Modern Control Theory
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课程概要

- State Observers（5.5）
  - Full order state observers (5-5,Page 189-195)
  - Minimum order observers(5-5, Page 195-199)

- Observer based feedback controller design
- Decoupling of Control System

State Observers

基本概念

（1）状态反馈需要系统所有的状态信息，但系统的全部状态不一定都能直接测得；

（2）状态重构问题，采用输出y和输入u，重新构造状态 $\hat{x}$，使之与系统的真实状态x等价；

证明其可行性： $\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$ 完全能观 ⇨ 可根据y确定系统的初始状态x_0

⇨ $$x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$ ⇨

x(t)可以从y和u求得。（出发点）

State Observers

构造方法：

（1）开环状态观测器

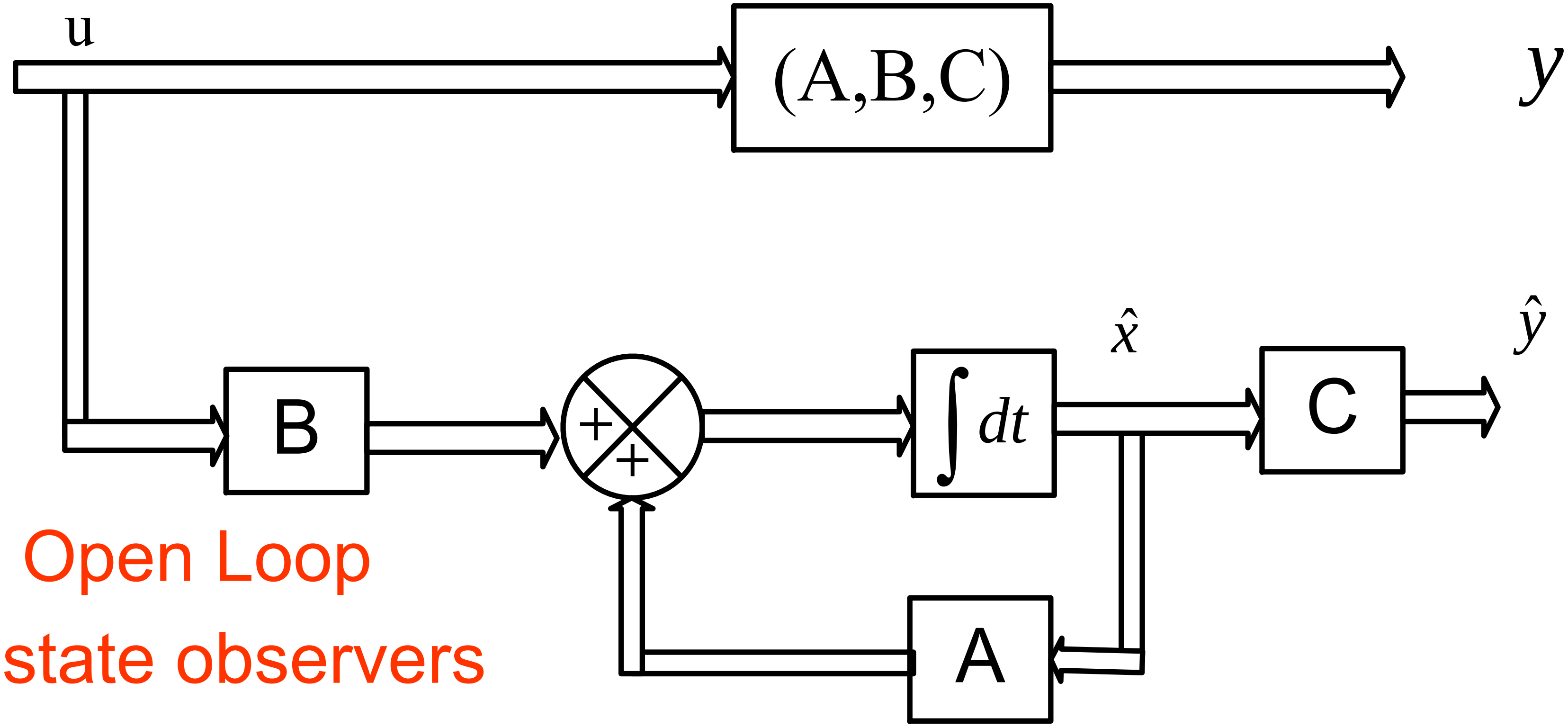
人为构造一个动态系统，以原系统的输入作为其输入，且两个系统在结构和参数基本相同。

$$\dot{\hat{x}} = A\hat{x} + Bu \qquad \dot{x} = Ax + Bu$$

$$\hat{y} = C\hat{x} \qquad y = Cx$$

$$\dot{x} - \dot{\hat{x}} = A(x - \hat{x}) \Rightarrow x - \hat{x} = e^{At}(x_0 - \hat{x}_0)$$

State Observers



Open Loop

state observers

结论1：由于很难使二者零状态相同，因此构造出的$\hat{x}$不可能和原系统等价。

结论2： $\lim_{t\to\infty}[x(t)-\hat{x}(t)]=0$ 要求？

State Observers

（2）另一种构造方法

$$y = Cx$$

$$\dot{y} = C\dot{x} = CAx + CBu$$

$$\ddot{y} = C\ddot{x} = CA^2x + CABu + CB\dot{u}$$

$$\dots$$

$$y^{(n-1)} = CA^{n-1}x + CA^{n-2}Bu + CA^{n-3}B\dot{u} + \dots + CBu^{(n-2)}$$

$$\begin{bmatrix} y \\ \dot{y} - CBu \\ \ddot{y} - CABu + CB\dot{u} \\ \dots \\ y^{(n-1)} - CA^{n-2}Bu - CA^{n-3}B\dot{u} - CBu^{(n-2)} \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \vdots \\ \vdots \\ CA^{n-1} \end{bmatrix} x$$

State Observers

结论：由于x(t)是由输入和输出测量值及各阶导数组成，导致干扰放大，因此观测器无法准确。

（3）对方法(1)改进，增加 $x-\hat{x}$ 的反馈，通常用 $y-\hat{y}$ 来代替。

$$\dot{\hat{x}} = A\hat{x} + Bu + G(y-\hat{y}) = A\hat{x} + Bu + Gy - GC\hat{x}$$

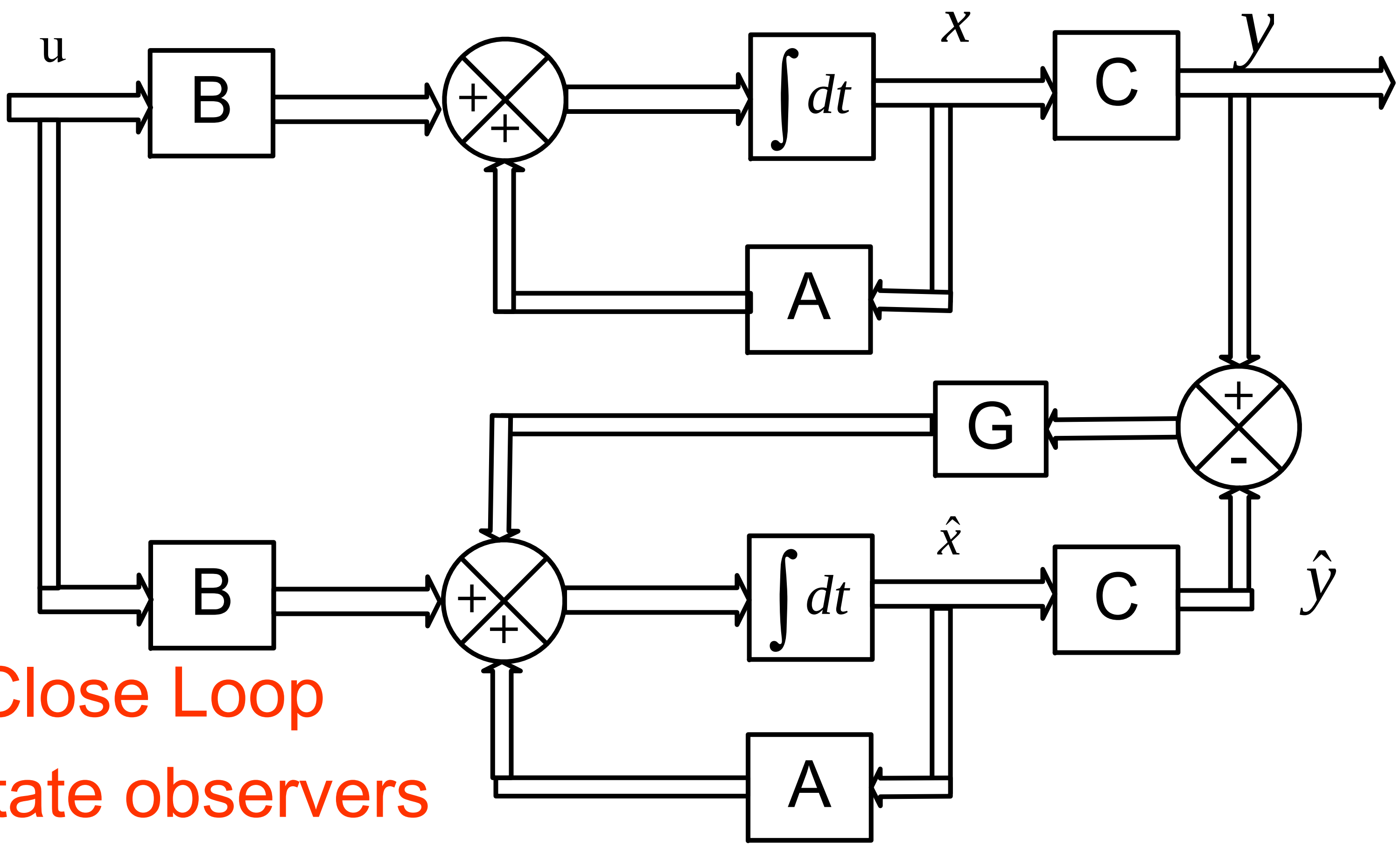
$$\dot{x} = Ax + Bu$$

$$\dot{\hat{x}} - \dot{x} = A(\hat{x}-x) - GC(\hat{x}-x)$$

$$= (A-GC)(\hat{x}-x) \quad \Rightarrow \quad (\hat{x}-x) = e^{(A-GC)}(\hat{x}_0 - x_0)$$

可通过G的设计加快逼近的速度

State Observers



Close Loop

state observers

State Observers

- initial error e(0) is not necessary equal to zero.
- The convergence speed is determined by the pole placement of A-GC;
- Normally the poles of A-GC is far away from I-axis of s-place.

State Observers

状态观测器存在定理

线性定常系统 $\Sigma_o=(A,B,C)$ 的状态观测器存在的充要条件是 Σ_o 的不能观子系统为渐进稳定。

证明：设 Σ_o 不能完全能观，则对其进行能观性分解，设已经进行了能观性分解，形成如下型式：

$$x=\begin{bmatrix} x_o \\ \hline x_{\bar{o}} \end{bmatrix}\begin{matrix} n_1 \\ \\ (n-n_1) \end{matrix} \qquad A=\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \qquad B=\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \qquad C=\begin{bmatrix} C_1 & 0 \end{bmatrix}$$

（A_{11},B_1,C_1)-能观子系统

（A_{22},B_2,0）-不能观子系统。

State Observers

根据状态观测器方程，G=[G$_1$,G$_2$]^T,则：

$$\dot{\hat{x}} = (A - GC)\hat{x} + Bu + GCx$$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}} = \begin{bmatrix} \dot{x}_o - \dot{\hat{x}}_o \\ \dot{x}_{\bar{o}} - \dot{\hat{x}}_{\bar{o}} \end{bmatrix} =$$

$$\begin{bmatrix} A_{11}x_o + B_1 u \\ A_{21}x_o + A_{22}x_{\bar{o}} + B_2 u \end{bmatrix} - \begin{bmatrix} (A_{11} - G_1C_1)\hat{x}_o + B_1 u + G_1C_1x_o \\ (A_{21} - G_2C_1)\hat{x}_o + A_{22}\hat{x}_{\bar{o}} + B_2 u + G_2C_1x_o) \end{bmatrix}$$

$$= \begin{bmatrix} (A_{11} - G_1C_1)(x_o - \hat{x}_o) \\ (A_{21} - G_2C_1)(x_o - \hat{x}_o) + A_{22}(x_{\bar{o}} - \hat{x}_{\bar{o}}) \end{bmatrix}$$

State Observers

确定使 $\hat{x}$ 渐进于 x 的条件

$$x_o - \hat{x}_o = e^{(A_{11} - G_1 C_1)t} [x_o(0) - \hat{x}_o(0)]$$

$$x_{\bar{o}} - \hat{x}_{\bar{o}} = e^{(A_{22})t} [x_{\bar{o}}(0) - \hat{x}_{\bar{o}}(0)] + \cdots$$

结论：不能观的部分渐进稳定才可通过G使观测值逼近x。

The Derivation of G

状态观测器定义

设线性定常系统（A,B,C）的状态是不能直接测量的，如果动态系统 $\hat{\Sigma}(A,B,C)$ 以原系统 Σ 的输入和输出作为它的输入量，且其输出满足如下的等价指标：

$$\lim_{t\to\infty}[x(t)-\hat{x}(t)]=0$$

则称动态系统 $\hat{\Sigma}$ 是系统 Σ 的状态观测器

(1) 观测器以原系统的输入和输出作为输入

(2) 原系统应该是完全能观测的，或者不能观部分是渐进稳定的

The Derivation of G

状态观测器$\hat{\Sigma}(A,B,C)$ 应该有较宽的频带，以使观测器可足够快的逼近x

状态观测器 $\hat{\Sigma}(A,B,C)$ 还应该有较窄的频带应抵抗干扰，应此这是矛盾的一对指标；

状态观测器 $\hat{\Sigma}(A,B,C)$ 应该近可能简单，即具有尽可能低的维数。

例题：已知系统

$$\dot{x}=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}x+\begin{bmatrix}1\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}2 & -1\end{bmatrix}x$$

设计状态观测器使其极点为－10，－10。

The Derivation of G

Step1. 化为能观标准II型

$$T_{o2}^{-1}=\begin{bmatrix} e_1 & e_2 & \cdots & e_n \end{bmatrix} = \begin{bmatrix} 1 & a_{n-1} & \cdots & a_2 & a_1 \\ 0 & 1 & \cdots & a_3 & a_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & a_{n-1} \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} CA^{n-1} \\ CA^{n-2} \\ \vdots \\ CA \\ C \end{bmatrix}$$

$$\overline{A}=T_{o2}^{-1}AT_{o2}=\begin{bmatrix} 0 & 0 & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ & 1 & \cdots & 0 & -a_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -a_{n-1} \end{bmatrix}$$

$$\overline{C}=CT_{o2}=\begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\overline{b}=T_{o2}^{-1}b=\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \end{bmatrix}$$

The Derivation of G

Step2. 引入反馈阵

$$\overline{G}=\begin{bmatrix}\overline{g}_0\\ \overline{g}_1\\ \vdots\\ \overline{g}_{n-1}\end{bmatrix} \quad \overline{A}-\overline{G}\,\overline{C}=\begin{bmatrix}0 & \cdots & \cdots & 0 & -(a_0+\overline{g}_0)\\ 1 & 0 & \cdots & 0 & -(a_1+\overline{g}_1)\\ 0 & 1 & \cdots & 0 & -(a_2+\overline{g}_2)\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & \cdots & 1 & -(a_{n-1}+\overline{g}_{n-1})\end{bmatrix}$$

$$f(\lambda)=\left|\lambda I-(\overline{A}-\overline{G}\,\overline{C})\right|$$

$$=\lambda^n+(a_{n-1}+\overline{g}_{n-1})\lambda^{n-1}+\cdots+(a_1+\overline{g}_1)\lambda+(a_0+\overline{g}_0)$$

The Derivation of G

Step3. 由期望极点得到期望特征多项式

$$f^*(\lambda) = \lambda^n + a_{n-1}{}^*\lambda^{n-1} + \cdots + a_1{}^*\lambda + a_0{}^*$$

$$= \prod_{i=1}^{n} (\lambda - \lambda_i^*)$$

Step4. 比较特征多项式系数

$$\overline{G} = \begin{bmatrix} a_0^* - a_0 & a_1^* - a_1 & \cdots & a_{n-1}^* - a_{n-1} \end{bmatrix}^T$$

The Derivation of G

Step5. 变换回原系统下

$$G = T_{o2}\overline{G}$$

Example:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Design a full order state observer and the desired eigenvalues of observer matrix are

$$A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mu_1 = -10 \ \ \mu_2 = -10$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The Derivation of G

Step1: examine the observability

$$N = \begin{bmatrix} C^* & A^* C^* \end{bmatrix}$$

$$N = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Obviously rank (N) =2

Step2: Transform to observable Canonical form, *Q=I*

Find a_i and a_i^*

$$|sI - A| = \begin{vmatrix} s & -20.6 \\ -1 & s \end{vmatrix} = s^2 - 20.6$$

$a_1 = 0 \quad a_0 = -20.6$

The Derivation of G

$$(s-\mu_1)(s-\mu_2) = s^2 + 20s + 100 \Rightarrow \begin{aligned} a_0^* &= 100 \\ a_1^* &= 20 \end{aligned}$$

Step3: Find *G* $G = Q\begin{bmatrix} a_0^* - a_0 \\ a_1^* - a_1 \end{bmatrix} = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$

Step4: The State Observer Representation ★

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu + G(y - \tilde{y})$$

$$= (A - GC)\tilde{x} + Bu + Gy$$

Step5: The Analog computer Implementation

Direct Substitution Approach to Obtain State Observer Gain Matrix K_e

- A simple method for n<=3

For a 3 order system, suppose $G = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \end{bmatrix}$

- Substituting of matrix K into the desired characteristic polynomial:

$$|sI - A + GC| = (s-\mu_1)(s-\mu_2)(s-\mu_3)$$

- By equating the coefficients of the like powers of s on both sides to determine G

Direct Substitution Approach to Obtain State Observer Gain Matrix G

Example:

$$\dot{x} = Ax + Bu \quad A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$y = Cx \qquad \mu_1 = -10 \quad \mu_2 = -10$$

$$G = \begin{bmatrix} g_0 \\ g_1 \end{bmatrix} \qquad \left| sI - A + GC \right| = 0$$

$$\left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} g_0 \\ g_1 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \right| = \begin{vmatrix} s & -20.6 + g_0 \\ -1 & s + g_1 \end{vmatrix}$$

$$= s^2 + g_1 s - 20.6 + g_0 = 0 \qquad G = \begin{bmatrix} g_0 \\ g_1 \end{bmatrix} = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

Matlab Solution

U=-Kx针对单输入系统，具有指定的极点p.

K=acker(A’,C’,P)

A’:A 的转置；

C’：C的转置；

P：制定的闭环极点

求出的K’＝G

Matlab Solution

$$\dot{x} = Ax + Bu \quad \dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad \mu_1 = -10 \quad \mu_2 = -10$$

$$y = Cx \qquad y = \begin{bmatrix} 2 & 0 \end{bmatrix} x$$

```
A=[0 1;-2 -3];
b=[0;1]
C=[2,0];
ob=obsv(A,C)
rankob=rank(ob)
rankob =    2
```

```
p1=[-10,-10];
a1=A';b1=C';c1=b';
k=acker(a1,b1,p1)
k =
    8.5000   23.5000
```

Matlab Solution

```
>> G=k'

G =

    8.5000

   23.5000
```

Comments on Selecting Best G

- Normally, the eigenvalues of G should be two to five times faster than the controller poles to make sure the observer estimation error converges to zero quickly.
- A large G will amplify the measurement noise, so the observer poles should be slower than two times the controller poles to make sure the bandwidth of the system will become lower and smooth the noise.
- Best G will make a compromise between speed response and sensitivity to disturbance and noises

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Minimum Order Observer

系统的状态降阶观测器

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

T变换阵

$$\begin{bmatrix} \dot{x}_1 \\ \hline \dot{x}_2 \end{bmatrix} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \begin{bmatrix} x_1 \\ \hline x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix} u$$

$$y = \left[0 \mid I\right] \begin{bmatrix} x_1 \\ \hline x_2 \end{bmatrix} = x_2$$

C为满秩；

x_1为n-m维向量，x_2为m维向量

A_{11}：（n-m）×（n-m）; A_{12}：（n-m）× m; B1: (n-m）× r

A_{21}：（m）×（n-m）； A_{22}：m× m; B2:m×r

Minimum Order Observer

在能检测型式下，x2已经输出为y，则只需要估计x1的n-m维向量，该控制器就是原系统的降阶观测器。

$$\dot{x}_1 = A_{11}x_1 + (A_{12}y + B_1 u)$$
$$\dot{y} = A_{21}x_1 + A_{22}y + B_2 u$$

只考虑x1

$$\dot{x}_1 = A_{11}x_1 + (A_{12}y + B_1 u)$$
$$y_1 = A_{21}x_1 = \dot{y} - A_{22}y - B_2 u$$

状态观测器为

Minimum Order Observer

$$\dot{\hat{x}}_1 = (A_{11} - HA_{21})\hat{x}_1 + (A_{12}y + B_1u) + Hy_1$$

H为(n-m）× m维矩阵，观测器的输出就是x1

$$\dot{\hat{x}}_1 = (A_{11} - HA_{21})\hat{x}_1 + (A_{12}y + B_1u) + H\dot{y} - H(A_{22}y + B_2u)$$

$$z_1 = \hat{x}_1 - Hy$$

$$\dot{z}_1 = \dot{\hat{x}}_1 - H\dot{y}$$

$$\dot{z}_1 = (A_{11} - HA_{21})z_1 + (B_1 - HB_2)u + [(A_{11} - HA_{21})H + A_{12} - HA_{22}]y$$

Minimum Order Observer

$$\hat{x}_1 = z_1 + Hy$$

例题：设系统

$$\dot{x} = \begin{bmatrix} 0 & 0 & 4 \\ 1 & 0 & -12 \\ 1 & 1 & 5 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$

试选择状态反馈矩阵G，使极点配置为－3，－4

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Observer Based Controller Design

利用状态观测器实现状态反馈的系统

基本概念：采用全维状态观测器输出系统的状态，并利用重构的状态对原系统反馈。

原系统：
$$\dot{x} = Ax + Bu$$
$$y = Cx$$

状态观测器：
$$\dot{\hat{x}} = (A - GC)\hat{x} + Bu + GCx$$

反馈控制律：
$$u = v + K\hat{x}$$

Observer Based Controller Design

代入整理，得

$$\dot{x} = Ax + BK\hat{x} + Bv$$

$$\dot{\hat{x}} = GCx + (A - GC + BK)\hat{x} + Bv$$

$$y = Cx$$

写成矩阵型式：

$$\begin{bmatrix} \dot{x} \\ \hline \dot{\hat{x}} \end{bmatrix} = \left[\begin{array}{c|c} A & BK \\ \hline GC & A - GC + BK \end{array}\right] \begin{bmatrix} x \\ \hline \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ \hline B \end{bmatrix} v$$

$$y = \left[\begin{array}{c|c} C & 0 \end{array}\right] \begin{bmatrix} x \\ \hline \dot{x} \end{bmatrix}$$

Observer Based Controller Design

基本特性

1.闭环极点设计的分离性

闭环极点包括直接状态反馈系统 Σ_K 的极点和观测器 Σ_G 的极点两部分，二者独立，相互分离。

考虑如下非奇异变换：

$$T=\left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right] \Rightarrow T^{-1}=\left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right]^{-1}=\left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right]$$

Observer Based Controller Design

变换后，有

$$\begin{bmatrix} x \\ \hline \tilde{x} \end{bmatrix} = \left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right] \begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} x \\ \hline x - \hat{x} \end{bmatrix}$$

$$\bar{A}_1 = T^{-1} A_1 T = \left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right] \left[\begin{array}{c|c} A & BK \\ \hline GC & A - GC + BK \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right]$$

$$= \left[\begin{array}{c|c} A + BK & BK \\ \hline 0 & A - GC \end{array}\right]$$

Observer Based Controller Design

$$\left[\begin{array}{c} x \\ \hline \tilde{x} \end{array}\right] = \left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right] \left[\begin{array}{c} x \\ \hat{x} \end{array}\right] = \left[\begin{array}{c} x \\ \hline x - \hat{x} \end{array}\right]$$

$$\overline{B}_1 = T^{-1} B_1 = \left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right] \left[\begin{array}{c} B \\ \hline B \end{array}\right] = \left[\begin{array}{c} B \\ \hline 0 \end{array}\right]$$

$$\overline{C}_1 = C_1 T = \left[C \mid 0\right] \left[\begin{array}{c|c} I & 0 \\ \hline I & -I \end{array}\right] = \left[C \mid 0\right]$$

$$\dot{x} = (A + BK)x + Bv + BK\tilde{x}$$

$$\dot{\tilde{x}} = (A - GC)\tilde{x}$$

$$y = Cx$$

Observer Based Controller Design

$$\det(sI-\bar{A}_1)=\det\left[\begin{array}{c|c} sI-(A+BK) & -BK \\ \hline 0 & sI-(A-GC) \end{array}\right]$$

$$=\det[sI-(A+BK)]\bullet\det[sI-(A-GC)]$$

结论：由观测器构成的状态反馈系统，闭环极点等于直接状态反馈（A+BK）的极点和状态观测器（A-GC）的极点之和。故状态反馈阵和观测器反馈阵可分别进行设计。

2.传递函数矩阵的不变性

Observer Based Controller Design

$$Q=\left[\begin{array}{c|c} R & s \\ \hline 0 & T \end{array}\right] \Rightarrow Q^{-1}=\left[\begin{array}{c|c} R^{-1} & -R^{-1}sT^{-1} \\ \hline 0 & T^{-1} \end{array}\right]$$

$$\Downarrow$$

$$W(s)=\overline{C}_1[sI-\overline{A}_1]^{-1}\overline{B}_1$$

$$=[C \quad 0]\left[\begin{array}{c|c} [sI-(A+BK)]^{-1} & [sI-(A+BK)]^{-1}\bullet BK\bullet[sI-(A-GC)]^{-1} \\ \hline 0 & [sI-(A-GC)]^{-1} \end{array}\right]\begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$=C[sI-(A+BK)]^{-1}B$$

Observer Based Controller Design

结论：观测器的极点由闭环零点对消，因此系统是不完全能控的，但是不能控的部分是估计误差，因此不影响系统的正常工作。

3.观测器反馈与直接状态反馈的等效性

$$\dot{x} = Ax + BK\hat{x} + Bv$$

$$\dot{\hat{x}} = GCx + (A - GC + BK)\hat{x} + Bv$$

$$y = Cx$$

通过选择A-GC的极点，可使t—>∞，x的估计值等于x。

Observer Based Controller Design

4.观测器反馈与带补偿器的输出反馈系统等效。

例题：

设受控系统的传递函数为 $W_0(s)=\dfrac{1}{s(s+6)}$

用状态反馈系统将闭环系统配置在－4±j6, 并实现上述反馈的全维和降维观测器（设极点为－10，－10）

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Decoupling of Control System

系统解耦问题

使输入和输出相互关联的多变量系统实现一个输出对应一个输入，一个输入也仅能对应一个输出，该问题称为解耦问题。

问题描述

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$W_0(s) = C(sI - A)^{-1}B = \begin{bmatrix} W_{11}(s) & & & 0 \\ & W_{22}(s) & & \\ & & \ddots & \\ 0 & & & W_{mm}(s) \end{bmatrix}$$

Decoupling of Control System

解耦后的系统，可被看作是一组相互独立的单变量系统，控制规模下降，系统简单。

方法1：采用串连前馈补偿器的方法，使组合系统的输入、输出呈现解耦形态，将使系统的维数增加。

$W_0(s)$----待解耦系统的传递函数阵

$W_d(s)$----前馈补偿器的传递函数阵

组合阵： $W(s)=W_0(s)W_d(s)$

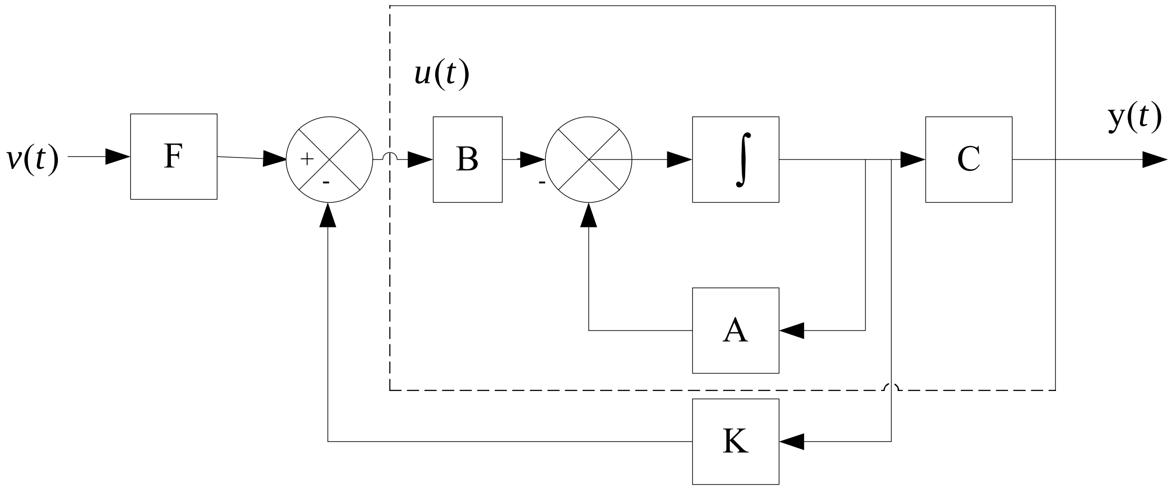
Decoupling of Control System

原系统满秩，则可直接求

$$W_d(s) = W_0(s)^{-1} W(s)$$

方法2、状态反馈解耦

问题描述：



Decoupling of Control System

K:m×n实常数反馈矩阵

F:m×m实常数非奇异矩阵

通过设计K、F使输入v和输出y解耦。

几个定义：

1.d_i:整数。满足下式：　$c_i A^l B \neq 0 \quad (l = 0,1 \ldots, m-1)$

(c_i:C的第i行向量)的最小的 l

Decoupling of Control System

例题：

已知系统 $\Sigma_0(A,B,C)$

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 3 & 0 & 0 & 2\\ 0 & 0 & 0 & 1\\ 0 & -2 & 0 & 0\end{bmatrix}\qquad B=\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 0\\ 0 & 1\end{bmatrix}\qquad C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

试计算di(i=1,2)

Decoupling of Control System

2)根据di定义下列矩阵

$$D=\begin{bmatrix} c_1A^{d_1} \\ c_2A^{d_2} \\ \vdots \\ c_mA^{d_m} \end{bmatrix} \qquad E=DB=\begin{bmatrix} c_1A^{d_1}B \\ c_2A^{d_2}B \\ \vdots \\ c_2A^{d_2}B \end{bmatrix}$$

$$L = DA = \begin{bmatrix} c_1A^{d_1+1} \\ c_2A^{d_2+1} \\ \vdots \\ c_mA^{d_m+1} \end{bmatrix}$$

Decoupling of Control System

2)试计算上题的D、E、L。

能解耦性判据

受控系统 $\Sigma_0=(A,B,C)$ 采用状态反馈能解耦的充要条件是M×M维矩阵E为非奇异。

$$\det E=\det\begin{bmatrix} c_1A^{d_1}B \\ c_2A^{d_2}B \\ \vdots \\ c_2A^{d_2}B \end{bmatrix}\neq 0$$

Decoupling of Control System

3)积分型解耦系统

定理：若系统 $\Sigma_0=(A,B,C)$是状态反馈能解耦的，则闭环系统

$$\dot{x}=A_p x+B_p v=(A+BK)x+BFv$$

$$y=C_p x=Cx$$

是一个积分型解耦系统，其中 $K=-E^{-1}L$

$$F=E^{-1}$$

Decoupling of Control System

闭环传递函数为:

$$W_0(s)=C[sI-(A+BK)]^{-1}BF=\begin{bmatrix} \dfrac{1}{s^{(d_1+1)}} & & & 0 \\ & \dfrac{1}{s^{(d_2+1)}} & & \\ & & \ddots & \\ 0 & & & \dfrac{1}{s^{(d_m+1)}} \end{bmatrix}$$

$$K=-E^{-1}L$$

即每个子系统相当于（di+1）阶子系统。

Decoupling of Control System

例题：试求上例的解耦系统

4)能解耦标准型

解耦系统 $\hat{\Sigma} = (\hat{A}, \hat{B}, \hat{C})$ 具有如下型式

$$\hat{A} = \begin{array}{c} \left[\begin{matrix} A_1 & & 0 \\ & \ddots & \\ 0 & & A_m \end{matrix}\right] \begin{matrix} p_1 \\ \\ p_m \end{matrix} \end{array} \quad \hat{B} = \begin{array}{c} \left[\begin{matrix} b_1 & & 0 \\ & \ddots & \\ 0 & & b_m \end{matrix}\right] \begin{matrix} p_1 \\ \\ p_m \end{matrix} \end{array} \quad \hat{C} = \begin{array}{c} \left[\begin{matrix} c_1 & & 0 \\ & \ddots & \\ 0 & & c_m \end{matrix}\right] \begin{matrix} 1 \\ \\ 1 \end{matrix} \\ \begin{matrix} p_1 & \quad\quad & p_m \end{matrix} \end{array}$$

$$p_i = d_i + 1, i = 1,2\ldots,m; p_1 + \ldots + p_m = n$$

Decoupling of Control System

$$A_i = \begin{bmatrix} 0 & I_{d_i} \\ 0 & 0 \end{bmatrix} \qquad b_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \qquad C_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

$$\overbrace{}^{}\!\!\! 1 \qquad \overbrace{d_i}$$

Decoupling of Control System

则称解耦系统 $\hat{\Sigma} = (\hat{A}, \hat{B}, \hat{C})$ 为解耦标准型

且极点配置可以在子系统独立进行。

例题：对上例解耦系统配置极点为－1，－1，－1，－1。