第 3 章作业参考答案

P118/1(1)：

基变量	x_1	x_2	x_3	x_4	右端项
x_3	1	4	1	0	80
x_4	2	3	0	1	90
检验数	9	16	0	0	0

基变量	x_1	x_2	x_3	x_4	右端项
x_2	1/4	1	1/4	0	20
x_4	5/2	0	-3/4	1	30
检验数	5	0	-4	0	-320

基变量	x_1	x_2	x_3	x_4	右端项
x_2	0	1	2/5	-1/5	14
x_1	1	0	-3/5	4/5	24
检验数	0	0	-1	-4	-440

$\boldsymbol{x}^* = (24,14,0,0)^T$， $z^* = -440$。

P118/1(2)：化为极小化问题。

基变量	x_1	x_2	x_3	x_4	右端项
x_3	2	3	1	0	6
x_4	-1	1	0	1	1
检验数	1	3	0	0	0

基变量	x_1	x_2	x_3	x_4	右端项
x_3	5	0	1	-3	3
x_2	-1	1	0	1	1
检验数	4	0	0	-3	-3

基变量	x_1	x_2	x_3	x_4	右端项
x_1	1	0	1/5	-3/5	3/5
x_2	0	1	1/5	2/5	8/5
检验数	0	0	-4/5	-3/5	-27/5

$\boldsymbol{x}^* = (3/5,8/5,0,0)^T$， $z^* = 27/5$。

P119/1(4)：

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	

x_5	1	1	1	0	1	0	0	4
x_6	4	-1	1	2	0	1	0	6
x_7	-1	1	2	3	0	0	1	12
	-3	5	2	1	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_2	1	1	1	0	1	0	0	4
x_6	5	0	2	2	1	1	0	10
x_7	-2	0	1	3	-1	0	1	8
	-8	0	-3	1	-5	0	0	-20

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_2	1	1	1	0	1	0	0	4
x_6	19/3	0	4/3	0	5/3	1	-2/3	14/3
x_4	-2/3	0	1/3	1	-1/3	0	1/3	8/3
	-22/3	0	-10/33	0	-14/3	0	-1/3	-68/3

$\boldsymbol{x}^* = (0,4,0,8/3,0,14/3,0)^T$， $z^* = -68/3$。

P119/2(2)：

	x_1	x_2	x_3	x_4	x_5	
x_3	1	1	1	0	0	5
x_4	-1	1	0	1	0	0
x_5	6	2	0	0	1	21
	2	1	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	
x_3	0	2/3	1	0	-1/6	3/2
x_4	0	4/3	0	1	1/6	7/2
x_1	1	1/3	0	0	1/6	7/2
	0	1/3	0	0	-1/3	-7

	x_1	x_2	x_3	x_4	x_5	
x_2	0	1	3/2	0	-1/4	9/4
x_4	0	0	-2	1	1/2	1/2

x_1	1	0	-1/2	0	1/4	11/4
	0	0	-1/2	0	-1/4	-31/4

$\boldsymbol{x}^* = (11/4, 9/4, 0, 1/2, 0)^T$， $z^* = 31/4$。

P119/2(4)：

		1 x_1	-3 x_2	1 x_3	x_4	x_5	y	
1	x_3	2	-1	1	0	0	0	8
M	y	2	1	0	-1	0	1	2
0	x_5	1	2	0	0	1	0	10
		2M+1	M+2	0	-M	0	0	2M+8

		1 x_1	-3 x_2	1 x_3	x_4	x_5	y	
1	x_3	0	-2	1	1	0	-1	6
1	x_1	1	1/2	0	-1/2	0	1/2	1
0	x_5	0	3/2	0	1/2	1	-1/2	9
		0	3/2	0	1/2	0	-M-1/2	7

	x_1	x_2	x_3	x_4	x_5	
x_3	4	0	1	-1	0	10
x_2	2	1	0	-1	0	2
x_5	-3	0	0	2	1	6
	-3	0	0	2	0	4

	x_1	x_2	x_3	x_4	x_5	
x_3	5/2	0	1	0	1/2	12
x_2	1/2	1	0	0	1/2	5
x_4	-3/2	0	0	1	1/2	3
	0	0	0	0	-1	-2

$\boldsymbol{x}^* = (0, 5, 12, 3, 0)^T$， $z^* = -2$。

P119/2(8)：

	2 x_1	-3 x_2	0 x_3	0 x_4	0 x_5	M y_1	M y_2	

M y_1	2	-1	-1	-1	0	1	0	3
M y_2	1	-1	1	0	-1	0	1	2
	3M-2	-2M+3	0	-M	-M	0	0	5M

	2 x_1	-3 x_2	0 x_3	0 x_4	0 x_5	M y_1	M y_2	
2 x_1	1	-1/2	-1/2	-1/2	0	1/2	0	3/2
M y_2	0	-1/2	3/2	1/2	-1	-1/2	1	1/2
	0	-1/2M+2	2/3M-1	1/2M-1	-M	-3/2M+1	0	1/2M+3

	2 x_1	-3 x_2	0 x_3	0 x_4	0 x_5	M y_1	M y_2	
2 x_1	1	-2/3	0	-1/3	-1/3	1/3	1/3	5/3
0 x_3	0	-1/3	1	1/3	-2/3	-1/3	2/3	1/3
	0	5/3	0	-2/3	-2/3	2/3-M	2/3-M	10/3

$\boldsymbol{x}^* = (5/3, 0, 1/3)^T$， $z^* = 10/3$。

P120/6：

令 $\boldsymbol{d} = \begin{pmatrix} \boldsymbol{d}_B \\ \boldsymbol{d}_N \end{pmatrix}$， $\boldsymbol{d}_B = -\boldsymbol{y}_j = -B^{-1}\boldsymbol{p}_j$， $d_j = 1, d_k = 0, k \in I_N \setminus \{j\}$，则

$$A\boldsymbol{d} = (B\ N)\begin{pmatrix} \boldsymbol{d}_B \\ \boldsymbol{d}_N \end{pmatrix} = B\boldsymbol{d}_B + N\boldsymbol{d}_N = -\boldsymbol{p}_j + \boldsymbol{p}_j = 0,\ \ \boldsymbol{d} \geq 0,$$

因此 $\boldsymbol{d}$ 是可行域的方向。

假设 $\boldsymbol{d}$ 不是可行域的极向，则存在可行域的不同方向 $\boldsymbol{d}^1, \boldsymbol{d}^2 : A\boldsymbol{d}^1 = A\boldsymbol{d}^2 = 0, \boldsymbol{d}^1, \boldsymbol{d}^2 \geq 0$ 和 $a_1, a_2 > 0$，使 $\boldsymbol{d} = a_1\boldsymbol{d}^1 + a_2\boldsymbol{d}^2$，则

$$0 = d_k = a_1 d_k^1 + a_2 d_k^2 \Rightarrow d_k^1 = d_k^2 = 0, \forall k \in I_N \setminus \{j\}$$

$$0 = A\boldsymbol{d}^1 = B\boldsymbol{d}_B^1 + d_j^1\boldsymbol{p}_j \Rightarrow \boldsymbol{d}_B^1 = -d_j^1 B^{-1}\boldsymbol{p}_j = d_j^1\boldsymbol{y}_j$$

因此 $\boldsymbol{d}^1 = d_j^1\boldsymbol{d}$，同理 $\boldsymbol{d}^2 = d_j^2\boldsymbol{d}$，因此 $\boldsymbol{d}^1, \boldsymbol{d}^2$ 同方向，矛盾。因此 $\boldsymbol{d}$ 是可行域的极向。