Modern Control Theory
Spring 2017

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Outline of Today's Lecture

- Administrative Issues
- Introduction of Control Systems(Chapter 1)
- Concept of State Space Modeling

Lecture Time

<table>
<tr><th>Time</th><th>Monday</th><th>Tuesday</th><th>Wednesday</th><th>Thursday</th><th>Friday</th></tr>
<tr><td>8:00 ~ 8:45</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>8:55 ~ 9:40</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>10:00 ~ 10:45</td><td></td><td></td><td></td><td></td><td rowspan="2">Office Hour</td></tr>
<tr><td>10:55 ~ 11:40</td><td></td><td></td><td></td><td></td></tr>
<tr><td>12:00 ~ 12:45</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>12:55 ~ 13:40</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>14:00 ~ 14:45</td><td></td><td></td><td></td><td></td><td rowspan="3">Lecture Time</td></tr>
<tr><td>14:55 ~ 15:40</td><td></td><td></td><td></td><td></td></tr>
<tr><td>16:00 ~ 16:45</td><td></td><td></td><td></td><td></td></tr>
<tr><td>16:55 ~ 17:40</td><td></td><td></td><td></td><td></td><td></td></tr>
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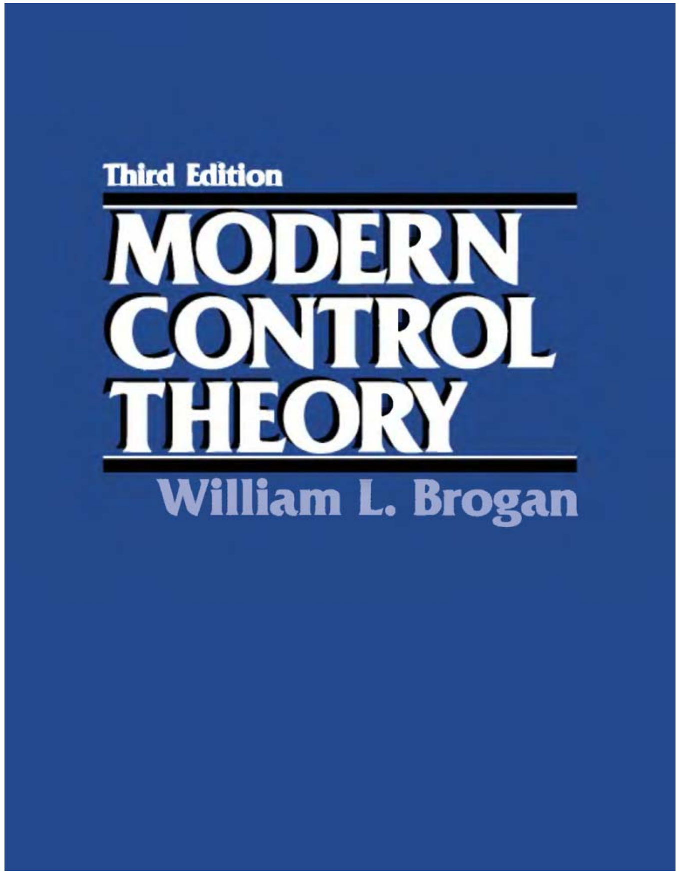
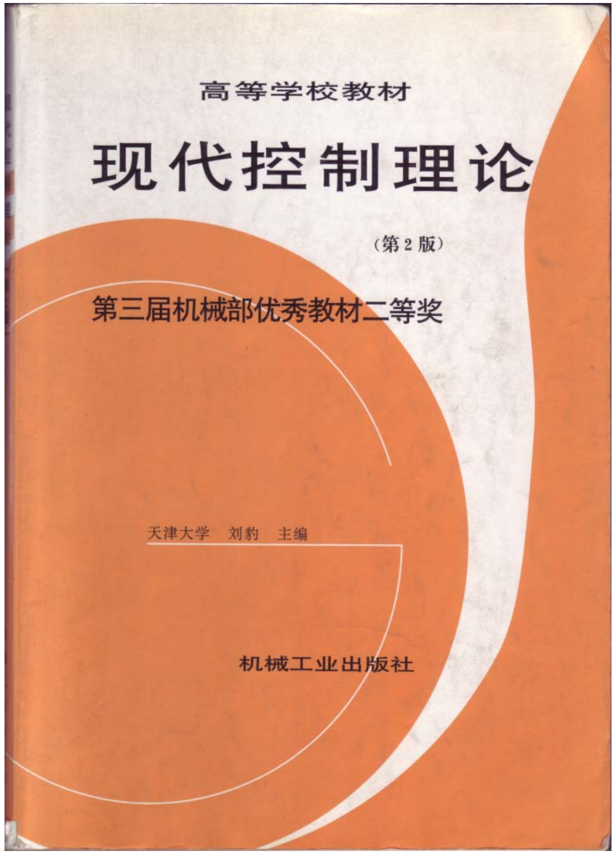
Textbook

- William L. Brogan, Modern Control Theory, 3rd edition, Prentice Hall.



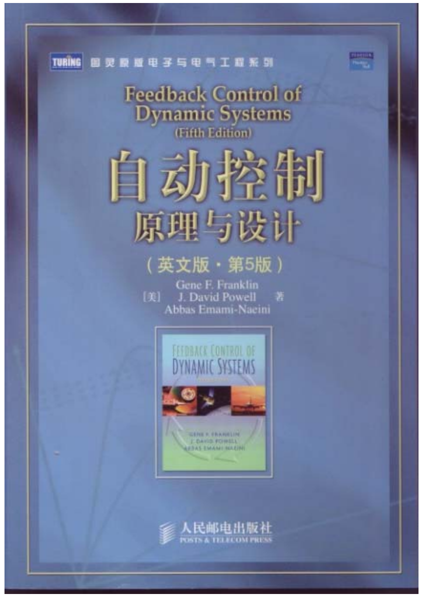
Textbook

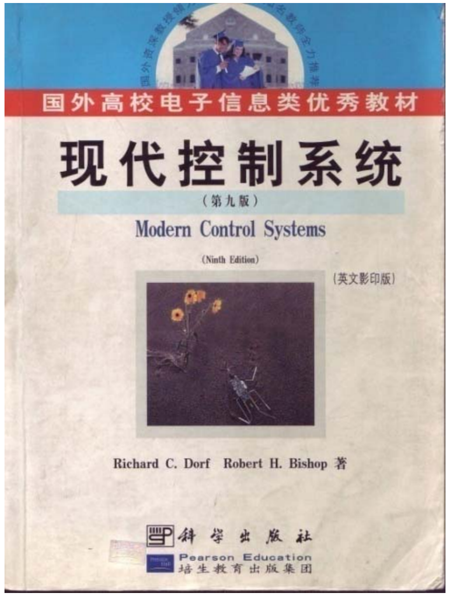
- 天津大学 刘豹 《现代控制理论》第二版（其他版本也可）.



Reference

- Gene F. Franklin, Feedback Control of Dynamic Systems, 5th edition, Prentice Hall.
- Richard C. Dorf, Robert H. Bishop, Modern Control Systems, 11th edition, Prentice Hall
- 王显正,陈正航，王旭咏，控制理论基础。科学出版社，2002。





Course Outline

1. **State space modeling method (9 hours)**
   - The definition of state space modeling
   - SS modeling from Physical systems
   - Analog Computer Implementation
   - The transformation between TF and SS models
   - The SS modeling of discrete system
   - Linearization of nonlinear systems

Course Outline (cont.)

2. The usage and introduction of MATLAB (or LABVIEW) (2 hours)

- the introduction to MATLAB (or LABVIEW)
- the usage of MATLAB(or LABVIEW)

3. The solution for SS model (3 hours)

- Homogeneous solution of LTI system
- Nonhomogeneous solution of LTI system
- The solution of discrete system
- Transformation from SS model to discrete model

Course Outline (cont.)

4. The controllability and observability of LTI system (3 hours)

- The definition of controllability and observability
- The conditions of complete state controllability and observability
- The Principle of Duality
- The controllable and observable canonical form.
- The realization of TF matrix.

5. Stability and Lyapunov method of control systems (3 hours)

- advantages of state space
- analysis of the state equations
- solutions to the state space equation
- Lyapunov stability analysis
- The application of Lyapunov methods for LTI system

Course Outline (cont.)

6. The design of LTI control system (10 hours)

- the basic structure of linear feedback control systems
- pole placement method
- The stabilization technique
- The decoupling technique
- State observer
- Feedback control system based on state observer

7. Optimal Control (6 hours)

- Statement of the Optimal Control
- Dynamic Programming
- Dynamic Programming Approach to Continuous-Time Optimal Control

8. Case study of control system design (12 hours)

- 12 Team works (Presentation)

Administrative Issues

- FTP site:
- ftp://public.sjtu.edu.cn

  Username: zhangweijun

  Password: public

- Download Lecture and homework1

Course Info.

Office Hours: 10:00-12:00pm, Friday

- Teaching Assistant
  - Shaowei Wang(王少伟)

    M:15821858298

    Email:1140803665@qq.com

Course Info.

- **Grading:**
  - 1 Team Work Presentation 50%
  - Homework 10%

  - Final Exam 40%

    Close book with one reference paper (Letter-size or A4-size)

Guideline for Course Studying

- It is not permissible for students to do the following:
  - Copy homework or exam papers from another student.
  - Talk to other students during exams/quizzes

- It is permissible for students to do the following
  - Discuss homework assignments with other students, as long as the final submitted work is directly and individually generated by yourself.

Outline

- Administrative Issues
- Introduction of Control Problem of Mechanical System
- Concept of State Space Modeling

State Space Representation

- Two standard forms for linear dynamic systems
  - Transfer function form (linear only)
  - State space form (linear and nonlinear, multiple inputs & Output, time invariant & time varying)
- A collection of **first order** ordinary differential equations (time domain)
- More suitable to find the solution of differential equation with the use of digital computer.

Basic Concept of State Spaces

- State variables: A minimum set of variables, $x_1(t),\ldots x_n(t)$, such that knowledge of these variables at any time t_0, plus information of the input signals, $u(t)$, $t \geq t_0$, are sufficient to determine the state of the system at any time $t>t_0$.
- State equations: collection of first order ODEs that represent system I/O relationship.

$$\frac{dx_1(t)}{dt} = f_1(x_1,\ldots x_n,u_1,\ldots,u_m)$$

$$\vdots$$

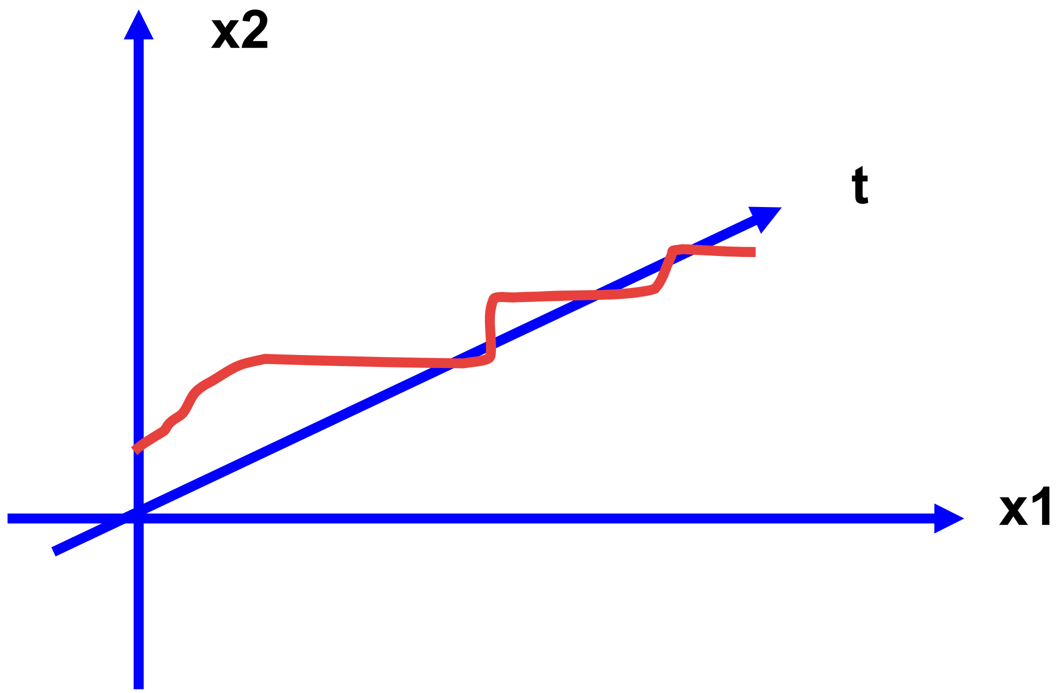
$$\frac{dx_n(t)}{dt} = f_n(x_1,\ldots x_n,u_1,\ldots,u_m)$$

Basic Concept of State Spaces

- **<u>State vector</u>: If n state variables are needed to completely describe the behavior of a given system, then those state variables can be considered the n components of a vector called a state vector.**
- **<u>State Space</u>: The n-dimensional space whose coordinates axes consist of the x1-axis, x2-axis,…, xn-axis is called a state space. Any state can be represented by a point in the state space.**

Basic Concept of State Spaces

- **The Evolution of State Vector Curve**



Construct State Space Representation

- Identify Input(s), output(s) and number of state variables of the system
- Key point:

  # of states = # of energy storage elements

- Identify first order ODE of each state (i.e., to find the time derivative of each state).

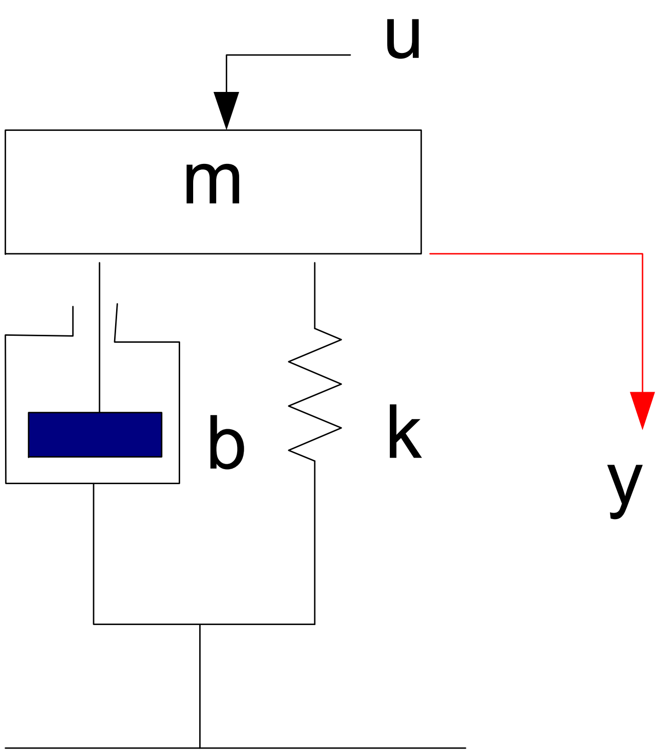
$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

Exceptions

- Two of the same type of elements connected together are treated as one element.

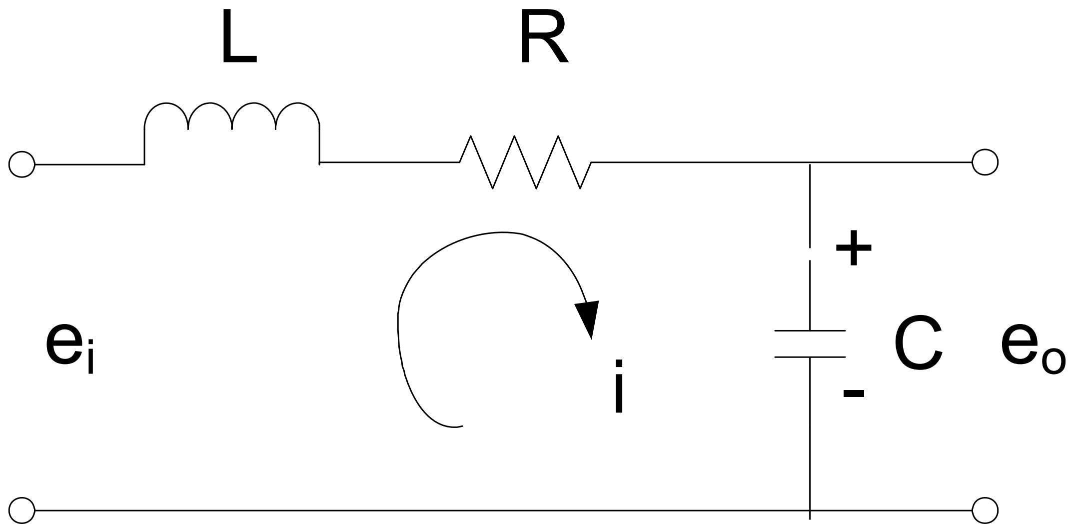
- For example, two massed moving together, two springs in series or in parallel are treated as one element (mass or spring).

Example



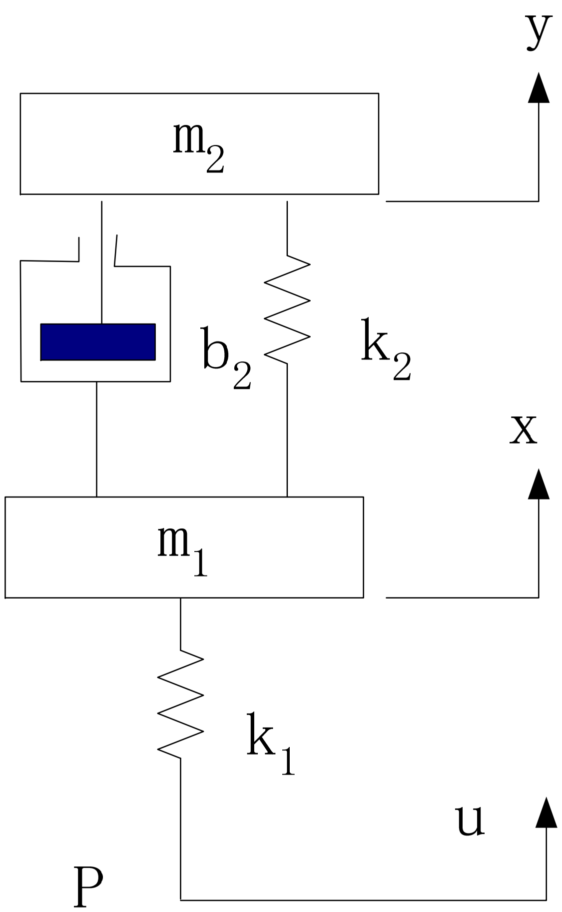
The displacement y is measured from the equilibrium position in the absence of the external force. y is the output and the external force u is the input. Obtain the State Space Representation of the system.

Example



The circuit consists of a resistance R (in ohms) and a capacitance C (in farads). Obtain a state-space representation of the system.

Example



-Consider the front suspension system of a motorcycle. A simplified version is shown in Figure. Point P is the contact point with the ground.

- Input: the vertical displacement u
- Output: displacement y

$$x_1 = x, x_2 = \dot{x}, x_3 = y, x_4 = \dot{y}$$

State Space Modeling

State Space equations

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + ... + b_{1r}u_r$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n + b_{21}u_1 + b_{22}u_2 + ... + b_{2r}u_r$$

$$...$$

$$\dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n + b_{n1}u_1 + b_{n2}u_2 + ... + b_{nr}u_r$$

Output equations

$$y_1 = c_{11}x_1 + c_{12}x_2 + ... + c_{1n}x_n + d_{11}u_1 + d_{12}u_2 + ... + d_{1r}u_r$$

$$y_m = a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n + d_{m1}u_1 + d_{m2}u_2 + ... + d_{mr}u_r$$

State Space Modeling

Matrix Description

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ b_{21} & b_{22} & \cdots & b_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nr} \end{bmatrix}$$

State Space Modeling

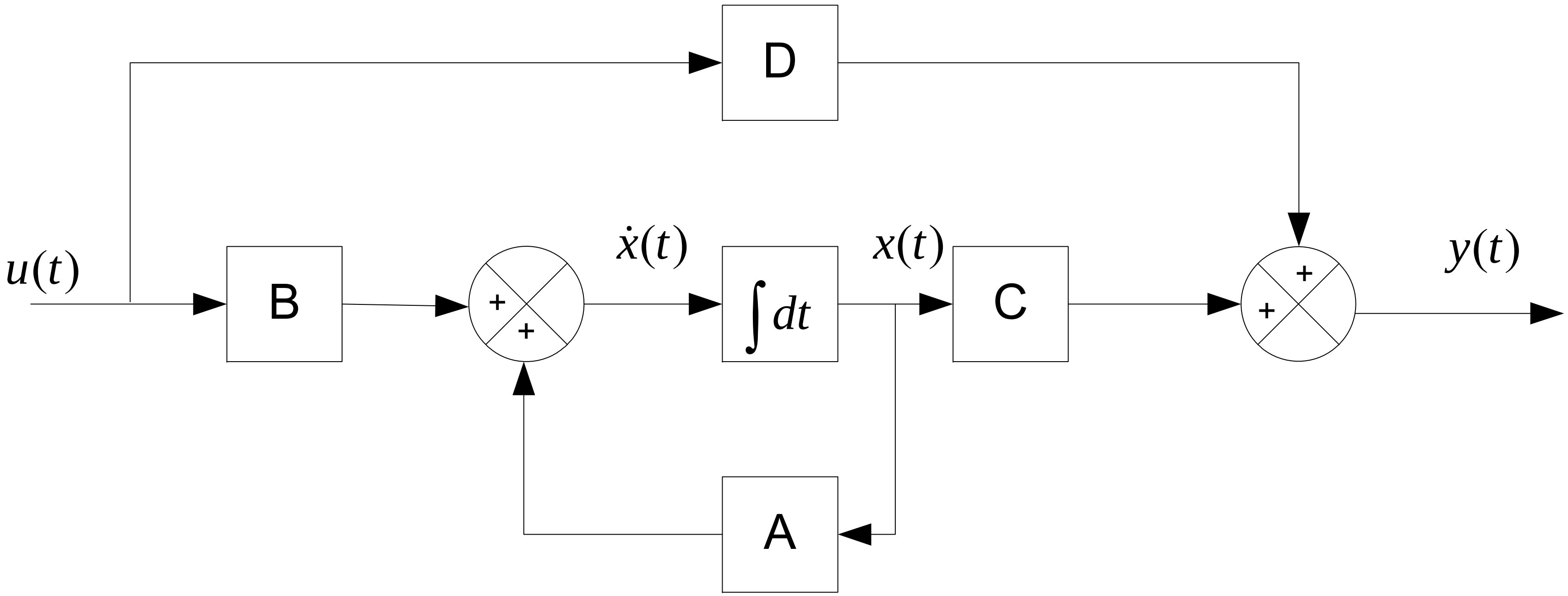
Matrix Description

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1r} \\ d_{21} & d_{22} & \cdots & d_{2r} \\ \vdots & \vdots & \vdots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mr} \end{bmatrix}$$

Block Diagram of State Space equations



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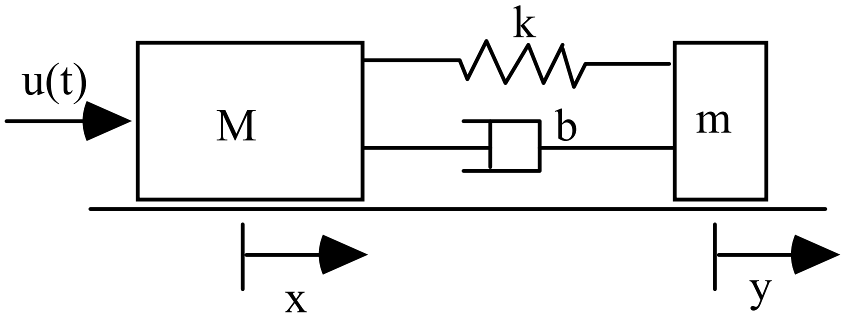


$$M\ddot{x} + b(\dot{x} - \dot{y}) + k(x - y) = u$$

$$m\ddot{y} + b(\dot{y} - \dot{x}) + k(y - x) = 0$$

$$\frac{d}{dt}\begin{bmatrix} \dot{x} \\ \dot{y} \\ x - y \end{bmatrix} = \underbrace{\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \dot{x} \\ \dot{y} \\ x - y \end{bmatrix} + \underbrace{\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}}_{\mathbf{B}} [u]$$

Output Equations

$$\underline{y} = C\underline{x}(t) + D\underline{u}(t)$$

C and D depend on the user selected output signal(s), purely determined algebraically. No dynamics (do not depend on the matrices A and B)

May require change of state definitions

$$\frac{d}{dt}\begin{bmatrix} \dot{x} \\ \dot{y} \\ x-y \end{bmatrix} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ x-y \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \end{bmatrix}\begin{bmatrix} u \end{bmatrix}$$

Relationship Between SS and TF

$$\frac{d\underline{x}(t)}{dt} = A\underline{x}(t) + B\underline{u}(t)$$

$$\underline{y} = C\underline{x}(t) + D\underline{u}(t)$$

$$\Rightarrow \quad \begin{aligned} s \cdot \mathbf{X}(s) &= \mathbf{A} \cdot \mathbf{X}(s) + \mathbf{B} \cdot \mathbf{U}(s) \\ \mathbf{Y}(s) &= \mathbf{C} \cdot \mathbf{X}(s) + \mathbf{D} \cdot \mathbf{U}(s) \end{aligned}$$

$$\Rightarrow \quad \mathbf{Y}(s) = \left[\mathbf{C}\left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D} \right] \cdot \mathbf{U}(s)$$

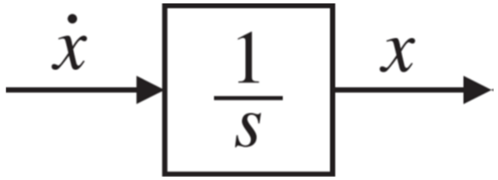
$$\Rightarrow \quad \boxed{\mathbf{G}(s) = \mathbf{C}\left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D}}$$

Poles of the system are the roots of the characteristic equation det(s*I*-A)=0. Therefore, the eigenvalues of the system matrix A, are the poles of the system.

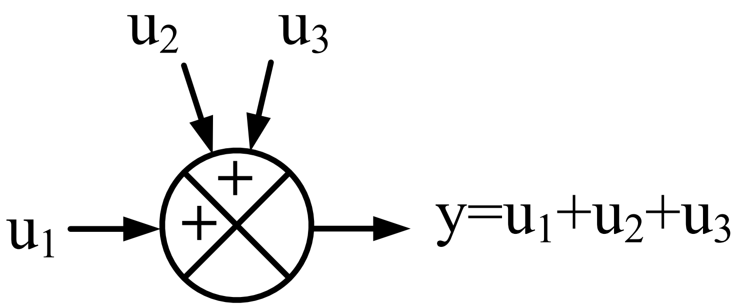
Analog Computer Implementation

- # of First order equations = # integrator
- The output of integrator = state variable
- Connecting the integrator, summer and potentiometer

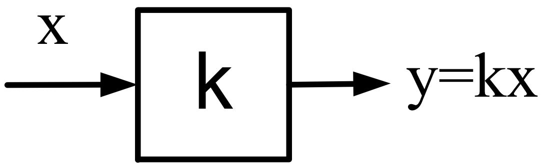
integrator



Summer



potentiometer



Analog Computer Implementation

Example: Build the Analog Computer Implementation for the system below

System1

$$\dot{x} = ax + bu$$

$$y = x$$

System2

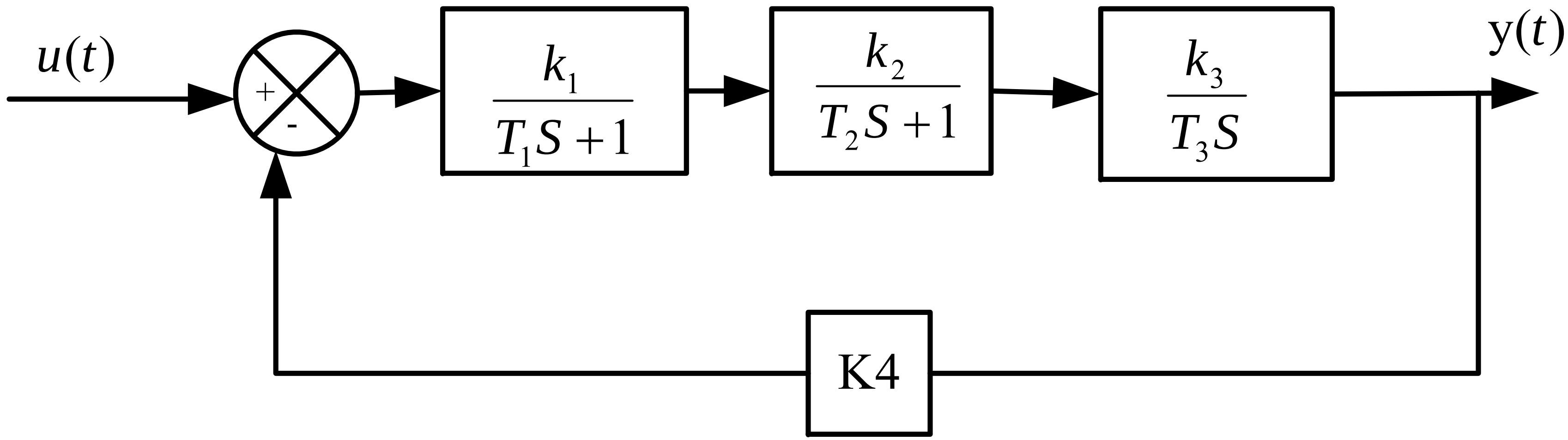
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_0 & a_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} u$$

$$y = [1 \quad 1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

From Block Diagram to SS

- Using the series of first-order terms

Example: build the SS representation from Block diagram below



From Block Diagram to SS

- Dividing the 2nd term into the collection of first-order terms

Example: build the SS representation from Block diagram below