1. Consider the convergent binocular imaging system shown in Fig. 1. The cameras and all the points are in the $y = 0$ plane. The image planes are perpendicular to their respective camera axes. Find the disparity corresponding to the point P. (Hint: The perpendicular distance between any pint (x_0, z_0) and a line given by $Ax + Bz + C = 0$ is $(Ax_0 + Bz_0 + C)/\sqrt{A^2 + B^2}$. (15 points)

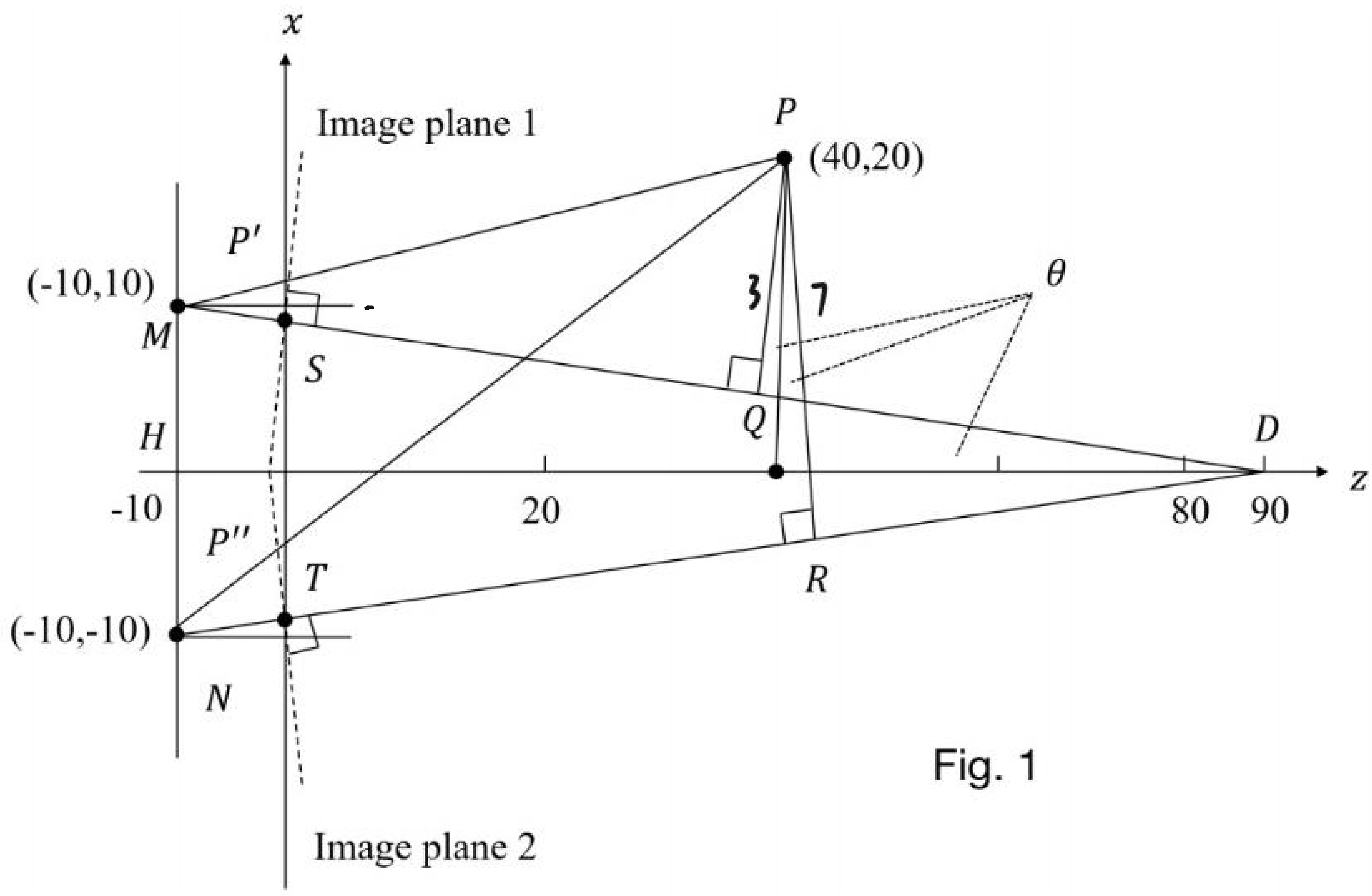

Fig. 1

line MD: $\begin{cases} \frac{A}{C}10 - \frac{B}{C}10 + 1 = 0 \\ 0 + \frac{B}{C}90 + 1 = 0 \end{cases} \Rightarrow \begin{cases} \frac{A}{C} = -\frac{1}{90} - \frac{9}{90} = -\frac{1}{9} \\ \frac{B}{C} = -\frac{1}{90} \end{cases}$ $\therefore -\frac{1}{9}x - \frac{1}{90}z + 1 = 0$. MD

Line ND: $\begin{cases} -\frac{A}{C}10 - \frac{B}{C}10 + 1 = 0 \\ 0 + \frac{B}{C}90 + 1 = 0 \end{cases} \Rightarrow \begin{cases} \frac{A}{C} = \frac{1}{10} + \frac{1}{90} = \frac{1}{9} \\ \frac{B}{C} = -\frac{1}{90} \end{cases}$ $\therefore \frac{1}{9}x - \frac{1}{90}z + 1 = 0$. ND

so $d_{P-MD} = \dfrac{-\frac{1}{9}\times 20 - \frac{1}{90}\times 40 + 1}{\sqrt{\frac{1}{9^2} + \frac{1}{90^2}}} = \dfrac{-\frac{2}{3} + 1}{\frac{1}{9}} = 3$

$d_{P-ND} = \dfrac{\frac{1}{9}\times 20 - \frac{1}{90}\times 40 + 1}{\sqrt{\frac{1}{9^2} + \frac{1}{90^2}}} = \dfrac{\frac{7}{9}}{\frac{1}{9}} = 7$

assume $Q(z_1, x_1)$, $R(z_2, x_2)$

$\begin{cases} -\frac{1}{9}x_1 - \frac{1}{90}z_1 + 1 = 0 \\ \frac{x_1 - 20}{z_1 - 40} = 10 \end{cases}$ $\begin{cases} \frac{1}{9}x_2 - \frac{1}{90}z_2 + 1 = 0 \\ \frac{x_2 - 20}{z_2 - 40} = -10 \end{cases}$

solve $\begin{cases} x_1 = \frac{520}{101} = 5.15 \\ z_1 = 38.5 \end{cases}$ $\begin{cases} x_2 = -\frac{480}{101} = -4.75 \\ z_2 = 42.5 \end{cases}$

Because the scope of MD and ND is 0.1 which very small,

so approximately, $P'P'' \simeq P''T - P'S$

$\therefore \frac{P'S}{PQ} = \frac{MS}{MQ} = \frac{10}{10 + z_1}$, $P'S = \frac{10}{10 + 38.5} \times 3 = 0.62$

$\frac{P''T}{PR} = \frac{NT}{NR} = \frac{10}{10 + z_2}$, $P''T = \frac{10}{10 + 42.5} \times 7 \simeq 1.33$

$\therefore P'P'' = 1.33 - 0.62 = 0.71$

2. Consider the binocular stereo imaging system shown in Fig. 2, find the disparity, $x_d = |x_1 - x_2|$, for the point P located at (10, 20, 10). (15 points)

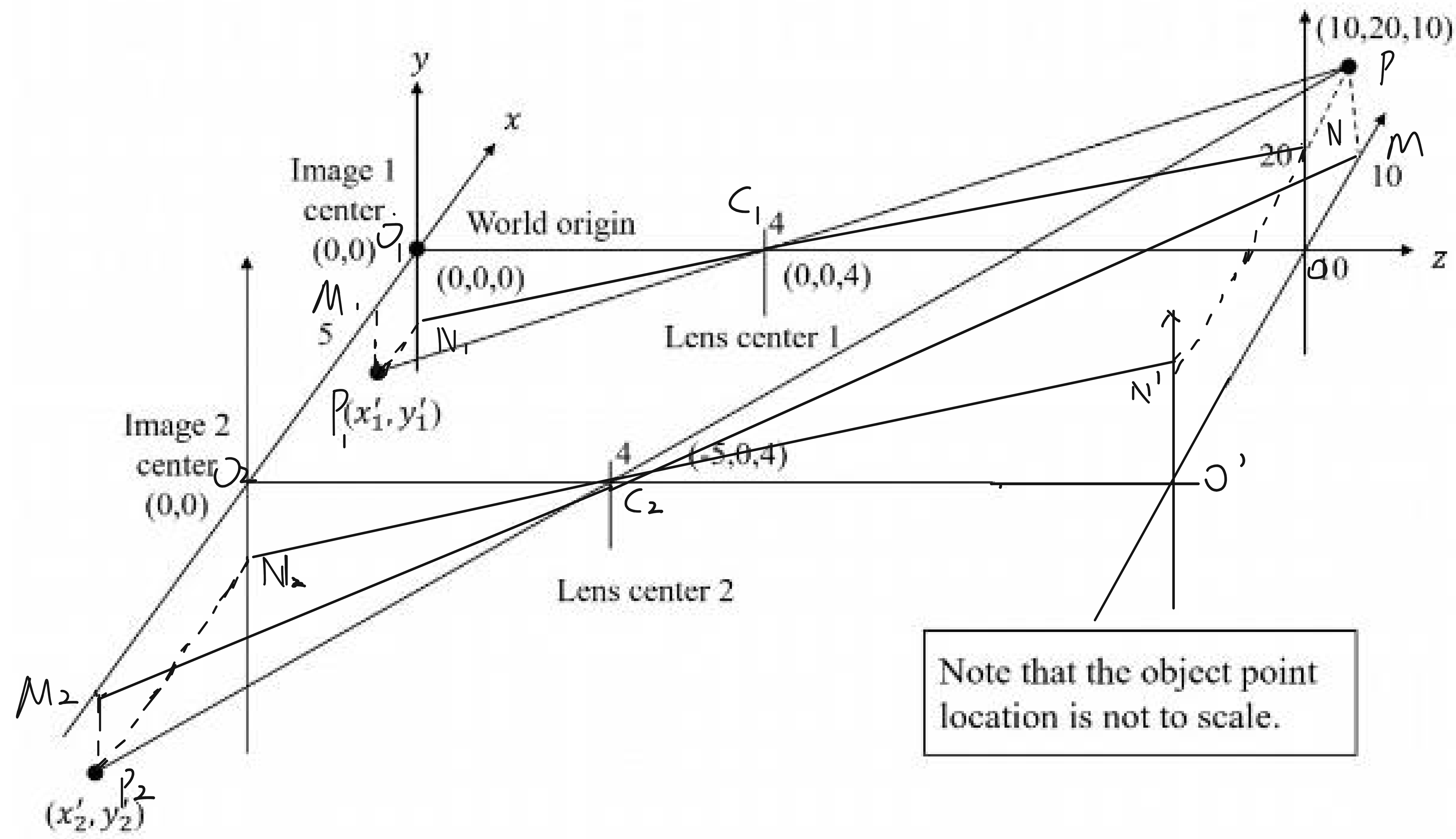

Fig. 2

We can get: $\triangle O_1C_1M_1 \cong \triangle OC_1M$ $\quad \triangle O_1C_1N_1 \cong \triangle OC_1N$

$\triangle O_2C_2M_2 \cong \triangle O'C_2M$ $\quad \triangle O_2C_2N_2 \cong \triangle O'C_2N'$

$$\therefore \begin{cases} \dfrac{O_2N_2}{O'N'} = \dfrac{O_2C_2}{C_2O'} \\ \dfrac{O_2M_2}{O'M} = \dfrac{O_2C_2}{C_2O'} \end{cases} \qquad \begin{cases} \dfrac{O_1N_1}{ON} = \dfrac{O_1C_1}{C_1O} \\ \dfrac{O_1M_1}{OM} = \dfrac{O_1C_1}{C_1O} \end{cases}$$

solve $\begin{cases} x_2' = 15 \times \frac{4}{6} = 10 \\ y_2' = 20 \times \frac{4}{6} = \frac{40}{3} \end{cases} \qquad \begin{cases} x_1' = 10 \times \frac{4}{6} = \frac{20}{3} \\ y_1' = 20 \times \frac{4}{6} = \frac{40}{3} \end{cases}$

$\therefore X_d = |X_2' - X_1'| = \frac{10}{3}$

3. Use the definition of disparity to characterize the accuracy of stereo reconstruction as a function of baseline and depth. (20 points)

Use the figure in Problem 2 as the example:

$O_1C_1 = O_2C_2 = f$ (focal length)

$O_1O_2 = OO' = b$ (baseline)

$C_1O = C_2O' = d$ (depth)

$$\begin{cases} \dfrac{O_1C_1}{C_1O} = \dfrac{O_1M_1}{OM} \\ \dfrac{O_2C_2}{C_2O'} = \dfrac{O_2M_2}{O'M} \end{cases}$$

$\therefore O_2M_2 - O_1M_1 = O'M \cdot \frac{f}{d} - OM \cdot \frac{f}{d} = OO' \frac{f}{d} = \frac{bf}{d}$

Let Δ = disparity $= O_2M_2 - O_1M_1 = \frac{bf}{d}$. $(\Delta)_{min} = 1$, while f and b are pre-set.

so $(d)_{max} = bf$ $\quad (d)_{min} = \frac{bf}{(\Delta)_{max}}$, $(\Delta)_{max}$ depends on the extent of pixel plane in the x direction

4. E is a 3×3 matrix and can be expressed as $t^{\wedge}R$

$t=[t_1\ t_2\ t_3]^T$ $t^{\wedge}=T=\begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$ is a skew-symmetrical matrix

$\therefore$ $t^{\wedge}$ can be decomposed as $Q^T\begin{bmatrix} 0 & \phi & 0 \\ -\phi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}Q$, Q is unitary matrix

$\therefore$ $E^TE = R^T(t^{\wedge})^T(t^{\wedge})R = (QR)^T\begin{bmatrix} \phi^2 & 0 & 0 \\ 0 & \phi^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}(QR)$, while both Q and R are unitary matrix

so the eigenvalues of E^TE is 0 and other two equal positive number

Q.E.D

5. 见另外PDF文件"25.pdf"