Tutorial 1: Graphs, networks and game theory

Thomas de Graaff

Version: November, 2024

Introduction

Intended Learning Outcome: In this tutorial you are asked to apply techniques as learned during the lectures and on DataCamp, specifically using the `igrahp` package. The ultimate goal is that you can visualise graphs from data in a meaningful way, derive network characteristics and apply game theoretic techniques.

Assessment Participation in the tutorial is advised, because its grade is part of your final grade. Activities are done in groups of two students (to be formed randomly in the first week of the course). As a group, you are supposed to write your answers *very concisely*. As a group of two, you get a uniform grade for the report. Note that the grade for the report is only valid in the academic year in which the tutorial is made.

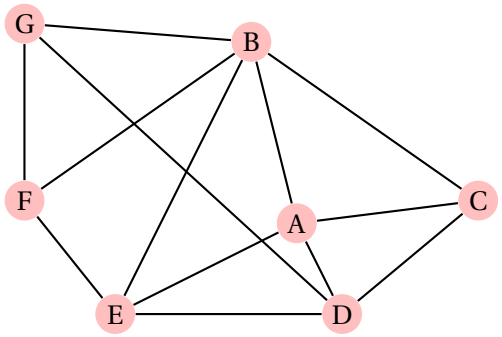
Rules & deadline You have to submit the assignments on Canvas in `.pdf` format before the start of Friday's tutorial class on November, 8 (using the assignment Inbox on Canvas).

Important guidelines:

- Provide the full names and student numbers of the group members in your submission title.
- Hand in your assignment in `.pdf` format.

Assignments

1. Note the network below



   The clustering coefficient is the probabilty that two random neighbors of a node have an edge as well. What is now the:

   a) clustering coefficient of *B*?

   b) clustering coefficient of *G*?

2. Find all pure strategy Nash equilibria in the game below. In the payoff matrix below the rows correspond to player A's strategies and the columns correspond to player B's strategies. The first entry in each box is player A's payoff and the second entry is player B's payoff.

		Player B	
		L	R
Player A	U	1,4	6,5
	D	3,3	4,2

3. Two identical firms—let's call them firm 1 and firm 2—must decide simultaneously and independently whether to enter a new market and what product to produce if they do enter the market. Each firm, if it enters, can develop and produce either product A or product B. If both firms enter and produce product A they each lose ten million dollars. If both firms enter and both produce product B, they each make a profit of five million dollars. If both enter and one produces A while the other produces B, then they each make a profit of ten million dollars. Any firm that does not enter makes a profit of zero. Finally, if one firm does not enter and the other firm produces A it makes a profit of fifteen million dollars, while if the single entering firm produces B it makes a profit of thirty million dollars. You are the manager of firm 1 and you have to choose a strategy for your firm.

   a) Set this situation up as a game with two players, firms 1 and 2, and three strategies for each firm: produce A, produce B or do not enter.

   b) One of your employees argues that you should enter the market (although he is not sure what product you should produce) because no matter what firm 2 does, entering and producing product B is better than not entering. Evaluate this argument.

c) Another employee agrees with the person in part (b) and argues that as strategy A could result in a loss (if the other firm also produces A) you should enter and produce B. If both firms reason this way, and thus enter and produce product B, will their play of the game form a Nash equilibrium? Explain.

d) Find all the pure strategy Nash equilibria of this game.

e) Another employee of your firm suggests merging the two firms and deciding co-operatively on strategies so as to maximize the sum of profits. Ignoring whether this merger would be allowed by the regulators do you think its a good idea? Explain.

4. The USairports data in the `igraphdata` package gives the network of passanger flights between airports in the United States. The data set was compiled based on flights in December, 2010. This network is directed and edge directions correspond to flight directions. Each edge is specific to a single carrier aircraft type. Multiple carriers between the same two airports are denoted by multiple edges. With the statistical program `R`, do the following:[1]

   a) Read in the US airports network from the `igraphdata` package as indicated in the template. Plot the data as nicely as you can given the techniques learned on DataCamp.

   b) What is the mean distance of this network? Show whether this deviates from an average random network by sampling a 1,000 times from a random network with the same number of nodes and the same probability to be connected. Can the null-hypothesis of a random network be rejected?

   c) Calculate the betweennessness measure for each node. What is the largest one? Plot the graph, using the betweenness as vertex size.

   d) Calculate the out-degree for each node. What is the largest one? Plot the graph, using the out-degree as vertex size.

   e) Argue why the two plots are so different.

5. In their recent paper König et al. (2017) analyse the various factions involved in the great war of Africa. To do so, they constructed a database reflecting the parties as nodes and the link between them denote whether they are enemies (with weight -1) or allies (with weight $+1$). For this assignment, download their paper and only read the introduction and 3.1: The Historical Context. The rest falls outside the scope of this class (although many terms should read as familiar). Michael König kindly shared this network for us. Our purpose in this assignment is to *redo* Figure 2 in their article and see whether we can improve insight by creating groups and clusters within this network using `igraph` commands:[2]

   a) To plot the importance of nodes we use the eigencentrality measure. Plot the network as nicely as you can. Can you figure out how to indicate whether ties are allies of enemies?

   b) Now separate this network into clusters. You can use one of the clustering methods explained on DataCamp. However, there is as well a clustering mechanism (`cluster.spinglass()`), that can take into account negative and positive weights. Describe shortly (10–15 lines) the clusters that emerge. Do they make sense?

[1] A template script to start with this assignment can be found on Canvas.
[2] A template script to start with this assignment can be found on Canvas.

References

König, M. D., D. Rohner, M. Thoenig, and F. Zilibotti (2017). "Networks in conflict: Theory and evidence from the great war of africa". In: *Econometrica* 85.4, pp. 1093–1132.