UNIT-4

Number system, Conversion, Binary arithmetic
Number system - Binary, decimal, octal, hexadecimal
Conversion - Binary to decimal, decimal to binary, octal to decimal, decimal to octal,
octal to binary, binary to octal, hexadecimal to binary, binary to hexadecimal,
hexadecimal to Decimal, decimal to hexadecimal, hexadecimal to octal, octal to
hexadecimal
Binary arithmetic – Addition, subtraction

What are the number systems in Computer?

Number systems are the technique to represent numbers in the computer system architecture, every value that you are saving or getting into/from computer memory has a defined number system.

Computer architecture supports following number systems.

1. **Binary number system**
2. **Octal number system**
3. **Decimal number system**
4. **Hexadecimal (hex) number system**

- **1) Binary Number System**

  A Binary number system has only two digits that are **0 and 1**. Every number (value) represents with 0 and 1 in this number system. The base of binary number system is 2, because it has only two digits.
- Digits: 0,1
- Symbols :- 2
- Base :- 2
- Bi = 2

- **2) Octal number system**

  Octal number system has only eight (8) digits from **0 to 7**. Every number (value) represents with 0,1,2,3,4,5,6 and 7 in this number system. The base of octal number system is 8, because it has only 8 digits.

- Digits: 0,1,2,3,4,5,6,7
- Symbols :- 8
- Base :- 8
- Octa = 8 $(123)_8$ $(128)_8$ it is not ocatal number

- **3) Decimal number system**
- Decimal number system has only ten (10) digits from **0 to 9**. Every number (value) represents with 0,1,2,3,4,5,6, 7,8 and 9 in this number system. The base of decimal number system is 10, because it has only 10 digits.
- Digits: 0,1,2,3,4,5,6,7,8,9
- Symbols :- 10
- Base :- 10
- Deca = 10 $(125)_8$ $(125)_{10}$

$(10)_8$ $(10)_{10}$ $(10)_2$

- 4) **Hexadecimal number system**

  A Hexadecimal number system has sixteen (16) alphanumeric values from **0 to 9** and **A to F**. Every number (value) represents with 0,1,2,3,4,5,6, 7,8,9,A,B,C,D,E and F in this number system. The base of hexadecimal number system is 16, because it has 16 alphanumeric values. Here **A is 10**, **B is 11**, **C is 12**, **D is 13**, **E is 14** and **F is 15**.

- Digits: 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
- Symbols :- 16
- Base :- 16
- Deca = 10 Hexa = 6

  $(10)_{16}$ $(A)_{16}$

Number System Conversions:-

- There are three types of conversion:

1. **Decimal Number System to Other Base (2,8,16)**
   [for example:
   Decimal to Binary Number System
   Decimal to Octal Number System
   Decimal to Hexa Decimal Number System
1. **Other Base to Decimal Number System**
   [for example:
   Binary Number System to Decimal Number System
   Octal Number System to Decimal Number System
   Hexa Decimal Number System to Decimal Number System
1. **Other Base to Other Base**
   [for example:
   Binary Number System to Hexadecimal Number System
   Binary to Octal Number System
   Octal to Hexa Decimal Number System
   Hexa Decimal to Octal Number System
   Octal to Binary Number System
   Hexa Decimal to Binary Number System

Decimal to Binary Number System

Decimal Number System to Other Base

To convert Number system from **Decimal Number System** to **Any Other Base** is quite easy; you have to follow just two steps:

A)

Divide the Number (Decimal Number) by the base of target base system (in which you want to convert the number: Binary (2), octal (8) and Hexadecimal (16)).

B)

Write the remainder from step 1 as a Least Signification Bit (LSB) to Step last as a Most Significant Bit (MSB).

(1) Decimal to Binary number system

- In this conversion , the decimal number is dividing by 2 progressively until 0 is obtained in the quotient. And remainders are arranged in reversed order or bottom to top. This method is called 'double dabble' method. The step is shown bellow.
  - Divide the decimal number by 2. because the 2 is a binary's base /radix.
  - Record the remainder in right side.
  - Divide the quotient of the previous division by the 2.
  - Repeat the above step untill the quotient becomes the less then 2.

- Example:- (Interger)
- Convert $(127)_{10}$ to $(\ ?\ )_2$

2	127	1
2	63	1
2	31	1
2	15	1
2	7	1
2	3	1
	1	1

bottom to top

Then answer is 1111111

- Example:- (Floating Number)
- Convert $(127.5625)_{10}$ to $(\ ?\ )_2$

2	127	1
2	63	1
2	31	1
2	15	1
2	7	1
2	3	1
	1	1

bottom to top

0.5625	*	2	=	1.25	1
0.125	*	2	=	0.25	0
0.25	*	2	=	0.5	0
0.5	*	2	=	1.0	1

top to bottom

The answer is 1111111.1001

(2) Decimal to Octal number system

- In this conversion , the decimal number is dividing by 8 progressively until 0 is obtained in the quotient. And remainders are arranged in reversed order or bottom to top. This method is called ‘Octal dabble’ method. The step is shown bellow.
  - Divide the decimal number by 8. because the 8 is a binary’s base /radix.
  - Record the remainder in right side.
  - Divide the quotient of the previous division by the 8.
  - Repeat the above step until the quotient becomes the less then 8.

- Example:- (Interger)
- Convert $(127)_{10}$ to $(\ ?\ )_8$

8	127	7
8	15	7
8	8	1

bottom to top

Then answer is 177

- Example:- (Floating Number)
- Convert $(127.685)_{10}$ to $(?)_2$

8	127	7
8	15	7
8	1	1

bottom to top

0.685	*	8	=	5.48	5
0.48	*	8	=	3.84	3
0.84	*	8	=	6.72	6
0.72	*	8	=	5.76	5
0.76	*	8	=	6.08	6

top to bottom

The answer is 177.53656

(3)Decimal to Hexa -decimal number system

- In this conversion , the decimal number is dividing by 16 progressively until 0 is obtained in the quotient. And remainders are arranged in reversed order or bottom to top. This method is called ‘Hexa dabble’ method. The step is shown bellow.
  - Divide the decimal number by 16. because the 16 is a binary’s base /radix.
  - Record the remainder in right side.
  - Divide the quotient of the previous division by the 16.
  - Repeat the above step until the quotient becomes the less then 16.

- Example:- (Interger)
- Convert $(127)_{10}$ to $(\ ?\ )_8$

16	127	15(F)
16	7	7

bottom to top

Then answer is 7F

- Example:- (Floating Number)
- Convert $(127.685)_{10}$ to $(\ ? \ )_{16}$

16	127	15(F)
16	7	7

bottom to top

$0.685 * 16 = 10.96$ 10(A)

$0.96 * 16 = 15.36$ 15(F)

$0.36 * 16 = 5.76$ 5

$0.76 * 16 = 12.16$ 12 (C)

$0.16 * 16 = 2.56$ 2

top to bottom

The answer is 7F.AF5C2

- **(4)Binary to Decimal Number System**

  For the evaluation of the binary number we must know the relative position of each bit. For this purpose we count the position of binary bits from the right side as shown below:

Suppose binary number = 1 0 1 0 1

Bit position n^{th} = 5 4 3 2 1

The decimal value of n^{th} bit = **n^{th} bit * 2^{n-1}**

The values of all bits are added to give the values of the binary number.

- Example:-
- $(11101)_2$ to $(\ ?\ )_{10}$

1 1 1 0 1 binary number

5 4 3 2 1 n^{th} position

Step:- Decimal number = n^{th} bit * 2^{n-1}

$$= 1 * 2^{5-1}$$

$$= 1 * 2^4$$

$$= 1 * 16$$

$$= 16$$

$= 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0$

$= 1 * 16 + 1 * 8 + 1 * 4 + 0 * 2 + 1 * 1$

$= 16 + 8 + 4 + 0 + 1$

$= 29$

- Example:-
- $(11101.101)_2$ to $(\ ?\ )_{10}$

1	1	1	0	1 .	1	0	1	binary number
5	4	3	2	1	0	-1	-2	n^{th} position

Step:- Decimal number = n^{th} bit * 2^{n-1}

$$= 1 * 2^{5-1}$$

$$= 1 * 2^4$$

$$= 1 * 16$$

$$= 16$$

$= 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 + 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3}$

$= 1 * 16 + 1 * 8 + 1 * 4 + 0 * 2 + 1 * 1 + 1 * ½ + 0 * ¼ + 1 * 1/8$

$= 16 + 8 + 4 + 0 + 1 + 0.5 + 0 + 0.125$

$= 29.625$

- **Repeat Example:-**
- $(11101.101)_2$ to $(?)_{10}$

1 1 1 0 1 . 1 0 1 binary number

4 3 2 1 0 -1 -2 -3 n^{th} position (modify)

Step:- Decimal number = n^{th} bit * $2^{nth\ position}$

$$= 1 * 2^4$$

$$= 1 * 16$$

$$= 16$$

$= 1 * 2^4 + 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 + 1 * 2^{-1} + 0 * 2^{-2} + 1 * 2^{-3}$

$= 1 * 16 + 1 * 8 + 1 * 4 + 0 * 2 + 1 * 1 + 1 * ½ + 0 * ¼ + 1 * 1/8$

$= 16 + 8 + 4 + 0 + 1 + 0.5 + 0 + 0.125$

$= 29.625$

- **(5)Octal to Decimal Number System**

  For the evaluation of the binary number we must know the relative position of each bit. For this purpose we count the position of binary bits from the right side as shown below:

Suppose octal number = 5 6 7 2 3

Bit position n^{th} = 5 4 3 2 1

The decimal value of n^{th} bit = $\mathbf{n^{th}}$ **bit** * $\mathbf{8^{n-1}}$

The values of all bits are added to give the values of the binary number.

- Example:-
- $(564)_8$ to $(\ ?\ )_{10}$

5 6 4 binary number

3 2 1 n^{th} position

Step:- Decimal number = n^{th} bit * 8^{n-1}

$$= 5 * 8^{3-1}$$

$$= 5 * 8^2$$

$$= 5 * 64$$

$$= 320$$

$= 5 * 8^2 + 6 * 8^1 + 4 * 8^0$

$= 5 * 64 + 6 * 8 + 4 * 1$

$= 320 + 48 + 4$

$= 372$

- **Example:-**
- $(564.25)_8$ to $(?)_{10}$

5 6 4 . 2 5 binary number

3 2 1 0 -1 n^{th} position

Step:- Decimal number = n^{th} bit * 8^{n-1}

$= 5 * 8^{3-1}$

$= 5 * 8^2$

$= 5 * 64$

$= 320$

$= 5 * 8^2 + 6 * 8^1 + 4 * 8^0 + 2 * 8^{-1} + 5 * 8^{-2}$

$= 5 * 64 + 6 * 8 + 4 * 1 + 2 * 1/8 + 5 * 1/64$

$= 320 + 48 + 4 + 2 * 0.125 + 5 * 0.015625$

$= 372 + 0.250 + 0.078125$

$= 372.328125$

- **Example (Repeat):-**
- $(564.25)_8$ to $(\ ? \ )_{10}$

5 6 4 . 2 5 binary number

2 1 0 -1 -2 n^{th} position

Step:- Decimal number = n^{th} bit * $8^{nth\ position}$

$= 5 * 8^2$

$= 5 * 64$

$= 320$

$= 5 * 8^2 + 6 * 8^1 + 4 * 8^0 + 2 * 8^{-1} + 5 * 8^{-2}$

$= 5 * 64 + 6 * 8 + 4 * 1 + 2 * 1/8 + 5 * 1/64$

$= 320 + 48 + 4 + 2 * 0.125 + 5 * 0.015625$

$= 372 + 0.250 + 0.078125$

$= 372.328125$

- **(6)Hexa Decimal to Decimal Number System**

  For the evaluation of the binary number we must know the relative position of each bit. For this purpose we count the position of binary bits from the right side as shown below:

Suppose Hexa-decimal number = A 6 F

Bit position n^{th} = 3 2 1

The decimal value of n^{th} bit = **n^{th} bit * 16^{n-1}**

The values of all bits are added to give the values of the binary number.

- Example:-
- $(A6F)_{16}$ to $(\ ?\ )_{10}$

A 6 F binary number

3 2 1 n^{th} position

Step:- Decimal number = n^{th} bit * 16^{n-1}

$= A * 16^{3-1}$

$= A * 16^{2}$

$= 10 * 256$

$= 2560$

$= A * 16^2 + 6 * 16^1 + F * 16^0$

$= 10 * 256 + 6 * 16 + 15 * 1$

$= 2560 + 96 + 15$

$= 2671$

- **Example:-**
- $(A6F.B5)_{16}$ to $(\ ?\ )_{10}$

A 6 F . B 5 binary number

3 2 1 0 -1 n^{th} position

Step:- Decimal number = n^{th} bit * 16^{n-1}

$= A * 16^{3-1}$

$= A * 16^2$

$= 10 * 256$

$= 2560$

$= A * 16^2 + 6 * 16^1 + F * 16^0 + B * 16^{-1} + 5 * 16^{-2}$

$= 10 * 256 + 6 * 16 + 15 * 1 + 11 * 1/16 + 5 * 1/256$

$= 2560 + 96 + 15 + 11/16 + 5/256$

$= 2671 + 0.6875 + 0.0195313$

$= 2671.707$

- **Example(Repeat):-**
- $(A6F.B5)_{16}$ to $(?)_{10}$

A 6 F . B 5 binary number

2 1 0 -1 -2 n^{th} position

Step:- Decimal number = n^{th} bit * $16^{nth\ position}$

$= A * 16^{2}$

$= A * 16^{2}$

$= 10 * 256$

$= 2560$

$= A * 16^2 + 6 * 16^1 + F * 16^0 + B * 16^{-1} + 5 * 16^{-2}$

$= 10 * 256 + 6 * 16 + 15 * 1 + 11 * 1/16 + 5 * 1/256$

$= 2560 + 96 + 15 + 11/16 + 5/256$

$= 2671 + 0.6875 + 0.0195313$

$= 2671.707$

- **(7)Binary number to Octal Number System:-**

The conversion of binary number to octal number is very easy. The base of the octal number is 8. as $8 = 2^3$, for binary to octal conversion groups of 3 bits are formed in the binary number from the right. After forming the groups each 3 bits binary groups is replaced by its octal equivalent. The following octal table is used for this conversion.

Octal number	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Example:-

$(1010110101)_2$ to $(\ ? \ )_8$

First groups of 3 bits are formed in the binary number from the right

Group of 3 bits = 1 010 110 101

Right to left

Octal number = 1 2 6 5 **(see in octal table)**

The answer is 1265

- Example(float):-

$(1010110.10110)_2$ to $(\ ? \ )_8$

First groups of 3 bits are formed in the binary number from right before the decimal point and after the decimal point from left.

Group of 3 bits = 1 010 110 . 101 10

Right to left   left to right

Octal number = 1 2 6 5 2

The answer is $(126.52)_8$

- **(8)Binary number to Hexa decimal Number System:-**

The conversion of binary number to hexa decimal number is very easy. The base of the hexa decimal number is 16. as $16 = 2^4$, for binary to hexa decimal conversion groups of 4 bits are formed in the binary number from the right. After forming the groups each 4 bits binary groups is replaced by its hexa decimal equivalent. The following hexa decimal table is used for this conversion.

Hexa decimal number	Binary	Hexa decimal number	Binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Example:-

$(1010110101)_2$ to $(\ ?\ )_{16}$

First groups of 4 bits are formed in the binary number from the right

Group of 3 bits = 10 1011 0101

Right to left

Hexa Dec. number = 2 B 5 **(see in hexa table)**

The answer is $(2B5)_{16}$

- Example(float):-

$(1010110.101101)_2$ to $(\ ? \ )_{16}$

First groups of 4 bits are formed in the binary number from right before the decimal point and after the decimal point from left.

Group of 3 bits = 101 0110 . 1011 0100

Right to left   left to right

Hexa number = 5 6 B 4

The answer is $(56.B4)_{16}$

- **(9)Hexa decimal to binary number system.**

  In this hexadecimal number to a binary number conversion, each digit of the given hexadecimal number is converted to its 4 –bit binary number. In this conversion we are use hexadecimal table. we write 4 bit binary number equivalent to hexadecimal number using hexadecimal table.

example.:- $(1265\,.\,A5)_{16}$ to $(\ ?\ )_2$

1	2	6	5	.	A	5
0001	0010	0110	0101		1010	0101

The answer is $(0001001001100101.10100101)_2$

- **(1)Octal to binary number system.**

  In this Octal number to a binary number conversion, each digit of the given Octal number is converted to its 3 –bit binary number. In this conversion we are use octal table. we write 3 bit binary number equivalent to Octal number using Octal table.

example.:- $(1265\ .\ 25)_8$ to $(\ ?\ )_2$

1	2	6	5	.	2	5
↓	↓	↓	↓		↓	↓
001	010	110	101		010	101

The answer is $(001010110101.010101)_2$

- **(11) octal to hexadecimal number system:-**

In this conversion, first we are converting the octal number to binary number then this binary number to Hexadecimal number.

EXAMPLE:- $(1265.71)_8$ TO $(\ ? \ )_{16}$

First we are converting octal to binary number

1	2	6	5	.	7	1	Hexadecimal number
001	010	110	101	.	111	001	Binary Number

The binary number is 001010110101.111001

Then binary to Hexadecimal number

Groups of 4 bit	=	0010	1011	0101	.	1110	01
Hexadecimal values	=	2	B	5	.	E	4

The answer is $(2B5.E4)_{16}$

- **(12) hexadecimal to octal number system:-**

In this conversion, first we are converting the hexadecimal number to binary number then this binary number to octal number.

EXAMPLE:- $(A2C.A2)_{16}$ TO $(\ ?\ )_8$

First we are converting hexadecimal to binary number

A	2	C	.	A	2	Hexadecimal number
1010	0010	1100	.	1010	0010	Binary Number

The binary number is 101000101100 .10100010

Then binary to octal number

Groups of 3 bit =	101	000	101	100	.	101	000	10
Octal values =	5	0	5	4	.	5	0	4

The answer is $(5054.504)_8$

Binary Addition

- Rules for binary addition are given below in table:

	Rules for binary addition			
	Number		Sum A + B	Carry
	A	B		
	0	0	0	0
	0	1	1	0
	1	0	1	0
	1	1	0	1
1	1	1	1	1

- Example:- Add 101011 and 110110

Carry	1	1	1	1	1		
		1	0	1	0	1	1
	+	1	1	0	1	1	0
	1	1	0	0	0	0	1

- Example:- Add 100111 and 10011

Carry				1	1	1	
		1	0	0	1	1	1
	+		1	0	0	1	1
		1	1	1	0	1	0

Binary Subtraction:-

- Rules for binary addition are given below in table:

Number		Subtraction A - B	Borrow
A	B		
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Note: Borrow up to previous 1

- Example:- Subtract 11010 from 1101011

Borrow		1					
	1	1	0	1	0	1	1
-			1	1	0	1	0
	1	0	1	0	0	0	1

- Example:- Subtract 100.0011 from 110.1100

Borrow					1	1		
	1	1	0	.	1	1	0	0
-	1	0	0	.	0	0	1	1
	0	1	0	.	1	0	0	1

Complement:-

- The complement is used to represent the negative of a binary, octal , decimal and hexadecimal number. In this section we shall study eight types of complement, these are listed below.
- 1's complement
- 2's complement
- 7's complement
- 8's complement
- 9's complement
- 10's complement
- 15's complement
- 16's complement

The 1's. & 2's , 7's & 8's , 9's & 10's , 15's & 16's are respectively used for binary, octal , decimal , or hexadecimal.

- **1’s complement:-**

  The one complement of a number is found by the two methods. These two methods are discussed in below.

Method-1:-

- The one complement of a number is found by changing all the 0’s to 1’s and all the 1’s to 0’s.
- This complemented value represents the negative of the original number.
- **Example**:- find 1’s complement of 1011011

original number	1011011
1’s complement	0100100

- **Method-2:-**

- The 1's complement of a binary number is obtained by subtracting each bit of the number from 1. that's why we are called this system is 1's complement.
- Example:- find 1's complement from of 101101101

$$
\begin{array}{rccccccccc}
 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
- & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\
\hline
 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{array}
\quad \text{1's complement}
$$

- **2’s complement:-**

  The 2’s complement of a number is found by the two methods. These two methods are discussed in below:

Method -1 :-

- First number is converted into 1’s complement then add 1 in 1’s complement result. Then the result is your 2’s complement.
- The 2’s complement of binary number = 1’s complement + 1
- Example:- 1011011
- 2’s complement = 1’s complement + 1
- Here 1’s complement = 0100100
- Now add 1 in 1’s complement 0100100 + 1
- = 0100101 is 2’s complement

- **Method-2:-**

- Starting at the least significant bit and working from right to left , copy each bit down up to end including the first 1 bit encountered then complement the remaining bits.
- Example:- find 2’s complement of 10101001101000

- 10101001101000 original number
- 01010110011000 2’s complement
- copy 1 bit encountered