This is only the first question of the project, related to problem modeling. The second part pertains to substituting an alternative, the third part involves solving the first two sections using the Lingo software, and the fourth part is related to sensitivity analysis and plotting.

Atlas Car Rental Company, with a fleet consisting of one type of car, provides car rental services in the cities of Isfahan, Mashhad, Shiraz, and Ahvaz. The services are offered through centers in these four cities for both pick-up and return of cars.

The company's planning department has predicted the daily demand for car rentals at each of its centers, as shown in Table 1. The company is not necessarily aiming to meet all available demands. All car rental centers of the company are closed on Fridays, and no new car rentals are registered on this day. Cars rented to customers on this day are not returned either.

	ISFAHAN	**MASHHAD**	**SHIRAZ**	**AHVAZ**
SATURDAY	100	250	95	160
SUNDAY	150	143	195	99
MONDAY	135	80	242	55
TUESDAY	83	225	111	96
WEDNESDAY	120	210	70	115
THURSDAY	230	98	124	80

Cars are rented for one, two, or three days, and they must be returned to the center where they were received or any other designated center at the beginning of the day following the rental period. For example, if a car is rented on Tuesday for two days, it must be returned to a center on Thursday morning. Similarly, if a car is rented on Thursday for three days, it must be returned to a center on Monday morning.

Based on past data, the company has observed that the rental duration does not depend on the location where the car is rented or returned. In 55% of cases, cars are rented for one day, 20% for two days, and 25% for three days.

The current prediction results regarding the rental status of cars, including the origin center, destination for pick-up, and destination for return, are presented in Table 2. For example, 60% of rented cars from Isfahan are returned to Isfahan, and 20% are returned to the Mashhad center.

Destination				Origin
Ahvaz	Shiraz	Mashhad	Isfahan	
10	10	20	60	Isfahan
5	25	55	15	Mashhad
11	54	20	15	Shiraz
53	27	12	8	Ahvaz

From an operational and execution standpoint, renting each car incurs costs of 20, 25, and 30 monetary units for one, two, and three days, respectively. The costs associated with capital depreciation, storage, and service for each car are 15 monetary units per week.

Exchange of healthy cars between the four rental centers is possible, regardless of their distance. A car is not rented on the day it is scheduled to be exchanged between two centers. The exchange cost for each car between different centers is provided in Table 3.

Destination				Origin
Ahvaz	Shiraz	Mashhad	Isfahan	
50	30	20	-	Isfahan
35	15	-	20	Mashhad
25	-	15	30	Shiraz
-	25	35	50	Ahvaz

Typically, 10% of cars rented by customers sustain damage. In such cases, the customer must pay 100 monetary units for the damage. Insurance covers the incurred damage. The damaged car must be transferred to a repair center, incurring transfer costs similar to those for transferring a healthy car. If the damaged car is transferred to a repair center in a different city, it takes one day. If the car is in a city where a repair center is available, the transfer cost and time will be zero. After the transfer, repairing a damaged car takes one day.

Repairing damaged cars is possible only in two centers, Mashhad and Shiraz, with capacities of 12 and 20 cars per day, respectively. After repair, the car will be available for rental in the city where it was repaired, or it may be transferred to another city. Thus, a damaged car delivered to a non-repair center on Tuesday will be sent to a repair center on Tuesday, repaired on Wednesday, and available for rental from Thursday in the city where it was repaired.

The rental price for each car is determined based on the number of rental days and whether it is returned to the same center where it was rented. Details are provided in Table 4.

	RETURN IN SAME CENTER	**RETURN IN OTHER CENTER**
1-DAY	50	70
2-DAY	70	100
3-DAY	120	150

If a car is rented on Thursday and returned on Saturday morning, a 20 monetary unit discount will apply. Due to the company's closure on Fridays, this rental will be considered a one-day rental.

For simplicity, it is assumed that the following conditions are met at the beginning of each day:

- Customers return cars whose rental period has ended on time.
- Damaged cars are sent to the repair center.
- Cars transferred from other centers will be received on time.
- Transfer and rental of cars are carried out at the beginning of the day.
- In repair centers, repaired cars will be available for rental.

Solution (By Sajjad Abed):

The given text appears to be a set of mathematical expressions and variables related to a car rental problem. Here's the translation:

"Considering that the centers are closed on Fridays, and car rentals on Fridays are not counted, we consider the week to be equivalent to 6 days.

S_i: The number of healthy cars available in the morning of day i in the city of Shiraz, available for rent or transfer to another city.

M_i: The number of healthy cars available in the morning of day i in the city of Mashhad, available for rent or transfer to another city.

E_i: The number of healthy cars available in the morning of day i in the city of Isfahan, available for rent or transfer to another city.

A_i: The number of healthy cars available in the morning of day i in the city of Ahvaz, available for rent or transfer to another city.

$i \in \{1,2,3,4,5,6\}$

X_(N,P,i): The number of healthy cars transferred from city N to city P on day i, where: $i \in \{1,2,3,4,5,6\}$ $N,P \in \{E,S,M,A\}$ $N \neq P$

J_(N,i,j): The number of cars rented in city N on day i for j days. $N \in \{E,S,M,A\}$ $i \in \{1,2,3,4,5,6\}$ $j \in \{1,2,3\}$

D_(N,i): The number of healthy cars received by city N from customers on day i. $i \in \{1,2,3,4,5,6\}$ $N \in \{E,S,M,A\}$

T_(N,i): The number of damaged cars kept in city N after the morning of day i. $i \in \{1,2,3,4,5,6\}$ $N \in \{E,S,M,A\}$

Y_(N,P,i): The number of damaged cars sent for repair from city N to repair center P on day i. (If N=P, the car arrives at the repair center on the same day and incurs no sending cost.) $i \in \{1,2,3,4,5,6\}$ $N,P \in \{E,S,M,A\}$

(Note: The destination city initially includes only S and M, but due to the second part of the question, Isfahan and Ahvaz are also considered as destination cities. However,

these values, with the parking capacity constraint mentioned later, initially take a value of zero in the first part.)

F_(N,P,i,j): The number of cars rented from city N on day i of the week for j days and returned to city P. This value is a fraction of the variable J_(N,i,j), and this ratio can be observed from Table 2. N,P ϵ {E,S,M,A} i ϵ {1,2,3,4,5,6} j ϵ {1,2,3}

The variable F is defined separately because the number of cars customers return to each city must be an integer value.

All the defined variables are greater than or equal to zero and are integers."

The Model:

(Description of all constraints are available in Persian version)

$$
\begin{aligned}
Max\ Z_1 = {} & 50\sum_i (0.6J_{E,i,1}) + (0.55J_{M,i,1}) + (0.54J_{S,i,1}) + (0.53J_{A,i,1}) \\
& +70\sum_i (0.6J_{E,i,2}) + (0.55J_{M,i,2}) + (0.54J_{S,i,2}) + (0.53J_{A,i,2}) \\
& +120\sum_i (0.6J_{E,i,3}) + (0.55J_{M,i,3}) + (0.54J_{S,i,3}) + (0.53J_{A,i,3}) \\
& +70\sum_i (0.4J_{E,i,1}) + (0.45J_{M,i,1}) + (0.46J_{S,i,1}) + (0.47J_{A,i,1}) \\
& +100\sum_i (0.4J_{E,i,2}) + (0.45J_{M,i,2}) + (0.46J_{S,i,2}) + (0.47J_{A,i,2}) \\
& +150\sum_i (0.4J_{E,i,3}) + (0.45J_{M,i,3}) + (0.46J_{S,i,3}) + (0.47J_{A,i,3}) \\
& +10\sum_{N\in U}\sum_i\sum_j J_{N,i,j} \\
& -20\times\left(\sum_{N\in U} J_{N,6,1}\right) \\
& -\left(20\sum_{N\in U}\sum_i J_{N,i,1} + 25\sum_{N\in U}\sum_i J_{N,i,2} + 30\sum_{N\in U}\sum_i J_{N,i,2}\right) \\
& -15[\,S_1 + M_1 + E_1 + A_1 \\
& +Y_{E,S,1} + Y_{M,S,1} + Y_{A,S,1} + Y_{E,M,1} + Y_{S,M,1} + Y_{A,M,1} + Y_{M,E,1} + Y_{S,E,1} + Y_{A,E,1} \\
& +Y_{E,A,1} + +Y_{M,A,1} + Y_{S,A,1} \\
& +Y_{E,S,6} + Y_{S,S,1} + Y_{M,S,6} + Y_{A,S,6} + Y_{E,M,6} + Y_{S,M,6} + Y_{M,M,1} + Y_{A,M,6} + Y_{E,E,1} \\
& +Y_{S,E,6} + Y_{M,E,6} + Y_{A,E,6} + Y_{S,A,6} + Y_{E,A,6} + Y_{M,A,6} + Y_{A,A,1}
\end{aligned}
$$

$$+\sum_{N} J_{N,6,2} + \sum_{N} J_{N,6,3} + \sum_{N} J_{N,5,3} \; + \; \sum_{N} T_{N,1}]$$

$$-[20\sum_{i}(X_{E,M,i} + Y_{E,M,i}) + 30\sum_{i}(X_{E,S,i} + Y_{E,S,i}) + 50\sum_{i}(X_{E,A,i} + Y_{E,A,i})$$

$$+20\sum_{i}(X_{M,E,i} + Y_{M,E,i}) + 15\sum_{i}(X_{M,S,i} + Y_{M,S,i}) + 35\sum_{i}(X_{M,A,i} + Y_{M,A,i})$$

$$+30\sum_{i}(X_{S,E,i} + Y_{S,E,i}) + 15\sum_{i}(X_{S,M,i} + Y_{S,M,i}) \; + 25\sum_{i}(X_{S,A,i} + Y_{S,A,i})$$

$$+50\sum_{i}(X_{A,E,i} + Y_{A,E,i}) + 35\sum_{i}(X_{A,M,i} + Y_{A,M,i}) + 25\sum_{i}(X_{A,S,i} + Y_{A,S,i})]$$

Subject to

$$= S_{i-1} - \sum_{j=0}^{3} J_{S.i-1,j} + D_{S,i} - X_{S,\{E.A.M\},i-1} + X_{\{E.A.M\},S,i-1} + Y_{\{E.A.M\},S,i-2} + Y_{S,S,i-1} \quad S_i$$

$$= A_{i-1} - \sum_{j=0}^{3} J_{A.i-1,j} + D_{A,i} - X_{A,\{E.S.M\},i-1} + X_{\{E.S.M\},A,i-1} + Y_{\{E.M,S\},A,i-2} + Y_{A,A,i-1} \quad A_i$$

$$= M_{i-1} - \sum_{j=0}^{3} J_{M.i-1,j} + D_{M,i} - X_{M,\{E.A.S\},i-1} + X_{\{E.A.S\},M,i-1} + Y_{\{E.A,S\},M,i-2} + Y_{M,M,i-1} \quad M_i$$

$$\boldsymbol{E_i = E_{i-1} - \sum_{j=0}^{3} J_{E.i-1,j} + D_{E,i} - X_{E,\{S.A.M\},i-1} + X_{\{S.A.M\},E,i-1} + Y_{\{M.A,S\},E,i-2} + Y_{E,E,i-1}}$$

$J_{S,1,1} \leq 52.25$

$J_{S,1,2} \leq 19$

$J_{S,1,3} \leq 23.75$

$J_{S,2,1} \leq 107.25$

$J_{S,2,2} \leq 39$

$J_{S,2,3} \leq 48.77$

$J_{S,3,1} \leq 229.9$

$J_{S,3,2} \leq 48.4$

$J_{S,3,3} \leq 60.5$

$J_{S,4,1} \leq 61.05$

$J_{S,4,2} \leq 22.2$

$J_{S,4,3} \leq 27.75$

$J_{S,5,1} \leq 38.5$

$J_{S,5,2} \leq 14$

$J_{S,5,3} \leq 17.5$

$J_{S,6,1} \leq 68.2$

$J_{S,6,2} \leq 24.8$

$J_{S,6,3} \leq 31$

$J_{A,1,1} \leq 88$

$J_{A,1,2} \leq 32$

$J_{A,1,3} \leq 40$

$J_{A,2,1} \leq 54.45$

$J_{A,2,2} \leq 19.8$

$J_{A,2,3} \leq 24.75$

$J_{A,3,1} \leq 30.25$

$J_{A,3,2} \leq 11$

$J_{A,3,3} \leq 13.75$

$J_{A,4,1} \leq 52.8$

$J_{A,4,2} \leq 19.2$

$J_{A,4,3} \leq 24$

$J_{A,5,1} \leq 63.25$

$J_{A,5,2} \leq 23$

$J_{A,5,3} \leq 28.75$

$J_{A,6,1} \leq 44$

$J_{A,6,2} \leq 16$

$J_{A,6,3} \leq 20$

$J_{M,1,1} \leq 137.5$

$J_{M,1,2} \leq 50$

$J_{M,1,3} \leq 62.5$

$J_{M,2,1} \leq 78.65$

$J_{M,2,2} \leq 28.6$

$J_{M,2,3} \leq 35.75$

$J_{M,3,1} \leq 44$

$J_{M,3,2} \leq 16$

$J_{M,3,3} \leq 20$

$J_{M,4,1} \leq 123.75$

$J_{M,4,2} \leq 45$

$J_{M,4,3} \leq 56.25$

$J_{M,5,1} \leq 115.5$

$J_{M,5,2} \leq 42$

$J_{M,5,3} \leq 52.5$

$J_{M,6,1} \leq 53.9$

$J_{M,6,2} \leq 19.6$

$J_{M,6,3} \leq 24.5$

$J_{E,1,1} \leq 55$

$J_{E,1,2} \leq 20$

$J_{E,1,3} \leq 25$

$J_{E,2,1} \leq 82.5$

$J_{E,2,2} \leq 30$

$J_{E,2,3} \leq 37.5$

$J_{E,3,1} \leq 74.24$

$J_{E,3,2} \leq 27$

$J_{E,3,3} \leq 33.75$

$J_{E,4,1} \leq 45.65$

$J_{E,4,2} \leq 16.6$

$J_{E,4,3} \leq 20.75$

$J_{E,5,1} \leq 65$

$J_{E,5,2} \leq 24$

$J_{E,5,3} \leq 30$

$J_{E,6,1} \leq 126.5$

$J_{E,6,2} \leq 46$

$J_{E,6,3} \leq 57.5$

$$D_{S,1} = 0.9\,[\,0.54(J_{S,1,6} + J_{S,2,5} + J_{S,3,4}) + 0.1(J_{E,1,6} + J_{E,2,5} + J_{E,3,4}) + 0.27(J_{A,1,6} + J_{A,2,5} + J_{A,3,4}) + 0.25(J_{M,1,6} + J_{M,2,5} + J_{M,3,4})]$$

$$D_{S,2} = 0.9\,[\,0.54(J_{S,1,1} + J_{S,2,6} + J_{S,3,5}) + 0.1(J_{E,1,1} + J_{E,2,6} + J_{E,3,5}) + 0.27(J_{A,1,1} + J_{A,2,6} + J_{A,3,5}) + 0.25(J_{M,1,1} + J_{M,2,6} + J_{M,3,5})]$$

$$D_{S,3} = 0.9\,[\,0.54(J_{S,1,2} + J_{S,2,1} + J_{S,3,6}) + 0.1(J_{E,1,2} + J_{E,2,1} + J_{E,3,6}) + 0.27(J_{A,1,2} + J_{A,2,1} + J_{A,3,6}) + 0.25(J_{M,1,2} + J_{M,2,1} + J_{M,3,6})]$$

$$D_{S,4} = 0.9\,[\,0.54(J_{S,1,3} + J_{S,2,2} + J_{S,3,1}) + 0.1(J_{E,1,3} + J_{E,2,2} + J_{E,3,1}) + 0.27(J_{A,1,3} + J_{A,2,2} + J_{A,3,1}) + 0.25(J_{M,1,3} + J_{M,2,2} + J_{M,3,1})]$$

$$D_{S,5} = 0.9\,[\,0.54(J_{S,1,4} + J_{S,2,3} + J_{S,3,2}) + 0.1(J_{E,1,4} + J_{E,2,3} + J_{E,3,2}) + 0.27(J_{A,1,4} + J_{A,2,3} + J_{A,3,2}) + 0.25(J_{M,1,4} + J_{M,2,3} + J_{M,3,2})]$$

$$D_{S,6} = 0.9\,[\,0.54(J_{S,1,5} + J_{S,2,4} + J_{S,3,3}) + 0.1(J_{E,1,5} + J_{E,2,4} + J_{E,3,3}) + 0.27(J_{A,1,5} + J_{A,2,4} + J_{A,3,3}) + 0.25(J_{M,1,5} + J_{M,2,4} + J_{M,3,3})]$$

$$D_{A,1} = 0.9\,[\,0.11(J_{S,1,6} + J_{S,2,5} + J_{S,3,4}) + 0.1(J_{E,1,6} + J_{E,2,5} + J_{E,3,4}) + 0.53(J_{A,1,6} + J_{A,2,5} + J_{A,3,4}) + 0.05(J_{M,1,6} + J_{M,2,5} + J_{M,3,4})]$$

$$D_{A,2} = 0.9\,[\,0.11(J_{S,1,1} + J_{S,2,6} + J_{S,3,5}) + 0.1(J_{E,1,1} + J_{E,2,6} + J_{E,3,5}) + 0.53(J_{A,1,1} + J_{A,2,6} + J_{A,3,5}) + 0.05(J_{M,1,1} + J_{M,2,6} + J_{M,3,5})]$$

$$D_{A,3} = 0.9\,[\,0.11(J_{S,1,2} + J_{S,2,1} + J_{S,3,6}) + 0.1(J_{E,1,2} + J_{E,2,1} + J_{E,3,6}) + 0.53(J_{A,1,2} + J_{A,2,1} + J_{A,3,6}) + 0.05(J_{M,1,2} + J_{M,2,1} + J_{M,3,6})]$$

$$D_{A,4} = 0.9\,[\,0.11(J_{S,1,3} + J_{S,2,2} + J_{S,3,1}) + 0.1(J_{E,1,3} + J_{E,2,2} + J_{E,3,1}) + 0.53(J_{A,1,3} + J_{A,2,2} + J_{A,3,1}) + 0.05(J_{M,1,3} + J_{M,2,2} + J_{M,3,1})]$$

$$D_{A,5} = 0.9\,[\,0.11(J_{S,1,4} + J_{S,2,3} + J_{S,3,2}) + 0.1(J_{E,1,4} + J_{E,2,3} + J_{E,3,2}) + 0.53(J_{A,1,4} + J_{A,2,3} + J_{A,3,2}) + 0.05(J_{M,1,4} + J_{M,2,3} + J_{M,3,2})]$$

$$D_{A,6} = 0.9\,[\,0.11(J_{S,1,5} + J_{S,2,4} + J_{S,3,3}) + 0.1(J_{E,1,5} + J_{E,2,4} + J_{E,3,3}) + 0.53(J_{A,1,5} + J_{A,2,4} + J_{A,3,3}) + 0.05(J_{M,1,5} + J_{M,2,4} + J_{M,3,3})]$$

$$D_{M,1} = 0.9[\,0.2(J_{S,1,6} + J_{S,2,5} + J_{S,3,4}) + 0.2(J_{E,1,6} + J_{E,2,5} + J_{E,3,4}) + 0.12(J_{A,1,6} + J_{A,2,5} + J_{A,3,4}) + 0.55(J_{M,1,6} + J_{M,2,5} + J_{M,3,4})]$$

$$D_{M,2} = 0.9\,[\,0.2(J_{S,1,1} + J_{S,2,6} + J_{S,3,5}) + 0.2(J_{E,1,1} + J_{E,2,6} + J_{E,3,5}) + 0.12(J_{A,1,1} + J_{A,2,6} + J_{A,3,5}) + 0.55(J_{M,1,1} + J_{M,2,6} + J_{M,3,5})]$$

$$D_{M,3} = 0.9\,[\,0.2(J_{S,1,2} + J_{S,2,1} + J_{S,3,6}) + 0.2(J_{E,1,2} + J_{E,2,1} + J_{E,3,6}) + 0.12(J_{A,1,2} + J_{A,2,1} + J_{A,3,6}) + 0.55(J_{M,1,2} + J_{M,2,1} + J_{M,3,6})]$$

$$D_{M,4} = 0.9\,[\,0.2(J_{S,1,3} + J_{S,2,2} + J_{S,3,1}) + 0.2(J_{E,1,3} + J_{E,2,2} + J_{E,3,1}) + 0.12(J_{A,1,3} + J_{A,2,2} + J_{A,3,1}) + 0.55(J_{M,1,3} + J_{M,2,2} + J_{M,3,1})]$$

$$D_{M,5} = 0.9\,[\,0.2(J_{S,1,4} + J_{S,2,3} + J_{S,3,2}) + 0.2(J_{E,1,4} + J_{E,2,3} + J_{E,3,2}) + 0.12(J_{A,1,4} + J_{A,2,3} + J_{A,3,2}) + 0.55(J_{M,1,4} + J_{M,2,3} + J_{M,3,2})]$$

$$D_{M,6} = 0.9\,[\,0.2(J_{S,1,5} + J_{S,2,4} + J_{S,3,3}) + 0.2(J_{E,1,5} + J_{E,2,4} + J_{E,3,3}) + 0.12(J_{A,1,5} + J_{A,2,4} + J_{A,3,3}) + 0.55(J_{M,1,5} + J_{M,2,4} + J_{M,3,3})]$$

$$D_{E,1} = 0.9\,[\,0.15(J_{S,1,6} + J_{S,2,5} + J_{S,3,4}) + 0.6(J_{E,1,6} + J_{E,2,5} + J_{E,3,4}) + 0.08(J_{A,1,6} + J_{A,2,5} + J_{A,3,4}) + 0.15(J_{M,1,6} + J_{M,2,5} + J_{M,3,4})]$$

$$D_{E,2} = 0.9\,[\,0.15(J_{S,1,1} + J_{S,2,6} + J_{S,3,5}) + 0.6(J_{E,1,1} + J_{E,2,6} + J_{E,3,5}) + 0.08(J_{A,1,1} + J_{A,2,6} + J_{A,3,5}) + 0.15(J_{M,1,1} + J_{M,2,6} + J_{M,3,5})]$$

$$D_{E,3} = 0.9\,[\,0.15(J_{S,1,2} + J_{S,2,1} + J_{S,3,6}) + 0.6(J_{E,1,2} + J_{E,2,1} + J_{E,3,6}) + 0.08(J_{A,1,2} + J_{A,2,1} + J_{A,3,6}) + 0.15(J_{M,1,2} + J_{M,2,1} + J_{M,3,6})]$$

$$D_{E,4} = 0.9\,[\,0.15(J_{S,1,3} + J_{S,2,2} + J_{S,3,1}) + 0.6(J_{E,1,3} + J_{E,2,2} + J_{E,3,1}) + 0.08(J_{A,1,3} + J_{A,2,2} + J_{A,3,1}) + 0.15(J_{M,1,3} + J_{M,2,2} + J_{M,3,1})]$$

$$D_{E,5} = 0.9\,[\,0.15(J_{S,1,4} + J_{S,2,3} + J_{S,3,2}) + 0.6(J_{E,1,4} + J_{E,2,3} + J_{E,3,2}) + 0.08(J_{A,1,4} + J_{A,2,3} + J_{A,3,2}) + 0.15(J_{M,1,4} + J_{M,2,3} + J_{M,3,2})]$$

$$D_{E,6} = 0.9\,[\,0.15(J_{S,1,5} + J_{S,2,4} + J_{S,3,3}) + 0.6(J_{E,1,5} + J_{E,2,4} + J_{E,3,3}) + 0.08(J_{A,1,5} + J_{A,2,4} + J_{A,3,3}) + 0.15(J_{M,1,5} + J_{M,2,4} + J_{M,3,3})]$$

$$T_{S,1} = T_{S,6} + \frac{1}{9}D_{S,1} - Y_{S,S,1} - Y_{M,S,1} - Y_{A,S,6} - Y_{E,S,6}$$

$$T_{S,2} = T_{S,1} + \frac{1}{9}D_{S,2} - Y_{S,S,2} - Y_{M,S,1} - Y_{A,S,1} - Y_{E,S,1}$$
$$T_{S,3} = T_{S,2} + \frac{1}{9}D_{S,3} - Y_{S,S,3} - Y_{M,S,2} - Y_{A,S,2} - Y_{E,S,2}$$
$$T_{S,4} = T_{S,3} + \frac{1}{9}D_{S,4} - Y_{S,S,4} - Y_{M,S,3} - Y_{A,S,3} - Y_{E,S,3}$$
$$T_{S,5} = T_{S,4} + \frac{1}{9}D_{S,5} - Y_{S,S,5} - Y_{M,S,4} - Y_{A,S,4} - Y_{E,S,4}$$
$$T_{S,6} = T_{S,5} + \frac{1}{9}D_{S,6} - Y_{S,S,6} - Y_{M,S,5} - Y_{A,S,5} - Y_{E,S,5}$$

$$T_{M,1} = T_{M,6} + \frac{1}{9}D_{M,1} - Y_{M,M,1} - Y_{S,M,1} - Y_{A,M,6} - Y_{E,M,6}$$
$$T_{M,2} = T_{M,1} + \frac{1}{9}D_{M,2} - Y_{M,M,2} - Y_{S,M,1} - Y_{A,M,1} - Y_{E,M,1}$$
$$T_{M,3} = T_{M,2} + \frac{1}{9}D_{M,3} - Y_{M,M,3} - Y_{S,M,2} - Y_{A,M,2} - Y_{E,M,2}$$
$$T_{M,4} = T_{M,3} + \frac{1}{9}D_{M,4} - Y_{M,M,4} - Y_{S,M,3} - Y_{A,M,3} - Y_{E,M,3}$$
$$T_{M,5} = T_{M,4} + \frac{1}{9}D_{M,5} - Y_{M,M,5} - Y_{S,M,4} - Y_{A,M,4} - Y_{E,M,4}$$
$$T_{M,6} = T_{M,5} + \frac{1}{9}D_{M,6} - Y_{M,M,6} - Y_{S,M,5} - Y_{A,M,5} - Y_{E,M,5}$$

$$T_{A1} = T_{A,6} + \frac{1}{9}D_{A,1} - Y_{A,A,1} - Y_{M,A,1} - Y_{S,A,6} - Y_{E,A,6}$$
$$T_{A,2} = T_{A,1} + \frac{1}{9}D_{A,2} - Y_{A,A,2} - Y_{M,A,1} - Y_{S,A,1} - Y_{E,A,1}$$
$$T_{A,3} = T_{A,2} + \frac{1}{9}D_{A,3} - Y_{A,A,3} - Y_{M,A,2} - Y_{S,A,2} - Y_{E,A,2}$$
$$T_{A,4} = T_{A,3} + \frac{1}{9}D_{A,4} - Y_{A,A,4} - Y_{M,A,3} - Y_{S,A,3} - Y_{E,A,3}$$
$$T_{A,5} = T_{A,4} + \frac{1}{9}D_{A,5} - Y_{A,A,5} - Y_{M,A,4} - Y_{S,A,4} - Y_{E,A,4}$$
$$T_{A,6} = T_{A,5} + \frac{1}{9}D_{A,6} - Y_{A,A,6} - Y_{M,A,5} - Y_{S,A,5} - Y_{E,A,5}$$

$$T_{E,1} = T_{E,6} + \frac{1}{9}D_{E,1} - Y_{E,E,1} - Y_{S,E,1} - Y_{A,E,6} - Y_{M,E,6}$$
$$T_{E,2} = T_{E,1} + \frac{1}{9}D_{E,2} - Y_{E,E,2} - Y_{S,E,1} - Y_{A,E,1} - Y_{M,E,1}$$
$$T_{E,3} = T_{E,2} + \frac{1}{9}D_{E,3} - Y_{E,E,3} - Y_{S,E,2} - Y_{A,E,2} - Y_{M,E,2}$$
$$T_{E,4} = T_{E,3} + \frac{1}{9}D_{E,4} - Y_{E,E,4} - Y_{S,E,3} - Y_{A,E,3} - Y_{M,E,3}$$
$$T_{E,5} = T_{E,4} + \frac{1}{9}D_{E,5} - Y_{E,E,5} - Y_{S,E,4} - Y_{A,E,4} - Y_{M,E,4}$$
$$T_{E,6} = T_{E,5} + \frac{1}{9}D_{E,6} - Y_{E,E,6} - Y_{S,E,5} - Y_{A,E,5} - Y_{M,E,5}$$

$$Y_{S,S,i} + Y_{M,S,i-1} + Y_{A,S,i-1} + Y_{E,S,i-1} \leq 20$$

$$Y_{M,M,i} + Y_{S,M,i-1} + Y_{A,M,i-1} + Y_{E,M,i-1} \leq 12$$

$$Y_{E,E,i} + Y_{S,E,i-1} + Y_{A,E,i-1} + Y_{M,E,i-1} \leq 0$$

$$Y_{A,A,i} + Y_{S,A,i-1} + Y_{M,A,i-1} + Y_{E,A,i-1} \leq 0$$

$$S_1 - J_{S,1,1} - J_{S,1,2} - J_{S,1,3} - X_{S,E,1} - X_{S,A,1} - X_{S,M,1} \geq 0$$

$$S_2 - J_{S,2,1} - J_{S,2,2} - J_{S,2,3} - X_{S,E,2} - X_{S,A,2} - X_{S,M,2} \geq 0$$

$$S_3 - J_{S,3,1} - J_{S,3,2} - J_{S,3,3} - X_{S,E,3} - X_{S,A,3} - X_{S,M,3} \geq 0$$

$$S_4 - J_{S,4,1} - J_{S,4,2} - J_{S,4,3} - X_{S,E,4} - X_{S,A,4} - X_{S,M,4} \geq 0$$

$$S_5 - J_{S,5,1} - J_{S,5,2} - J_{S,5,3} - X_{S,E,5} - X_{S,A,5} - X_{S,M,5} \geq 0$$

$$S_6 - J_{S,1,6} - J_{S,6,2} - J_{S,6,3} - X_{S,E,6} - X_{S,A,6} - X_{S,M,6} \geq 0$$

$$A_1 - J_{A,1,1} - J_{A,1,2} - J_{A,1,3} - X_{A,E,1} - X_{A,S,1} - X_{A,M,1} \geq 0$$

$$A_2 - J_{A,2,1} - J_{A,2,2} - J_{A,2,3} - X_{A,E,2} - X_{A,S,2} - X_{A,M,2} \geq 0$$

$$A_3 - J_{A,3,1} - J_{A,3,2} - J_{A,3,3} - X_{A,E,3} - X_{A,S,3} - X_{A,M,3} \geq 0$$

$$A_4 - J_{A,4,1} - J_{A,4,2} - J_{A,4,3} - X_{A,E,4} - X_{A,S,4} - X_{A,M,4} \geq 0$$

$$A_5 - J_{A,5,1} - J_{A,5,2} - J_{A,5,3} - X_{A,E,5} - X_{A,S,5} - X_{A,M,5} \geq 0$$

$$A_6 - J_{A,1,6} - J_{A,6,2} - J_{A,6,3} - X_{A,E,6} - X_{A,S,6} - X_{A,M,6} \geq 0$$

$$E_1 - J_{E,1,1} - J_{E,1,2} - J_{E,1,3} - X_{E,A,1} - X_{E,S,1} - X_{E,M,1} \geq 0$$

$$E_2 - J_{E,2,1} - J_{E,2,2} - J_{E,2,3} - X_{E,A,2} - X_{E,S,2} - X_{E,M,2} \geq 0$$

$$E_3 - J_{E,3,1} - J_{E,3,2} - J_{E,3,3} - X_{E,A,3} - X_{E,S,3} - X_{E,M,3} \geq 0$$

$$E_4 - J_{E,4,1} - J_{E,4,2} - J_{E,4,3} - X_{E,A,4} - X_{E,S,4} - X_{E,M,4} \geq 0$$

$$E_5 - J_{E,5,1} - J_{E,5,2} - J_{E,5,3} - X_{E,A,5} - X_{E,S,5} - X_{E,M,5} \geq 0$$

$$E_6 - J_{E,1,6} - J_{E,6,2} - J_{E,6,3} - X_{E,A,6} - X_{E,S,6} - X_{E,M,6} \geq 0$$

$$M_1 - J_{M,1,1} - J_{M,1,2} - J_{M,1,3} - X_{M,A,1} - X_{M,S,1} - X_{M,E,1} \geq 0$$

$$M_2 - J_{M,2,1} - J_{M,2,2} - J_{M,2,3} - X_{M,A,2} - X_{M,S,2} - X_{M,E,2} \geq 0$$

$$M_3 - J_{M,3,1} - J_{M,3,2} - J_{M,3,3} - X_{M,A,3} - X_{M,S,3} - X_{M,E,3} \geq 0$$

$$M_4 - J_{M,4,1} - J_{M,4,2} - J_{M,4,3} - X_{M,A,4} - X_{M,S,4} - X_{M,E,4} \geq 0$$

$$M_5 - J_{M,5,1} - J_{M,5,2} - J_{M,5,3} - X_{M,A,5} - X_{M,S,5} - X_{M,E,5} \geq 0$$

$$M_6 - J_{M,1,6} - J_{M,6,2} - J_{M,6,3} - X_{M,A,6} - X_{M,S,6} - X_{M,E,6} \geq 0$$

$$S_i, E_i, A_i, M_i, X_{N,P,i}, J_{N,i,j}, Y_{N,P,i}, D_{N,i}, T_{N,i} \geq 0 \qquad Int$$

$(N, P \in \{E, S, M, A\} \qquad i \in \{1,2,3,4,5,6\} \qquad j \in \{1,2,3\})$

In $X_{N,P,i}$ we have $P \neq N$