

A summary of the project in English is available here:

The question posed to us requires reducing the cost of student transportation services by appropriately allocating them to schools. In the table below, the number of students in a given area is specified, along with the percentage of students in each grade. The cost of school transportation from each area to each school is also determined. Each grade can constitute 30 to 36 percent of the total population of a school. Students from a specific area and grade can be assigned to one school or another. Note that the dashed line in the table indicates that students from that area cannot be allocated to the specified school.

| Transportation cost per student | | | Area | Total students' population | Percentage of 7 th grade | Percentage of 8 th grade | Percentage of 9 th grade |
|---------------------------------|-------------|-------------|-------------------|----------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| Scholl no.1 | Scholl no.2 | Scholl no.3 | | | | | |
| 300 | 0 | 700 | 1 | 450 | 32 | 38 | 30 |
| - | 400 | 500 | 2 | 600 | 37 | 28 | 35 |
| 600 | 300 | 200 | 3 | 550 | 30 | 32 | 28 |
| 200 | 500 | - | 4 | 350 | 28 | 40 | 32 |
| 0 | - | 400 | 5 | 500 | 39 | 34 | 27 |
| 500 | 300 | 0 | 6 | 450 | 34 | 28 | 38 |
| 900 | 1100 | 1000 | Schools' capacity | | | | |

Solution (By Sajjad Abed):

Model:

$$\text{Min } Z = \sum_{i=1}^6 \sum_{k=1}^3 \left(C_{i,k} * \sum_{j=7}^9 S_{i,j,k} \right)$$

s. t.

$$\sum_{k=1}^3 S_{i,j,k} = P_{i,j} * N_i$$

for i in (1 – 6) , j in (7 – 9)

$$\sum_{i=1}^6 \sum_{j=7}^9 S_{i,j,k} \leq Cap_k$$

for k in (1 – 3)

$$0.3 * \sum_{i=1}^6 \sum_{j=7}^9 S_{i,j,k} \leq \sum_{i=1}^6 S_{i,j,k}$$

for k in (1 – 3) , j in (7 – 9)

$$\sum_{i=1}^6 S_{i,j,k} \leq 0.36 * \sum_{i=1}^6 \sum_{j=7}^9 S_{i,j,k}$$

for i in (1 – 6) , j in (7 – 9)

$$S_{2,j,1} = S_{4,j,3} = S_{5,j,2} = 0$$

for j in (7 – 9)

$$0 \leq S_{i,j,k}$$

$$S_{i,j,k} \text{ (int)}$$

In which:

Number of students from region i and grade j assigned to school k (Variable):

$S_{i,j,k}$

Cost of assigning a student from region i to school k (Parameter):

$C_{i,k}$

Percentage of students from region i studying in grade j (Parameter):

$P_{i,j}$

Number of students in region i (Parameter):

N_i

Capacity of school k (Parameter):

Cap_k

Additionally, indices i , j , and k represent the region, grade, and school, respectively.

Cplex code:

```

OR2_Project.mod  OR2_Project.dat  OR2_Project.ops
1 /*****
2  * OPL 12.9.0.0 Data
3  * Author: sabel
4  * Creation Date: Nov 19, 2021 at 4:31:53 PM
5  *****/
6
7 SheetConnection connex ("data.xlsx");
8
9 Cap from SheetRead (connex,"data!B3:B5");
10 P from SheetRead (connex,"data!E3:G8");
11 C from SheetRead (connex,"data!E11:G16");
12 N from SheetRead (connex,"data!B10:B15");
13
14 someSet to SheetWrite (connex, "Result!B2:E55");

```

| | | Percentage of students (P) | | | |
|--------|-------------|----------------------------|----------|----------|----------|
| School | Capacity | Area | School 1 | School 2 | School 3 |
| 1 | 900 | 1 | 0.32 | 0.38 | 0.3 |
| 2 | 1100 | 2 | 0.37 | 0.28 | 0.35 |
| 3 | 1000 | 3 | 0.3 | 0.32 | 0.38 |
| | | 4 | 0.28 | 0.4 | 0.32 |
| | | 5 | 0.39 | 0.34 | 0.27 |
| | | 6 | 0.34 | 0.28 | 0.38 |
| Area | Numbers (N) | Transportation Cost (C) | | | |
| Area | | Area | School 1 | School 2 | School 3 |
| 1 | 450 | 1 | 300 | 0 | 700 |
| 2 | 600 | 2 | 99 | 400 | 500 |
| 3 | 550 | 3 | 600 | 300 | 200 |
| 4 | 350 | 4 | 200 | 500 | 99 |
| 5 | 500 | 5 | 0 | 99 | 400 |
| 6 | 450 | 6 | 500 | 300 | 0 |

.mod code:

```
OR2_Project.mod  OR2_Project.dat  OR2_Project.ops
2  * OPL 12.9.0.0 Model
3  * Author: sabel
4  * Creation Date: Nov 19, 2021 at 4:31:53 PM
5  *****/
6  range I=1..6;
7  range J=7..9;
8  range K=1..3;
9  int Cap[K]=...;
10 float P[I][J]=...;
11 int N[I]=...;
12 int C[I][K]=...;
13 dvar int+ S[I][J][K];
14
15 minimize sum(i in I,k in K) (C[i][k]*sum(j in J)S[i][j][k]);
16 subject to{
17     forall(i in I,j in J)
18         sum(k in K) (S[i][j][k])=P[i][j]*N[i];
19     forall(k in K)
20         sum(i in I,j in J) (S[i][j][k]) <= Cap[k];
21     forall(j in J,k in K)
22         0.3*(sum(i in I,j in J) (S[i][j][k])) <= sum(i in I)(S[i][j][k]);
23     forall(j in J,k in K)
24         sum(i in I)(S[i][j][k]) <= 0.36*(sum(i in I,j in J) (S[i][j][k]));
25     forall(j in J)
26         S[2][j][1]==0;
27     forall(j in J)
28         S[4][j][3]==0;
29     forall(j in J)
30         S[5][j][2]==0;
31 };
32
33 tuple someTuple{
34 int a;
35 int b;
36 int c;
37 int value;
38 };
39 {someTuple} someSet = {<i,j,k,S[i][j][k]> | i in I, j in J, k in K};
```

Cplex output:

IBM ILOG CPLEX Optimization Studio

File Edit Navigate Search Run Window Help

OPL Projects 97104515-97104561

Run Configurations

OR2_Project.mod CPLEX

OR2_Project.ops

OR2_Project.dat

rdata view

Problem browser 97104515-97104561

Solution with objective 426.800

| Name | Value |
|------------------------|--|
| C | [300 0 700] [99 400 500] [600 300 200] ... |
| Cap | [900 1100 1000] |
| I | 1.6 |
| J | 7.9 |
| K | 1.3 |
| N | [450 600 550 350 500 450] |
| P | [[0.32 0.38 0.3] [0.37 0.28 0.35] [0.3 0.32 0. ... |
| Decision variables (1) | S |
| Result data (1) | someSet |

Outline 97104515-97104561

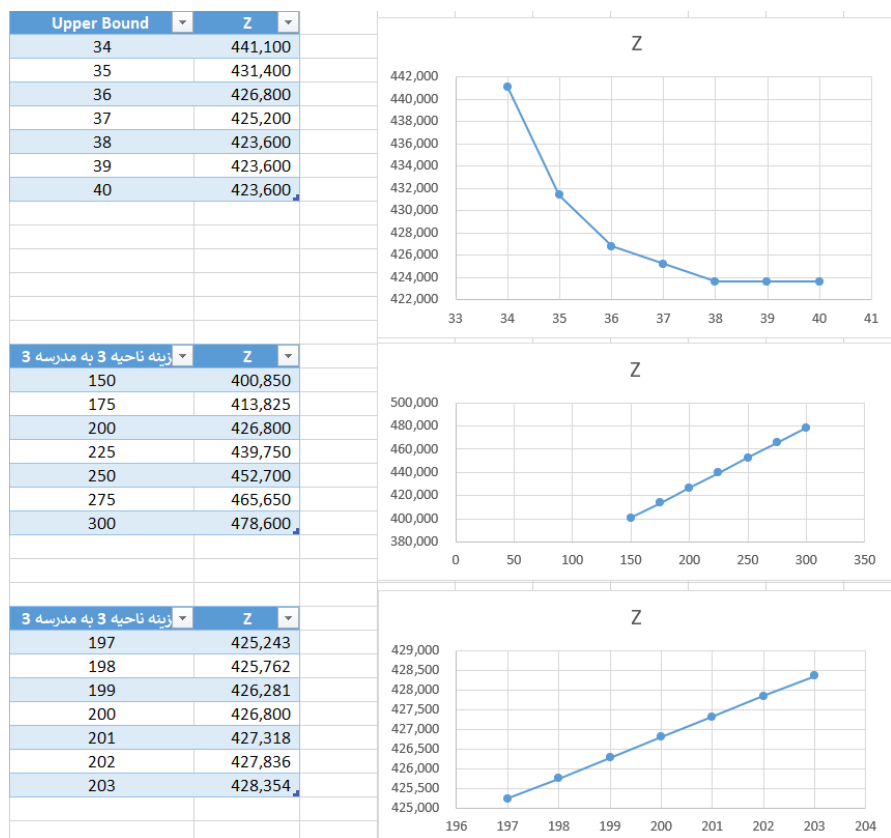
- using CPLEX
- Internal data (3)
 - I: range
 - J: range
 - K: range
- External data (4)
 - C: int[[I][K]]
 - Cap: int[K]
 - N: int[I]
 - P: float[[I][J]]
- Decision variables (1)
 - S: dvar int+[[I][J][K]]
- Objective: simple
- Constraints (7)
 - i in I, j in J
 - j in J
 - j in J
 - j in J
 - j in J, k in K
 - j in J, k in K
 - k in K
- Result types (1)
 - someTuple: tuple<int,int,int,value,int>
- Result data (1)
 - someSet: {someTuple}

Properties 97104515-97104561

| Property | Value |
|------------|----------------|
| Dimensions | 3 |
| Name | S |
| Total size | 54 ([6][3][3]) |

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Sensitive Analysis:



Shadow price:

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|--------|------|-------------|--------------|-----------------------|--------------------|--------------------|
| \$I\$4 | XA | 0 | -11.25 | 20 | 11.25 | 1.00E+30 |
| \$J\$4 | XB | 10 | 0 | 50 | 1.00E+30 | 3.333333333 |
| \$K\$4 | XC | 0 | -2.5 | 35 | 2.5 | 1E+30 |

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|--------|----------|-------------|--------------|----------------------|--------------------|--------------------|
| \$O\$5 | Milling | 50 | 0 | 60 | 1E+30 | 10 |
| \$P\$5 | Rounding | 65 | 0 | 100 | 1E+30 | 35 |
| \$Q\$5 | Drilling | 40 | 12.5 | 40 | 8 | 40 |

RESULT:

| Area | Grade | School | Capacity |
|------|-------|--------|----------|
| 1 | 7 | 1 | 0 |
| 1 | 7 | 2 | 144 |
| 1 | 7 | 3 | 0 |
| 1 | 8 | 1 | 0 |
| 1 | 8 | 2 | 171 |
| 1 | 8 | 3 | 0 |
| 1 | 9 | 1 | 0 |
| 1 | 9 | 2 | 135 |
| 1 | 9 | 3 | 0 |
| 2 | 7 | 1 | 0 |
| 2 | 7 | 2 | 222 |
| 2 | 7 | 3 | 0 |
| 2 | 8 | 1 | 0 |
| 2 | 8 | 2 | 168 |
| 2 | 8 | 3 | 0 |
| 2 | 9 | 1 | 0 |
| 2 | 9 | 2 | 210 |
| 2 | 9 | 3 | 0 |
| 3 | 7 | 1 | 0 |
| 3 | 7 | 2 | 0 |
| 3 | 7 | 3 | 165 |
| 3 | 8 | 1 | 0 |
| 3 | 8 | 2 | 0 |
| 3 | 8 | 3 | 176 |
| 3 | 9 | 1 | 12 |
| 3 | 9 | 2 | 20 |
| 3 | 9 | 3 | 177 |
| 4 | 7 | 1 | 98 |
| 4 | 7 | 2 | 0 |
| 4 | 7 | 3 | 0 |
| 4 | 8 | 1 | 140 |
| 4 | 8 | 2 | 0 |
| 4 | 8 | 3 | 0 |
| 4 | 9 | 1 | 112 |
| 4 | 9 | 2 | 0 |
| 4 | 9 | 3 | 0 |
| 5 | 7 | 1 | 195 |
| 5 | 7 | 2 | 0 |
| 5 | 7 | 3 | 0 |
| 5 | 8 | 1 | 170 |
| 5 | 8 | 2 | 0 |
| 5 | 8 | 3 | 0 |
| 5 | 9 | 1 | 135 |
| 5 | 9 | 2 | 0 |
| 5 | 9 | 3 | 0 |
| 6 | 7 | 1 | 0 |
| 6 | 7 | 2 | 0 |
| 6 | 7 | 3 | 153 |
| 6 | 8 | 1 | 0 |
| 6 | 8 | 2 | 0 |
| 6 | 8 | 3 | 126 |
| 6 | 9 | 1 | 0 |
| 6 | 9 | 2 | 0 |
| 6 | 9 | 3 | 171 |
| | | | |
| | | | |