第五讲

参数化时频变换

General Parameterized Time-frequency Transform

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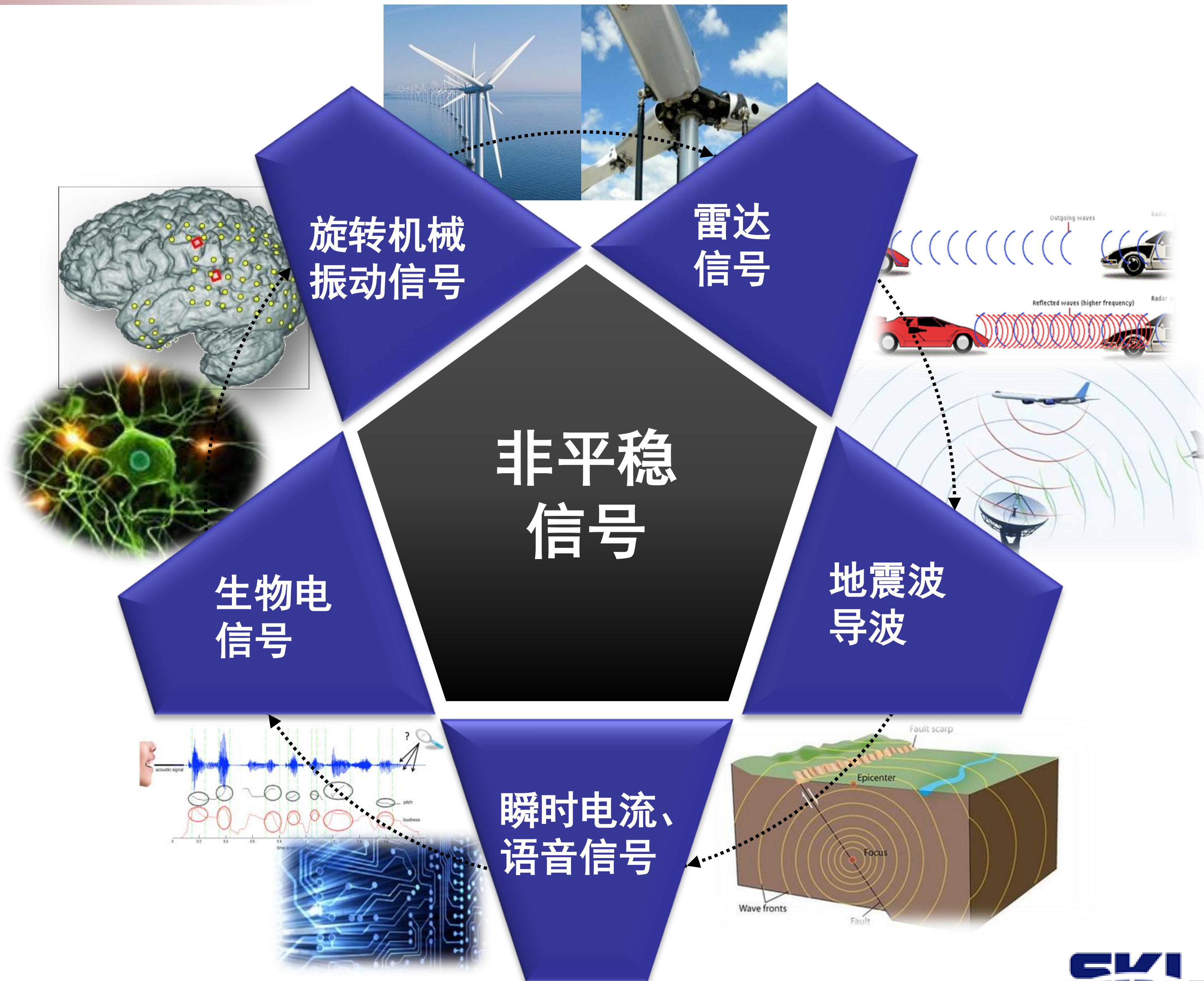
研究背景及意义


非平稳
信号
旋转机械
振动信号
雷达
信号
地震波
导波
瞬时电流、
语音信号
生物电
信号
Outgoing waves
Reflected waves (higher frequency)
Fault scarp
Epicenter
Focus
Wave fronts
Fault

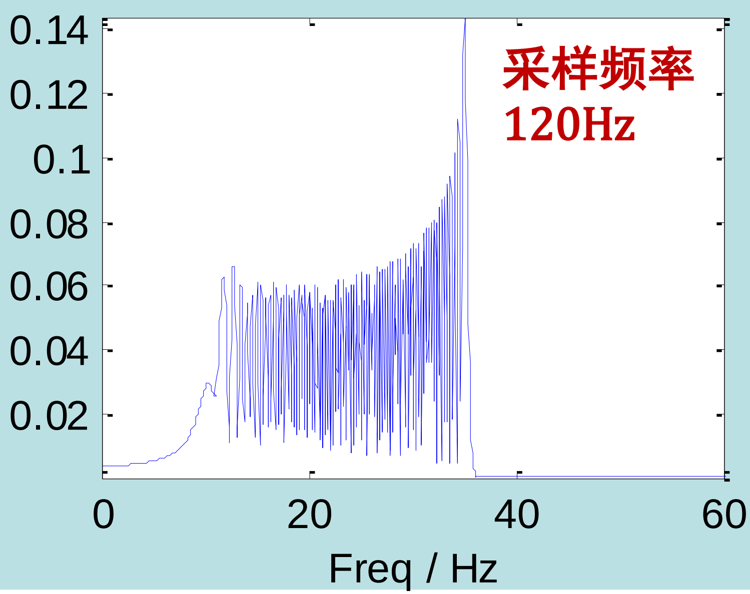
非平稳信号

特点： 非平稳信号的频率常随时间变化

例1：

$$s(t) = \sin\left(2\pi\left(10t + \frac{5}{4}t^2 + \frac{1}{9}t^3 - \frac{1}{120}t^4\right)\right) \quad (0 \le t \le 15)$$

$$f(t) = 10 + 2.5t + t^2/3 - t^3/30 \quad (\text{Hz})$$



Fourier变换

1) 能反映信号的频率范围
2) 不能反映频率随时间变化的变化规律

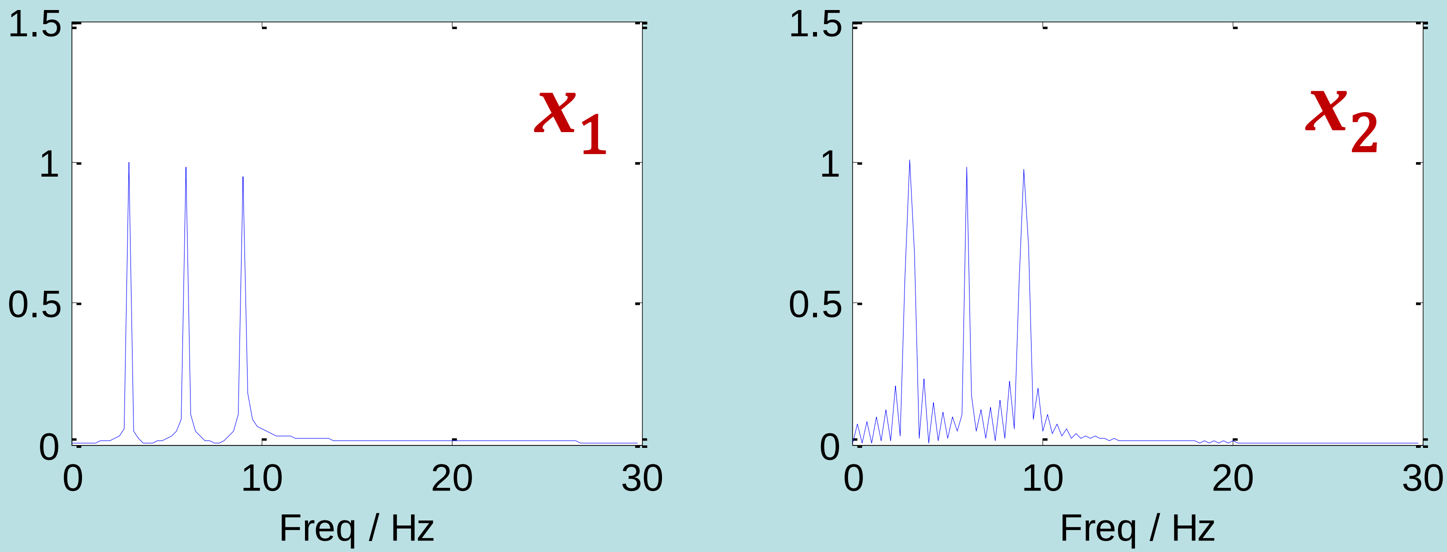
非平稳信号

例2：

$$x_1(t) = \sin(6\pi t) + \sin(12\pi t) + \sin(18\pi t) \qquad 0 \le t < 2\text{s}$$

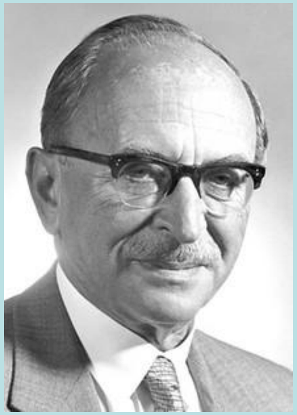
$$x_2(t) = \begin{cases} 2\sin(6\pi t) + \sin(12\pi t) & 0 \le t < 2\text{s} \\ \sin(12\pi t) + 2\sin(18\pi t) & 2 \le t \le 4\text{s} \end{cases}$$

Fourier变换

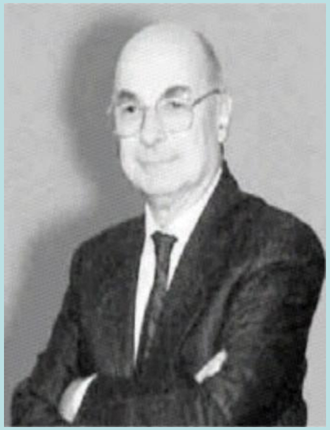


1) 准确反映信号所含频率分量
2) 不能反映频率分量存在的时间段

时频分析方法



短时傅立叶变换
(STFT)
D. Gabor 1946
Nobel Prize in Physics 1971



连续小波变换
(CWT)
J. Morlet 1984

非平稳信号

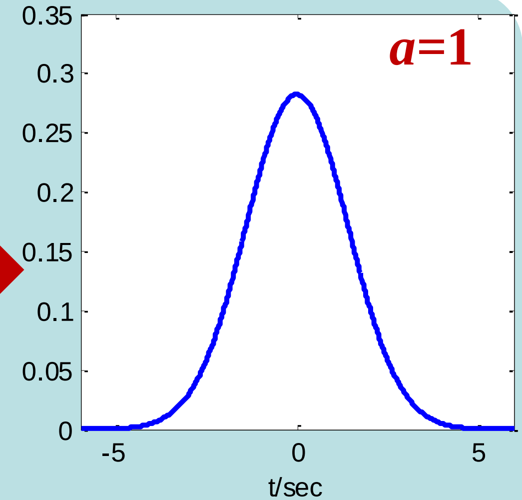
Wigner-Ville分布
(WVD)
E.P Wigner 1932
J. Ville 1948

线调频小波变换
(Chirplet)
S. Mann,
S. Haykin, 1991

短时傅立叶变换

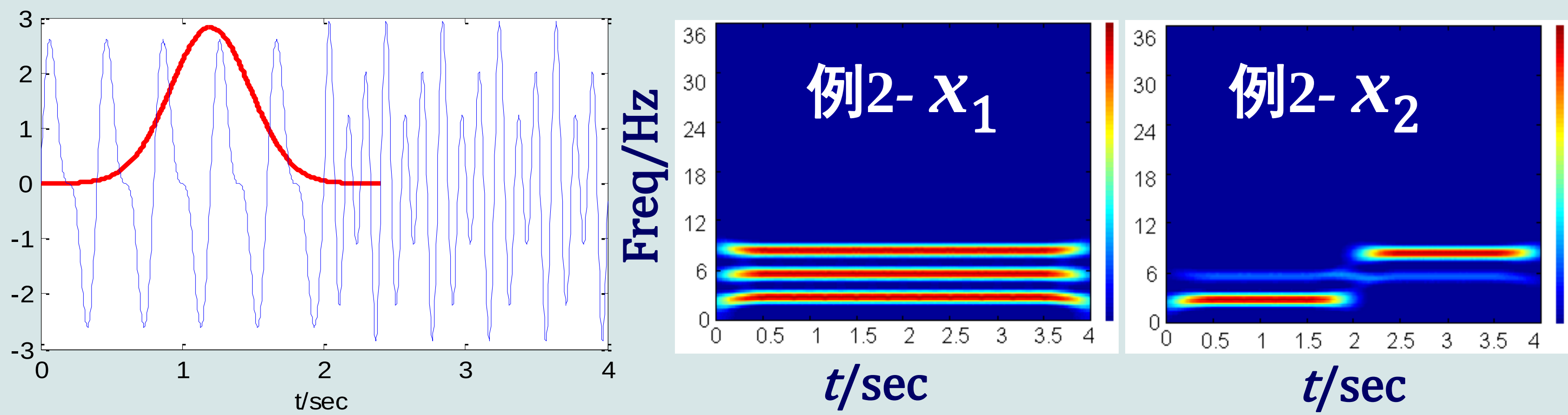
定义： $$\mathrm{STFT}_x(\tau,\omega)=\int_{-\infty}^{\infty} x(t)g(t-\tau)e^{-j\omega t}dt$$

窗函数 $$g_a(t)=\frac{1}{2\sqrt{\pi a}}e^{-t^2/4a}$$



本质： 加窗傅立叶变换

适用对象： 分段平稳信号



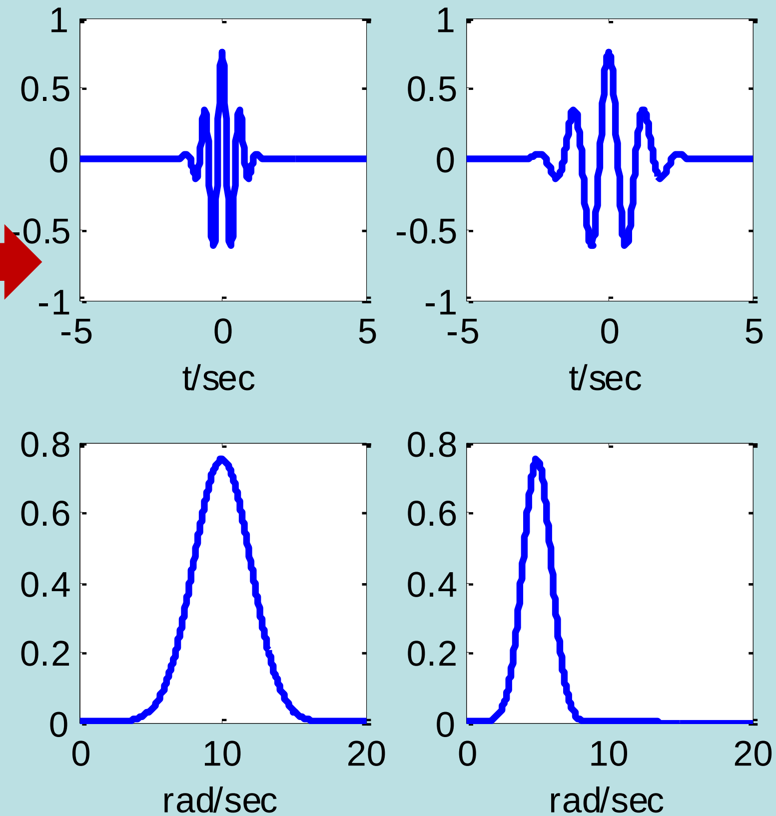
连续小波变换

定义：

$$CWT_x(a,b;\psi)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}x(t)\psi\left(\frac{t-b}{a}\right)dt$$

母波函数 $\psi(t)=\pi^{-1/4}e^{-j\omega_0 t}e^{-t^2/2}$

$\omega_0=5$



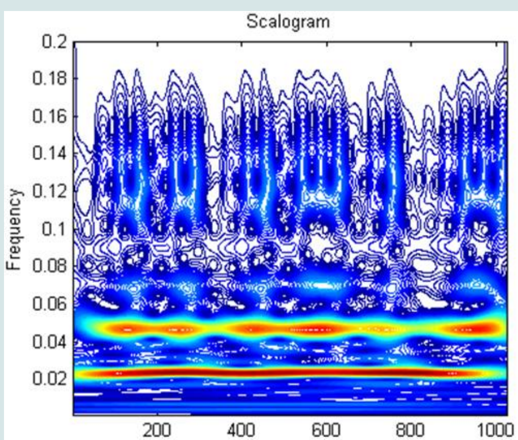
$a=0.5$　　$a=1$

本质： 变分辨率带通滤波

适用对象： 局部奇异性信号

示例

转子碰摩
故障典型
时频特征



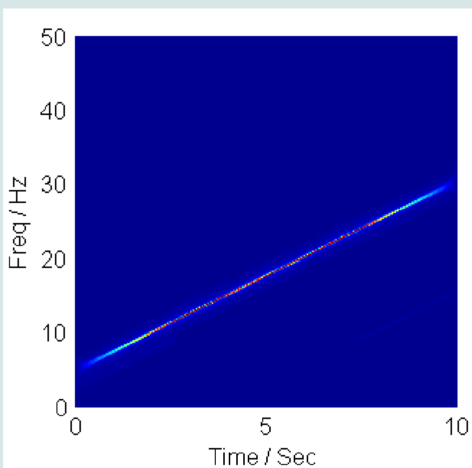
Wigner-Ville分布

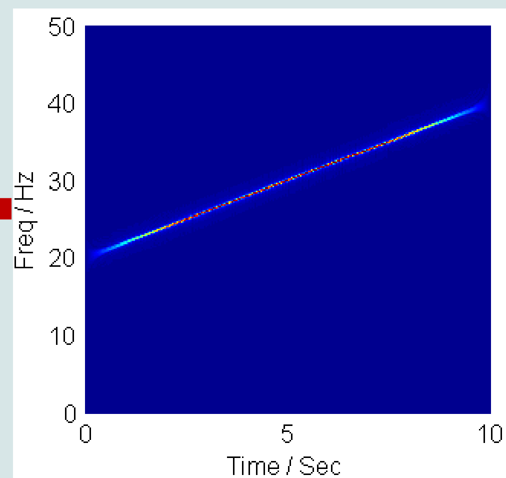
定义： $$WVD_x(t,\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}x^*\left(t-\frac{1}{2}\tau\right)x\left(t+\frac{1}{2}\tau\right)\mathrm{e}^{-j\tau\omega}\mathrm{d}\tau$$

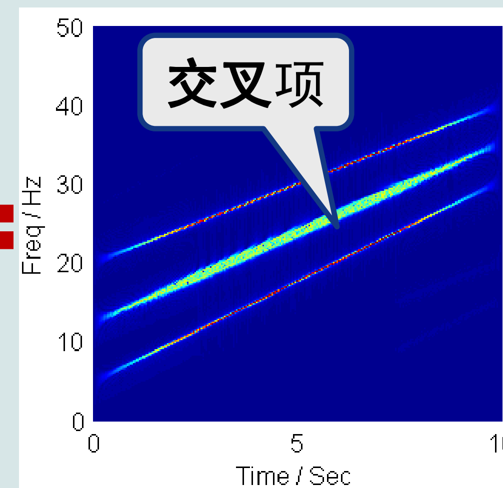
本质： 瞬时相关函数的傅立叶变换

适用对象： 单分量信号

示例







单分量信号 x_1 + 单分量信号 x_2 = 多分量信号 x

WVD(x_1) + WVD(x_2) ≠ WVD(x)

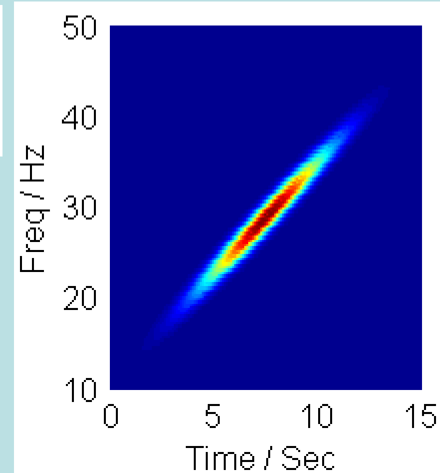
线调频小波变换

定义： $$CT_x(t,\omega,\alpha)=\int_{-\infty}^{+\infty}x(t)\Psi^*_{(t,\alpha)}(\tau)e^{-j\omega\tau}d\tau$$

调频窗函数 $$\Psi_{(t,\alpha)}(t)=g_a(t-\tau)e^{-j\frac{\alpha}{2}(t-\tau)^2}$$

本质： 加调频窗的傅立叶变换

适用对象： 线性调频信号



STFT

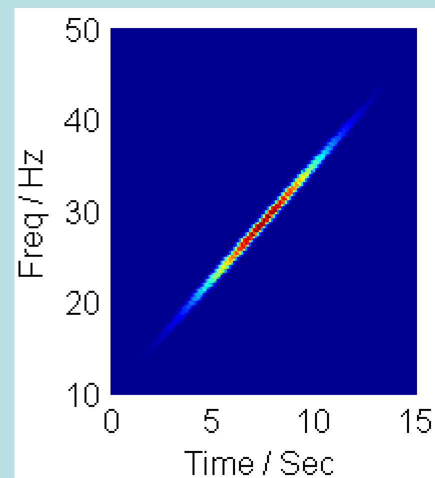
示例

脉冲

反射波

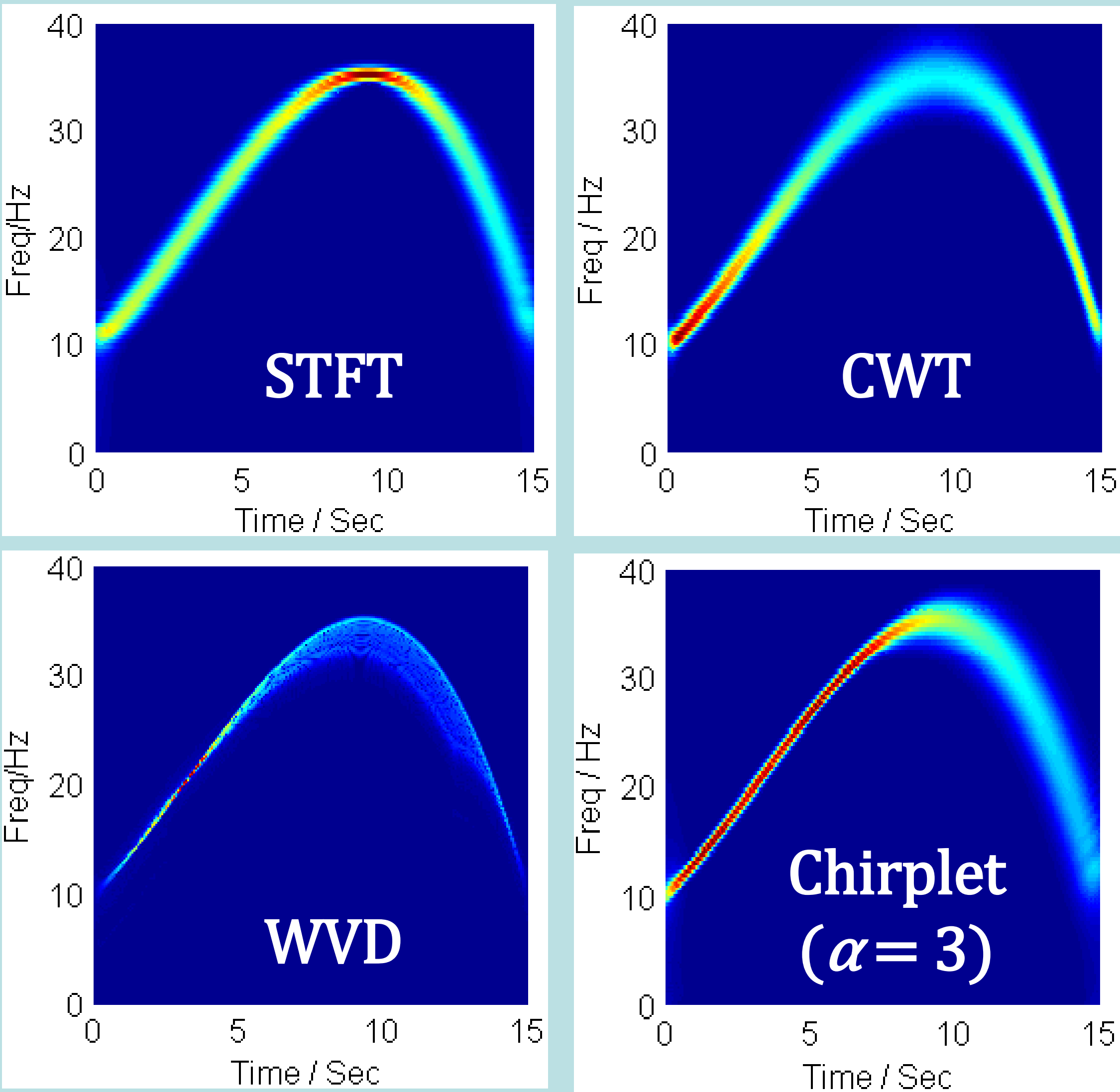
反射波信号简化模型

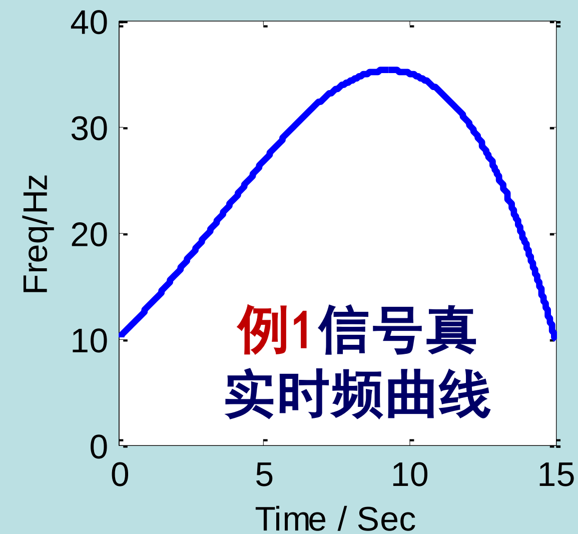
$$s(t)=\overline{A}e^{j\omega_c(t-\lambda)+j\gamma(t-\lambda)^2}$$



Chirplet

时频分析方法的不足



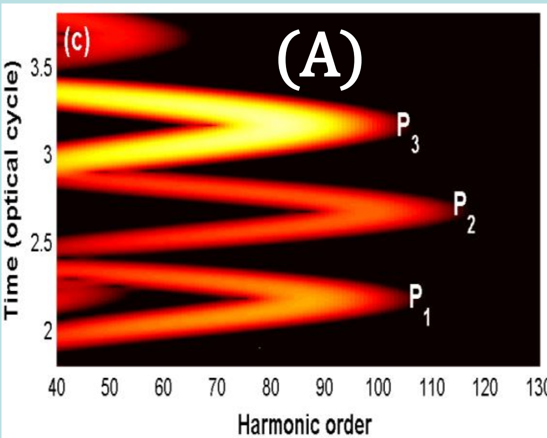


不足：

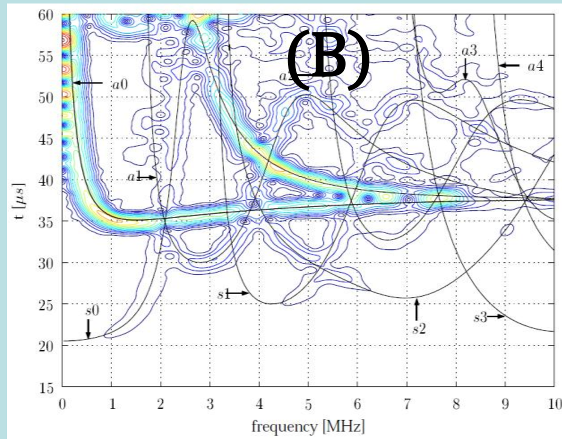
1) 不能正确反映分量的瞬时幅值
2) 集中性较差

非线性调频分量

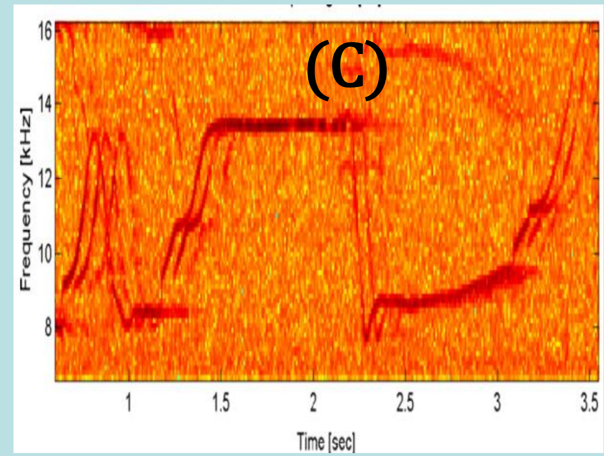
特点：频率是时间的非线性函数



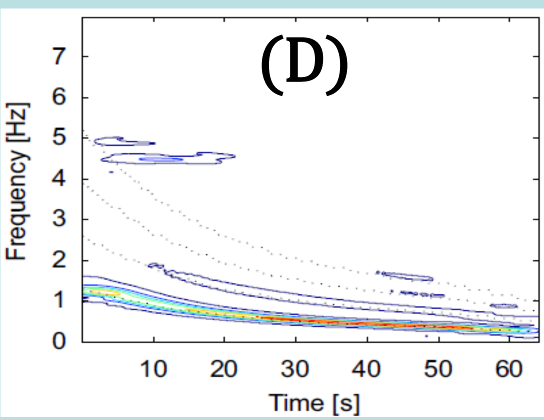
Opt Exp, 19(2011) 26174



J. Ac. Soc. Am. 107(2000), Pt.1

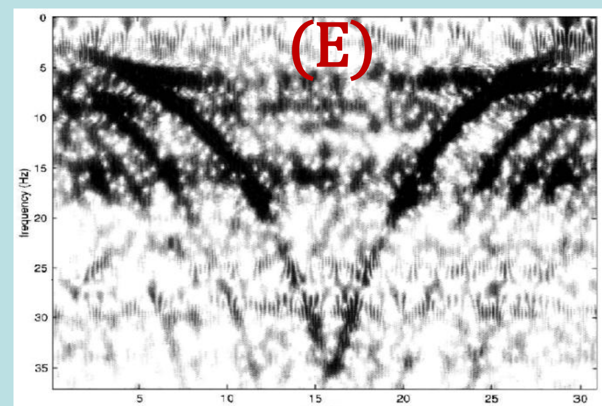


App. Ac. 71 (2010) 1070–1080



J S. Vib. 330 (2011)1225–1243

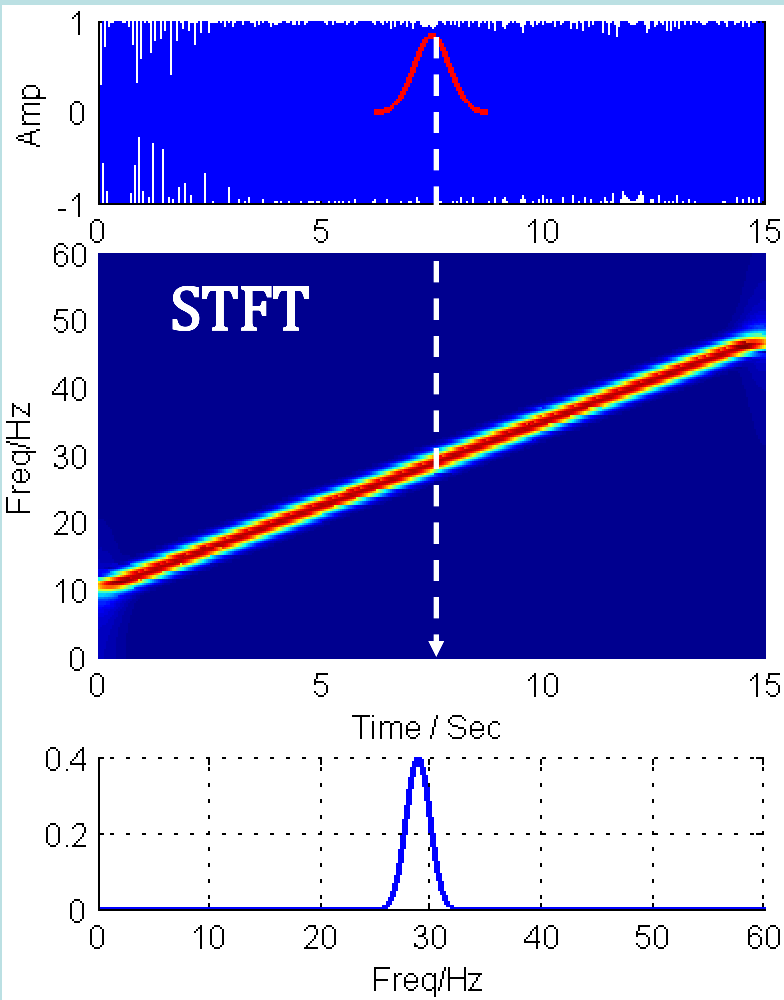
(A) 激光脉冲信号
(B) Lamb波信号
(C) 鲸鱼声波
(D) 水轮机停机振动信号
(E) 战斗机机翼测试信号



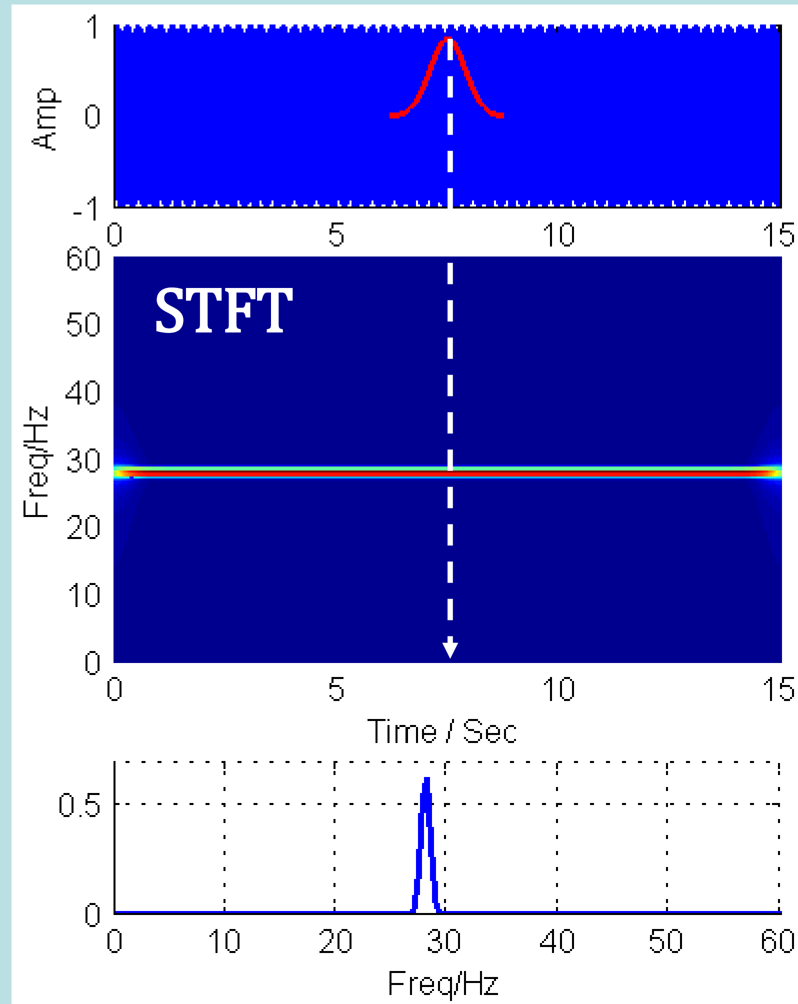
J. G. Con. DY 21(1998)375-382

参数化时频分析-原理

线调频信号



正弦信号



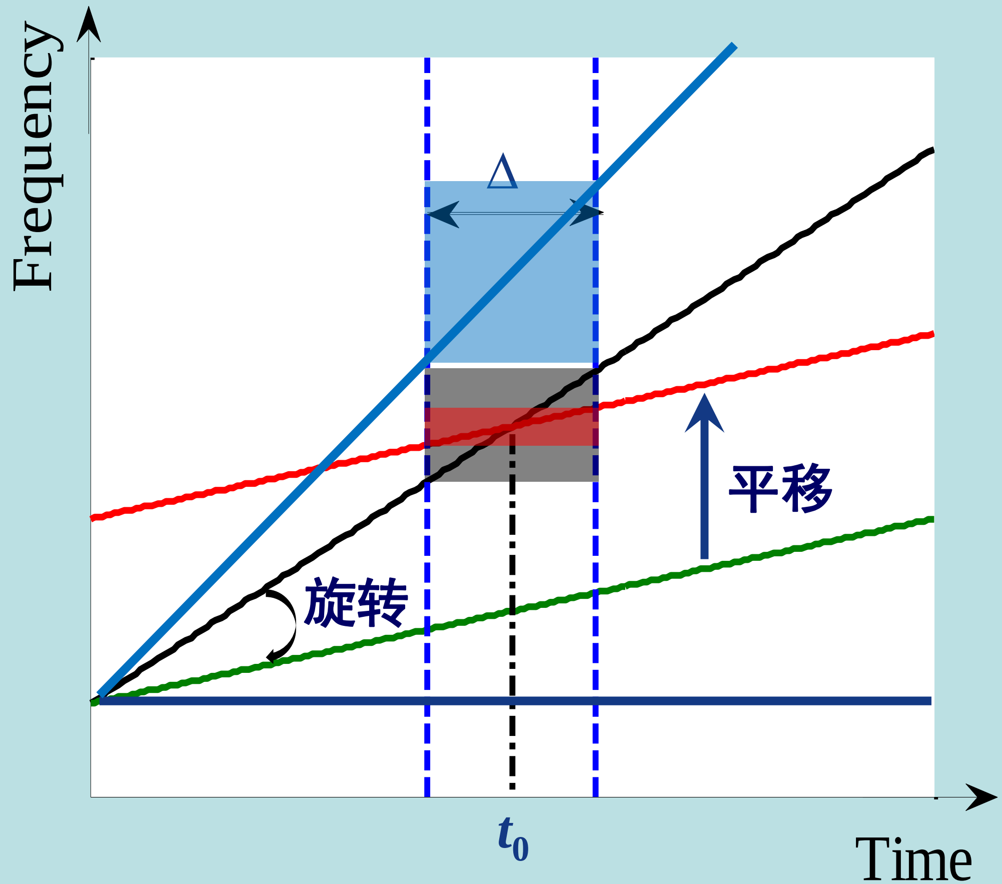
参数化时频分析-原理

Chirplet 定义新表达

$$CT_s(t,\omega,\alpha) = A(t)\int_{-\infty}^{+\infty}\overline{z}(\tau)g(\tau-t)e^{-j\omega\tau}d\tau$$

$$\begin{cases} \overline{z}(\tau)=s(\tau)\Phi_\alpha^R(\tau)\Phi_\alpha^M(\tau,t) \\ \Phi_\alpha^R(\tau)=e^{-j\alpha\tau^2/2} & \text{旋转算子} \\ \Phi_\alpha^M(\tau,t)=e^{j\alpha\tau t} & \text{平移算子} \\ A(t)=e^{-jt^2\alpha/2} \end{cases}$$

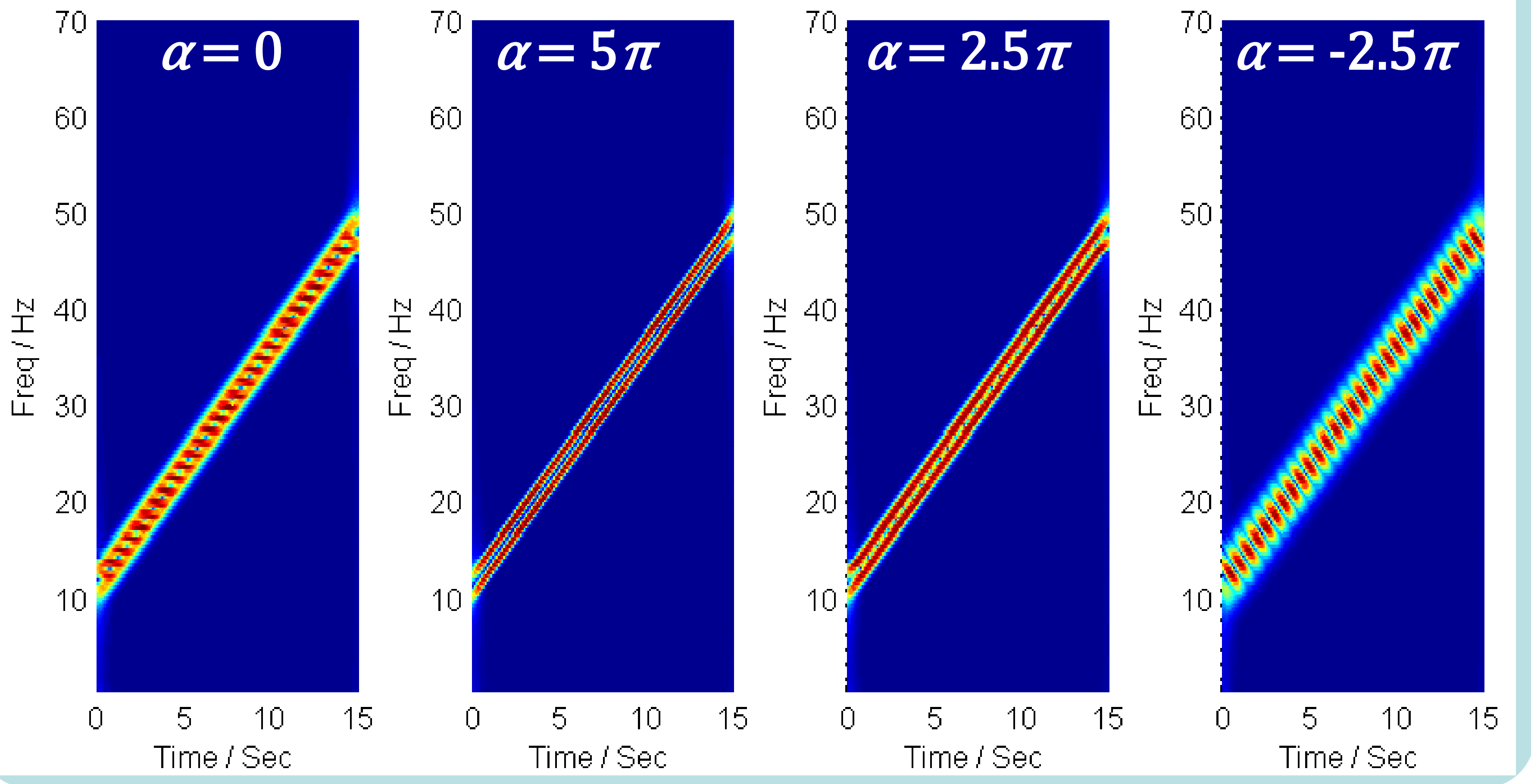
Chirplet 工作原理



参数化时频分析–原理

例3： $x(t)=\sin(2\pi(10+2.5t)t)+\sin(2\pi(12+2.5t)t)\quad(0\le t\le 15\text{s})$

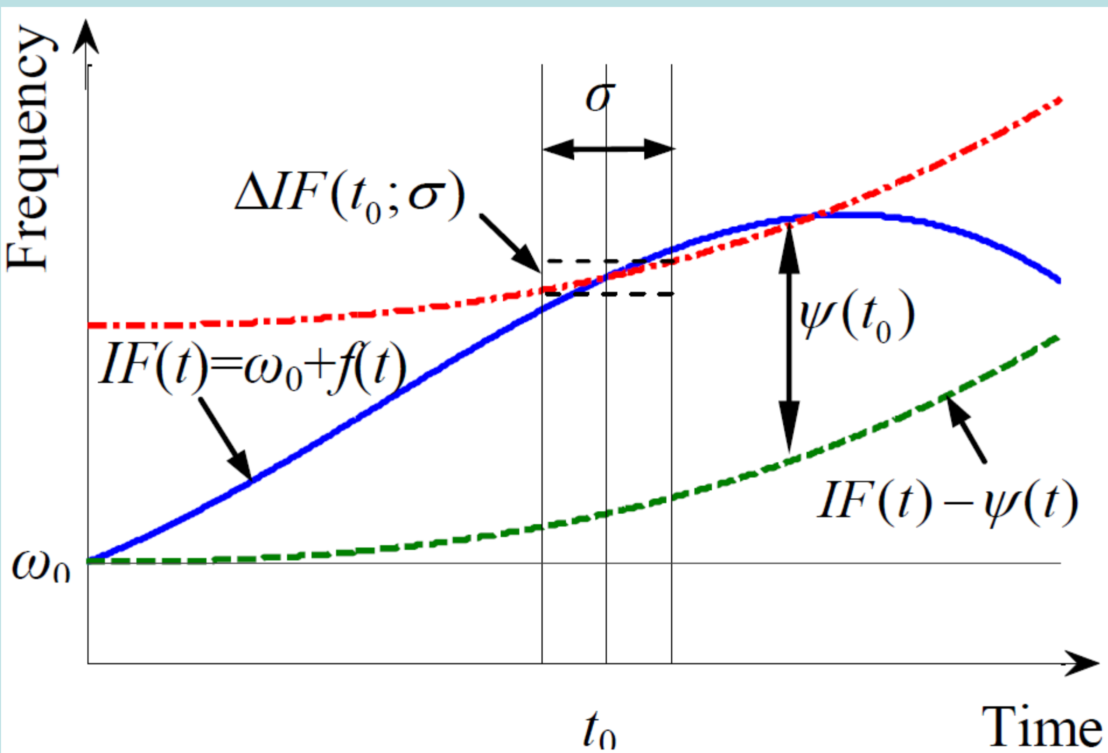
Chirp Rate = 5π / s



参数化时频分析-原理

信号模型： $$x(t)=e^{-j\left(\omega_0 t+\int_0^t f(\tau)d\tau\right)}$$

参数化时频分析原理图解



瞬时频率：

$$IF(t)=\omega_0+f(t)$$

旋转算子

$$\Phi^R(t)=e^{j\left(\int_0^t \psi(\tau)d\tau\right)}$$

平移算子

$$\Phi^M(\tau,t)=e^{-j\psi(\tau)t}$$

参数化时频分析–原理

定义

$$\mathrm{PTFT}_x(t,\omega,\psi) = \int_{-\infty}^{+\infty} \bar{z}(\tau) g(\tau - t) e^{-j\omega\tau} d\tau$$

$$\begin{cases} \bar{z}(t) = \Phi^R \Phi^M x(t) \\ \Phi^M(\tau,t) \quad = e^{-j\psi(\tau)t} \\ \Phi^R(t) = e^{j\left(\int_0^t \psi(\tau) d\tau\right)} \end{cases}$$

性质

线性可加

$$ax + by \to a\mathrm{PTFT}_x + b\mathrm{PTFT}_y$$

时移不变

$$x(t - t_0) \to \mathrm{PTFT}_x(t - t_0, \omega, \psi)$$

频移不变

$$x(t) e^{j\omega_0 t} \to \mathrm{PTFT}_x(t, \omega + \omega_0, \psi)$$

参数化时频分析-方法

多项式调频小波变换（PCT)

$$\psi(t)=\sum_{k=2}^{n+1}\alpha_{k-1}t^{k-1}$$

思想： 闭区间上的连续函数可用多项式函数一致逼近

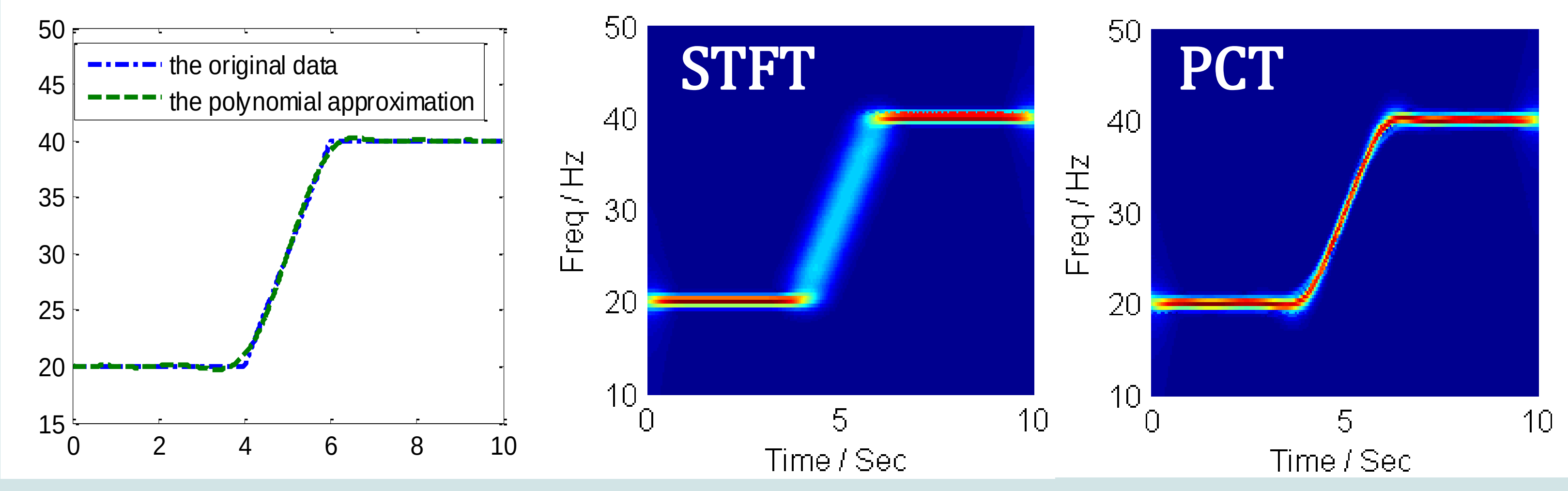
旋转算子

$$\Phi^{R}(t)=e^{j\left(\int_0^t \psi(\tau)d\tau\right)}$$

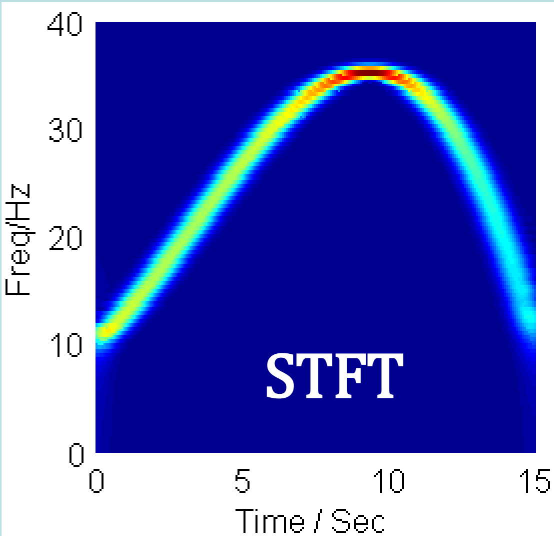
平移算子

$$\Phi^{M}(\tau,t)=e^{-j\psi(\tau)t}$$

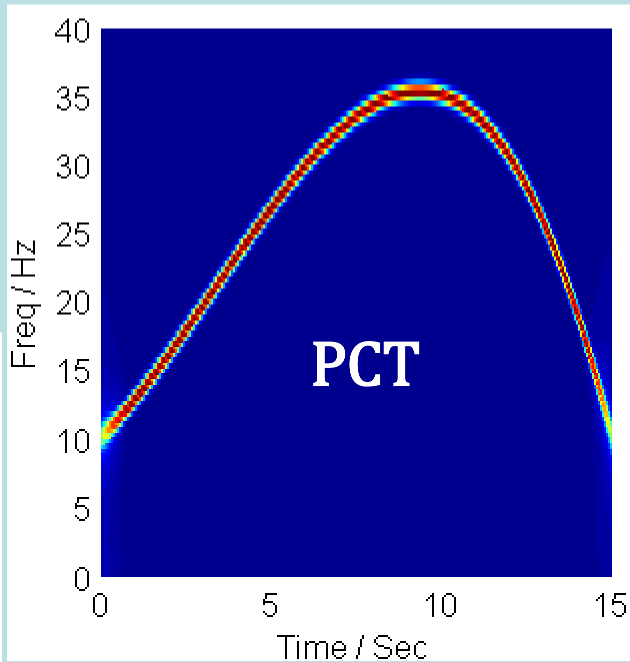
示例

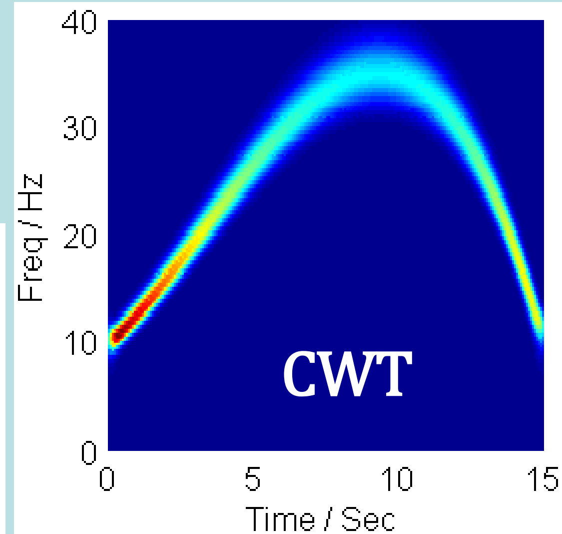


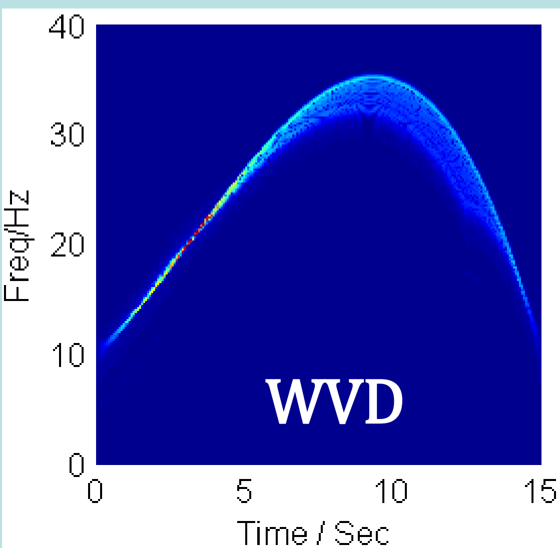
参数化时频分析-方法

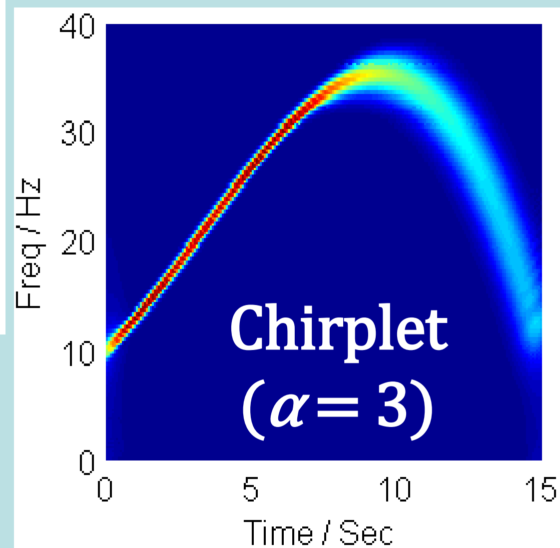


示例- PCT









参数化时频分析-方法

样条调频小波变换（SCT）

$$\psi(t)=\sum_{k=2}^{n+1}\lambda_k^i(t-t_i)^{k-1}$$

$$\text{if } t_i \le t \le t_{i+1},\quad i=1,\ldots,l-1$$

思想： 分段多项式逼近，避免Runge现象

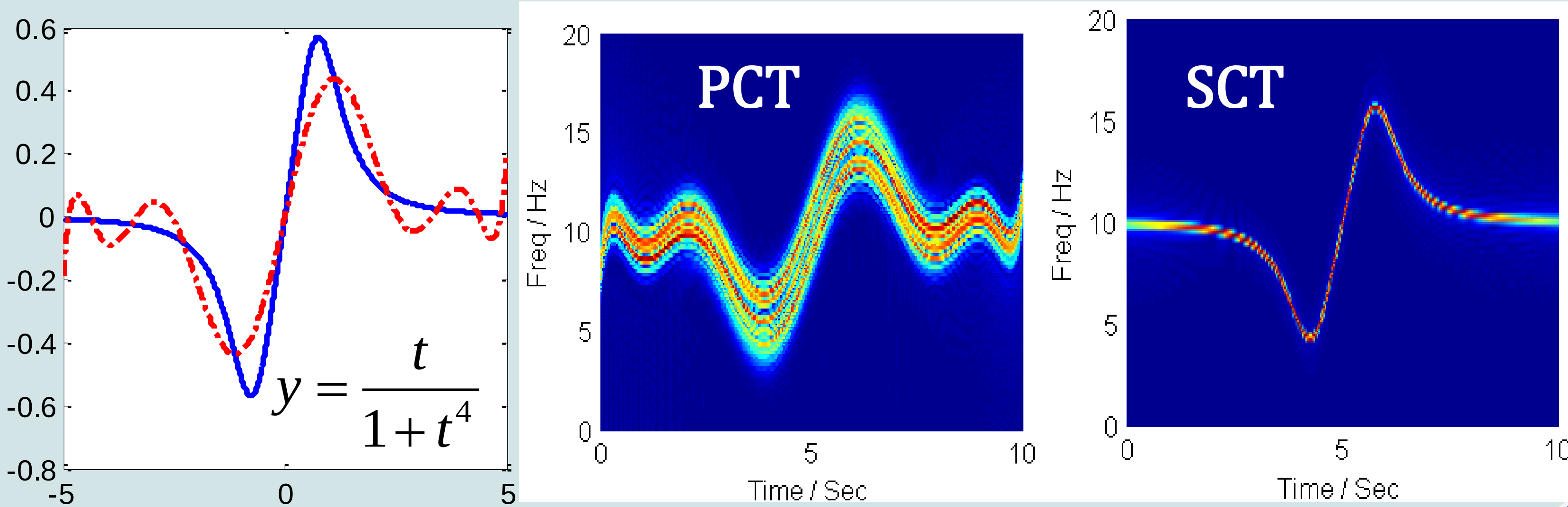
旋转算子

$$\Phi^R(t)=e^{j\left(\int_0^t \psi(\tau)d\tau\right)}$$

平移算子

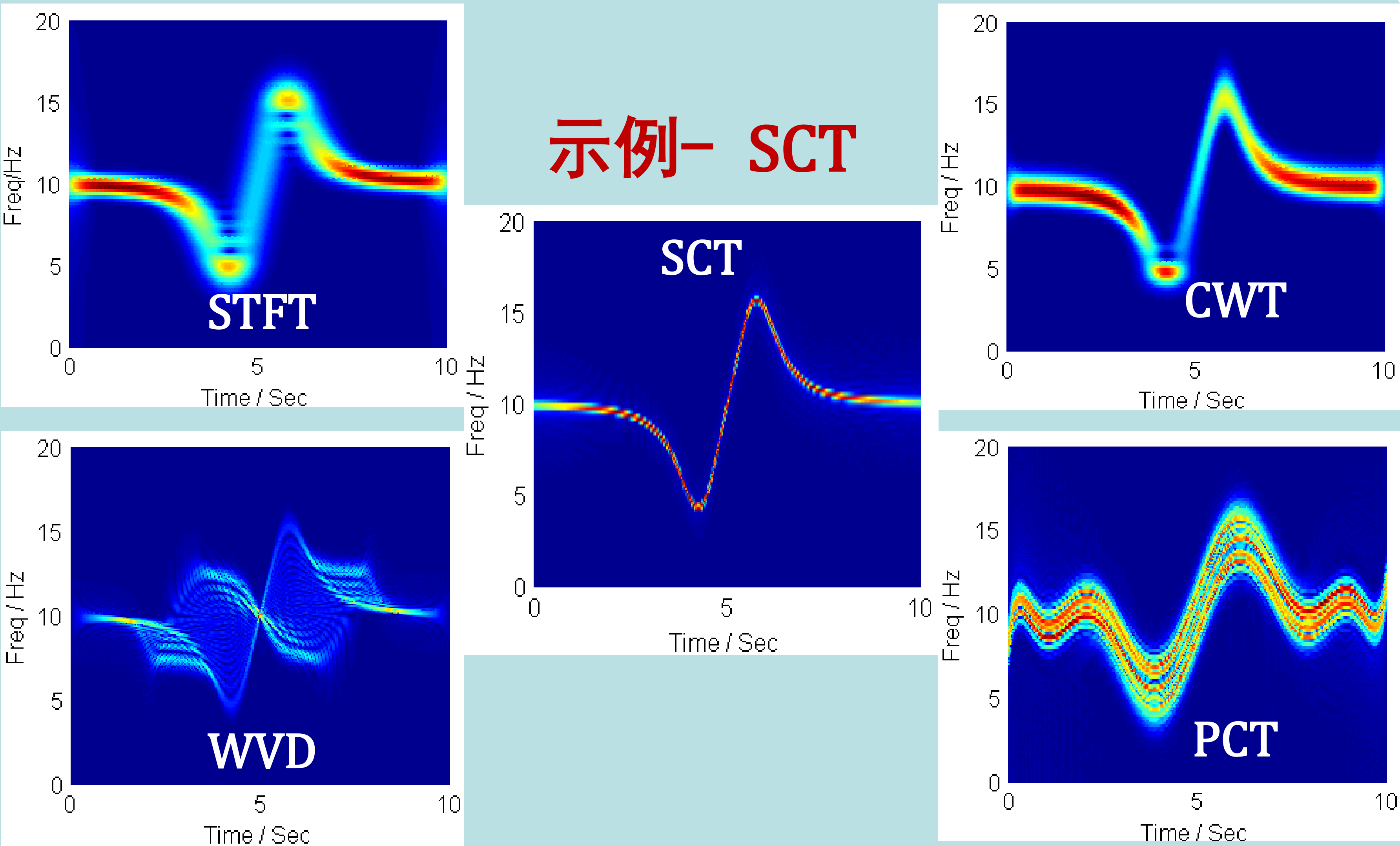
$$\Phi^M(\tau,t)=e^{-j\psi(\tau)t}$$

示例



参数化时频分析-方法

示例- SCT



参数化时频分析-方法

广义Warblet变换（GWT）

$$\psi(t)=\sum_{i=1}^{N}\Psi(\omega_i)e^{j\omega_i t}$$

思想： 任何可积函数都可用三角函数展开

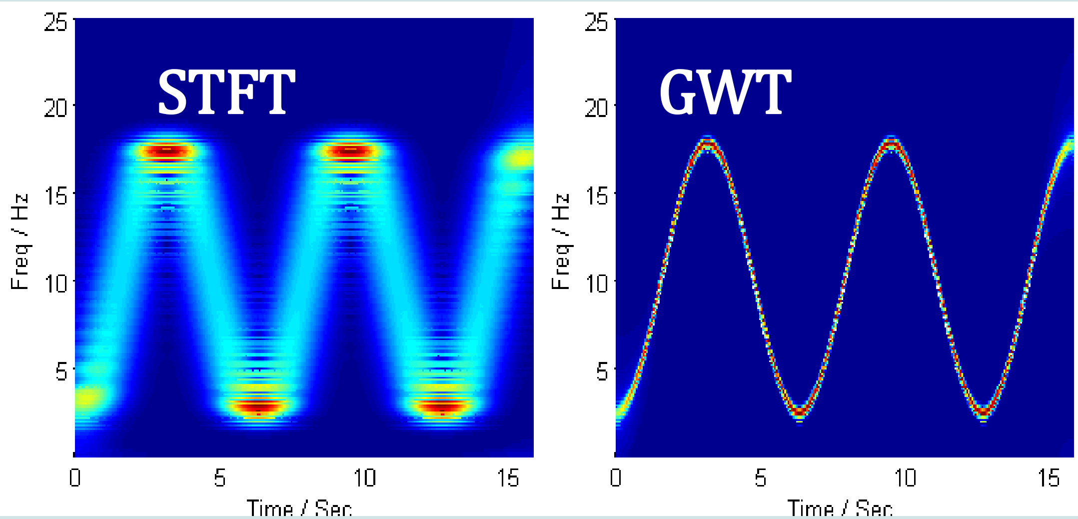
旋转算子

$$\Phi^R(t)=e^{j\left(\int_0^t \psi(\tau)d\tau\right)}$$

平移算子

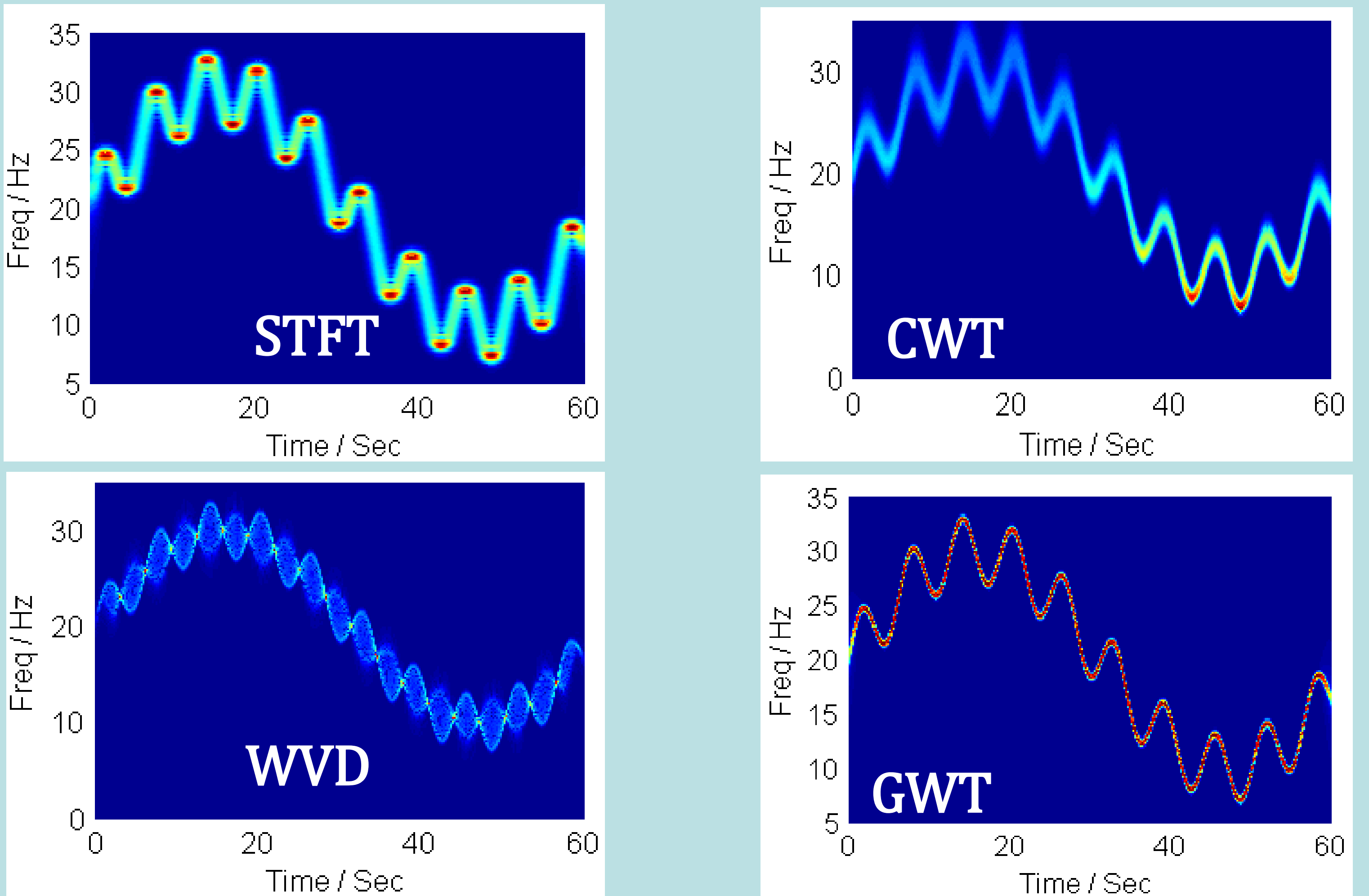
$$\Phi^M(\tau,t)=e^{-j\psi(\tau)t}$$

示例



参数化时频分析-方法

示例- GWT



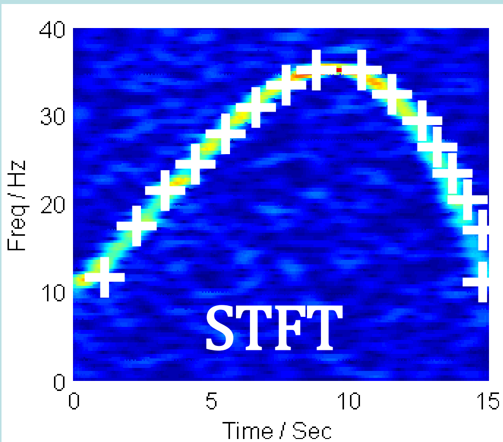
参数化时频分析-方法

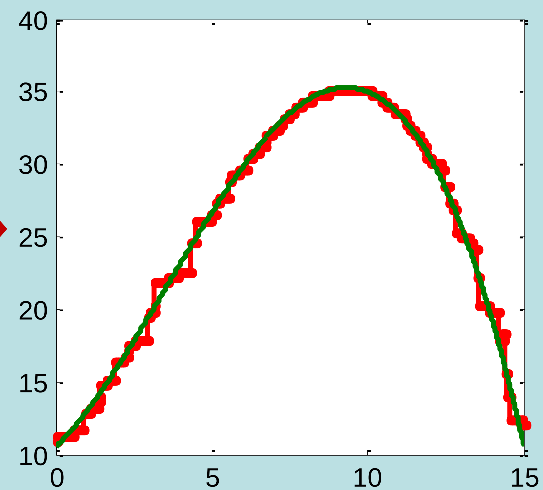
如何确定变换核参数

多项式调频小波变换	$\left[\alpha_1,\cdots,\alpha_n\right]$
样条调频小波变换	$\begin{bmatrix}\lambda_1^1,\cdots,\lambda_n^1\\ \vdots \\ \lambda_1^l,\cdots,\lambda_n^l\end{bmatrix}$
广义Warblet变换	$\left[\Psi_1,\cdots,\Psi_n\right]$

参数化时频分析-方法

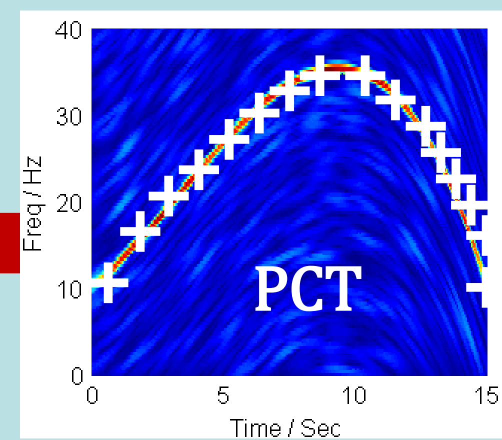
变换核参数估计-PCT

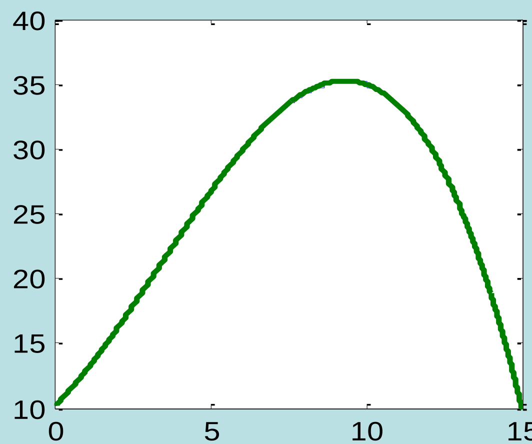




最小二乘法拟合

$$\left[\alpha_1^0, \cdots, \alpha_n^0\right]$$



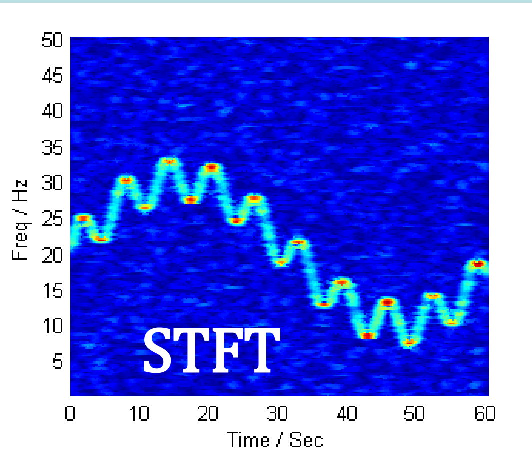


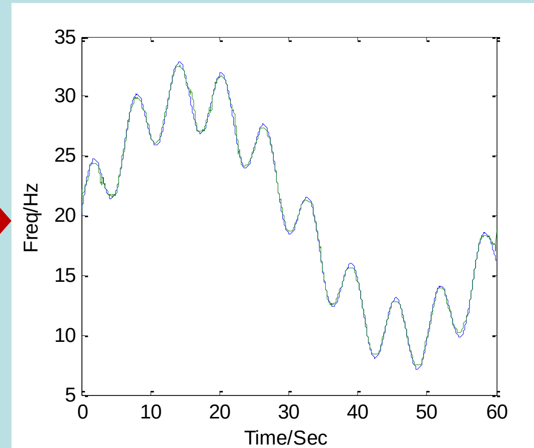
$$\left[\alpha_1^1, \cdots, \alpha_n^1\right]$$

最小二乘法拟合

参数化时频分析-方法

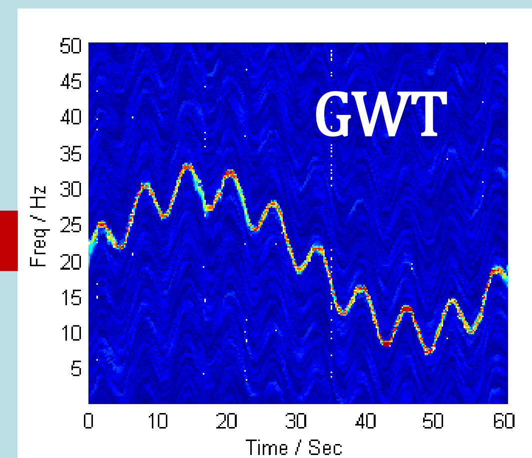
变换核参数估计-GWT

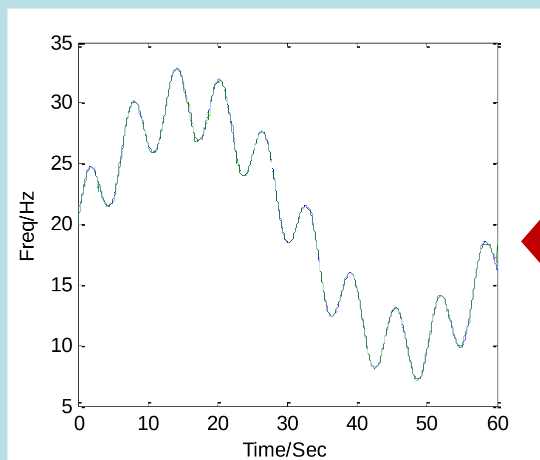




Fourier变换

$$\left[\Psi_1^0,\cdots,\Psi_n^0\right]$$





$$\left[\Psi_1^1,\cdots,\Psi_n^1\right]$$

Fourier变换

参数化时频分析-方法

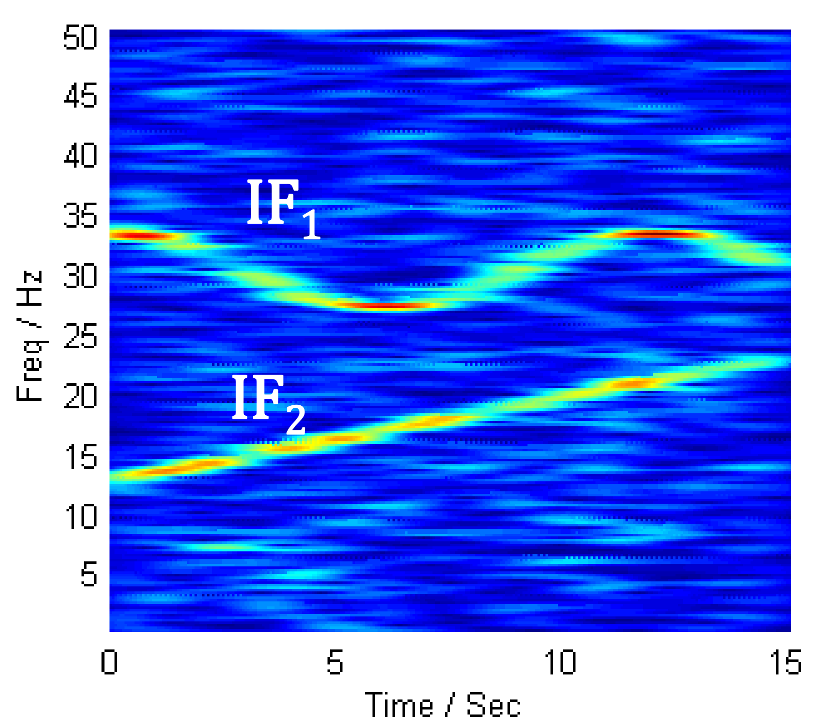
如何分析多频率分量信号？

示例- 多频率分量信号

$$s(t)=e^{j\pi\left[60t+12\sin\left(\frac{\pi t}{6}\right)\right]}+e^{j\left(0.7\pi t^2+25\pi t\right)}+n$$

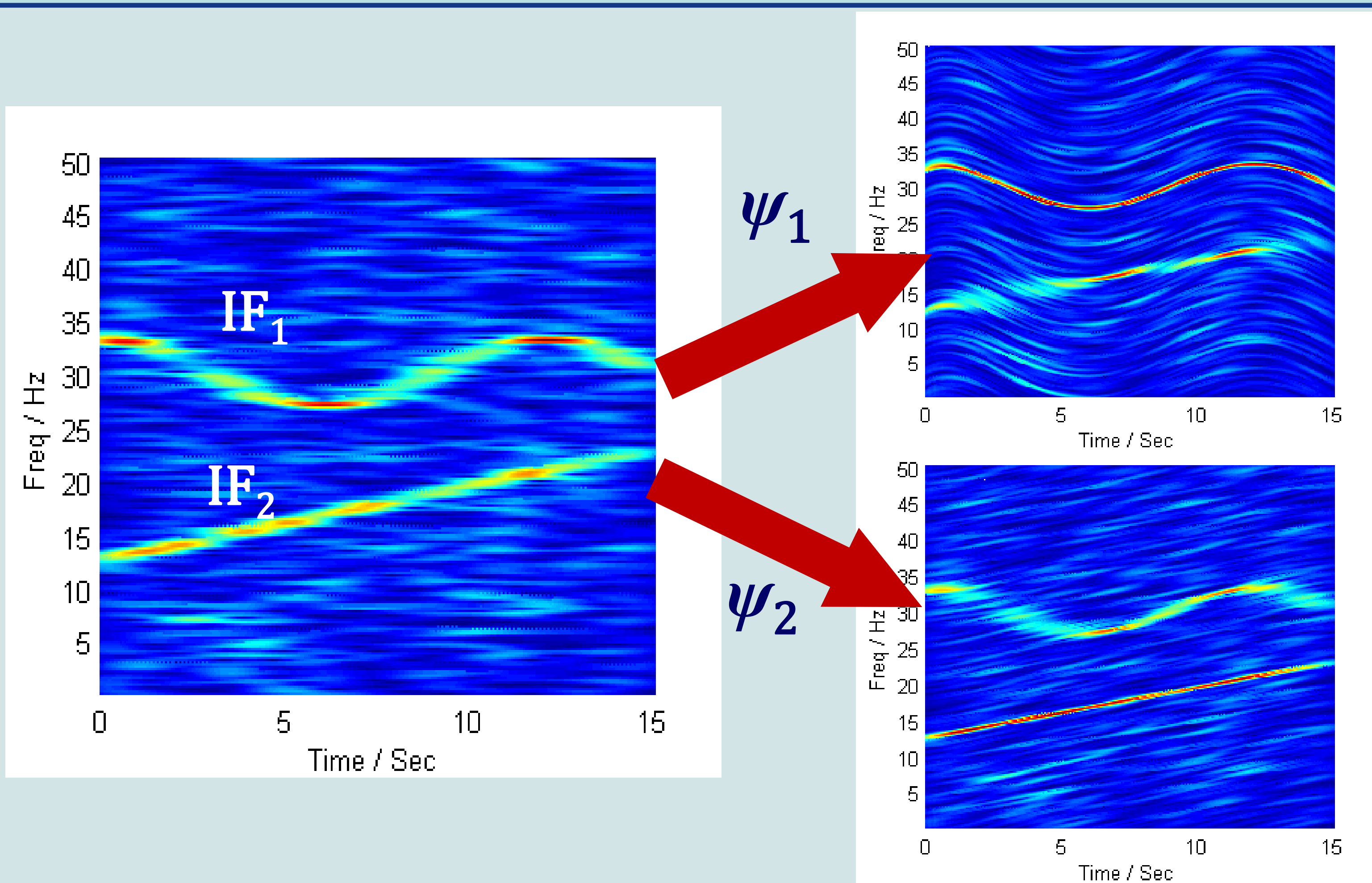
$$IF_1(t)=30+\pi\cos\left(\pi t/6\right)$$

$$IF_2(t)=0.7t+12.5$$



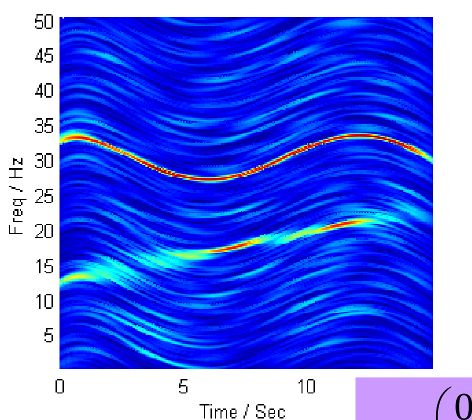
参数化时频分析-方法

多频率分量信号分析-时频融合法

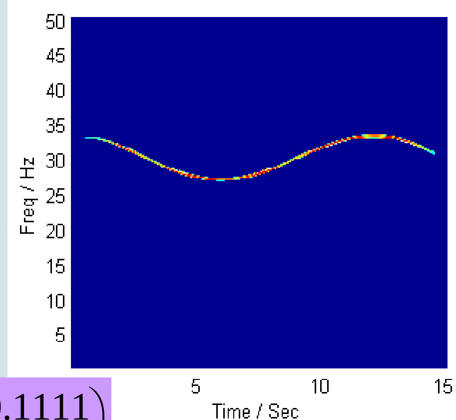


参数化时频分析-方法

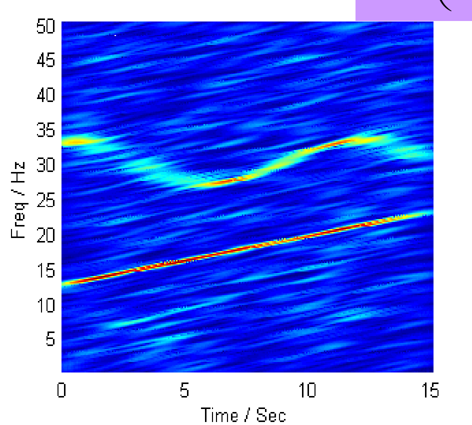
多频率分量信号分析-时频融合法



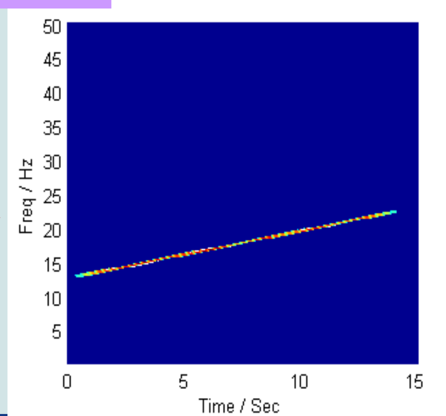
高通
滤波

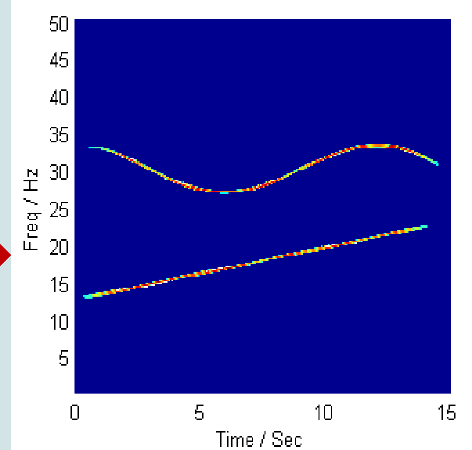


$$H=\begin{pmatrix}0.1111 & 0.1111 & 0.1111\\ 0.1111 & 0.8889 & 0.1111\\ 0.1111 & 0.1111 & 0.1111\end{pmatrix}$$

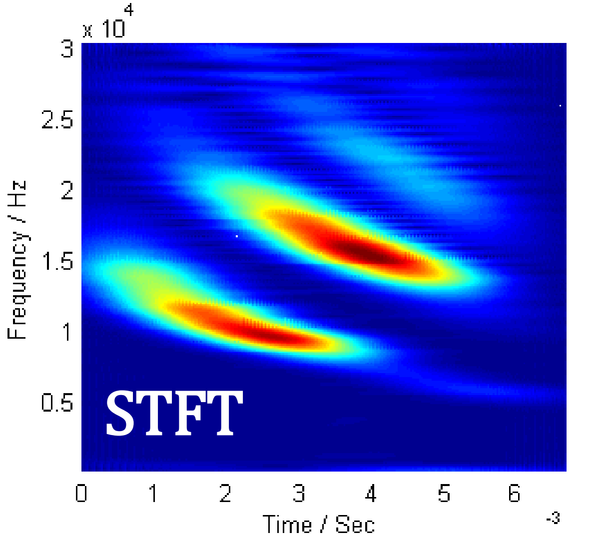


高通
滤波



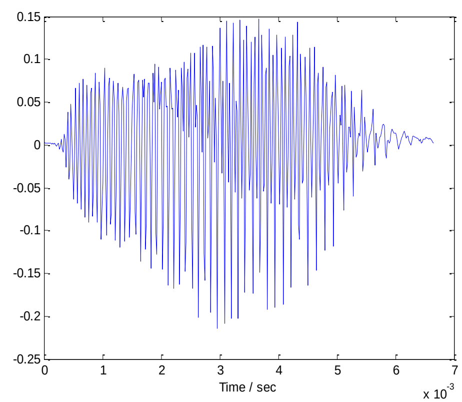


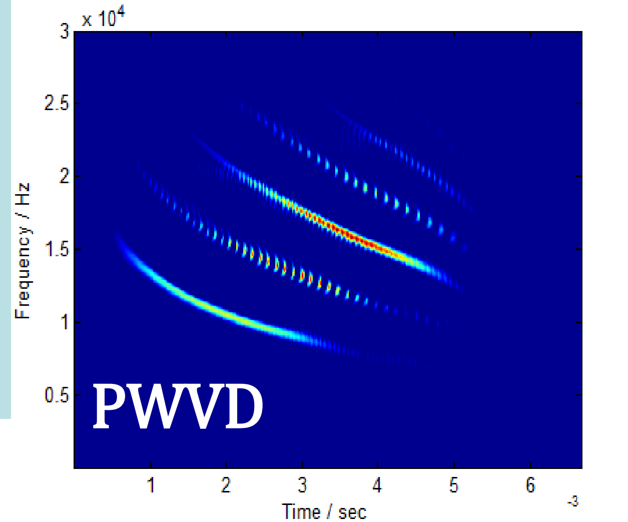
参数化时频分析-方法

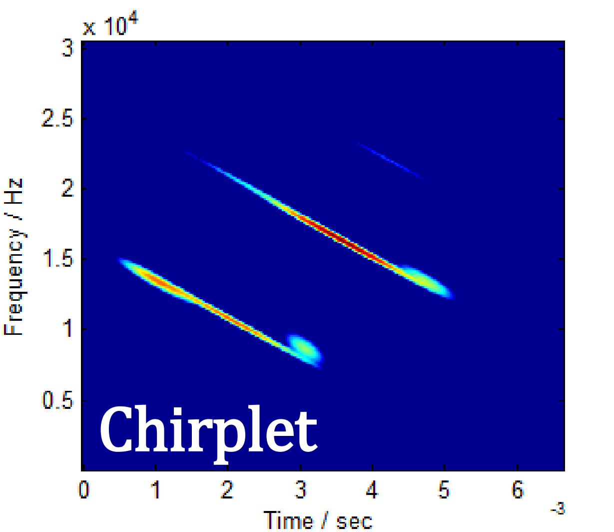


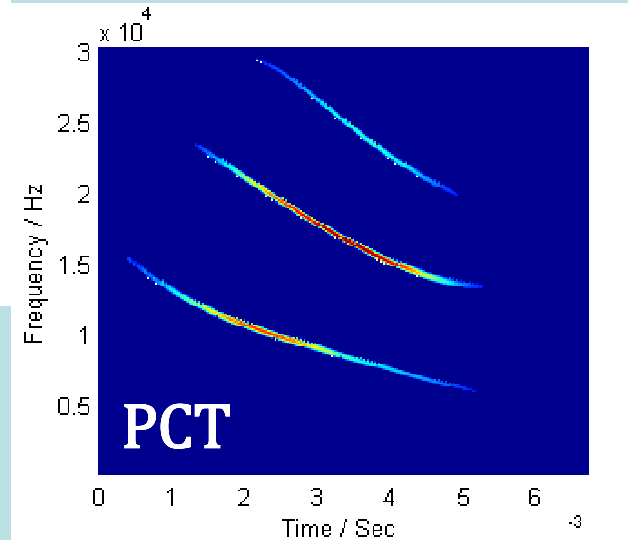
时频融合法示例

蝙蝠回波定位信号









参数化时频分析-方法

多频率分量信号分析-信号分解法

信号模型：

$$x(t)=\sum_{k=1}^{N} e^{-j\left(\omega_k t+\int_0^t f_k(\tau)d\tau+\phi_k\right)}+n$$

问题：

$$x(t)\approx\sum_{k=1}^{N} x_k(t)$$

$$x_k(t)=e^{-j\left(\omega_k t+\int_0^t \psi_k(\tau;\alpha_1^k,\cdots,\alpha_l^k)d\tau+\phi_k\right)}$$

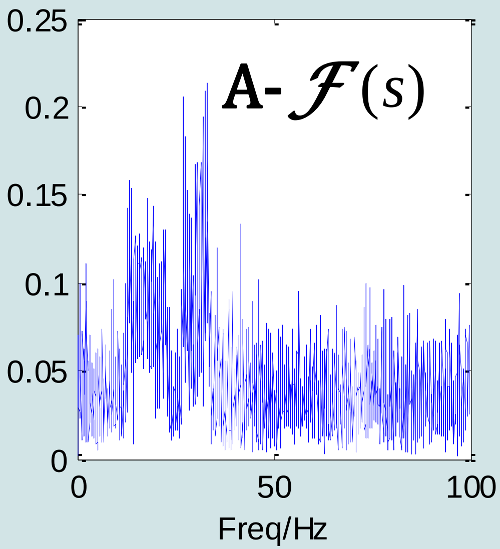
$$\psi_k\left(\tau;\alpha_1^k,\cdots,\alpha_l^k\right)\approx f_k(\tau)$$

参数化时频分析-方法

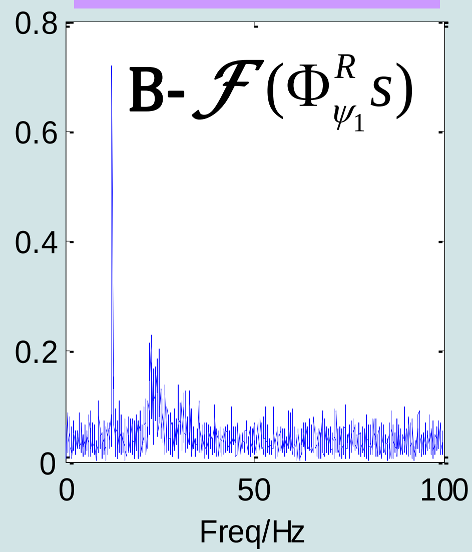
多频率分量信号分析-信号分解法

示例信号 $x(t) = e^{j\pi\left[60t+12\sin(\pi t/6)\right]} + e^{j\left(25\pi t+0.7\pi t^2\right)} + n$

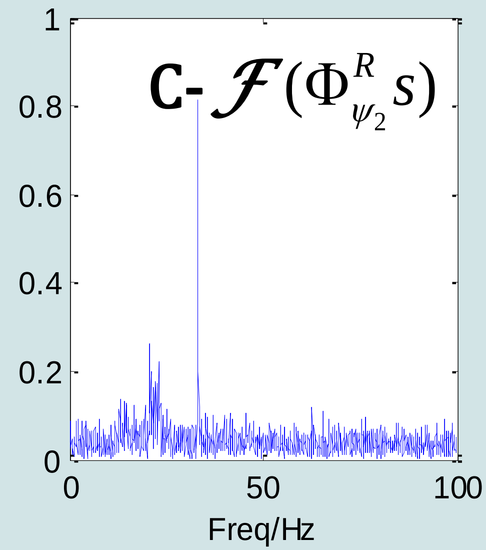
$\Phi_{\psi_0}^{R}(t) = 1$



$\Phi_{\psi_1}^{R}(t) = e^{-j\left(0.7\pi t^2\right)}$



$\Phi_{\psi_2}^{R}(t) = e^{-j12\pi\sin(\pi t/6)}$



$Ind = \int\left|\mathcal{F}\left(\Phi_{k}^{R} s\right)\right|^4 d\omega$

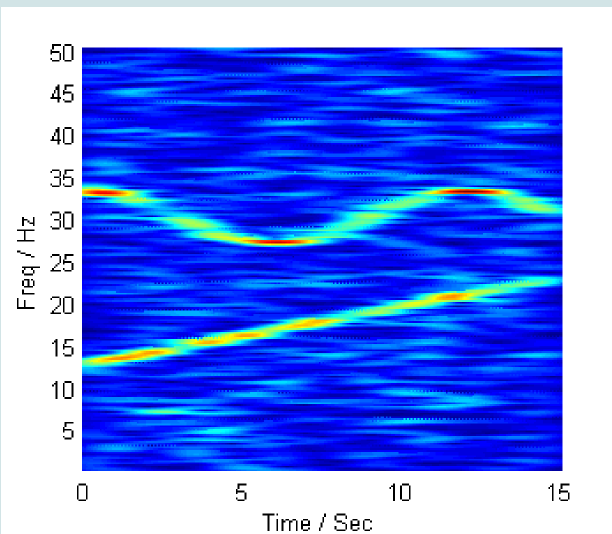
Ind: **(A)-0.1007 ; (B)-0.4650; (C)-0.6060**

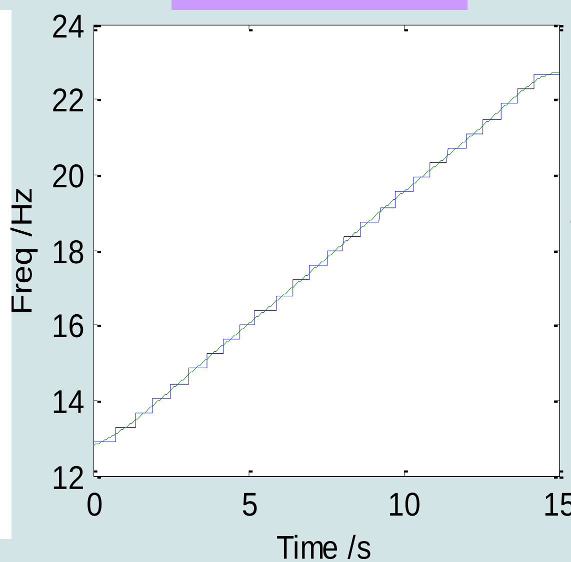
参数化时频分析-方法

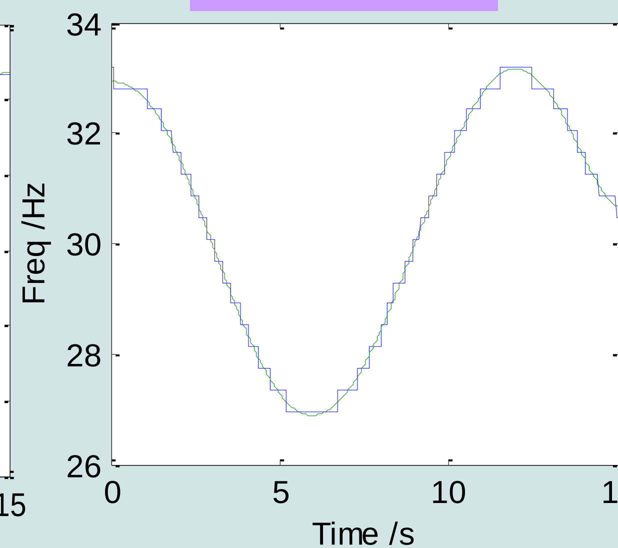
多频率分量信号分析-信号分解法

参数估计： $$\left(\tilde{\alpha}_1^k,\ldots,\tilde{\alpha}_l^k\right)=\arg\max_{\left(\alpha_1^k,\ldots,\alpha_l^k\right)}\left(\int\left|\mathcal{F}\left(\Phi_k^R s\right)\right|^4 d\omega\right)$$

初始值设定： $\left(\alpha_1^1,\cdots,\alpha_{l_1}^1\right)_0$ $\left(\alpha_1^2,\cdots,\alpha_{l_2}^2\right)_0$







估计算法：粒子群优化算法或遗传算法

参数化时频分析-方法

多频率分量信号分析-信号分解法

相位估计:

$$\left(\omega_k, \alpha_1^k, \cdots, \alpha_l^k\right) \rightarrow z(t) = \Phi_{\psi_k}^R x(t) \approx e^{j(\omega_k t + \phi_k)} + \Delta$$

相关分析:

$$y_k(t) = e^{j\omega_k t}$$

$$r_{z,y_k}(\tau) = \int y_k(t) z(t-\tau) dt$$

$$\tau_{\max} = \arg\max_{\tau}\left(\left|r_{z,y_k}(\tau)\right|\right)$$

$$\tau_{\max} \rightarrow \phi_k$$

参数化时频分析-方法

多频率分量信号分析-信号分解法

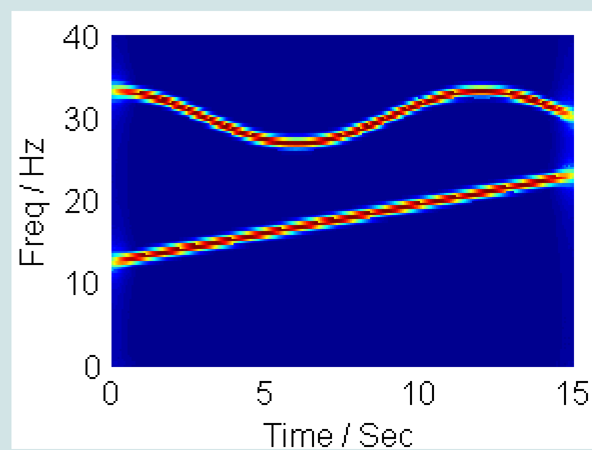
$$s(t)=e^{j\left[\pi\left(60t+12\sin(\pi t/6)\right)+0.5\right]}+e^{j\left(25\pi t+0.7\pi t^2+0.7\right)}$$

分量1

参数	估计值
ω	33.148935817873870
α_1	-0.040803408016272
α_2	-0.377328980216910
α_3	-0.027811922535334
α_4	0.016548259074640
α_5	-0.000659065850102
α_6	-0.000108174469633
α_7	0.000009353945751

分量2

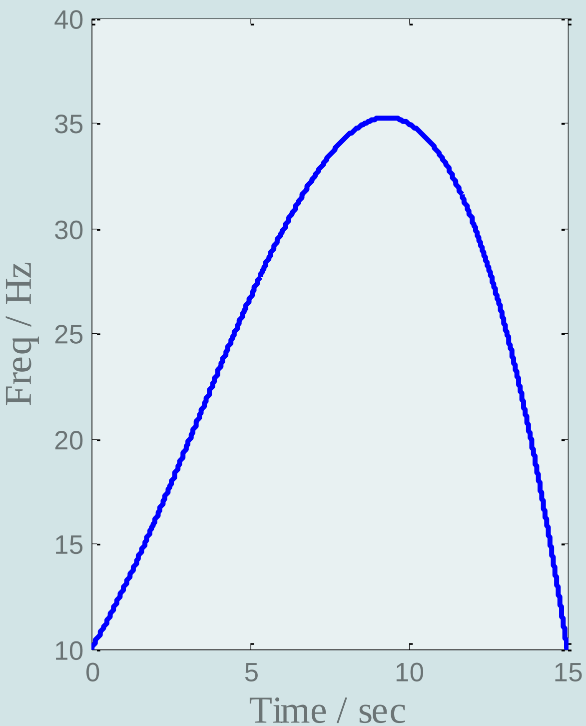
参数	估计值
ω	12.504644817794059
α_1	0.696761765522457
α_2	0.000257614908471

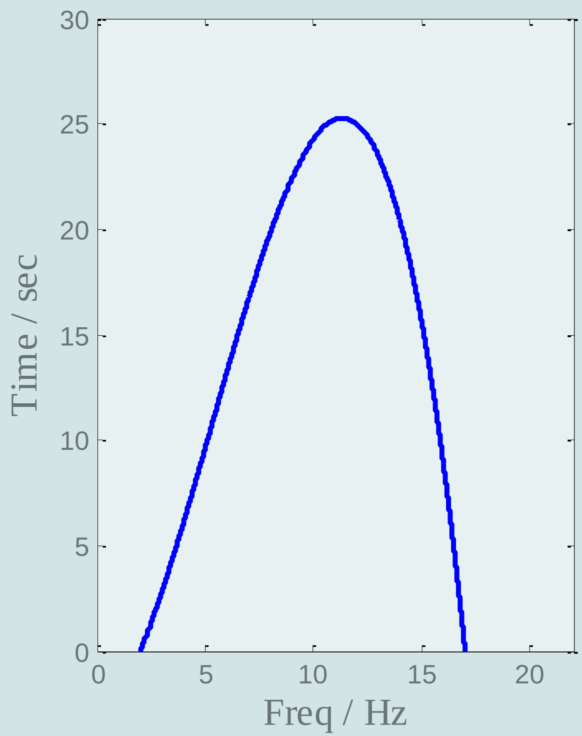


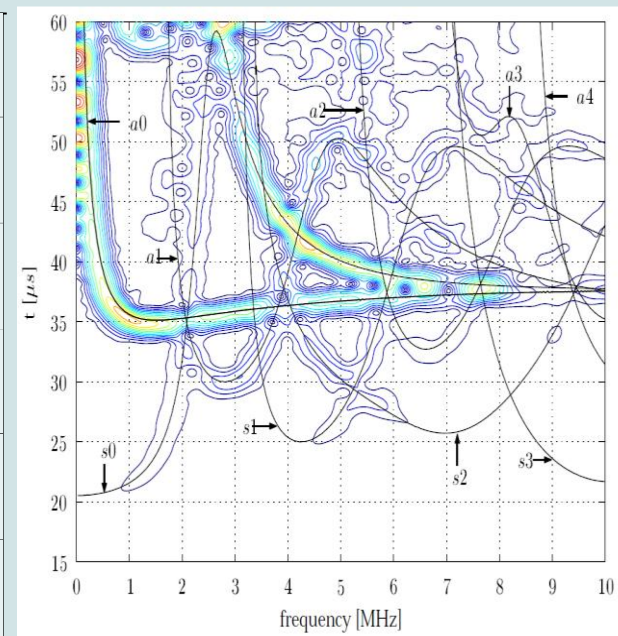
参数化时频分析-方法

如何刻画局部频率时延？

瞬时频率曲线　**局部频率时延曲线**　**例：频散曲线**







参数化时频分析-方法

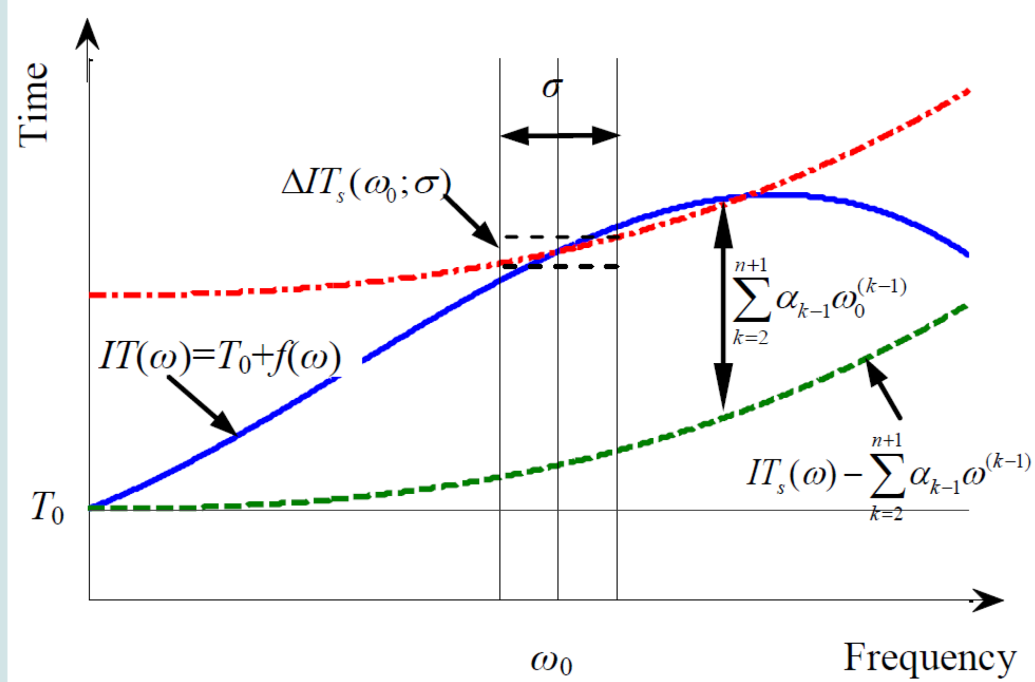
参数化频率时延分析

定义

$$\mathrm{PTFT}_x(\omega,t,\psi) = \int_{-\infty}^{+\infty} \overline{z}(\omega) g(\Omega-\omega) e^{j\Omega t} d\Omega$$

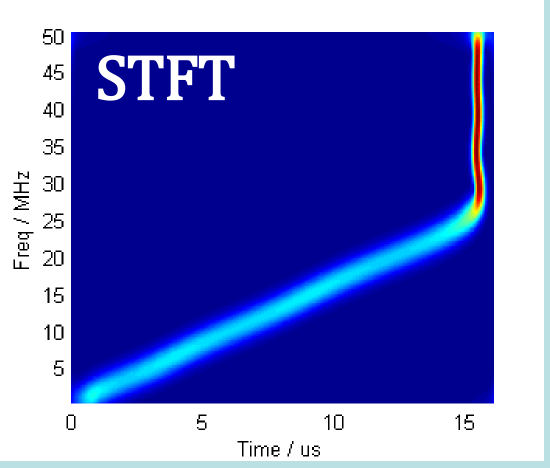
$$\begin{cases} \overline{z}(\omega) = \Phi^R \Phi^M X(\omega) \\ \Phi^M(\Omega,\omega) = e^{-j\psi(\Omega)\omega} \\ \Phi^R(\omega) = e^{j\left(\int_0^{\omega} \psi(\Omega) d\Omega\right)} \end{cases}$$

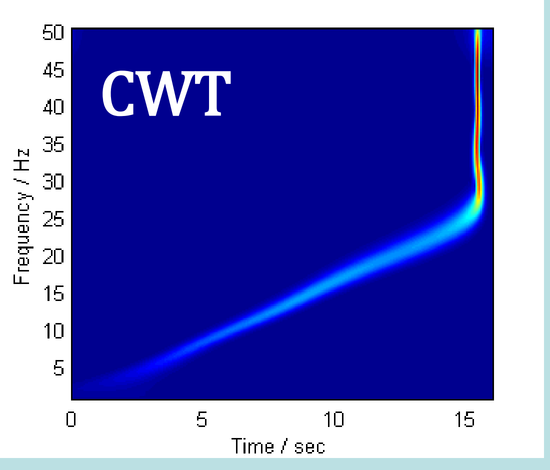
工作原理图解



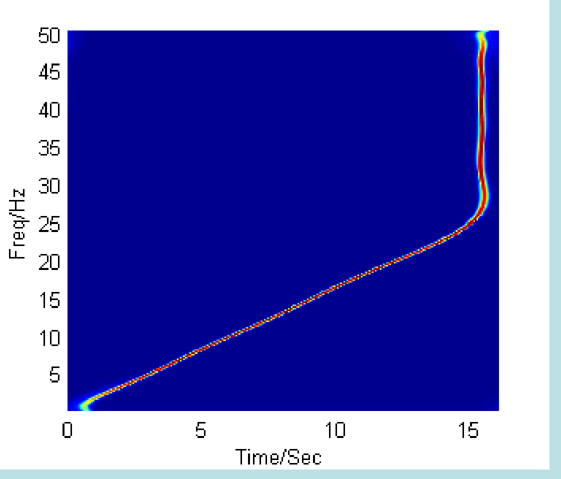
参数化时频分析-方法

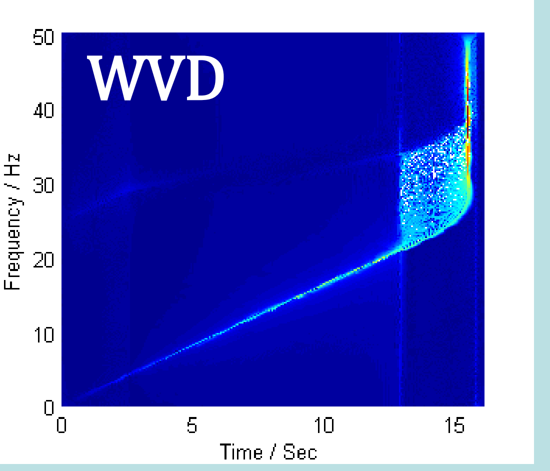
示例

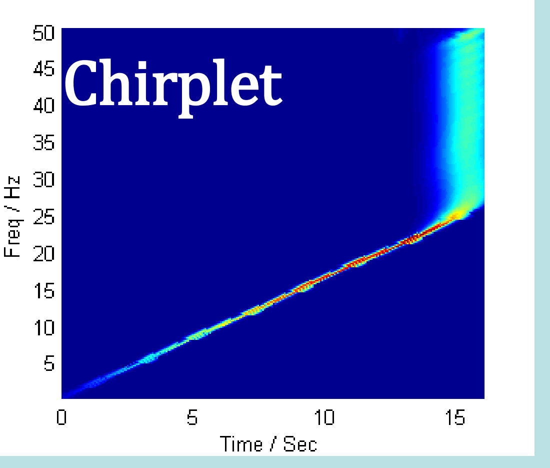




多项式频率时延变换



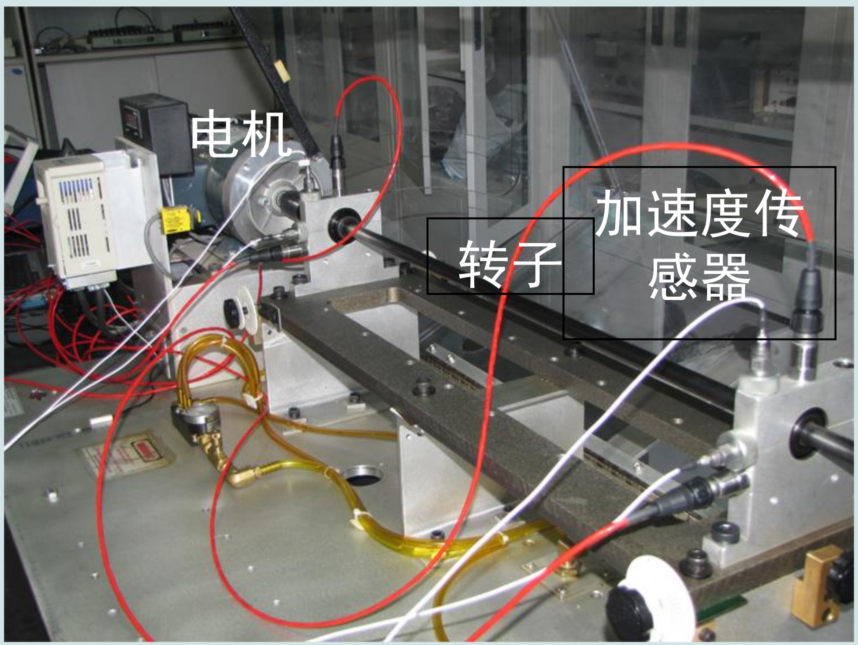




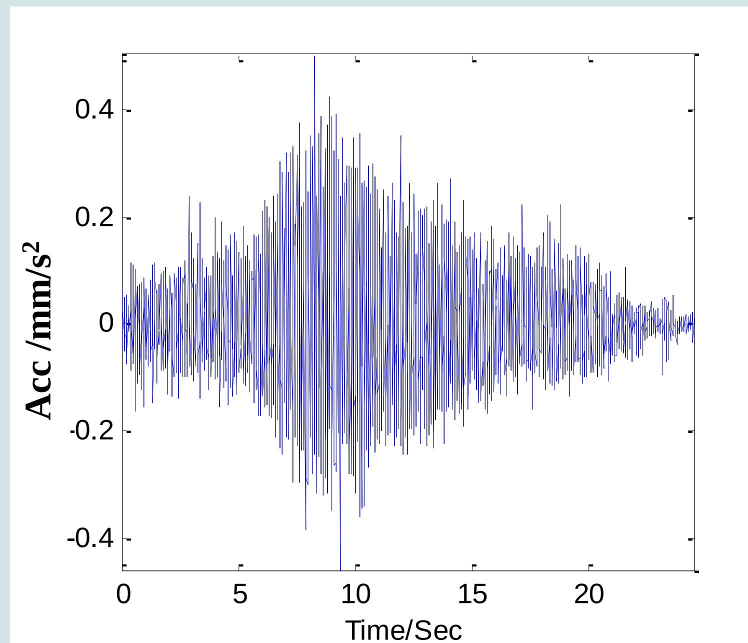
参数化时频分析–应用

应用一：转子瞬时转速估计

实验装置

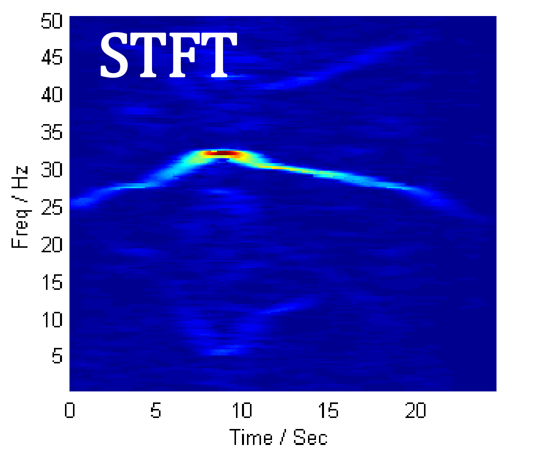


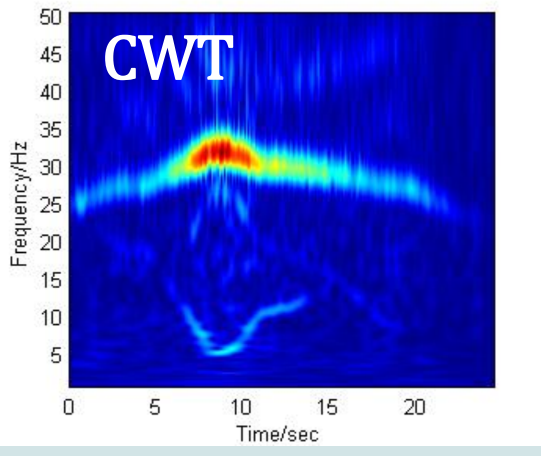
起停机过程信号



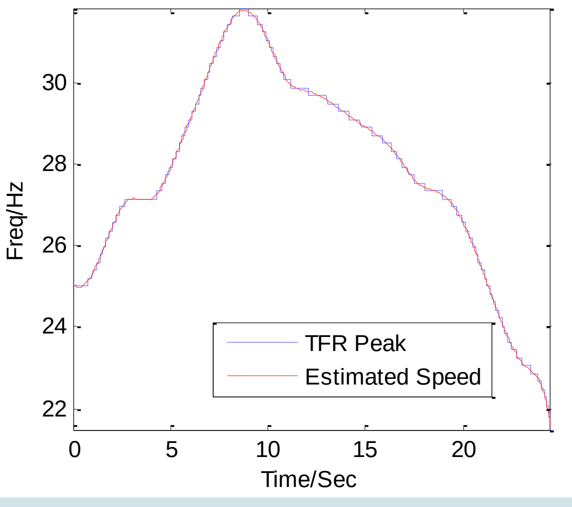
参数化时频分析-应用

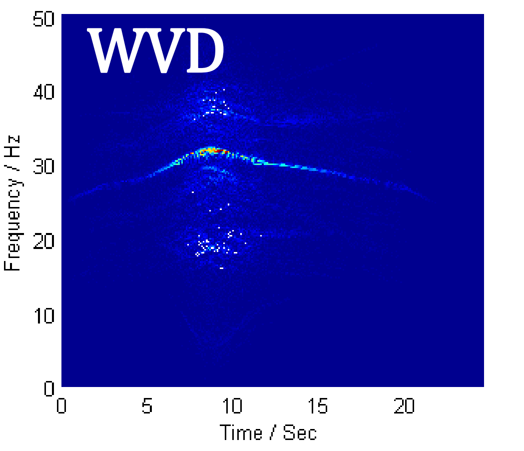
应用一：转子瞬时转速估计

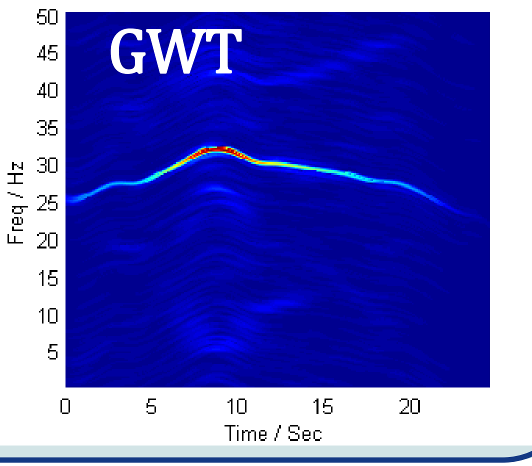




瞬时转速曲线



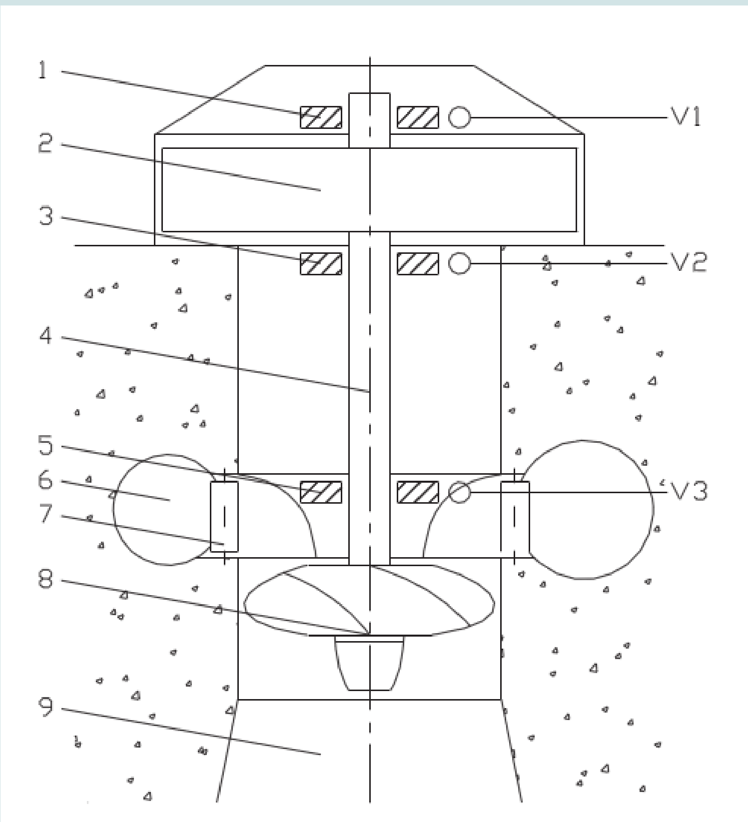




参数化时频分析-应用

应用二：水轮机振动信号精细时频特征提取

水轮机简图



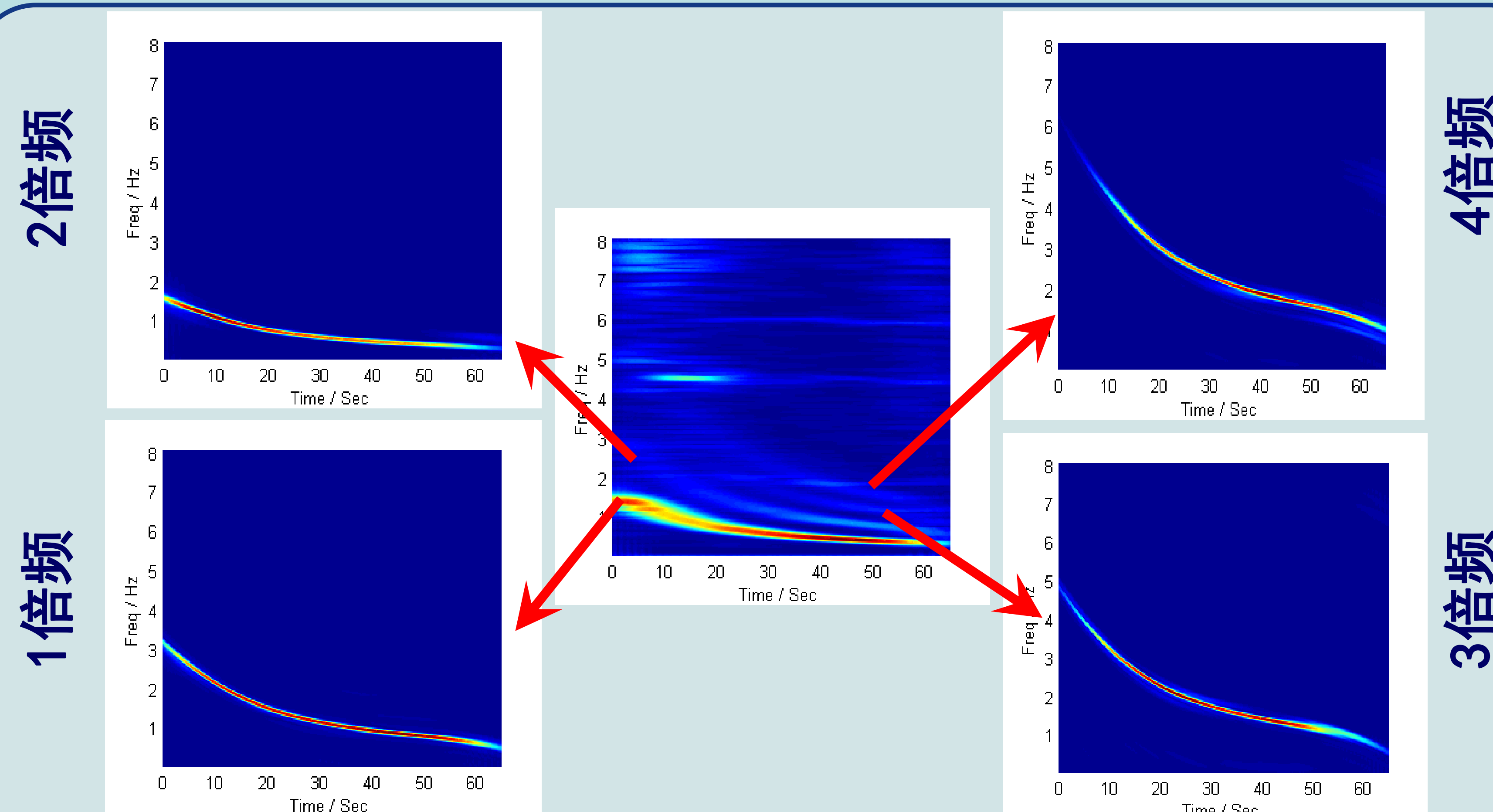
1— 上导轴承
2— 发电机转子
3— 推力轴承
4— 主轴
5— 水导轴承
6— 蜗壳
7— 导叶
8— 水轮机叶片
9— 尾水管

传感器测量位置

V1— 上导轴承
V2— 推力轴承
V3— 水导轴承

参数化时频分析-应用

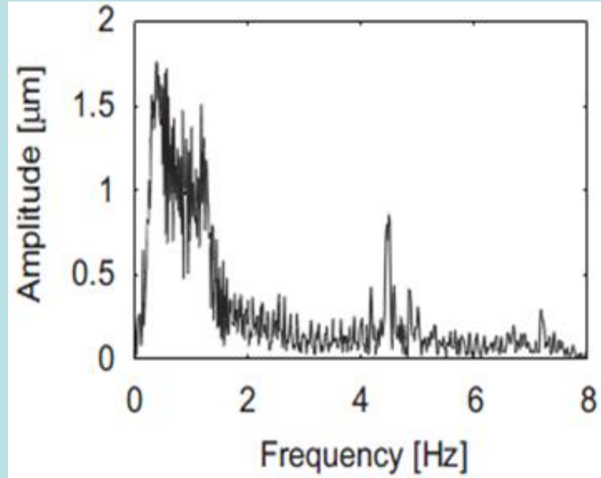
应用二：水轮机振动信号精细时频特征提取



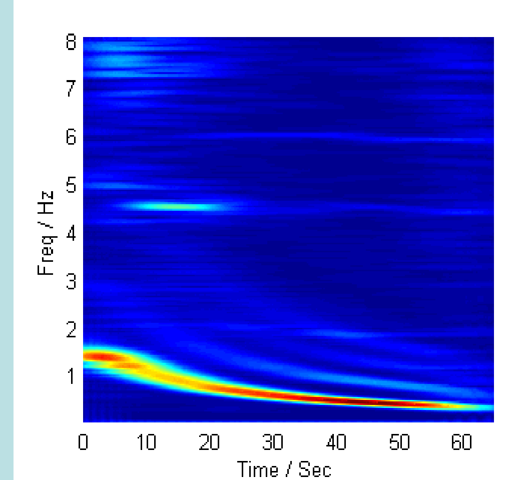
参数化时频分析–应用

应用二：水轮机振动信号精细时频特征提取

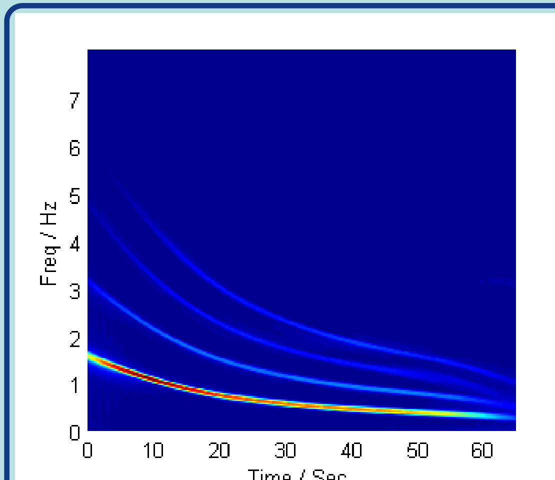
- ➢ 傅立叶变换频谱图**不能揭示**信号存在4个倍频分量；
- ➢ 短时傅立叶变换结果**模糊显示**信号存在4个倍频分量；
- ➢ 参数化时频分析结果**清晰显示**出了4个倍频分量的存在。

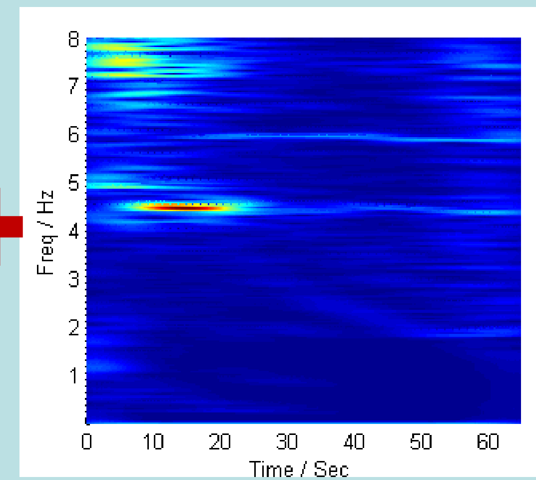


傅立叶变换



短时傅立叶变换





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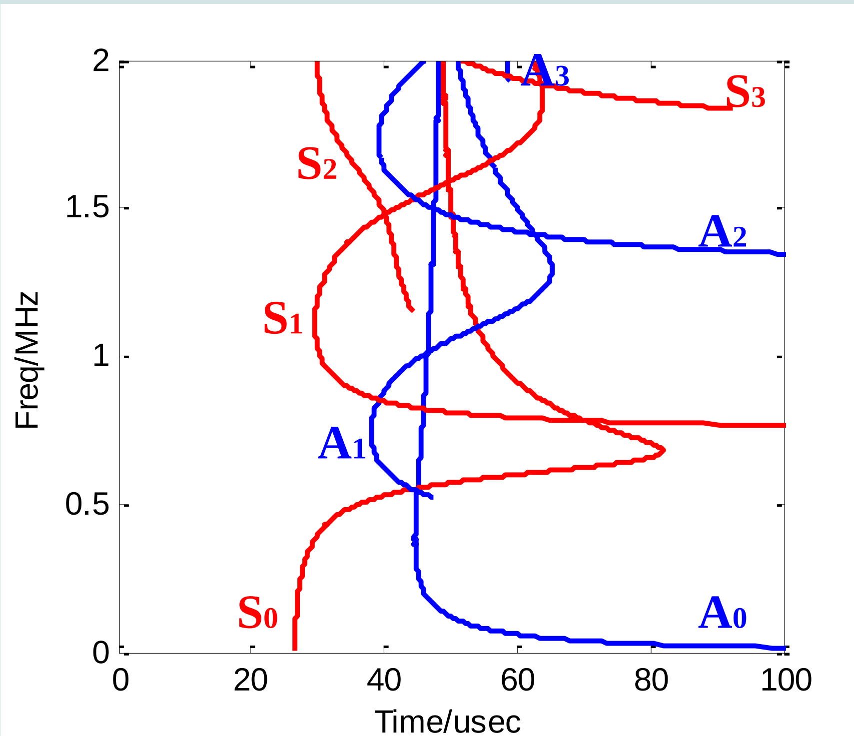
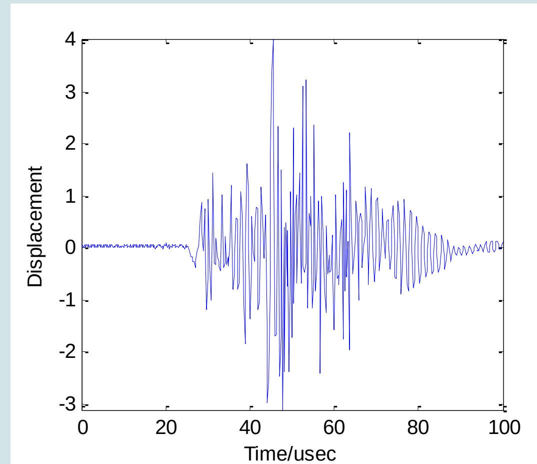
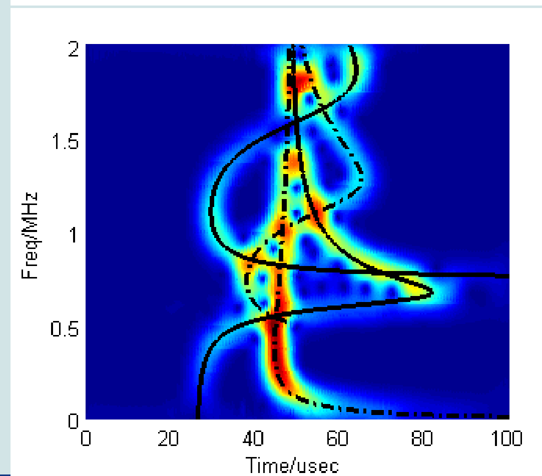
参数化时频分析–应用

应用三：Lamb波群延迟分析



群延迟曲线





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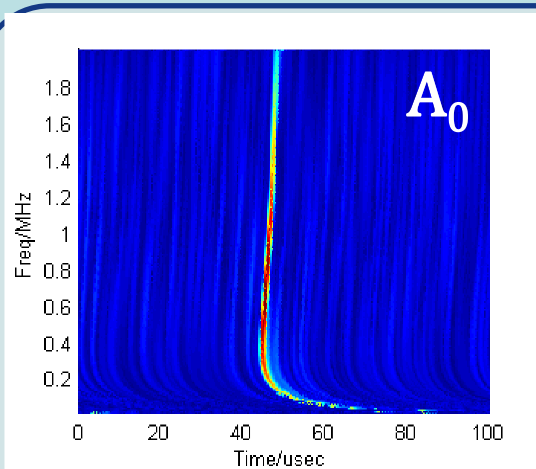
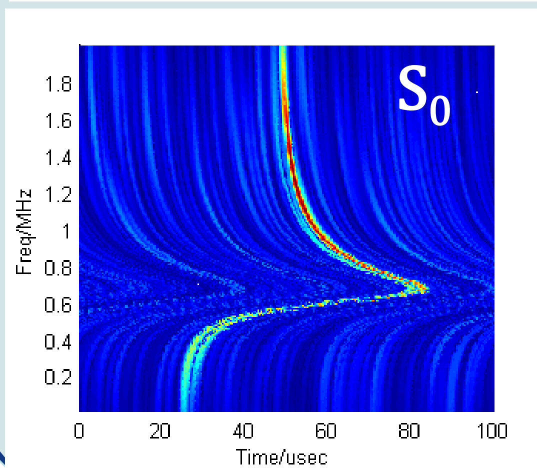
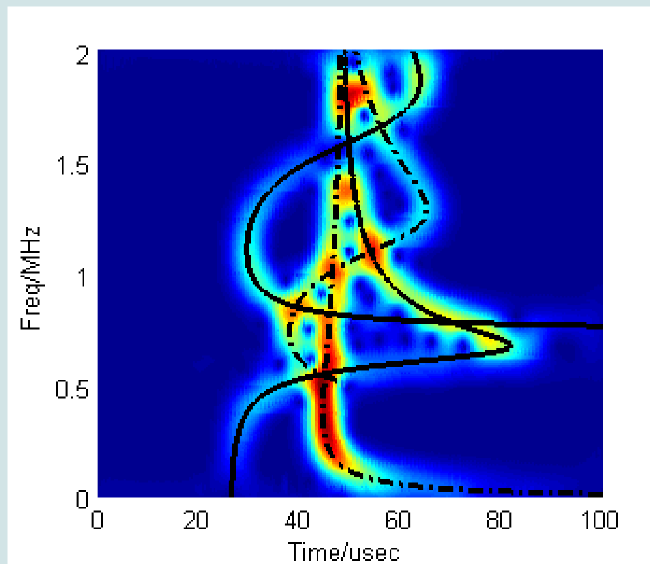
参数化时频分析–应用

应用三：Lamb波群延迟分析

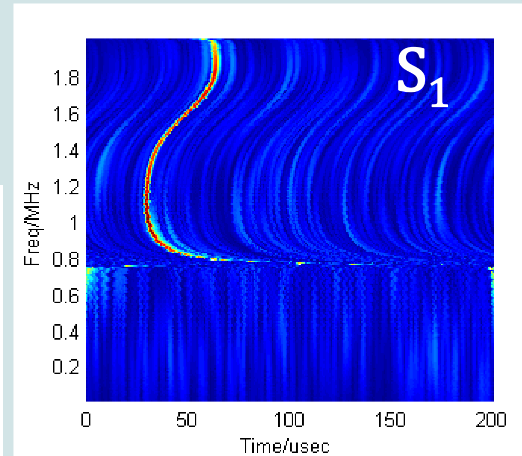


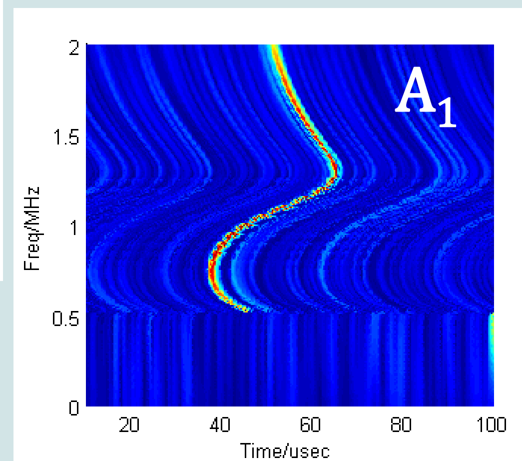


仿真信号



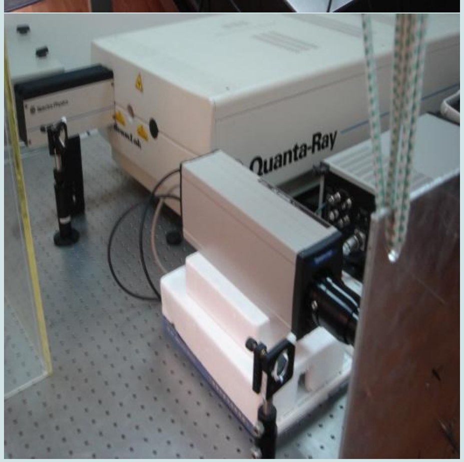
参数化频率时延分析

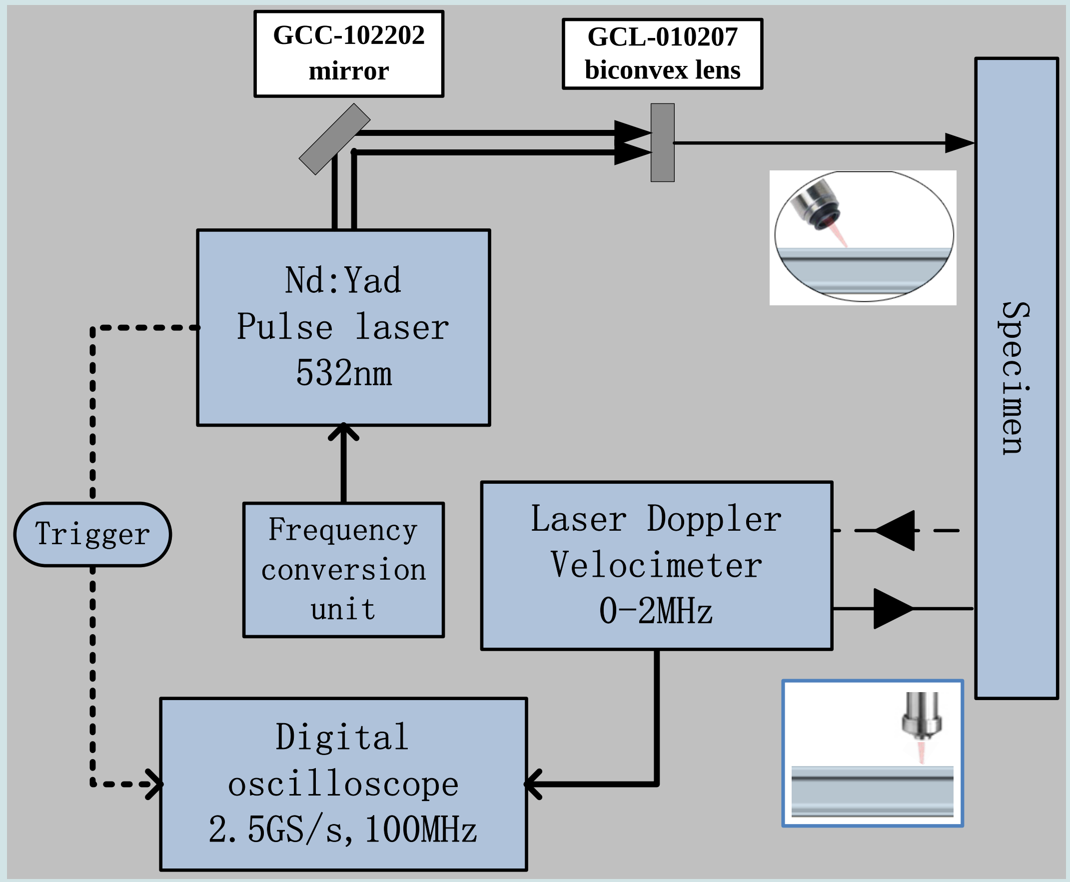




参数化时频分析-应用

应用三：Lamb波群延迟分析

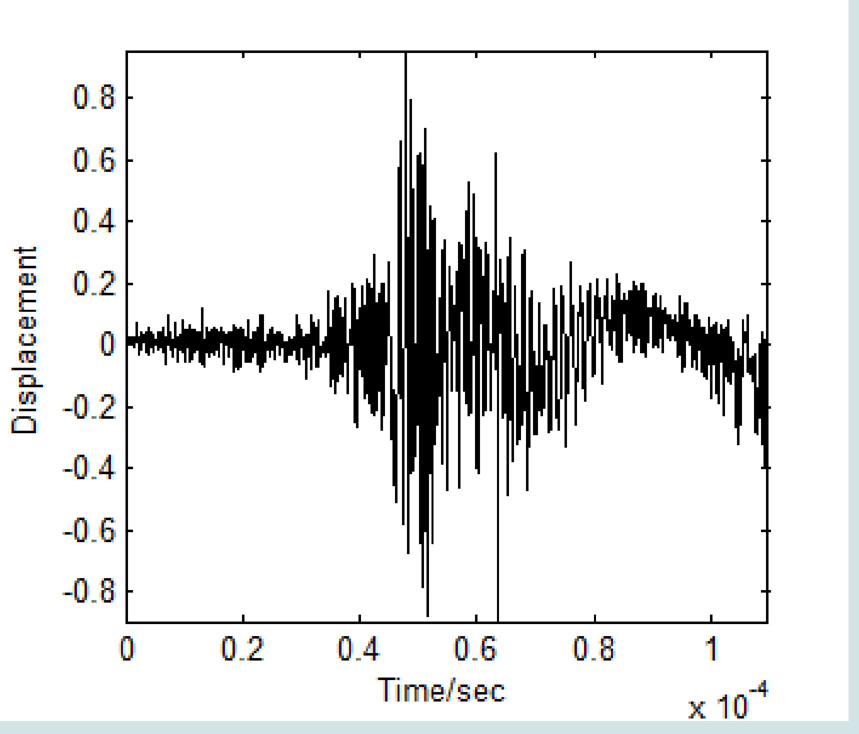




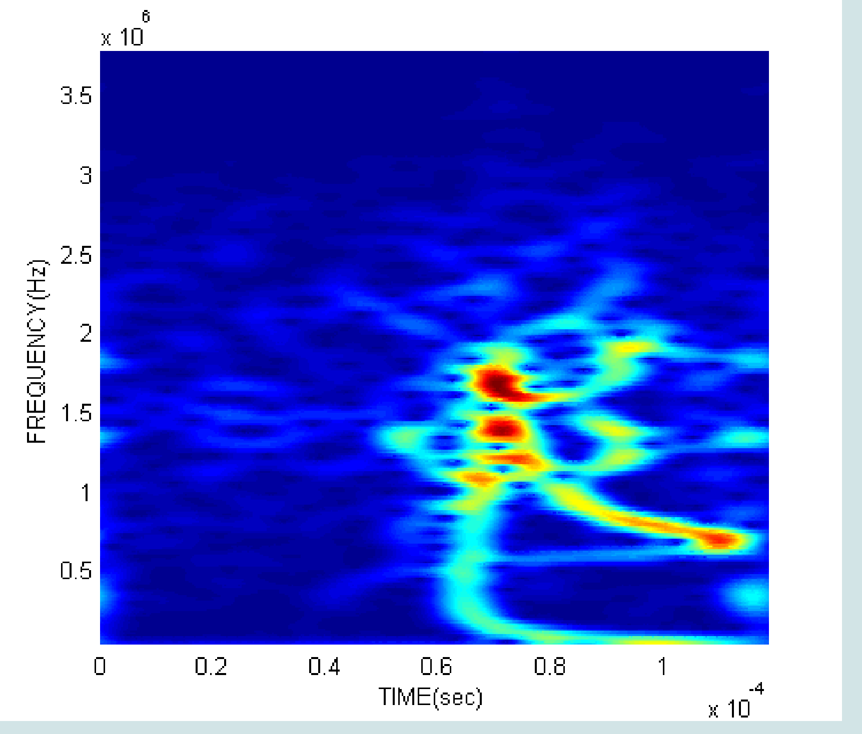
实验测试

参数化时频分析-应用

应用三：Lamb波群延迟分析



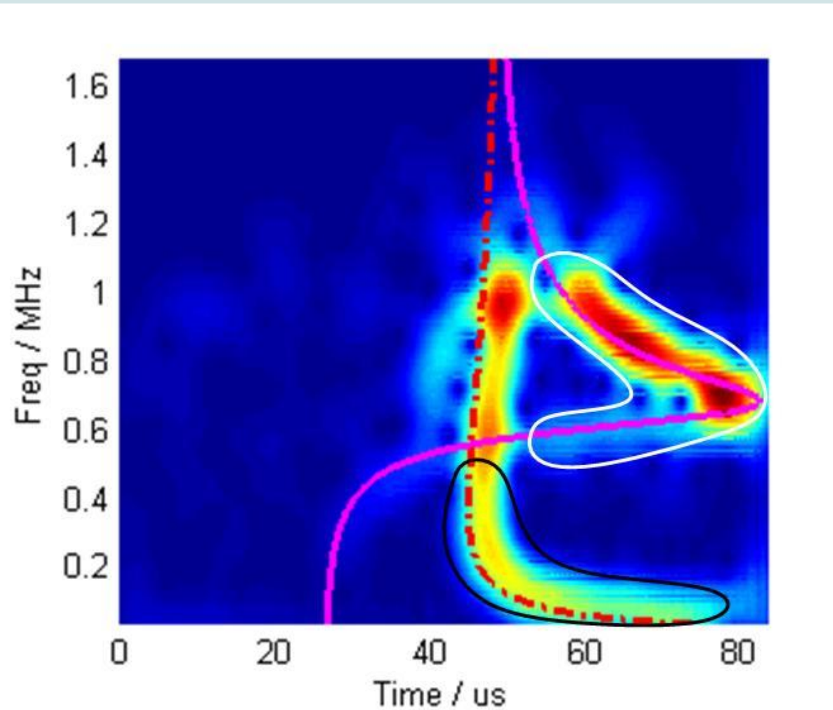
实验测试信号



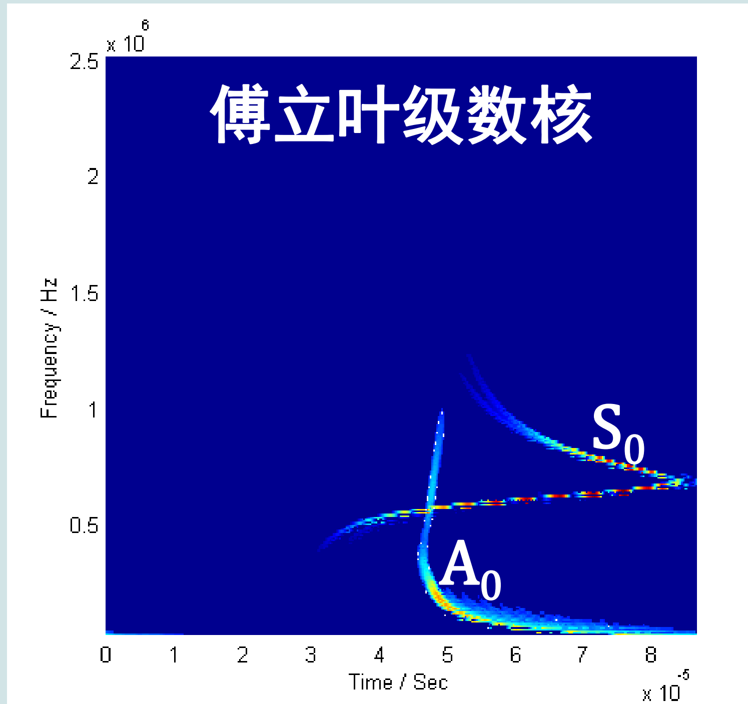
STFT

参数化时频分析–应用

应用三：Lamb波群延迟分析



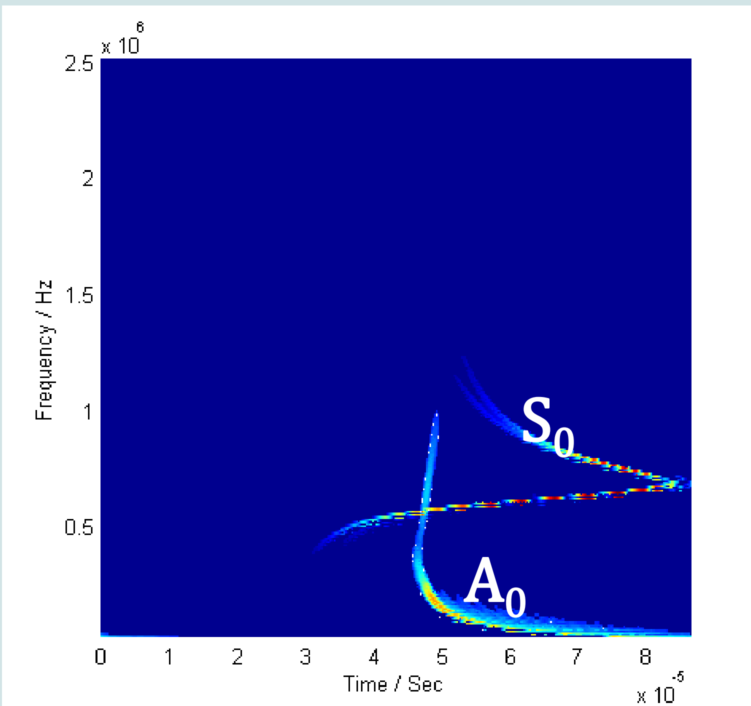
预处理后的STFT



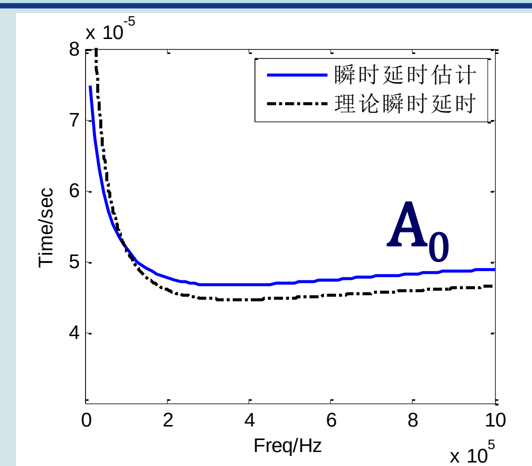
参数化频率时延分析结果

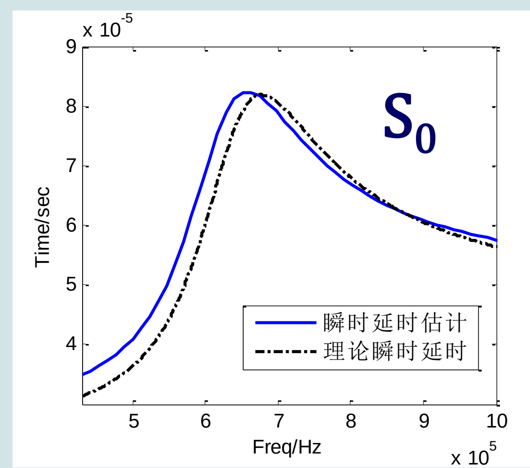
参数化时频分析–应用

应用三：Lamb波群延迟分析



参数化频率时延分析结果





谢谢聆听
欢迎交流