ECE594 Graduate paper

Electromagnetically induced transparency when viewed from a classical approach

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1 *Abstract*

By using the model for harmonic oscillations, quantum states can be more easily described. Resonance happens when the electric field from the outer electrons is modeled as a function of two springs. We can manipulate the position of electrons from their valence band (ground state) into a resonance that can induce transparency for a distinct quantized energy.

2 *Introduction*

Electromagnetically induced transparency (EIT) is a phenomenon that occurs mainly in radio wave propagation. Imagine having a material that reflects red and blue light, turning it the color purple in normal circumstances. Now imagine directly shining a high intensity beam of red light onto the material and no matter how intense the blue light is, it will no longer reflect. This is what EIT is like. The way this is described is by two springs connected in series, each spring representing an electron. These springs have different force distribution constants, which causes them to resonate at a single distinct frequency and to stand still to any stimuli within that energy level.

This can be modeled also through quantum mechanics and draws an interesting comparison. Due to the structure of the atom, energy is distinctly quantified into frequency bands. Some properties of these bands exist only due to meta-stability (Temperature and quantum fluctuations) and cause the material to reflect radiation it would otherwise be transparent to. We can move the electron through these frequency bands due to stimuli external stimuli and effectively render the entire material transparent.

Lastly the effect can be modeled through circuit theory and even tested in a lab at home. Most spring mechanics can often be easily and effectively modeled as a resonant circuit. This is because basic circuit components can represent either the integral or derivative of a given signal, which leads to a variety of applications and in this context the ability to test a given spring equation.

2.1 Uses of EIT

The key element behind EIT is the ability transmit incoming waves at will, a useful application of this would be for materials undetectable to radar. Other applications could be in optical switches, which have the potential to move faster than our current (GHz) switches right now.

Another property of EIT is electronically manipulating slow wave propagation. This can be used for propagation direction for antennas (Due to snells law and EM boundary conditions), and optical memory.

3 EIT described by quantum states

In an atom, there exists distinct quantized states for the electron to exist into, these states are often called energy or frequency bands.

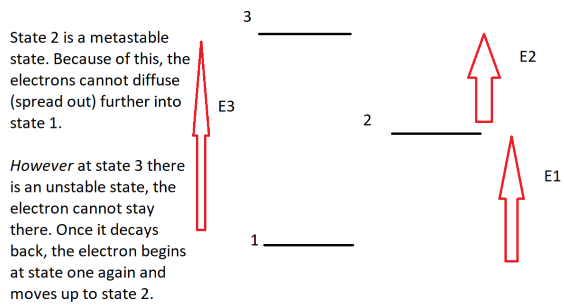

Figure 1: Electron states

Each frequency band can be modeled as a state of the outer electron in the atom and often these frequency bands are shifted due to temperature and quantum fluctuations. The way this works is imagining a meta-stable state ($|1>$ in figure 1) in between the outer band of an atom and the conduction band, this meta-stable state has enough energy (E1) in itself to shift the ground state (Zero electron energy, $|0>$) frequency response towards higher energy levels.

We exploit this meta-stable state by giving the material just enough energy to enter an unstable state, $|3>$, before instantly returning back down to the ground state. When at the ground state, the electron doesn't have enough energy to absorb (and reflect) the wave that it would otherwise absorb.

4 *EIT described using springs*

The outer elements of the atom can also be described as two springs operating in series. By manipulating these springs with different damping coefficients, we can achieve a point where one of the springs does not move.

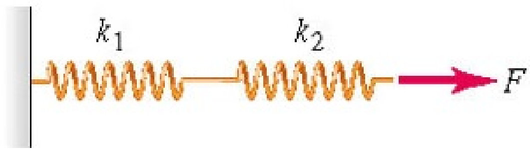

Figure 2: Coupled springs model of the atom.

We first start with the initial spring equation given at the beginning of the class
Particle 1:

$$\frac{dx_1{}^2}{d^2t} + \gamma_1 \frac{dx_1}{dt} + \omega^2 x_1 - \Omega^2 x_2 = \frac{F}{M} e^{-j\omega_s t}$$

Particle 2:

$$\frac{dx_2{}^2}{d^2t} + \gamma_2 \frac{dx_2}{dt} + \omega^2 x_2 - \Omega^2 x_1 = 0$$

Notice that the first equation has an electrical stimuli as well as the spring factor, while the second equation does not. This is because the first electron has a free range of motion while the second electron is **bounded between the first particle and what can be interpreted as an impenetrable wall**. As well, notice that there is an effect on the total energy of the system by the opposite particle. This effect is measured by the term $\Omega = \sqrt{\frac{K}{M}}$, this was described as the coupling frequency between the atom and the pumping oscillator.

Suppose we make the assumption that the position of x is modeled by a decaying wave, $e^{\lambda \cdot t}$ and plug it into the above equation.
Particle 1:
(1)
$$\lambda^2 \cdot N \cdot e^{\lambda \cdot t} + \gamma_1 \cdot \lambda \cdot N \cdot e^{\lambda \cdot t} + \omega^2 \cdot N \cdot e^{\lambda \cdot t} - \omega^2 \cdot N \cdot e^{\lambda_2 \cdot t} = \frac{F}{M} e^{-jw_s t}$$

(2) $\mathrm{N}_1 \cdot e^{\lambda_1 \cdot t} \left(\lambda^2 + \gamma_1 \lambda_1 + \omega^2\right) - \Omega_2^2 \cdot N_2 \cdot e^{\lambda_1 \cdot t} = \frac{F}{M} e^{-jw_s t}$
Supplementing $\lambda = -jw_s t$ so that all components of both springs respond to the stimulating frequency.
(3) $\mathrm{N}_1 \cdot e^{-jw_s \cdot t} \left(-\omega_s^2 + \gamma_1 \cdot -jw_s + \omega^2\right) - N_2 \cdot \Omega_2^2 = \frac{F}{M} e^{-jw_s t}$
(4) $\mathrm{N}_1 \cdot \left(-\omega_s^2 + \gamma_1 \cdot -jw_s + \omega^2\right) - N_2 \Omega_2^2 = \frac{F}{M}$
(5) $\mathrm{N}_1 = \frac{F + N_2 \Omega_2^2}{M \cdot (-\omega_s^2 + \gamma_1 \cdot -jw_s + \omega^2)}$

Using the same process, the equation for the second particle achieves the equality

(6) $N_2 = \frac{N_1 \cdot \omega^2}{(-\omega_s^2 + \gamma_2 \cdot -jw_s + \omega^2)}$
And so by introducing N_2 into equation (4),
(7) $N_1 \left(-\omega_s^2 + \gamma_1 \cdot -jw_s + \omega^2 - \frac{\Omega_1^2 \Omega_2^2}{(-\omega_s^2 + \gamma_2 \cdot -jw_s + \omega^2)}\right) = \frac{F}{M}$
(8)
$N_1 \left(\frac{\left(-\omega_s^2 + \gamma_1 \cdot -jw_s + \omega^2\right)\left(-\omega_s^2 + \gamma_2 \cdot -jw_s + \omega^2\right)}{(-\omega_s^2 + \gamma_2 \cdot -jw_s + \omega^2)} - \frac{\Omega_1^2 \Omega_2^2}{(-\omega_s^2 + \gamma_2 \cdot -jw_s + \omega^2)}\right) = \frac{F}{M}$
Now taking the liberty that the paper is correct in assuming that $\Omega_1 = \Omega_2$
(9) $N_1 = \frac{F\left(-\omega_s^2 + -jw_s \cdot \gamma_2 + \omega^2\right)}{M[(-\omega_s^2 + \gamma_1 \cdot -jw_s + \omega^2)(-\omega_s^2 + \gamma_2 \cdot -jw_s + \omega^2) - \Omega_1^2 \Omega_2^2]}$

This models the behavior of the electron when under the effect of an external wave.

The behavior when graphed shows a distinct point of resonance, namely when the loss coefficient $\gamma_1 \gg \gamma_2$, implying that the majority of the power is not lost on the first spring. We also know that $w = \sqrt{\frac{K}{M}}$, which allows us to plot the material as a function of w_s

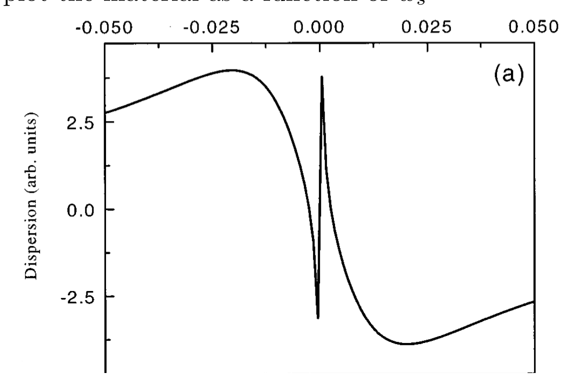

Figure 3: Signal dispersion vs ω, found during EIT [1]

5 *EIT as a circuit*

EIT can be modeled using the same spring equations mentioned before but described using circuit components. This is because basic components in circuit theory can be used to create a differential equation, such as the equations mentioned for the two particles in section 3.

We first start with the original equations listed as before, modelling the equality using Kirchoff's voltage law. Because Kirchoff's voltage law says that all voltage drops over a closed path *must* equal the voltage stimuli
Loop 1:

$$\frac{di_1{}^2}{d^2t} + \gamma_1 \frac{di_1}{dt} + \omega^2 i_1 - \Omega^2 i_2 = \frac{F}{M} e^{-j\omega_s t}$$

Loop 2:

$$\frac{di_2{}^2}{d^2t} + \gamma_2 \frac{di_2}{dt} + \omega^2 i_2 - \Omega^2 i_1 = 0$$

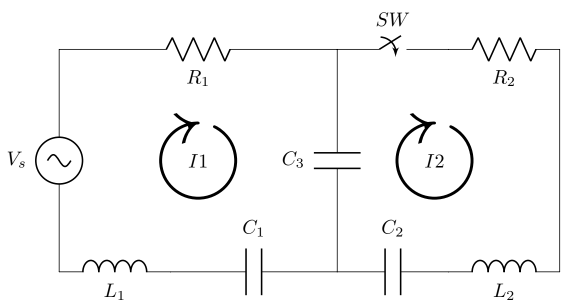

Figure 4: Circuit described for EIT

To reiterate the previous equations, this is their description in circuit theory, Loop 1:

$$(1) i_1 \cdot R_1 + \frac{1}{C_3} \int i_1 - i_2 dt + \frac{1}{C_1} \int i_1 dt + L_1 \frac{di_1}{dt} = V_s$$

By taking the derivative of both sides,

$$(2) \frac{di_1}{dt} \cdot R_1 + \frac{1}{C_3} (i_1 - i_2) + \frac{1}{C_1} i_1 + L_1 \frac{d^2 i_1}{dt^2} = \frac{dV_s}{dt}$$

Next we divide by L to achieve
(3)

$$\frac{di_1}{dx} \cdot \frac{R_2}{L} + \frac{1}{L} \left(\frac{1}{C_3} + \frac{1}{C_1} \right) i_1 - \frac{1}{C_3} i_2 + \frac{d^2 i_1}{dt^2} = \frac{1}{L_1} \frac{dV_s}{dt}$$

and replacing the equations with the equality,

and replacing the equations with the equality,
$\omega^2 = \frac{C_3 \cdot C_1}{L(C_3+C_1)}$, $\gamma_1 = \frac{R_1}{L_1}$, $\Omega^2 = \frac{1}{C3}$, $\frac{dV_s}{dt} = F \cdot V_s$
where F is an arbitrary constant.

We can replace the the equation back into its more familiar form once again
(4)

$$\frac{di_1{}^2}{d^2t} + \gamma_1 \frac{di_1}{dt} + \omega^2 i_1 - \Omega^2 i_2 = \frac{F}{M} e^{-j\omega_s t}$$

The second equation is not much more difficult.

6 *Applications of EIT*

EIT has a variety of applications as previously stated in the introduction.

6.1 Optical switching

The most important being for optical switching, which if done properly could pave new ways in optical computing. In recent experiments done at *Huazong University*, optical switches were made with an efficiency of 85% and a switching rate of 100GHz [2].

6.2 Quantum memory

Another application of EIT is slow wave propagation, slow wave propagation is the ability to change the index of refraction to a value so high that the wave has almost completely slowed down. Because energy cannot be created or destroyed, the specific values of the wave, such as amplitude and frequency, the wave can be used to contain quantum memory. The way this works is due to the transition frequency in the spring equations. Right during resonance, as seen on the next page, there is a change in the index of refraction.

The index of refraction, often notated by n, measures the speed of the wave with respect to the speed of light.

What fraction of the speed of light is what denotes n. As seen in figure 5, the index of refraction approaches 0 with respect to a single frequency while other frequencies see the effectively transparent medium. This means that the speed of light is effectively 0 for one single frequency and can be stored indefinitely.

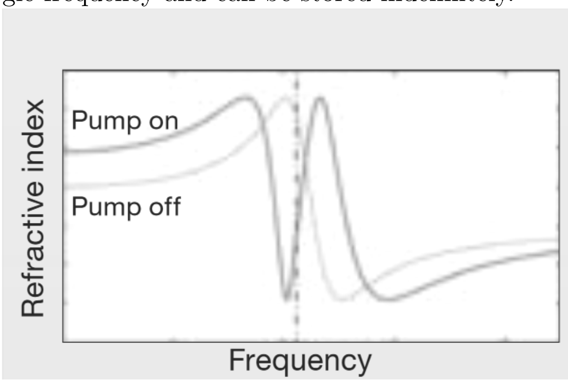

Figure 5: Index of refraction graph vs frequency. [4]

In an article published at the American Physical Society, quantum memory has been developed to the point of a storage efficiency of 1.5% and a retrieval efficiency of over 50%. [3]

6.3 Optical directivity

By utilizing the same properties that were described earlier, we can change the index of refraction and therefore the response towards light that the material has. This can be used to direct beams off into different directions using the optical switching behavior mentioned before and using the different index of refraction. Similar but not the exact same experiments have been done in the laboratory, where colors were routed in two different directions between boundary conditions.

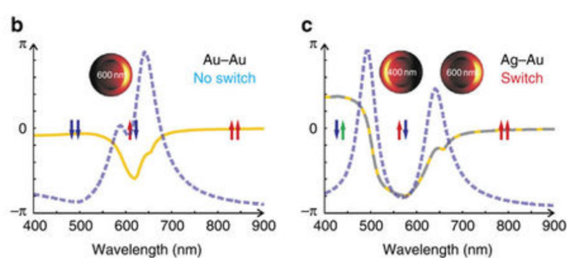

Figure 6: Phase changes at boundary conditions [5]

7 *Conclusion*

EIT is the ability to transmit light through a material despite it naturally reflecting it. While this has a variety of applications, it is also an interesting introduction to quantum physics due to its analog towards classical mechanics. It not only connected the beginnings of quantum mechanics with classical spring and circuit models but also details a great understanding of what is going on inside the quantum world.

However, some of these descriptions have their shortcomings, such as the circuit heavily relying on things such as the switch for the second loop to describe complex phenomena. As well the spring model does not take into account things such as relativity (In the case of high orbital atoms), quantum tunneling (How does EIT happen for valence electrons inside the D orbital, which is separated by a 'large' distance from the other orbitals?) and entanglement (What happens if the two electrons become entangled?). It will be very interesting to see if later on, these flaws in the model are addressed and we gain a full view of how EIT is described in its totality.

References

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