Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
A		2430065

Sea lampreys have a fluctuating sex ratio which can allow them to adjust to changing environmental conditions and resources. This adaptation can improve the species' overall survival and reproductive ability. It can also have a negative impact on population stability. Sea lampreys can improve resource consumption by altering their sex ratio. In periods of abundant resources, an imbalanced sex ratio favoring one gender, female lampreys, may improve reproductive efficiency. In areas of limited resources, changing the sex ratio may limit population growth and prevent resource overexploitation. Significant changes in the sex ratio might disrupt the ecosystem's equilibrium by impacting the populations of other species that interact with sea lampreys, which include large fish and parasitic organisms such as nematodes. Although our second model's effectiveness could not be tested due to a lack of data, we believe our first model accurately demonstrates the relationship between sea lamprey sex ratio and nutrient concentration availability.

The Impact of Variable Sex Ratios in Lamprey

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Abstract

In this paper our team presents three individual computerized models that examine the advantages and disadvantages of the ability of the sea lamprey species to alter its sex ratio depending on resource availability and also attempts to provide insights into the resulting interactions in an ecosystem. We first created a model to demonstrate the relationship between sea lamprey sex ratio and nutrient availability. We created a second model to demonstrate the relationship between the population of the lampreys and the population of the lamprey's prey. We then added to this model to make a revised version that interpolates the effect that the population of lampreys had on the ecological system. We were able to use data to determine the accuracy of our first sex-ratio model. Our model does not perfectly portray an ecosystem of lampreys, predators, prey, and parasites. We discuss why in our weaknesses section.

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1 Introduction

Sea lampreys are eel-like fish that lack fins and jaws. They have a cartilaginous skeleton and can grow 12 to 20 inches long. Sea lampreys are found in the Atlantic Ocean but spawn in freshwater rivers and lakes. Some sea lampreys can spend their entire life span in freshwater. Sea lampreys can also be dangerous to the environment and in many cases, a counteractive measure must be taken by humans [4]. Our team has decided to focus on the sea lampreys that live in freshwater. The sex ratio of sea lampreys varies based on external circumstances, including the environmental location. Lentic environments have a lower nutrition concentration than streams and fast-flowing bodies of water [2]. Low food availability results in slower growth rates and a higher proportion of around 78% of males in the population. Males account for around 56% of the population in areas where food is more readily accessible [2]. This allows us to assume that male sea lampreys still dominate their populations just significantly less or more. In order to determine how the sex ratio of the sea lampreys affects their population and the stability of the ecosystem, we created three models, influenced by several other models we have found[12].

2 Definitions

Throughout this paper we will be using the following terms:

1. **Filter feeding** - A method of obtaining food through suspended matter, food particles, or microscopic organisms from the water.
2. **Nutrient concentration**- The amount of essential elements or compounds present in a given quantity of a substance.
3. **Carrying capacity**- The maximum population size that a specific environment or ecosystem can maintain over time.
4. **Abiotic**- Non-living components of the ecosystem
5. **Biotic**- Components of the ecosystem that originate from living organisms.

3 Approach

Our goal is to create and analyze a model to understand how the sex ratio of sea lampreys affects ecological interactions.

To achieve this we will:

- Determine the relationships between population size, food availability, sex ratio, and predator population.
- Estimate the birth rate, death rate, starting population size, prey population, predator population, nutrient concentration in their habitat, and carrying capacity.

3.1 Logistic growth formula

Logistic growth is a type of population growth in which a population's increase is limited by environmental conditions. The logistic growth model is represented by the differential equation $dN/dt = rN((K-N)/K)$, where N is the population size, r is the growth rate, and K is carrying capacity. This equation represents the theory that when a population approaches its carrying capacity, the rate of increase slows and eventually reaches a stable equilibrium. Carrying capacity is determined by the limiting factors in an ecosystem. These include biotic factors such as food and abiotic factors such as water and air [12].

4 Assumptions

In creating our mathematical models pertaining to the population and sex ratios of lampreys, several assumptions must be made, in order to keep our findings consistent as well as usable.

- Most environmental conditions are kept the same. Items such as temperature, location, and seasonal conditions are constant due to a lack of available information on these factors.
- Sex ratio of lampreys is solely controlled by food availability.
- Availability of egg-laying territory is not an issue.
- Humans are not interfering with the lamprey, prey, or predator population in any way.
- The fertility rate of females is constant, in other words, every egg laid will hatch into a larva.
- All lampreys have the same lifespan without interference from predators.

5 Our Models

We first did research in order to form equations that would model our variables. Next, we used the technical computing software, Wolfram Mathematica, to create graphical representations of our equations. This was used to create a logistic population growth model and a model for the sea lamprey sex ratio's effect on other organisms in the ecosystem such as parasites

5.1 Sex Ratio Effect Model

Our first attempt at a model was to show the relationship between the sex ratio and food availability. Through research, we determined that lampreys decide their sex during the larval stage. One major factor in determining which sex is chosen is food availability. Due to the fact that larvae filter feed, we determined that nutrient concentration(mg/L) of the water was an important factor. Through modification of a logistic growth function to include nutrient concentration and sex ratio, we created this equation [6].

$$S = 100/(1 + e^{-(a\times N+b)})$$

Figure 1 shows the relationship between the sex ratio of lamprey males and the nutrient concentration using the above equation. We tested this model on a set of data from the Royal Society in a study that found that sex ratio is influenced by larval growth rates [3]. This model demonstrates an inverse relationship between the male sex ratio and nutrient concentration, correlating with the data set.

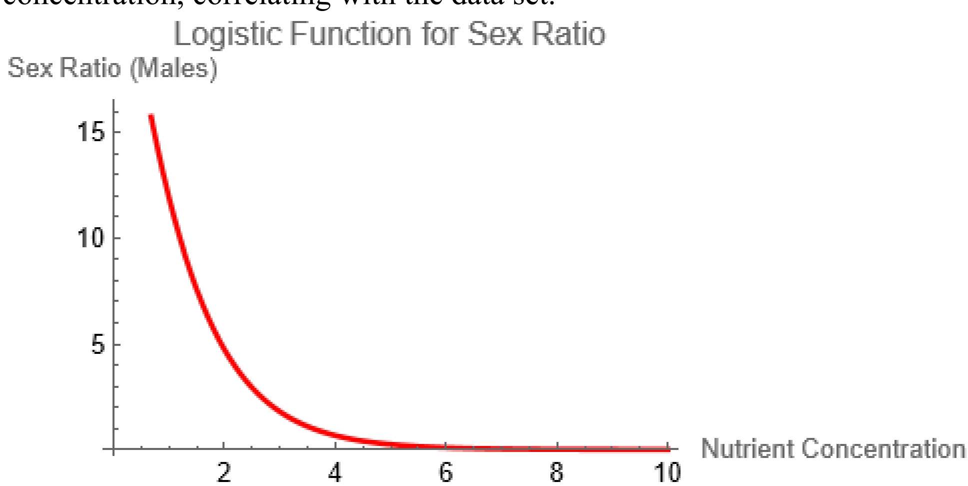

Figure 1: Logistic Function for Sex Ratio

5.2 Logistic Population Growth Model

Our original solution to the population of lampreys relative to the population of prey fell short due to the fact that it did not correctly account for the decrease of prey on account of an increase in lamprey and the prey deaths that would occur as a result. Our first equation was a logistic carrying capacity formula we had modified to include birth and death ratios of the lamprey population [6].

$$\frac{dP_1}{dt} = \frac{B}{D}P_1(t) \times \left(1 - \frac{P_1(t)}{K}\right) \times NM$$
$$\frac{dP_2}{dt} = \frac{B_2}{D_2} \times P_2(t) \times \left(1 - \frac{P_2(t)}{K_2}\right) - P_1(t) \times a$$

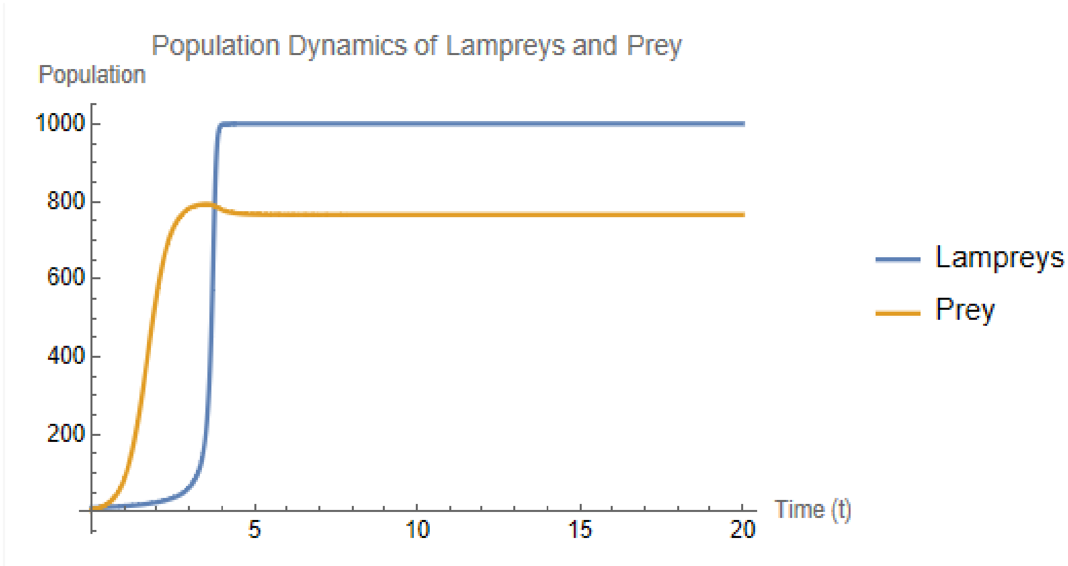

Figure 2: First attempt at Population Dynamics in a Lamprey ecosystem. The blue line represents the Lampreys and the orange line represents their prey.

Where in the first equation, P_1 is the population of Lamprey, B is the birth rate for lamprey, D is the death rate for lamprey, and K is the carrying capacity of lamprey, N is the nutrient concentration in the environment, and M is the male to female ratio of lampreys. In the second equation, P_2 is the population of the lamprey's prey, B_2 is the birth rate for prey, D_2 is the death rate for prey, K_2 is the carrying capacity of the prey, and a is the amount of prey each lamprey kills per t.

5.2.1 Revised Model

In our second model of the logistic population growth formula, we attempted to account for the fallbacks that the first model had by changing the growth rate to utilize the ratio of male to female lampreys, as more females would equate to greater birth rates.

$$\frac{dP_1}{dt} = (B \times P_1(t) - D_1 \times (P_1(t) - L)) \times P_1(t) \times (1 - \frac{P_1(t)}{P_2(t)}) \times NM$$
$$\frac{dp_2}{dt} = (\frac{B_2}{D_2} - (P_1(t)a)) \times P_2(t) \times (1 - \frac{P_2(t)}{K_2})$$

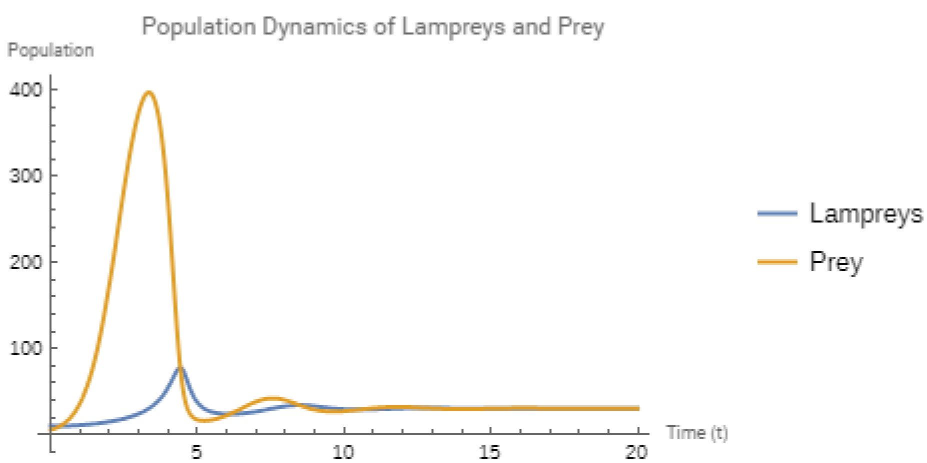

Figure 3: Population Dynamics of Lampreys and Prey in an Ecosystem. The blue line represents the Lampreys and the orange line represents their prey.

In this modified model, we allow all of the variables to be referencing the same phenomenon, however, this modification correctly induces the prey versus predator effect where prey increases as predators decrease and vice versa.

Lastly, we attempt to model the ecosystem similarly to before but with the addition of parasites [9]. Again we utilize a carrier capacity formula. However, this model is not effective at representing the role of parasites in an ecosystem. Parasites latch onto hosts and remain until the host has been severely damaged. Our model does not fully demonstrate this behavior.

$$\frac{dP_3}{dt} = \frac{B_3}{D_3} P_3(t)(1 - \frac{P_3}{K})NM$$

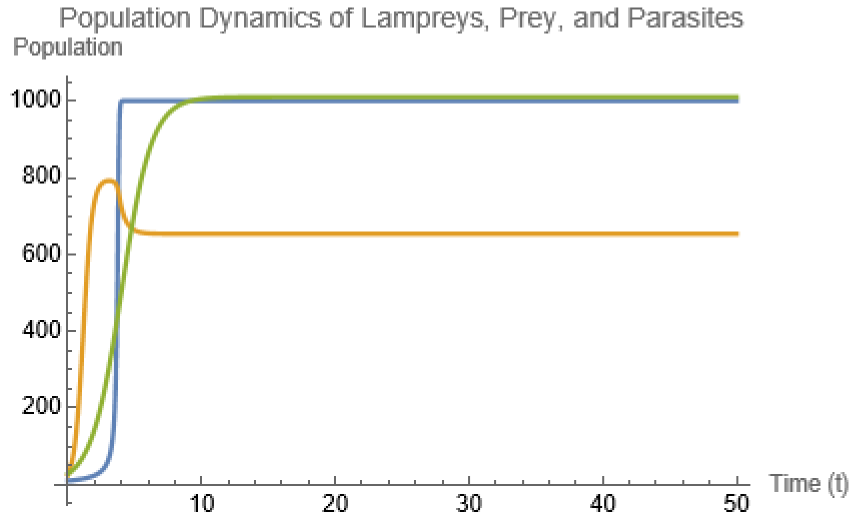

Figure 4: Population Dynamics with Lampreys, Prey and Parasites

6 Strengths and Weaknesses of Ecosystem Model

Strength
1. A large strength of our model is its simplicity. Due to the amount of assumptions made, we were able to create a model that is more generalized as well as more applicable to other scenarios.

Weaknesses
1. We were not able to find a real data set for lampreys in ecosystems that took into account sex ratios, nutrient concentration, and other factors such as predator and prey populations.
2. Many assumptions were made that allowed us to create a model for this situation. However, this means that we did not represent the total complexity of a lamprey ecosystem.
3. A weakness of our parasite model is that rather than a carrying capacity formula, an alternative solution would be preferable as the current model does not take into account how difficult it would be to infect their prey.

7 Sensitivity Analysis

When creating a mathematical model, it is helpful and important to show how adjusting certain values will contribute to understanding of relationships. In our graphs, we included sliders to adjust each variable's value within a set range of reasonable values.

7.1 Sex Ratio Model Sensitivity Analysis

In the first model, we explored the relationship between sex ratio and food availability. There were two other variables, *a* and *b*, which represented coefficients. The former of the two represents the rate of change of the curve and the latter represents a translation term along the x-axis. In Figure 5 below, we see the effects of making *a* less than 0 which causes a growth in the sex rate as nutrient concentration increases.

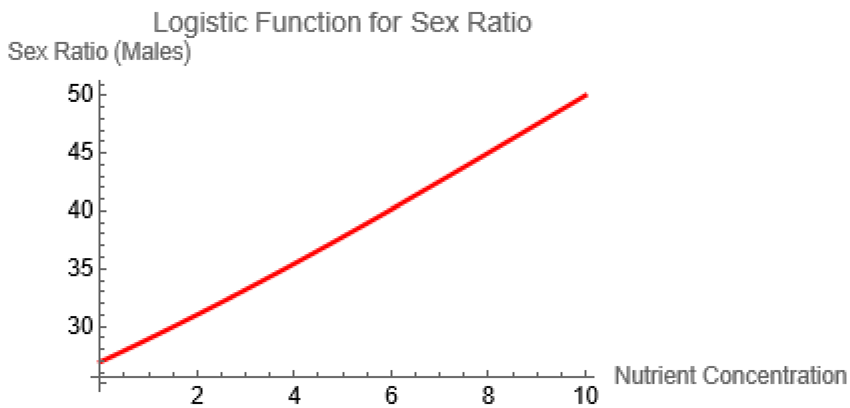

Figure 5: Sensitivity Analysis altering the value of *a* to a negative.

7.2 Population Dynamics Model Sensitivity Analysis

In this model we looked at the ecosystem in regards to the lamprey and prey relationship. The variable that is altered in the figure below is the death rate. When the death rate exceeds the birth rate, the graph displays a steady loss in population over time. Figure 6 below shows this.

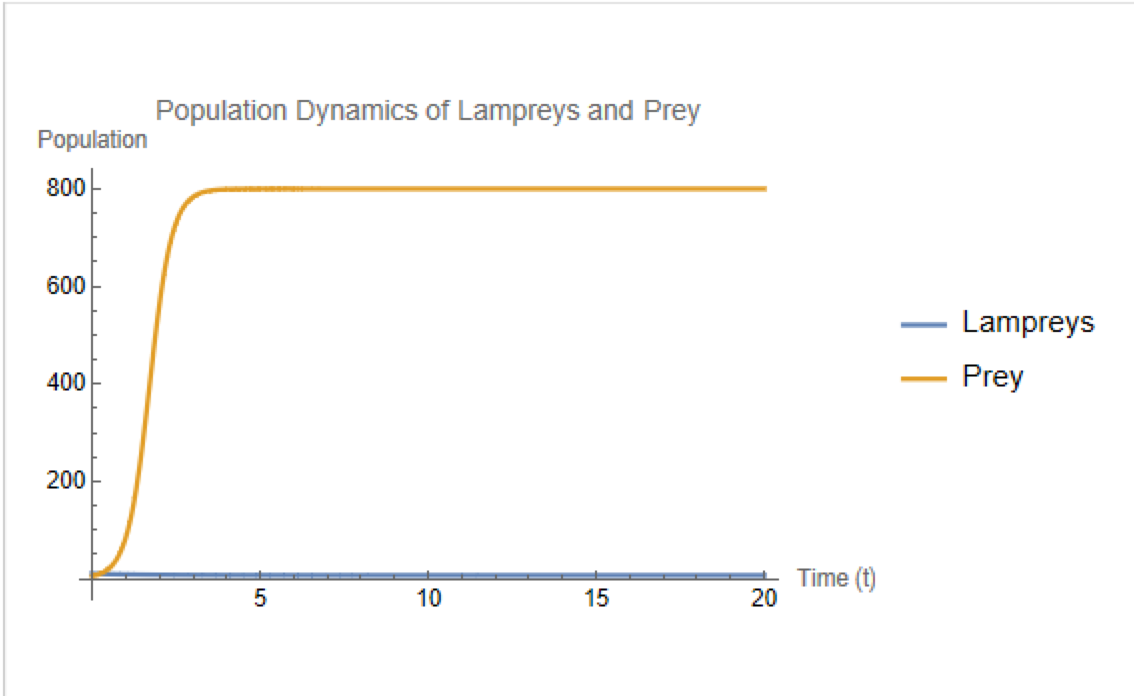

Figure 6: Sensitivity analysis altering death rates.

8 Conclusion

In conclusion, the sex ratio of the lampreys negatively affects the surrounding ecosystem because an increase in the number of female lampreys is directly associated with an increase in the population of lampreys themselves, which creates an imbalance in the prey-predator population. This imbalance leads to potential volatility in some species. In addition to this, we have found by use of our model that an increase of lamprey is beneficial for the parasites that may infect them, including ones such as nematodes.

9 References

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