0.6, 0.7) (0.4, 0.3, 0.2), \\ &\quad (0.7, 0.8, 0.9) (0.5, 0.35, 0.2), (0.6, 0.75, 0.9) (0.3, 0.2, 0.1)] \\ &= \max [(0.3, 0.25, 0.2), (0.4, 0.3, 0.2), (0.5, 0.35, 0.2), (0.3, 0.2, 0.1)] \\ C_{12} &= (0.5, 0.35, 0.2) \end{aligned}$$

$$\begin{aligned} C_{21} &= \max [ \min (0.4, 0.5, 0.6) (0.3, 0.15, 0), (0.5, 0.7, 0.9) (0.7, 0.55, 0.4), \\ &\quad (0.4, 0.5, 0.6) (0.3, 0.25, 0.2), (0.5, 0.6, 0.7) (0.6, 0.5, 0.4)] \\ &= \max [(0.3, 0.15, 0), (0.5, 0.55, 0.4), (0.3, 0.25, 0.2), (0.3, 0.2, 0.1)] \\ C_{21} &= (0.5, 0.55, 0.4) \end{aligned}$$

$$\begin{aligned} C_{22} &= \max [ \min (0.4, 0.5, 0.6) (0.3, 0.25, 0.2), (0.5, 0.7, 0.9) (0.4, 0.3, 0.2), \\ &\quad (0.4, 0.5, 0.6) (0.5, 0.35, 0.2), (0.5, 0.6, 0.7) (0.3, 0.2, 0.1)] \\ &= \max [ (0.3, 0.25, 0.2), (0.4, 0.3, 0.2), (0.4, 0.35, 0.2), (0.3, 0.2, 0.1)] \\ C_{22} &= (0.4, 0.35, 0.2) \end{aligned}$$

$$\begin{aligned} C_{31} &= \max [ \min (0.7, 0.8, 0.9) (0.3, 0.15, 0), (0.4, 0.6, 0.8) (0.7, 0.55, 0.4), \\ &\quad (0.7, 0.8, 0.9) (0.3, 0.25, 0.2), (0.4, 0.55, 0.7) (0.6, 0.5, 0.4)] \\ &= \max [(0.3, 0.15, 0), (0.4, 0.55, 0.4), (0.3, 0.25, 0.2), (0.4, 0.5, 0.4)] \\ C_{31} &= (0.4, 0.55, 0.4) \end{aligned}$$

$$\begin{aligned} C_{32} &= \max [ \min (0.7, 0.8, 0.9) (0.3, 0.25, 0.2), (0.4, 0.6, 0.8) (0.4, 0.3, 0.2), \\ &\quad (0.7, 0.8, 0.9) (0.5, 0.35, 0.2), (0.4, 0.55, 0.7) (0.3, 0.2, 0.1)] \end{aligned}$$

$$= \max [(0.3,0.25,0.2), (0.4,0.3,0.2), (0.5,0.35,0.2), (0.3,0.2,0.1)]$$

$$C_{32} = (0.5,0.35,0.2)$$

$$G_2 = (G_s)_{mem} (.) (J)(-) (G_0)_{mem} = \begin{matrix} & I_1 & I_2 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (0.6,0.55,0.4) & (0.5,0.35,0.2) \\ (0.5,0.55,0.4) & (0.4,0.35,0.2) \\ 0.4,0.55,0.4 & (0.5,0.35,0.2) \end{pmatrix} \end{matrix}$$

$$[(J)(-)(G_s)_{mem}] = \begin{matrix} & M_1 & M_2 & M_3 & M_4 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (0.3,0.25,0.2) & (0.5,0.4,0.3) & (0.3,0.2,0.1) & (0.4,0.25,0.1) \\ (0.6,0.5,0.4) & (0.5,0.3,0.1) & (0.6,0.5,0.4) & (0.5,0.4,0.3) \\ (0.3,0.2,0.1) & (0.6,0.4,0.2) & (0.3,0.2,0.1) & (0.6,0.45,0.3) \end{pmatrix} \end{matrix}$$

$$C_{11} = \max [\min (0.3,0.25,0.2) (0.7,0.85,1), (0.5,0.4,0.3) (0.3,0.45,0.6),$$

$$(0.3,0.2,0.1) (0.7,0.75,0.8), (0.4,0.25,0.1) (0.4,0.5,0.6)]$$

$$= \max [(0.3,0.25,0.2), (0.3,0.4,0.3), (0.3,0.2,0.1), (0.4,0.25,0.1)]$$

$$C_{11} = (0.4,0.4,0.3)$$

$$C_{12} = \max [\min (0.3,0.25,0.2) (0.7,0.75,0.8), (0.5,0.4,0.3) (0.6,0.7,0.8),$$

$$(0.3,0.2,0.1) (0.5,0.65,0.8), (0.4,0.25,0.1) (0.7,0.8,0.9)]$$

$$= \max [(0.3,0.25,0.2), (0.5,0.4,0.3), (0.3,0.2,0.1), (0.4,0.25,0.1)]$$

$$C_{12} = (0.5,0.4,0.3)$$

$$C_{21} = \max [\min (0.6,0.5,0.4) (0.7,0.75,0.8), (0.5,0.3,0.1) (0.6,0.7,0.8),$$

$$(0.6,0.5,0.4) (0.5,0.65,0.8), (0.5,0.4,0.3) (0.7,0.8,0.9)]$$

$$= \max [(0.6,0.5,0.4), (0.3,0.3,0.1), (0.6,0.5,0.4), (0.4,0.4,0.3)]$$

$$C_{21} = (0.6, 0.5, 0.4)$$

$$\begin{aligned} C_{22} &= \max [ \min (0.6, 0.5, 0.4) (0.7, 0.75, 0.8), (0.5, 0.3, 0.1) (0.6, 0.7, 0.8), \\ &\quad (0.6, 0.5, 0.4) (0.5, 0.65, 0.8), (0.5, 0.4, 0.3) (0.7, 0.8, 0.9)] \\ &= \max [ (0.6, 0.5, 0.4), (0.5, 0.3, 0.1), (0.5, 0.5, 0.4), (0.5, 0.4, 0.3)] \end{aligned}$$

$$C_{22} = (0.6, 0.5, 0.4)$$

$$\begin{aligned} C_{31} &= \max [ \min (0.3, 0.2, 0.1) (0.7, 0.85, 1), (0.6, 0.4, 0.2) (0.3, 0.45, 0.6), \\ &\quad (0.3, 0.2, 0.1) (0.7, 0.75, 0.8), (0.6, 0.45, 0.3) (0.4, 0.5, 0.6)] \\ &= \max [ (0.3, 0.2, 0.1), (0.3, 0.4, 0.2), (0.3, 0.2, 0.1), (0.4, 0.45, 0.3)] \end{aligned}$$

$$C_{31} = (0.4, 0.45, 0.3)$$

$$\begin{aligned} C_{32} &= \max [ \min (0.3, 0.2, 0.1) (0.7, 0.75, 0.8), (0.6, 0.4, 0.2) (0.6, 0.7, 0.8), \\ &\quad (0.3, 0.2, 0.1) (0.5, 0.65, 0.8), (0.6, 0.45, 0.3) (0.7, 0.8, 0.9)] \\ &= \max [ (0.3, 0.2, 0.1), (0.6, 0.4, 0.2), (0.3, 0.2, 0.1), (0.6, 0.45, 0.3)] \end{aligned}$$

$$C_{32} = (0.6, 0.45, 0.3)$$

$$G_3 = (J)(-)(G_s)_{mem} (.) (G_0)_{mem} = \begin{matrix} & I_1 & I_2 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (0.4, 0.4, 0.3) & (0.5, 0.4, 0.3) \\ (0.6, 0.5, 0.4) & (0.6, 0.5, 0.4) \\ (0.4, 0.45, 0.3) & (0.6, 0.45, 0.3) \end{pmatrix} \end{matrix}$$

$$G_4 = \max \{G_2, G_3\} = \begin{matrix} & I_1 & I_2 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (0.6, 0.55, 0.4) & (0.5, 0.4, 0.3) \\ (0.6, 0.55, 0.4) & (0.6, 0.5, 0.4) \\ (0.4, 0.55, 0.4) & (0.6, 0.45, 0.3) \end{pmatrix} \end{matrix}$$

$$G_5 = G_1 (-) G_4 = \begin{matrix} & I_1 & I_2 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (0.3, 0.2, 0.4) & (0.2, 0.35, 0.6) \\ (-0.2, -0.05, 0.2) & (-0.1, 0.2, 0.4) \\ (0.3, 0.25, 0.5) & (0.1, 0.3, 0.5) \end{pmatrix} \end{matrix}$$

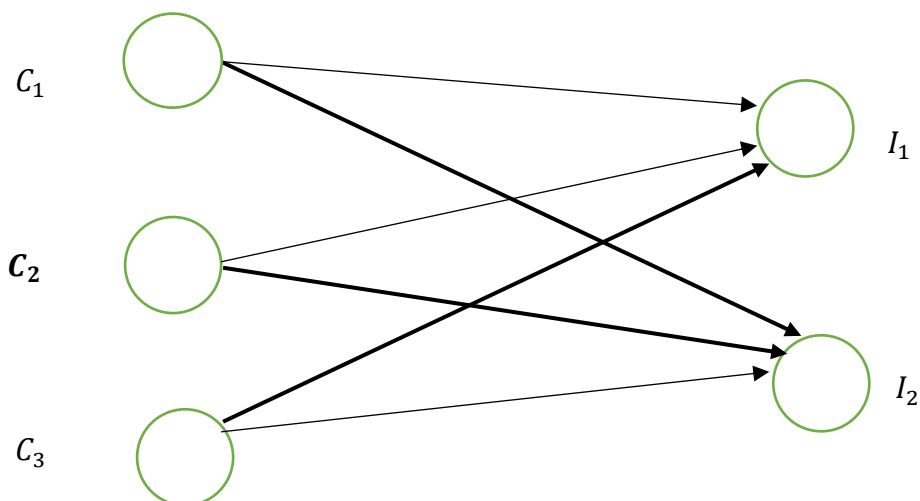
$$G_5 = G_1 (-) G_4 = \begin{matrix} & I_1 & I_2 \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} (0.3, 0.2, 0.4) & (0.2, 0.35, 0.6) \\ (-0.2, -0.05, 0.2) & (-0.1, 0.2, 0.4) \\ (0.3, 0.25, 0.5) & (0.1, 0.3, 0.5) \end{pmatrix} \end{matrix}$$

**Step 5:**

$$G_6 = \begin{matrix} & I_1 & I_2 & \text{Row } i = \max \text{ of } i^{th} \text{ row} \\ \begin{matrix} C_1 \\ C_2 \\ C_3 \end{matrix} & \begin{pmatrix} 0.3 & 0.38 & 0.38 \\ -0.01 & 0.16 & 0.16 \\ 0.35 & 0.3 & 0.35 \end{pmatrix} \end{matrix}$$

Diagram:

If we prove the above case study reveals in the form of graph network as follows.





In the above graph network, nodes and vertices denote the cases and illness, lengths and edges denote the assumption of illness to the cases. The darken edges denote the strong confirmation of illness to the cases.

## **CHAPTER 5**

### **APPLICATIONS ON FUZZY SYSTEM IN MEDICAL DIAGNOSIS**

#### **Aerospace Field:**

In aerospace, fuzzy logic is used in the following areas

- Altitude control of spacecraft
- Satellite altitude control
- Flow and mixture regulation in aircraft deicing vehicles

#### **Automotive Field:**

In automotive, fuzzy logic is used in the following areas

- Trainable fuzzy systems for idle speed control
- Shift scheduling method for automatic transmission
- Intelligent highway systems
- Traffic control
- Improving efficiency of automatic transmission

#### **Business Field:**

In business, fuzzy logic is used in the following areas

- Decision-making support systems
- Personnel evaluation in a large company

## **Defense Field:**

In defense, fuzzy logic is used in the following areas:

- Underwater target recognition
- Automatic target recognition of thermal infrared images
- Naval decision support aids
- Control of a hypervelocity interceptor
- Fuzzy set modeling of NATO decision making

## **Marine Field:**

In the marine field, fuzzy logic is used in following areas

- Autopilot for ships
- Optimal route selection
- Control of autonomous underwater vehicles
- Ship steering

## **Medical Field:**

In the medical field, fuzzy logic is used in the following areas

- Medical diagnostic support system
- Control of arterial pressure during anesthesia
- Multivariable control of anesthesia
- Modeling of neuropathological findings in Alzheimer's patients
- Radiology diagnoses
- Fuzzy inference diagnoses of diabetes and prostate cancer

## **Transportation Sector:**

In transportation, fuzzy logic is used in the following areas

- Train schedule control

- Railway acceleration
- Braking and stopping

## **Electronics:**

In electronics, fuzzy logic is used in the following areas

- Control of automatic exposure in video cameras
- Air conditioning systems
- Washing humidity in a clean room
- Machine timing
- Microwave ovens
- Vacuum cleaners

## **Finance:**

In the finance field, fuzzy logic is used in the following areas

- Banknote transfer control
- Fund management
- Stock market predictions

## **Industrial Sector:**

In industrial, fuzzy logic is used in the following areas

- Cement controls heat exchanger control
- Activated sludge wastewater treatment process control
- Water purification plant control
- Quantitative pattern analysis for industrial quality assurance
- Control of constraint satisfaction problems in structural
- Control of water purification plants

## **Manufacturing Sector:**

In the manufacturing industry, fuzzy logic is used in the following areas

- Optimization of cheese production
- Optimization of milk production

## **Securities:**

In securities, fuzzy logic is used in the following areas

- Decision systems for securities trading
- Various security appliances

## **Pattern Recognition and Classification**

In pattern recognition and classification, fuzzy logic is used in the following areas

- Fuzzy logic based speech recognition
- Fuzzy logic based
- Handwriting recognition
- Fuzzy logic based facial characteristic analysis
- Command analysis
- Fuzzy image search

## **Psychology Field:**

In psychology, fuzzy logic is used in the following areas

- Fuzzy logic based analysis of human behavior
- Criminal investigation and prevention based on fuzzy logic reasoning.

## **Fuzzy logic in Medical Field**

### **Fuzzy logic with Asthma Disease:**

- Asthma is a persistent lung issues influencing lungs because of restricted aviation routes.
- It is a sort of hazardous sickness making breathing issues a person.
- A framework to determine asthma by allotting boundaries to have fuzzy logic

### **Fuzzy logic with Diabetes Disease:**

- It is a kind of sickness coming about high blood glucose level in body
- A lot of sugar level in the body leads different issues like harming kidney and nerves.

### **Fuzzy logic with Cholera Disease:**

- Cholera is a bacterial disease mostly occurred after consumption of drinking contaminated water.
- It is a type of disease that can **lead** to dehydration, diarrhea and up to death if not tackle at right time.

### **Fuzzy logic with Breast Cancer:**

- Breast cancer is a kind of sickness caused because of irregularities found in bosom that shapes the cells.
- This infection viewed as the second generally deadliest in ladies when contrasted with cellular breakdown in the lungs.

### **Fuzzy logic with Liver Disease:**

- It is a sort of hepatic illness that makes liver forestall working and its working
- Most of variables of liver infection are because of alcoholic or hereditary nature.
- The most well-known kinds of liver infection are greasy liver.

### **Fuzzy logic with Dental Disease:**

- It is a kind of sickness that tainting encompassing teeth as tooth rot, periodontal infection, gum disease, dental plaque
- The goal of utilizing fuzzy logic dependent on 164 fuzzy principals as fuzzy extraction with 5 informative factors.
- In 2014 proposed a model of consolidating three distinct infections shrouded in one fuzzy master approach.

### **Fuzzy logic with Heart Disease:**

- It is a kind of sickness caused because of harm or blockage of veins in heart influencing less supplement and oxygen supply to heart organ.
- Various kind of heart illness are normal like vein issue, heart failure, cardiovascular breakdown etc...

## **Applications of hypertension in medical field:**

### **Data Science Transformation**

- Precision medicine data revolution
- Creation of precision cohorts to decipher heterogeneous treatment effect
- Artificial intelligent

### **Digital Transformation**

- Enable home blood pressure measurement

- Promote healthy behaviors

## **Population Science Transformation**

- Cross-sector approach
- Health in all policies

## **Health Care Deliver Transformation:**

- Better access to diagnosis and effective treatment
- Dissemination of standardized evidence based-treatment
- Coordinated, team-based care

## **Bio Medical Transformation:**

- Novel RNA, DNA cell-based therapies
  - RNAi
  - Gene editing
  - Regenerative medicine



## CONCLUSION

In our work we have defined medicine is one of the widely field in which the applicability if fuzzy logic theory was recognized quite early on natural language and also uses method instead of a specific algorithm and with more applications has flexibility in our knowledge.

This paper brings out and overview and reflection based on the literature on the fuzzy membership function in medical diagnosis and gives an insight are for further study to build and design different matrix models and methods of diagnosing the illness with fuzzy statistical distribution.

By the doctor knowledge we gather about the cases from the past 4 years of history and investigate above more cases with diabetes and hypertension. The knowledge provided by each of these sources carries with it varying degrees of and certainty. Hence in this paper we have analysis that the cases  $C_1$  and  $C_2$  suffer from diabetes whereas the cases  $C_3$  faces hypertension further our work is supported by a decision-making problem in medical diagnosis.

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