NPTEL ONLINE CERTIFICATION COURSES

Course Name: Fuzzy Logic and Neural Networks

Faculty Name: Prof. Dilip Kumar Pratihar

Department : Mechanical Engineering

Week 1

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Topic

Lecture 01: Introduction to Fuzzy Sets

CONCEPTS COVERED

Concepts Covered:

- Classical Set/Crisp Set
- Properties of Classical Set/Crisp Set
- Fuzzy Set
- Representation of Fuzzy Set

Classical Set/Crisp Set (A)

- Universal Set/Universe of Discourse (X): A set consisting of all possible elements

  Ex: All technical universities in the world
- Classical or Crisp Set is a set with fixed and well-defined boundary
- Example: A set of technical universities having at least five departments each

Representation of Crisp Sets

- $A=\{a_1,a_2,\ldots\ldots,a_n\}$
- $A=\{x|P(x)\}$, P: property
- **Using characteristic function**

$$\mu_A(x)=\begin{cases}1, & \text{if } x \text{ belongs to } A,\\ 0, & \text{if } x \text{ does not belong to } A.\end{cases}$$

Notations Used in Set Theory

- Φ : Empty/Null set
- $x \in A$: Element *x* of the Universal set X belongs to set A
- $x \notin A$: *x* does not belong to set A
- $A \subseteq B$: set A is a subset of set B
- $A \supseteq B$: set A is a superset of set B
- $A = B$: A and B are equal
- $A \neq B$: A and B are not equal

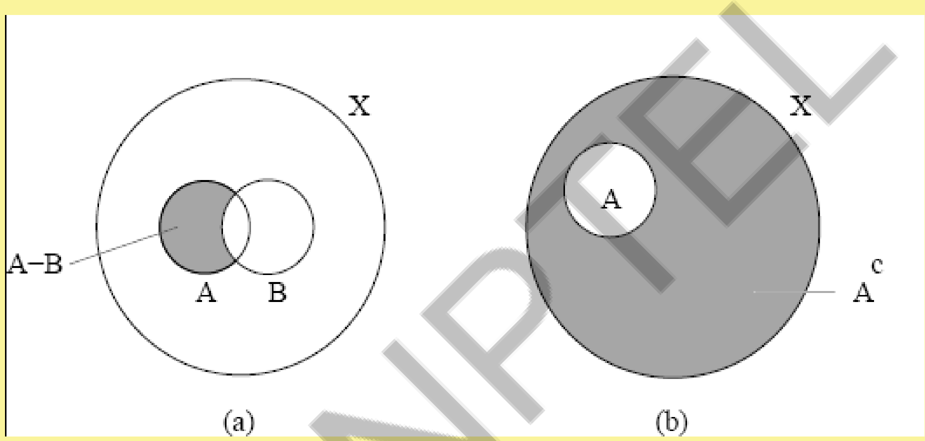
- $A \subset B$: **A is a proper subset of B**
- $A \supset B$: **A is a proper superset of B**
- $|A|$: **Cardinality of set A is defined as the total number of elements present in that set**
- $p(A)$: **Power set of A is the maximum number of subsets including the null that can be constructed from a set A**

Note: $|p(A)| = 2^{|A|}$

Crisp Set Operations

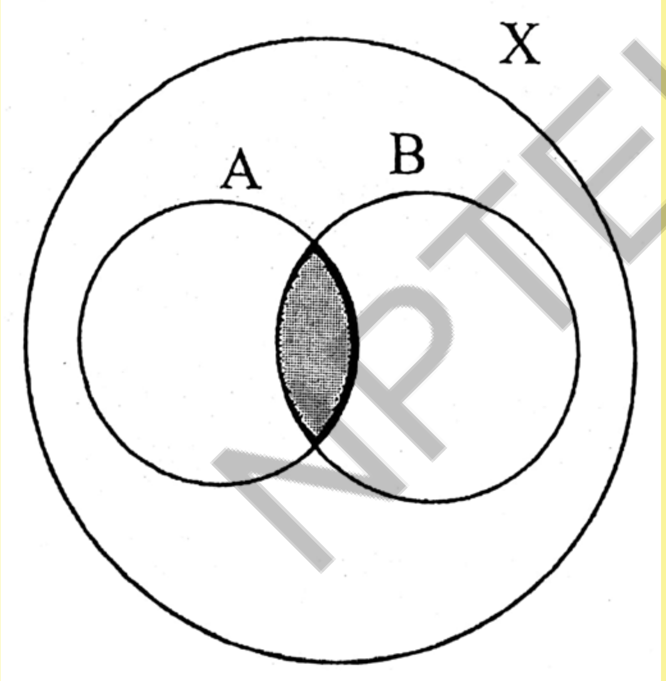
- Difference: $\boldsymbol{A - B = \{x | x \in A \text{ and } x \notin B\}}$

It is known as relative complement of set B with respect to set A

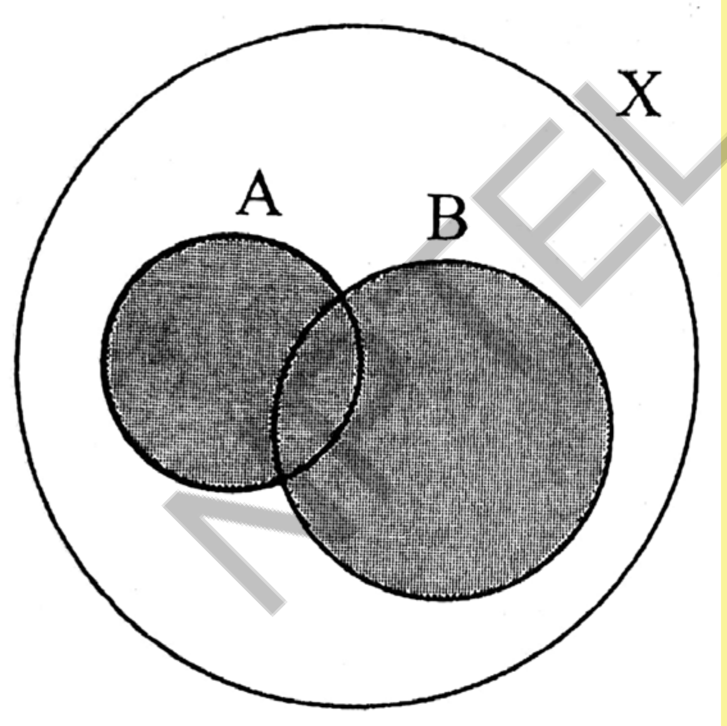


Absolute complement: $\boldsymbol{\bar{A} = A^C = X - A = \{x | x \in X \text{ and } x \notin A\}}$

- Intersection: $A \cap B = \{x | x \in A \; and \; x \in B\}$



- Union: $A \cup B = \{x|x \in A \; or \; x \in B\}$



Properties of Crisp Sets

1. Law of involution: $\bar{\bar{A}} = A$
2. Law of Commutativity: $A \cup B = B \cup A$; $A \cap B = B \cap A$
3. Associativity: $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$
4. Distributivity: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$; $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$;
5. Laws of Tautology: $A \cup A = A; A \cap A = A$
6. Laws of Absorption: $A \cup (A \cap B) = A$; $A \cap (A \cup B) = A$
7. Laws of Identity: $A \cup X = X$; $A \cap X = A$; $A \cup \Phi = A$; $A \cap \Phi = \Phi$
8. De Morgan's Laws: $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$; $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$
9. Law of contradiction: $A \cap \bar{A} = \Phi$
10. Law of excluded middle: $A \cup \bar{A} = X$

Fuzzy Sets

- Sets with imprecise/vague boundaries
- Introduced by Prof. L.A. Zadeh, University of California, USA, in 1965
- Potential tool for handling imprecision and uncertainties
- Fuzzy set is a more general concept of the classical set

Representation of a Fuzzy Set

$$A(x) = \{(x, \mu_A(x)), x \in X\}$$

Note:

Probability: Frequency of likelihood that an element is in a class

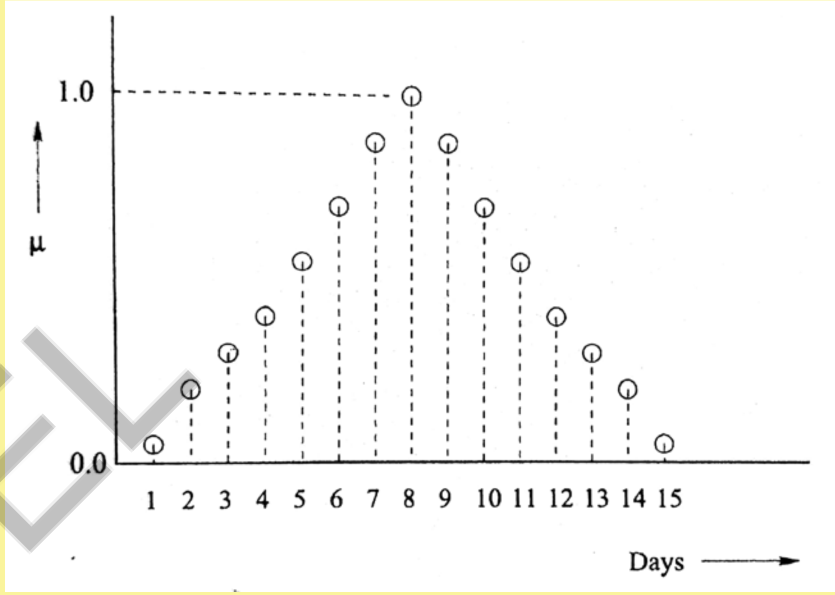
Membership: Similarity of an element to a class

Types of Fuzzy sets

1. Discrete Fuzzy set

$$A(x)=\sum_{i=1}^{n}\mu_A(x_i)/x_i,$$

n: Number of elements present in the set



2. Continuous Fuzzy set

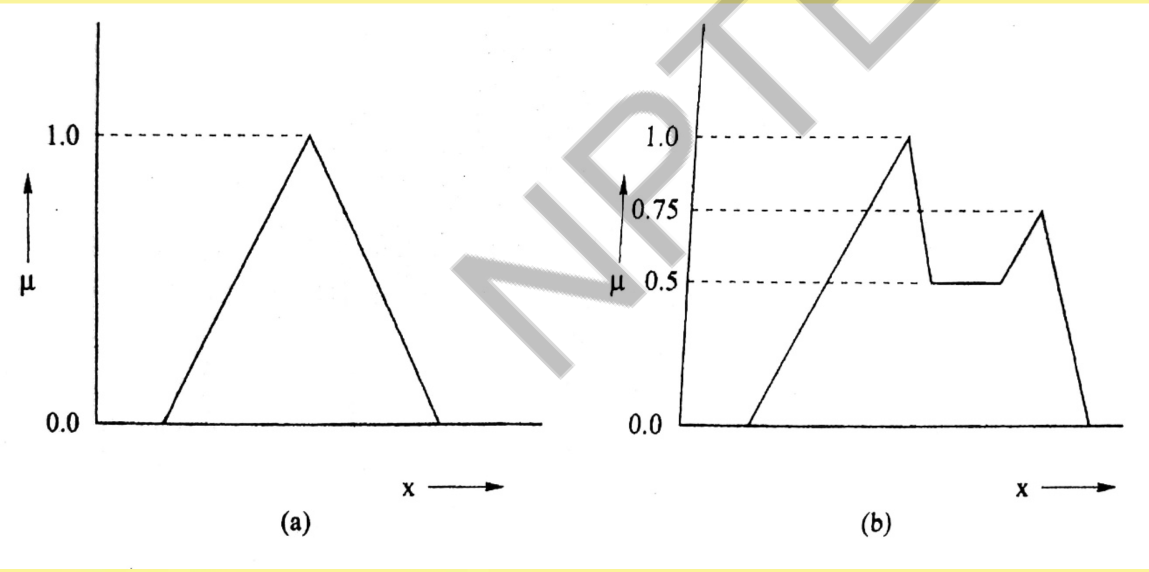
$$A(x)=\int_X \mu_A(x)/x$$

Convex vs. Non-Convex Membership Function Distribution

A fuzzy set A(*x*) will be convex, if

$$\mu_A\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$$

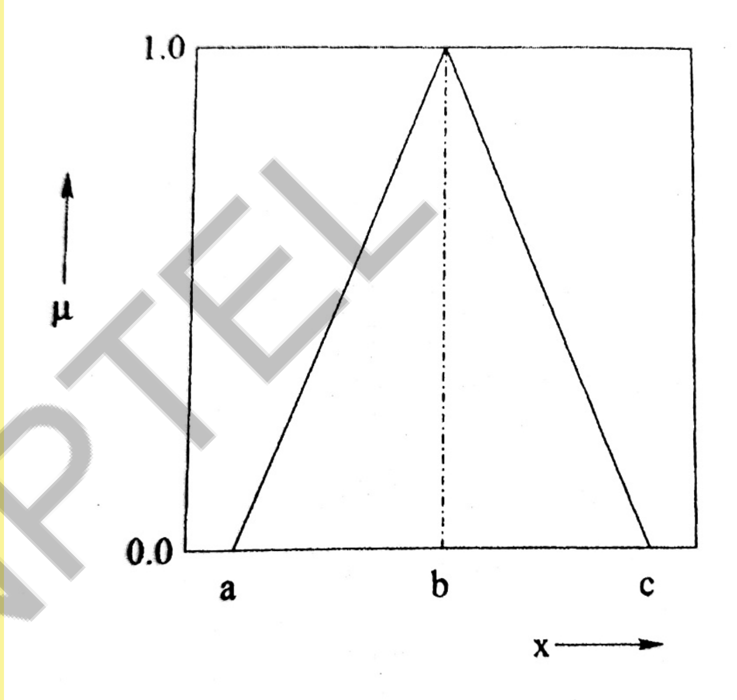
Where 0.0 ≤ λ ≤ 1.0



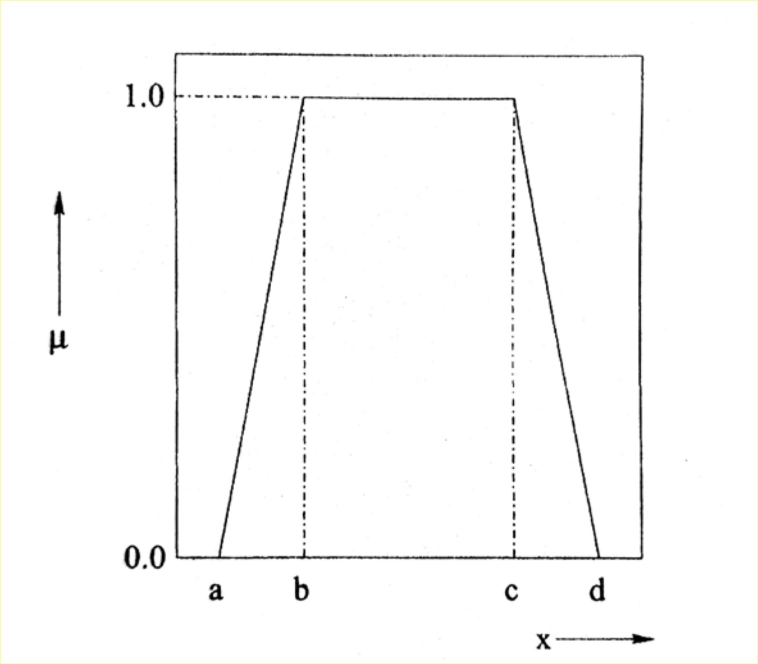
Various Types of Membership Function Distributions

1. Triangular Membership

$$\mu_{triangle} = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$

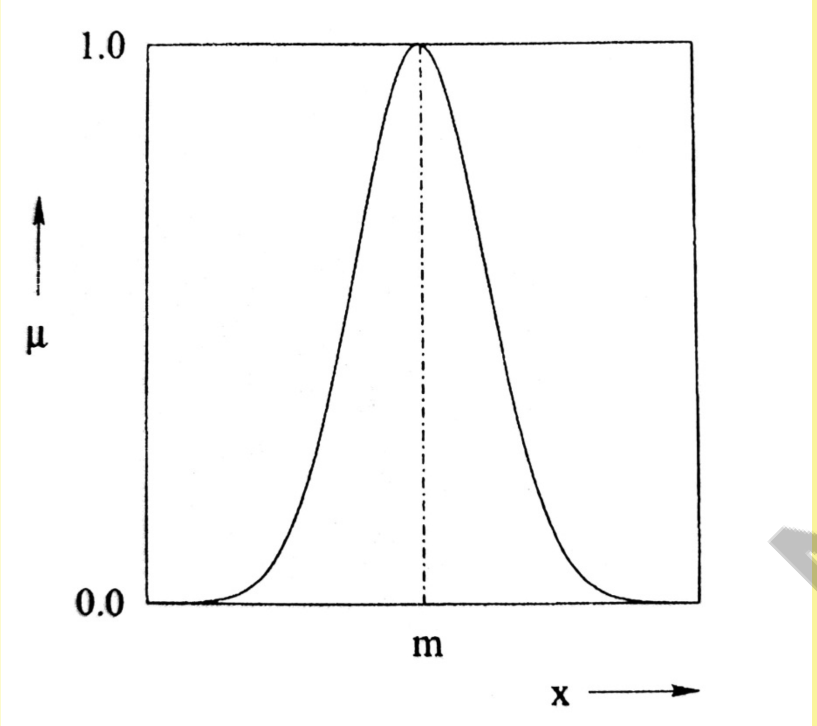


2. Trapezoidal Membership



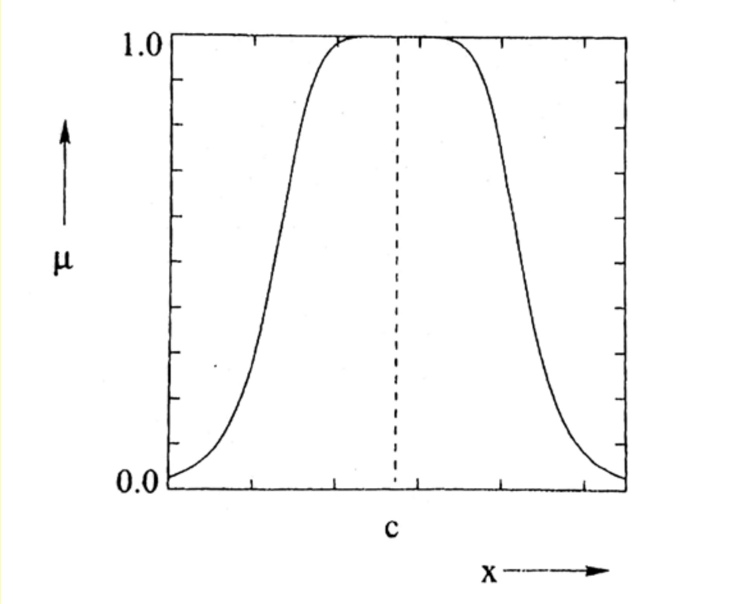
$$\mu_{trapezoidal} = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

3. Gaussian Membership



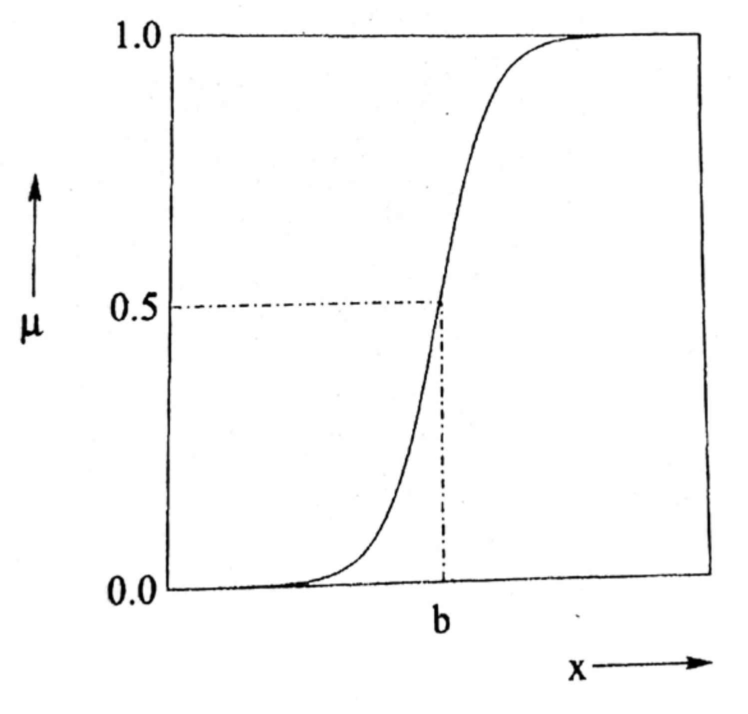
$$\mu_{Gaussian} = \frac{1}{e^{\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}}$$

4. Bell-shaped Membership Function



$$\mu_{Bell-shaped} = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

5. Sigmoid Membership



$$\mu_{Sigmoid} = \frac{1}{1+e^{-a(x-b)}}$$

References

Reference:

Pratihar D.K.: Soft Computing: Fundamentals and Applications, Narosa Publishing House, New-Delhi, 2014

Conclusion

Conclusion:

Classical Set/Crisp Set has been defined

Properties of Classical Set/Crisp Set has been explained

Fuzzy Set has been defined

Deals with representation of Fuzzy Set

NPTEL ONLINE CERTIFICATION COURSES

Course Name: FUZZY LOGIC AND NUERAL NETWORKS

Faculty Name: Prof. Dilip Kumar Pratihar

Department: Mechanical Engineering, IIT Kharagpur

Topic

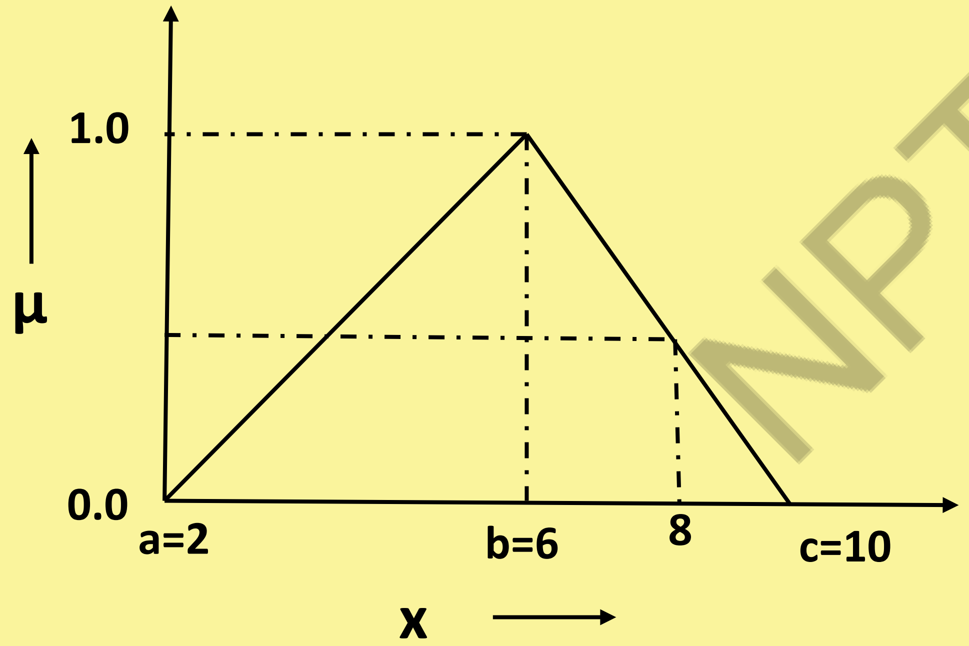
Lecture 02: Introduction to Fuzzy Sets (contd.)

Concepts Covered:

- A few terms of Fuzzy Sets
- Standard Operations in Fuzzy Sets
- Properties of Fuzzy Sets
- Fuzziness and Inaccuracy of Fuzzy Sets

Numerical Example

Triangular Membership: Determine μ, corresponding to x=8.0



$$\mu_{triangle} = max[min\left(\frac{x-a}{b-a},\frac{c-x}{c-b}\right),0]$$

$$= max[min\left(\frac{x-2}{6-2},\frac{10-x}{10-6}\right),0]$$

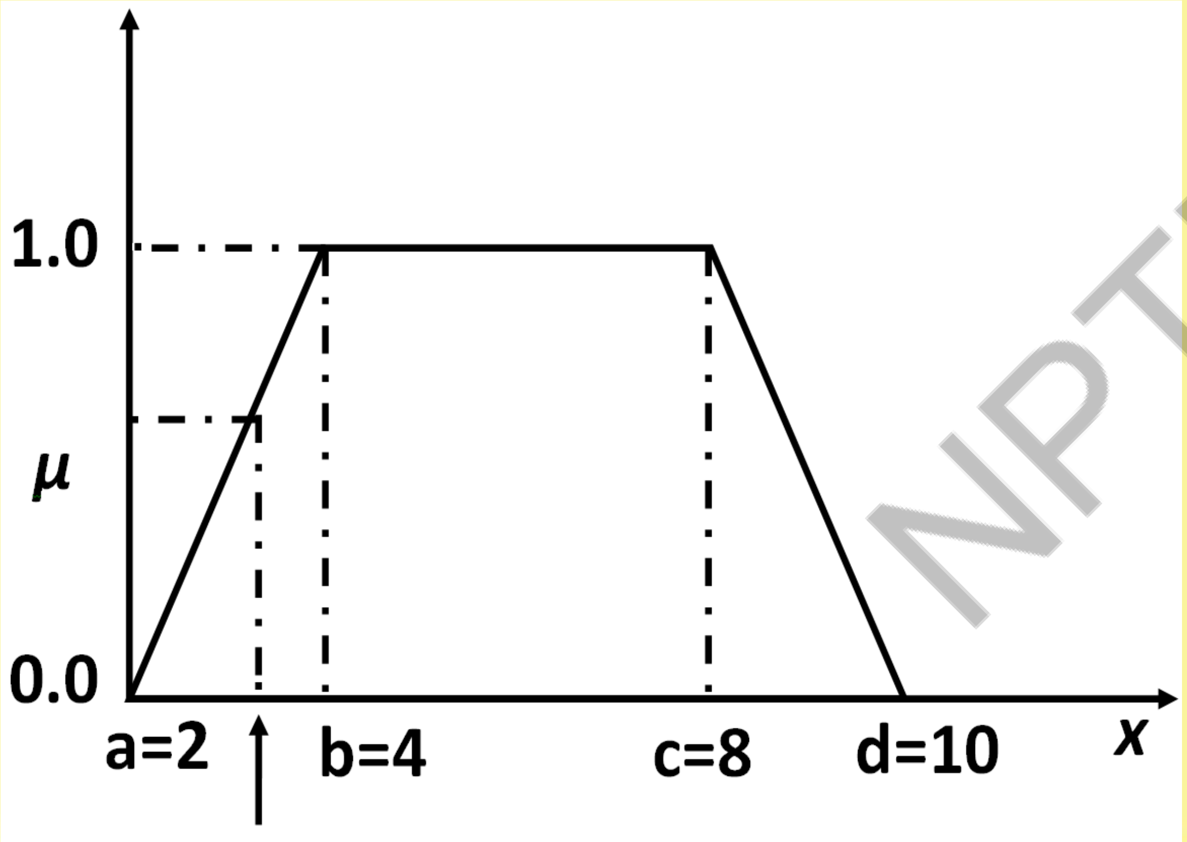
$$= max[min\left(\frac{x-2}{4},\frac{10-x}{4}\right),0]$$

We put, x=8.0

$$\mu_{triangle} = max\left[min\left(\frac{3}{2},\frac{1}{2}\right),0\right] = \frac{1}{2} = 0.5$$

Trapezoidal Membership

- Determine μ corresponding to x = 3.5



$$\mu_{trapezoidal} = \max\left[\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right]$$

$$= \max\left[\min\left(\frac{x-2}{4-2}, 1, \frac{10-x}{10-8}\right), 0\right]$$

$$= \max\left[\min\left(\frac{x-2}{2}, 1, \frac{10-x}{2}\right), 0\right]$$

- We put x = 3.5

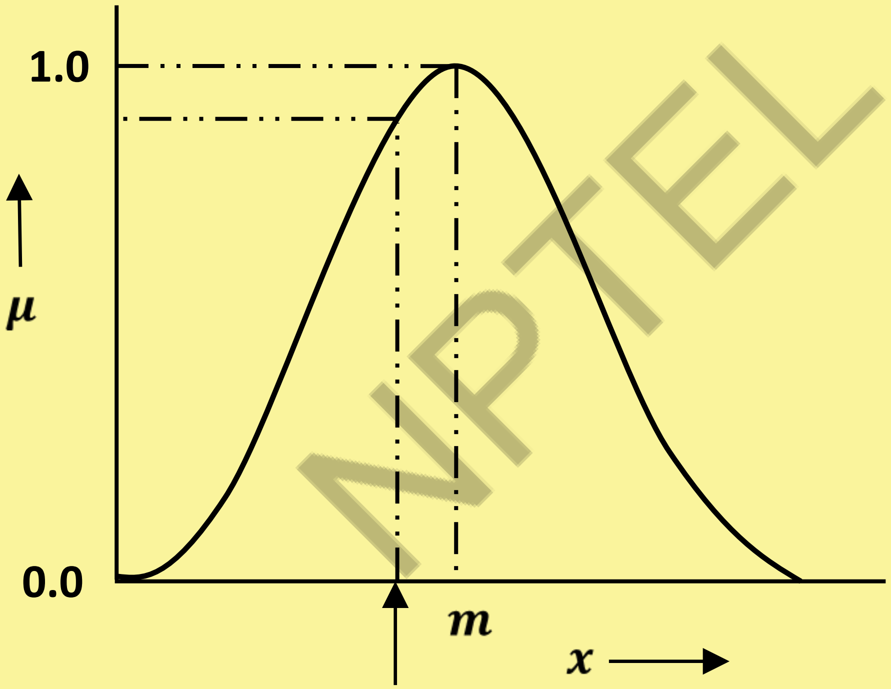
$$\mu_{trapezoidal} = \max\left[\min\left(\frac{1.5}{2}, 1, \frac{6.5}{2}\right), 0\right]$$

$$= \max[0.75, 0]$$

$$= 0.75$$

Gaussian Membership:

Determine μ corresponding to $x = 9.0$



$$\mu_{Gaussian} = \frac{1}{e^{\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}}$$

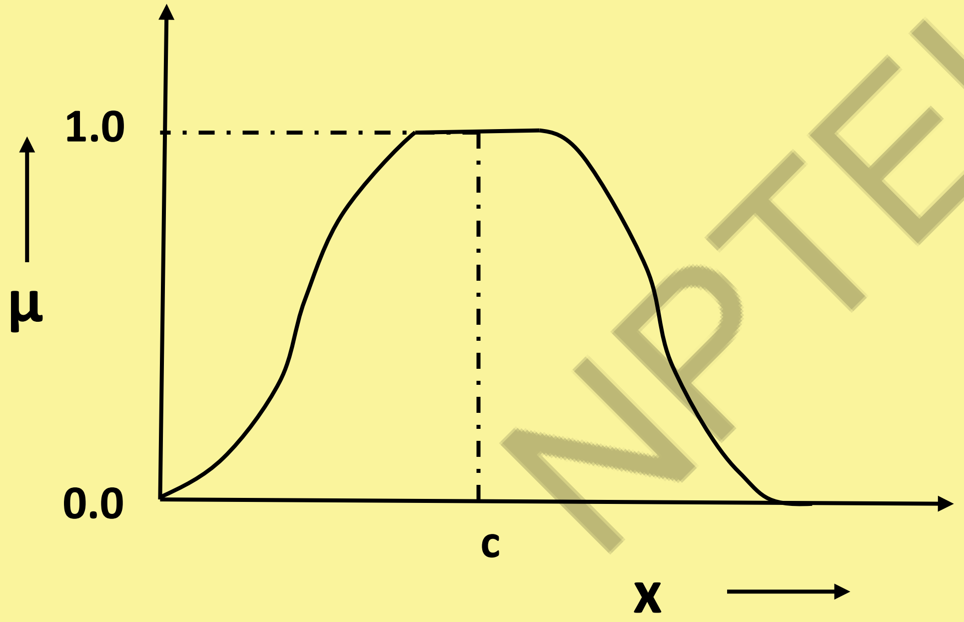
$$Take\ m = 10.0\ and\ \sigma = 3.0$$

$$\mu_{Gaussian} = \frac{1}{e^{\frac{1}{2}\left(\frac{x-10.0}{3.0}\right)^2}}$$

We put $x = 9.0$

$$\therefore\ \mu_{Gaussian} = \frac{1}{e^{\frac{1}{2}\left(\frac{9.0-10.0}{3.0}\right)^2}} = 0.9459$$

Bell-shaped Membership function: Determine μ corresponding to $x = 8.0$



$$\mu_{Bell-shaped} = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$

Take c=10.0, a=2.0, b=3.0

$$\mu_{Bell-shaped} = \frac{1}{1 + \left|\frac{x-10}{2}\right|^{6}}$$

We put x=8.0

$$\mu_{Bell-shaped} = \frac{1}{1 + \left|\frac{8-10}{2}\right|^{6}} = 0.5$$

Sigmoid Membership Function:

Determine μ corresponding to x = 8.0

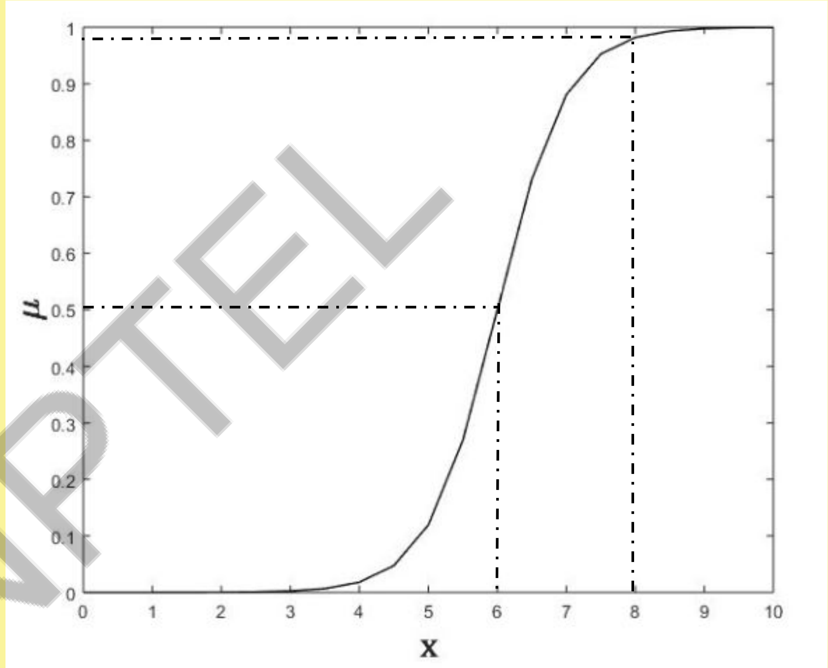
$$\mu_{\text{sigmoid}} = \frac{1}{1 + e^{-a(x-b)}}$$

Take b = 6.0; a = 2

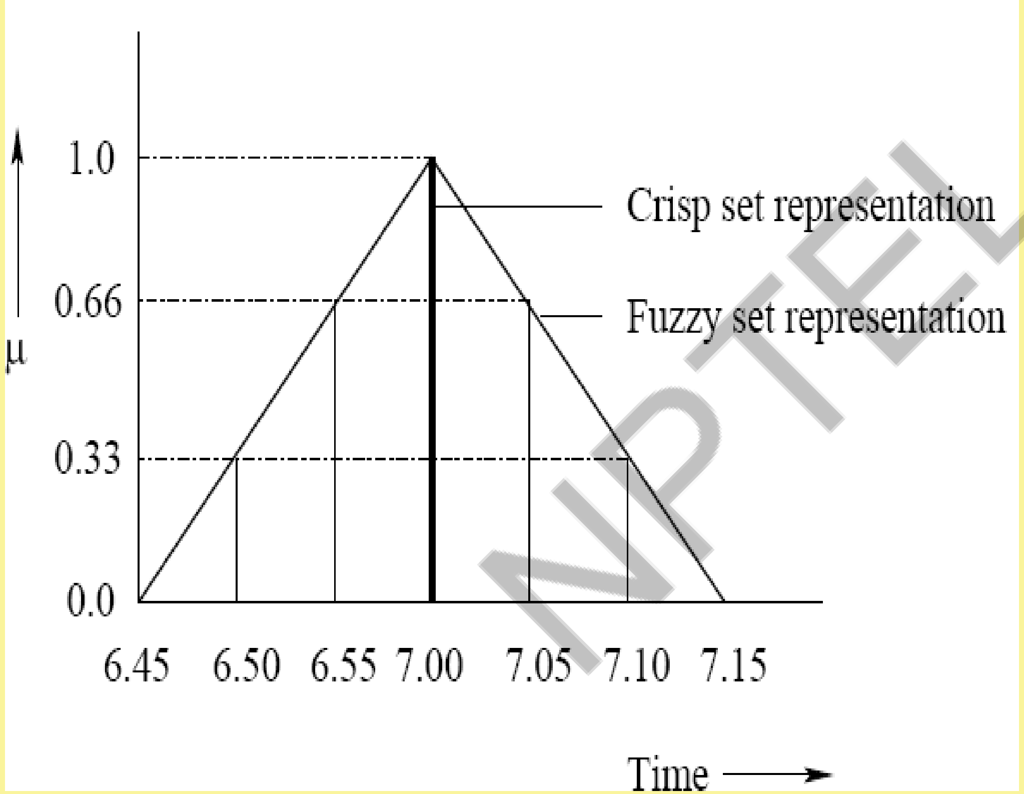
$$\mu_{\text{sigmoid}} = \frac{1}{1 + e^{-2(x-6.0)}}$$

we put x = 8.0

$$\mu_{\text{sigmoid}} = \frac{1}{1 + e^{-2\times 2.0}} = \frac{1}{1 + e^{-4}} = 0.98$$



Difference Between Crisp and Fuzzy Sets

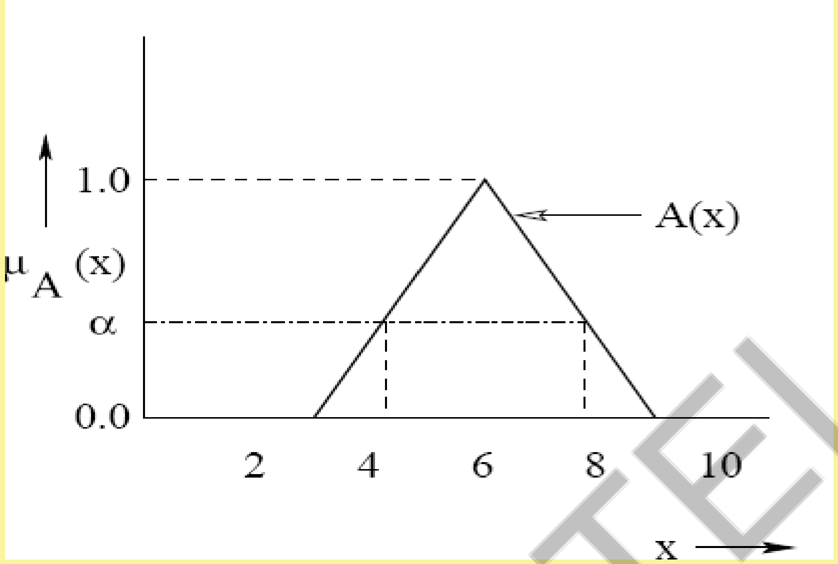


A Few Definitions in Fuzzy Sets

- **α-cut of a fuzzy set $\alpha_{\mu_A}(x)$**

A set consisting of elements *x* of the Universal set X, whose membership values are either greater than or equal to the value of α.

$$\alpha_{\mu_A}(x) = \{x | \mu_A(x) \geq \alpha\}$$



- **Strong α-cut of a Fuzzy Set**

$$\alpha^{+}_{\mu_A}(x) = \{x | \mu_A(x) > \alpha\}$$

Numerical Example

The membership function distribution of a fuzzy set is assumed to follow a Gaussian distribution with mean m = 100 and standard deviation σ =20 . Determine 0.6 – cut of this distribution.

Solution:

Gaussian distribution :

$$\mu = \frac{1}{e^{\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}}$$

where m : Mean ; σ : Standard deviation

By substituting the values of μ = 0.6, m = 100, σ =20 and taking log (ln) on both sides, we get

$$0.6 = \frac{1}{e^{\frac{1}{2}\left(\frac{x-100}{20}\right)^2}}$$

$$\Rightarrow e^{\frac{1}{2}\left(\frac{x-100}{20}\right)^2} = \frac{1}{0.6}$$

By taking ln

$$\ln\left(e^{\frac{1}{2}\left(\frac{x-100}{20}\right)^2}\right) = \ln(1.6667)$$

$$\Rightarrow x = (79.7846, 120.2153)$$

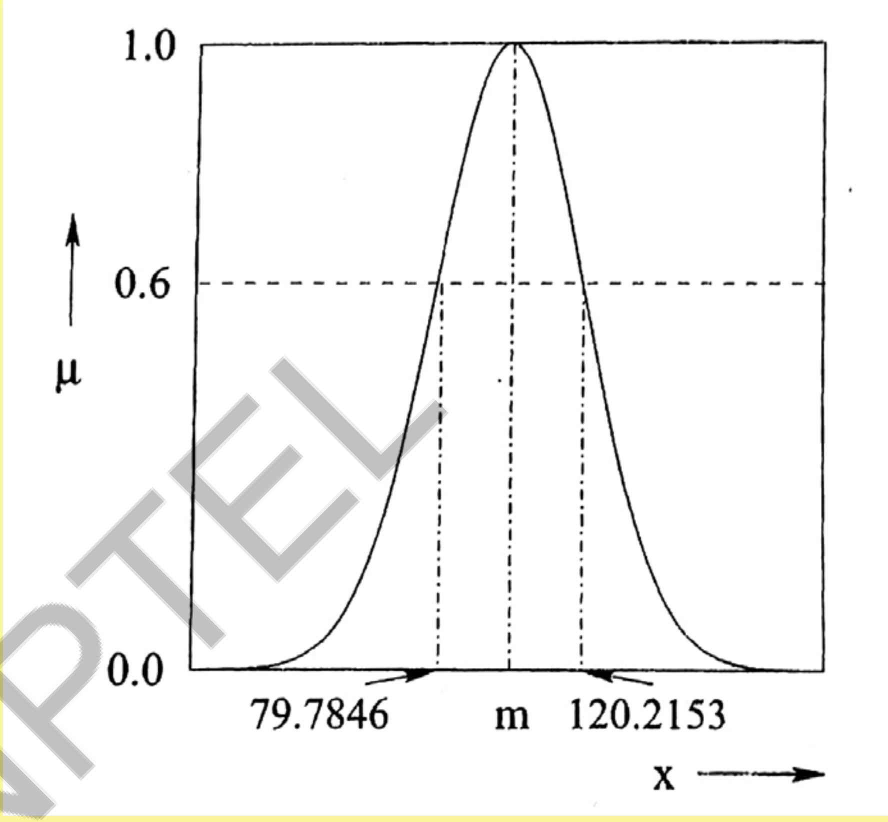


Figure : 0.6-cut of a fuzzy set.

- **Support of a Fuzzy Set A(x)**

It is defined as the set of all $x \in X$, such that $\mu_A(x) > 0$

$$\text{supp(A)} = \{x \in X | \mu_A(x) > 0\}$$

Note: Support of a fuzzy set is nothing but its Strong 0-cut

- **Scalar Cardinality of a Fuzzy Set A(x)**

$$|A(x)| = \sum_{x \in X} \mu_A(x)$$

Numerical Example

Let us consider a fuzzy set $A(x)$ as follows:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$

Scalar Cardinality $|A(x)| = 0.1 + 0.2 + 0.3 + 0.4 = 1.0$

- **Core of a Fuzzy Set A(x)**

  It is nothing but its 1-cut

- **Height of a Fuzzy Set A(x)**

  It is defined as the largest of membership values of the elements contained in that set.

- **Normal Fuzzy Set**

  For a normal fuzzy set, $h(A) = 1.0$

- **Sub-normal Fuzzy Set**

  For a sub-normal fuzzy set, $h(A) < 1.0$

Some Standard Operations in Fuzzy Sets

- **Proper Subset of a Fuzzy Set**

$$A(x) \subset B(x), if\ \mu_A(x) < \mu_B(x)$$

Numerical Example

Let us consider the two fuzzy sets, as follows:

$$A(x)=\{(x_1,0.1),(x_2,0.2),(x_3,0.3),(x_4,0.4)\}$$
$$B(x)=\{(x_1,0.5),(x_2,0.7),(x_3,0.8),(x_4,0.9)\}$$

As for all $x \in X$, $\mu_A(x) < \mu_B(x)$,

$A(x) \subset B(x)$, that is , $A(x)$ is the proper subset of $B(x)$

Some Standard Operations in Fuzzy Sets (contd.)

- **Equal fuzzy sets**

$$A(x) = B(x), if\ \mu_A(x) = \mu_B(x)$$

Numerical Example

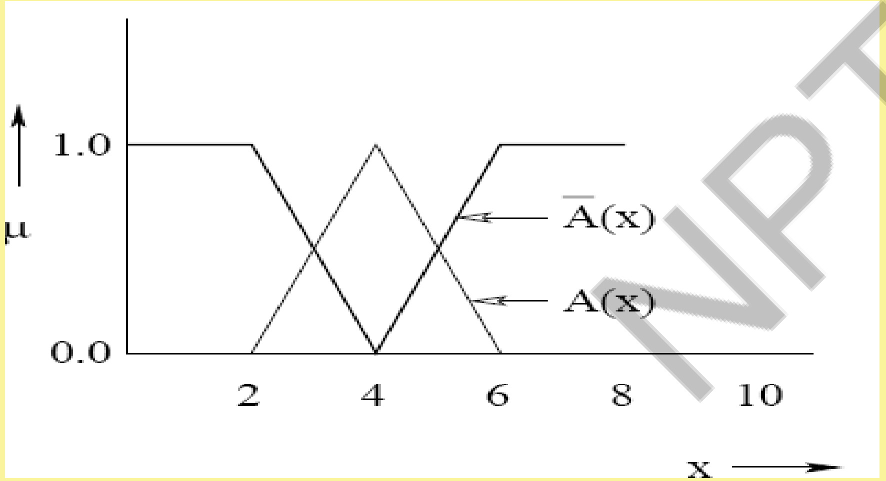
Let us consider the two fuzzy sets, as follows:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$
$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$

As for all $x \in X$, $\quad \mu_A(x) \neq \mu_B(x), \quad A(x) \neq B(x)$

- **Complement of a Fuzzy Set**

$$\bar{A}(x) = 1 - A(x)$$



Numerical Example

Let us consider a fuzzy set $A(x)$ as follows:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$

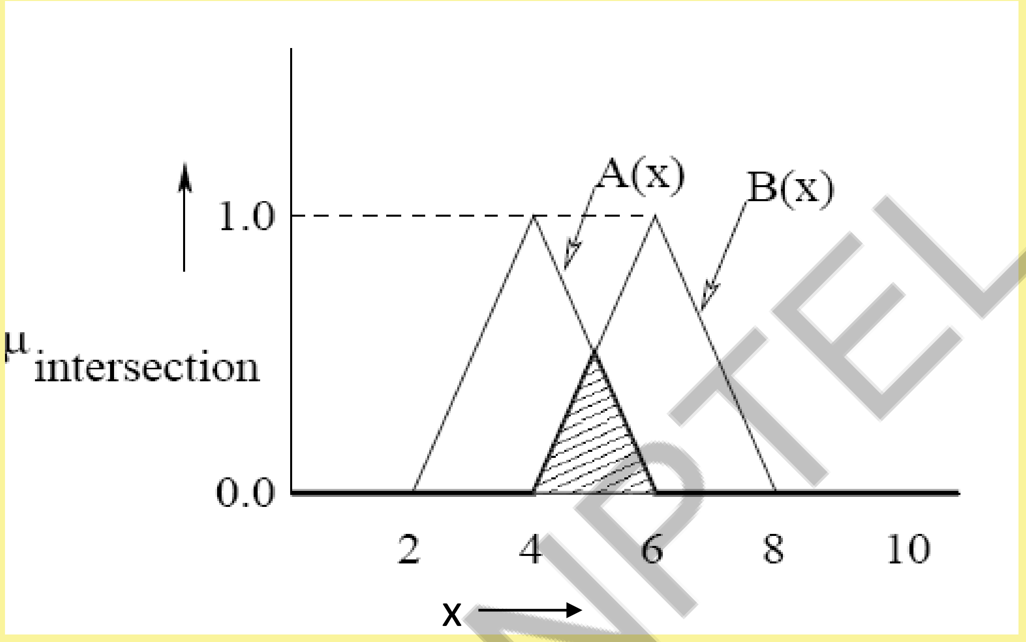
Complement $\bar{A}(x) = \{(x_1, 0.9), (x_2, 0.8), (x_3, 0.7), (x_4, 0.6)\}$

- **Intersection of Fuzzy Sets**

Intersection of two fuzzy sets A(x) and B(x) is denoted by $(A \cap B)(x)$

and its membership values are determined as follows :

$$\mu_{(A \cap B)}(x) = min\{\mu_A(x), \mu_B(x)\}$$



Note: Intersection is analogous to logical AND operation

Numerical Example

Let us consider the two fuzzy sets as follows:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$

$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$

Now, $\mu_{(A \cap B)}(x_1) = \min\{\mu_A(x_1), \mu_B(x_1)\} = \min\{0.1, 0.5\} = 0.1$

Similarly, $\mu_{(A \cap B)}(x_2) = \min\{0.2, 0.7\} = 0.2$

$$\mu_{(A \cap B)}(x_3) = \min\{0.3, 0.8\} = 0.3$$

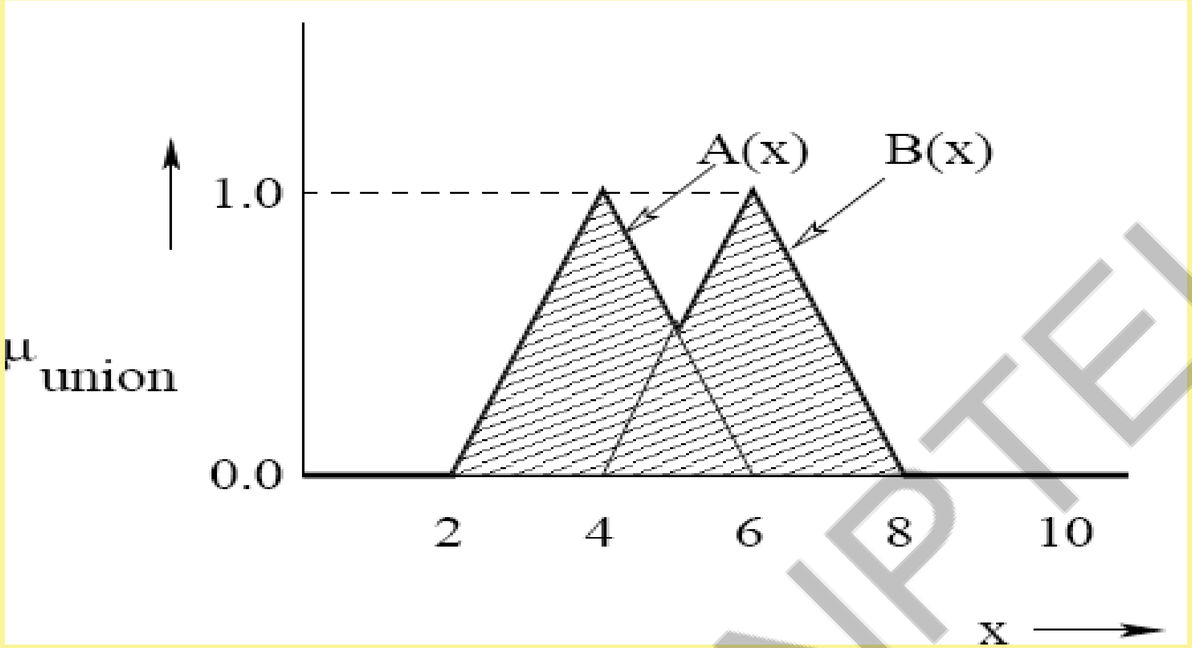
$$\mu_{(A \cap B)}(x_4) = \min\{0.4, 0.9\} = 0.4$$

- ## Union of Fuzzy Sets

Union of two fuzzy sets A(x) and B(x) is represented by: $(A \cup B)(x)$

and its membership value is determined as follows:

$$\mu_{(A \cup B)}(x) = max\{\mu_A(x), \mu_B(x)\}$$



Note: Union is analogous to logical OR operation

Numerical Example

Let us consider the following two fuzzy sets:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$
$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$

Now, $\mu_{(A \cup B)}(x_1) = \max\{\mu_A(x_1), \mu_B(x_1)\} = \max\{0.1, 0.5\} = 0.5$

Similarly, $\mu_{(A \cup B)}(x_2) = \max\{0.2, 0.7\} = 0.7$

$$\mu_{(A \cup B)}(x_3) = \max\{0.3, 0.8\} = 0.8$$
$$\mu_{(A \cup B)}(x_4) = \max\{0.4, 0.9\} = 0.9$$

- **Algebraic product of Fuzzy Sets**

$$A(x).B(x) = \{(x, \mu_A(x).\mu_B(x)), x \in X\}$$

Numerical Example

Let us consider the following two fuzzy sets:

$$A(x)=\{(x_1,0.1),(x_2,0.2),(x_3,0.3),(x_4,0.4)\}$$
$$B(x)=\{(x_1,0.5),(x_2,0.7),(x_3,0.8),(x_4,0.9)\}$$

$$A(x).B(x)=\{(x_1,0.05),(x_2,0.14),(x_3,0.24),(x_4,0.36)\}$$

- **Multiplication of a Fuzzy Set by a Crisp Number**

$$d.A(x) = \{(x, d \times \mu_A(x)), x \in X\}$$

Numerical Example

Let us consider a fuzzy set

$A(x)=\{(x_1,0.1),(x_2,0.2),(x_3,0.3),(x_4,0.4)\}$ and a crisp number $d = 0.2$

$d.A(x)=\{(x_1,0.02),(x_2,0.04),(x_3,0.06),(x_4,0.08)\}$

- **Power of a Fuzzy Set**

$A^P(x)$: p-th power of a fuzzy set $A(x)$ such that

$$\mu_{A^p}(x) = \{\mu_A(x)\}^p, x \in X$$

Concentration: p=2

Dilation: p=1/2

Numerical Example

Let us consider a fuzzy set

$A(x)=\{(x_1,0.1),(x_2,0.2),(x_3,0.3),(x_4,0.4)\}$ and power $p = 2$

$A^2(x)=\{(x_1,0.01),(x_2,0.04),(x_3,0.09),(x_4,0.16)\}$

- **Algebraic Sum of two Fuzzy Sets A(x) and B(x)**

$$A(x) + B(x) = \{(x, \mu_{A+B}(x)), x \in X\}$$

where

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)$$

Numerical Example

Let us consider the following two fuzzy sets:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$
$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$

$$\therefore A(x) + B(x) = \{(x_1, 0.55), (x_2, 0.76), (x_3, 0.86), (x_4, 0.94)\}$$

- **Bounded Sum of two Fuzzy Sets**

$$A(x) \oplus B(x) = \{(x, \mu_{A\oplus B}(x)), x \in X\}$$

where

$$\mu_{A\oplus B}(x) = min\{1, \mu_A(x) + \mu_B(x)\}$$

Numerical Example

Let us consider the following two fuzzy sets:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$
$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$

$$\therefore A(x) \oplus B(x) = \{(x_1, 0.6), (x_2, 0.9), (x_3, 1.0), (x_4, 1.0)\}$$

- **Algebraic Difference of two Fuzzy Sets**

$$A(x) - B(x) = \{(x, \mu_{A-B}(x)), x \in X\}$$

where

$$\mu_{A-B}(x) = \mu_{A \cap \bar{B}}(x)$$

Numerical Example

- Let us consider the following two fuzzy sets:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$
$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$
$$Now, \overline{B}(x) = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.2), (x_4, 0.1)\}$$

$$\therefore A(x) - B(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.2), (x_4, 0.1)\}$$

- **Bounded Difference of two Fuzzy Sets**

$$A(x)\Theta B(x) = \{(x, \mu_{A\Theta B}(x)), x \in X\}$$

where

$$\mu_{A\Theta B}(x) = max\{0, \mu_A(x) + \mu_B(x) - 1\}$$

Numerical Example

- Let us consider the following two fuzzy sets:

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$
$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$

$$A(x) \Theta B(x) = \{(x_1, 0.0), (x_2, 0.0), (x_3, 0.1), (x_4, 0.3)\}$$

- **Cartesian product of two Fuzzy Sets**

Two fuzzy sets A(x) defined in X
and B(y) defined in Y

Cartesian product of two fuzzy sets is denoted by A(x)×B(y), such that $\mu_{A\times B}(x,y) = min\{\mu_A(x), \mu_B(y)\}$

Numerical Example

- Let us consider the following two fuzzy sets:

$$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$$
$$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$$

$$\min(\mu_A(x_1), \mu_B(y_1)) = \min(0.2, 0.8) = 0.2$$
$$\min(\mu_A(x_1), \mu_B(y_2)) = \min(0.2, 0.6) = 0.2$$

$$\min(\mu_A(x_1), \mu_B(y_3)) = \min(0.2, 0.3) = 0.2$$
$$\min(\mu_A(x_2), \mu_B(y_1)) = \min(0.3, 0.8) = 0.3$$
$$\min(\mu_A(x_2), \mu_B(y_2)) = \min(0.3, 0.6) = 0.3$$
$$\min(\mu_A(x_2), \mu_B(y_3)) = \min(0.3, 0.3) = 0.3$$

$$\min(\mu_A(x_3), \mu_B(y_1)) = \min(0.5, 0.8) = 0.5$$
$$\min(\mu_A(x_3), \mu_B(y_2)) = \min(0.5, 0.6) = 0.5$$
$$\min(\mu_A(x_3), \mu_B(y_3)) = \min(0.5, 0.3) = 0.3$$
$$\min(\mu_A(x_4), \mu_B(y_1)) = \min(0.6, 0.8) = 0.6$$

$$\min(\mu_A(x_4), \mu_B(y_2)) = \min(0.6, 0.6) = 0.6$$
$$\min(\mu_A(x_4), \mu_B(y_3)) = \min(0.6, 0.3) = 0.3$$

$$\therefore A \times B = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix}$$

Composition of fuzzy relations

Let $A = [a_{ij}]$ and $B = [b_{jk}]$ be two fuzzy relations expressed in the matrix form.

Composition of these two fuzzy relations, that is, C is represented as follows:

$$C = A \circ B$$

In matrix form

$$[c_{ik}] = [a_{ij}] \circ [b_{jk}]$$

Where

$c_{ik} = \max[\min(a_{ij}, b_{jk})]$

Numerical Example

- Let us consider the following two Fuzzy relations:

$$A = [a_{ij}] = \begin{bmatrix} 0.2 & 0.3 \\ 0.5 & 0.7 \end{bmatrix}$$

$$B = [b_{jk}] = \begin{bmatrix} 0.3 & 0.6 & 0.7 \\ 0.1 & 0.8 & 0.6 \end{bmatrix}$$

- Elements of $[c_{ik}]$ matrix can be determined as follows:

$$c_{11} = \max[\min(a_{11}, b_{11}), \min(a_{12}, b_{21})]$$

$$= \max[\min(0.2, 0.3), \min(0.3, 0.1)]$$

$$= \max[0.2, 0.1]$$

$$= 0.2$$

$$c_{12} = \max\left[\min(a_{11}, b_{12}), \min(a_{12}, b_{22})\right]$$

$$= \max\left[\min(0.2, 0.6), \min(0.3, 0.8)\right]$$

$$= \max\left[0.2, 0.3\right]$$

$$= 0.3$$

$$c_{13} = \max[\min(a_{11}, b_{13}), \min(a_{12}, b_{23})]$$

$$= \max[\min(0.2, 0.7), \min(0.3, 0.6)]$$

$$= \max[0.2, 0.3]$$

$$= 0.3$$

$$c_{21} = \max\left[\min(a_{21}, b_{11}), \min(a_{22}, b_{21})\right]$$

$$= \max\left[\min(0.5, 0.3), \min(0.7, 0.1)\right]$$

$$= \max\left[0.3, 0.1\right]$$

$$= 0.3$$

$$c_{22} = \max[\min(a_{21}, b_{12}), \min(a_{22}, b_{22})]$$

$$= \max[\min(0.5, 0.6), \min(0.7, 0.8)]$$

$$= \max[0.5, 0.7]$$

$$= 0.7$$

$$c_{23} = \max\left[\min(a_{21}, b_{13}), \min(a_{22}, b_{23})\right]$$

$$= \max\left[\min(0.5, 0.7), \min(0.7, 0.6)\right]$$

$$= \max\left[0.5, 0.6\right]$$

$$= 0.6$$

$$\therefore C = \begin{bmatrix} 0.2 & 0.3 & 0.3 \\ 0.3 & 0.7 & 0.6 \end{bmatrix}$$

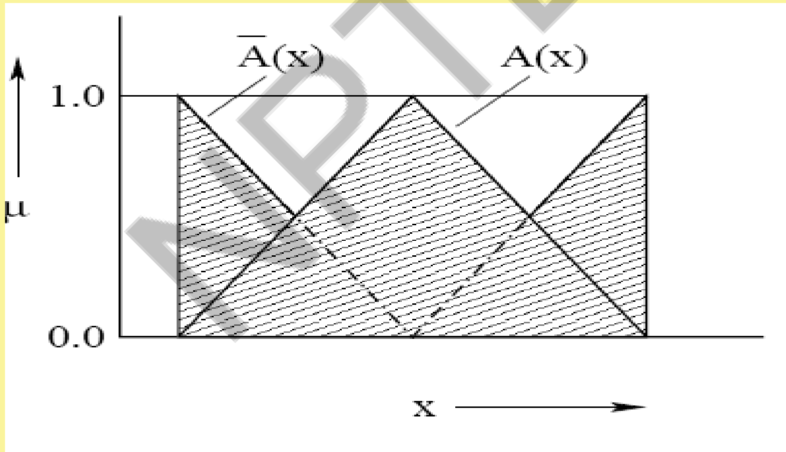
Properties of Fuzzy Set

Fuzzy sets follow the properties of crisp sets except the following two:

- **Law of excluded middle**

In crisp set, $A \cup \bar{A} = X$

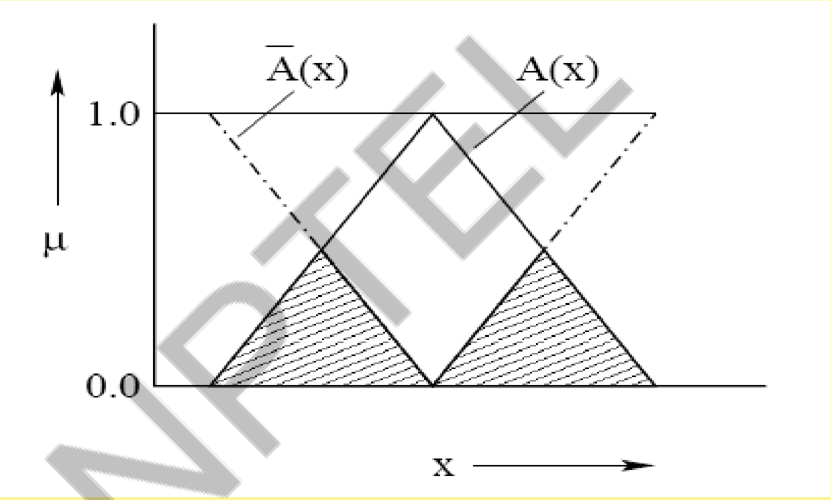
In fuzzy set, $A \cup \bar{A} \neq X$



- **Law of contradiction**

In crisp set, $A \cap \bar{A} = \emptyset$

In fuzzy set, $A \cap \bar{A} \neq \emptyset$



Measure of Fuzziness of Fuzzy Set

Entropy has been used to measure fuzziness of a fuzzy set.

Let $X = \{x_1, x_2 \dots, x_n\}$ be the discrete universe of discourse.

Entropy of a fuzzy set $A(x)$ is determined as follows:

$$H(A) = -\frac{1}{n}\sum_{i=1}^{n}[\mu_A(x_i)\log\{\mu_A(x_i)\} + \{1 - \mu_A(x_i)\}\log\{1 - \mu_A(x_i)\}]$$

Numerical Example

Let $A(x) = \{(x_1, 0.1), (x_2, 0.3), (x_3, 0.4), (x_4, 0.5)\}$.

Entropy

$$H(A) = -\frac{1}{4}[\{0.1 \times \log(0.1) + 0.9\log(0.9)\} + \{0.3\log(0.3) + 0.7\log(0.7)\} + \{0.4\log(0.4) + 0.6\log(0.6)\} + \{0.5\log(0.5) + 0.5\log(0.5)\}]$$

$$= 0.2499$$

Measure of Inaccuracy of Fuzzy Set

Let us consider two fuzzy sets: $A(x)$ and $B(x)$ defined in the same discrete universe of discourse $X = \{x_1, x_2 \dots, x_n\}$

Inaccuracy of fuzzy set $B(x)$ is measured with respect to the fuzzy set $A(x)$ as follows:

$$I(A;B) = -\frac{1}{n}\sum_{i=1}^{n}[\mu_A(x_i)\log\{\mu_B(x_i)\} + \{1 - \mu_A(x_i)\}\log\{1 - \mu_B(x_i)\}]$$

Numerical Example

$$A(x) = \{(x_1, 0.1), (x_2, 0.2), (x_3, 0.3), (x_4, 0.4)\}$$
$$B(x) = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0.8), (x_4, 0.9)\}$$

Inaccuracy of $B(x)$ with respect to $A(x)$,

$$I(A;B)$$
$$= -\frac{1}{4}[\{0.1 \times \log(0.5) + 0.9 \times \log(0.5)\}$$
$$+ \{0.2 \times \log(0.7) + 0.8 \times \log(0.3)\}$$
$$+ \{0.3 \times \log(0.8) + 0.7 \times \log(0.2)\} + \{0.4 \times \log(0.9)$$
$$+ 0.6 \times \log(0.1)\}$$

$$= 0.4717$$

References

References:

- Soft Computing: Fundamentals and Applications by D.K. Pratihar, Narosa Publishing House, New-Delhi, 2014
- Fuzzy Sets and Fuzzy Logic: Theory and Applications by G.J. Klir, B. Yuan, Prentice Hall, 1995

Conclusion

Conclusion:

- A few terms related to Fuzzy Sets have been defined
- Some standard Operations in Fuzzy Sets have been explained
- Properties of Fuzzy Sets have been explained
- Fuzziness and Inaccuracy of Fuzzy Sets are determined

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Thank you!