파이썬으로 배우는 데이터 구조

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학습 목표

BST의 다양한 메소드들을 이해하고 구현할 수 있다

Data Structures in Python
Chapter 7 - 2

- Binary Search Tree(BST)
- **BST Algorithms**
- AVL Tree
- AVL Algorithms

Agenda & Readings

- Binary Search Tree(BST) Algorithms
  - minimum() and maximum()
  - predecessor() and successor()
  - delete(), _delete()
  - Converting Binary Tree to BST
  - LCA(Lowest Common Ancestor)

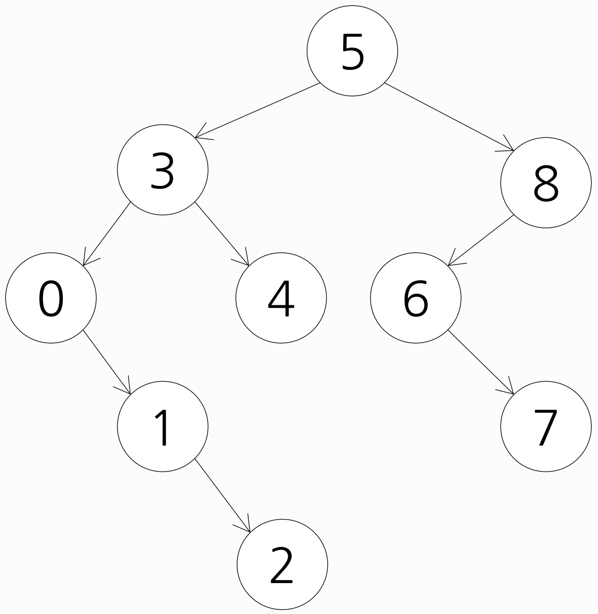
- Reference:
  - Problem Solving with Algorithms and Data Structures
  - Chapter 6 - Tree

minimum(), maximum():

- minimum() and maximum() returns the node with min or max key.
  - Note that the entire tree does not need to be searched.
  - The minimum key is located at the left most node, the maximum at the right most node.
  - Complexity of algorithm to find the maximum or minimum will be O(log N) in almost balanced binary tree. If tree is skewed, then we have worst case complexity of O(N).

```
def minimum(self, node = None):
    if node is None: node = self.root
    return self._minimum(node)

def _minimum(self, node):
    if node.left == None: return node
    return self._minimum(node.left)
```

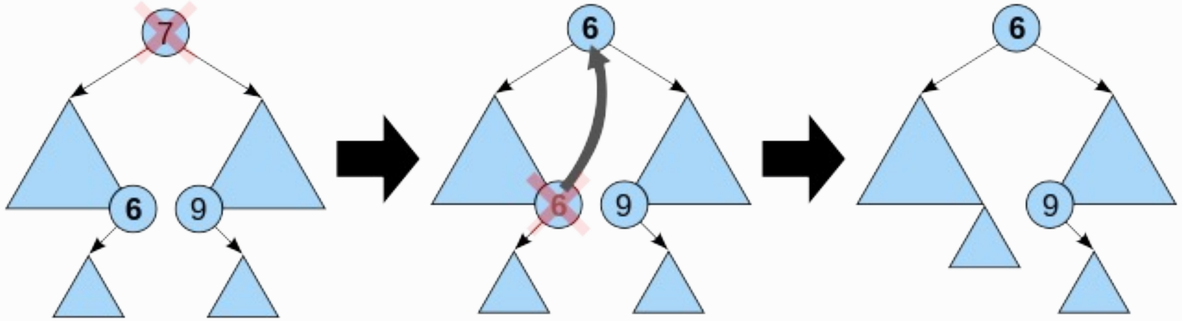


predecessor(), successor():

- Predecessor
  - The predecessor is **the largest node** that is smaller than the root (current node) - thus it is on the left branch of the Binary Search Tree, and the **rightmost leaf** (largest on the left branch).
- Successor
  - The successor is **the smallest node** that is bigger than the root/current - thus it is on the right branch of the BST, and also on **the leftmost leaf** (smallest on the right branch).
- Notice that either predecessor or successor has at most one child if any.
- Complexity of algorithm: O(log N) if balanced, and O(N) if the tree is skewed.

```
def successor(self, node = None):
    if node is None: node = self.root
    return self._successor(node)

def _successor(self, node):
    if node and node.right:
        return self._minimum(node.right)
    return None
```



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def _successor(self, node):
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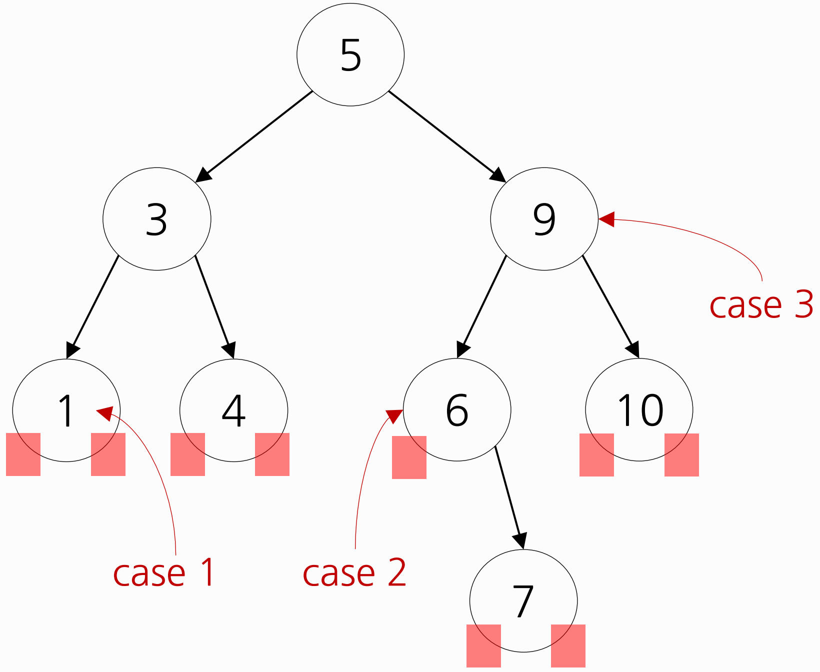
```

delete()

- When we delete a node, **three possibilities** arise depending on how many children the node to be deleted has:
  - Case 1: No child
  - Case 2: One child
  - Case 3: Two children



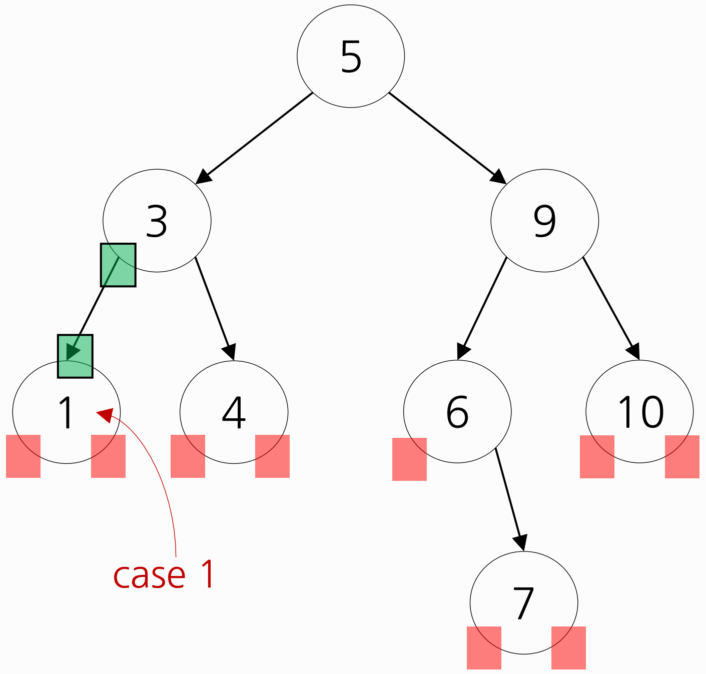

```
def _delete(self, node, key):
    if node is None: return node

    if key < node.key:
        node.left = self._delete(node.left, key)
    elif key > node.key:
        node.right = self._delete(node.right, key)
    else:     # key == node.key:
        if node.left and node.right: # two children
                two children case
        elif node.left or node.right: # one child
                one child case
        else:                          # no child
                no child case
    return node
```

find

delete()

- **Case 1:** No child - Simply return **None**.



delete()

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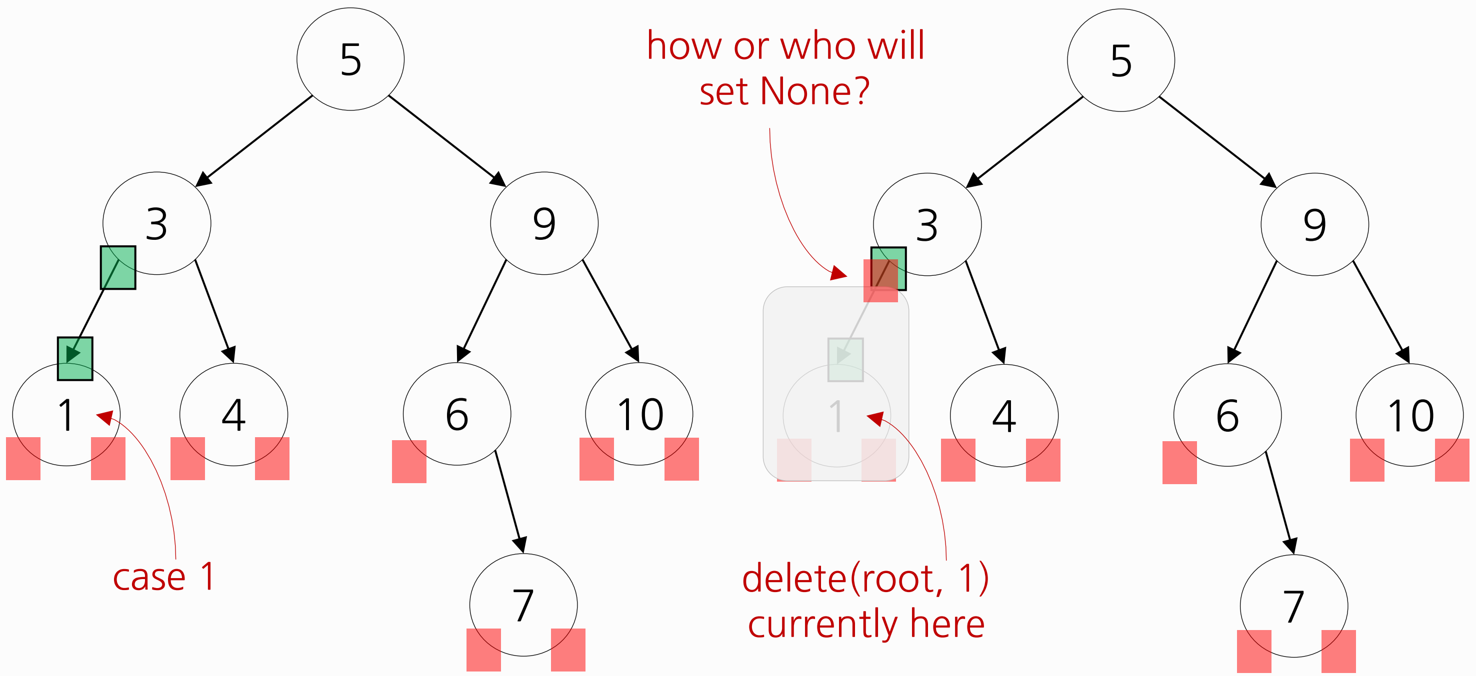

delete()

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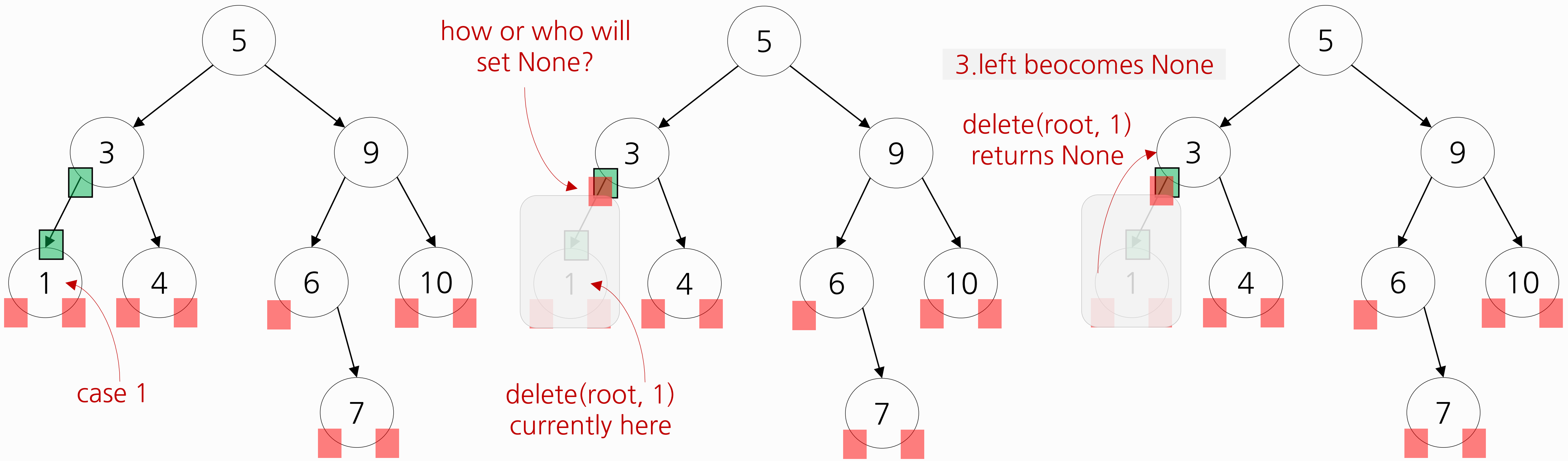

delete() Example

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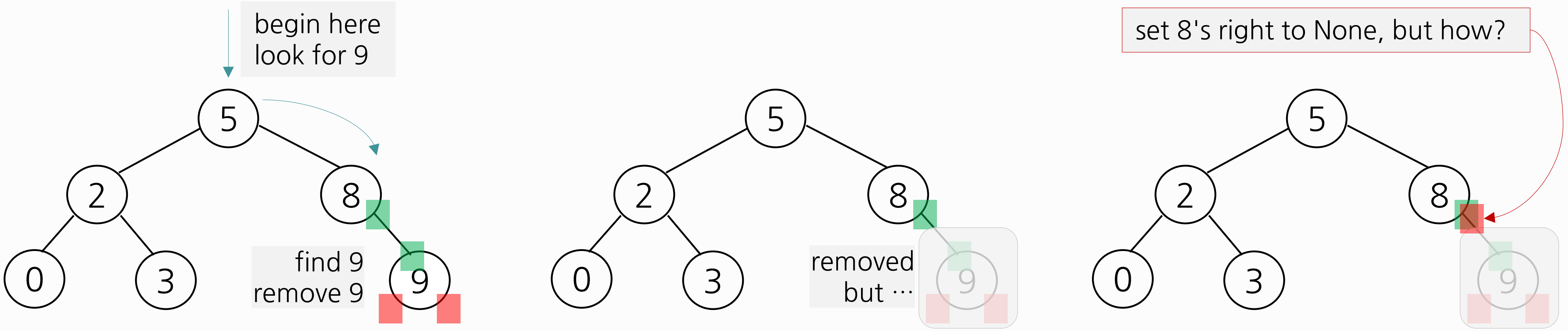
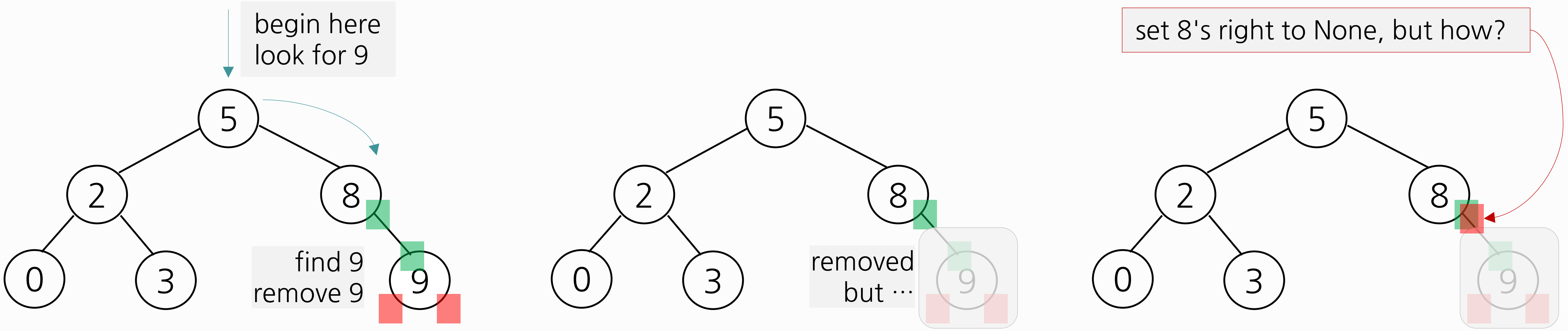


find

```
def _delete(self, node, key):
    ...
    elif key > node.key:
        node.right = self._delete(node.right, key)
    ...
    ...
    ...
    return node
```

delete() Example

- Case 1: No child - Simply return **None**.



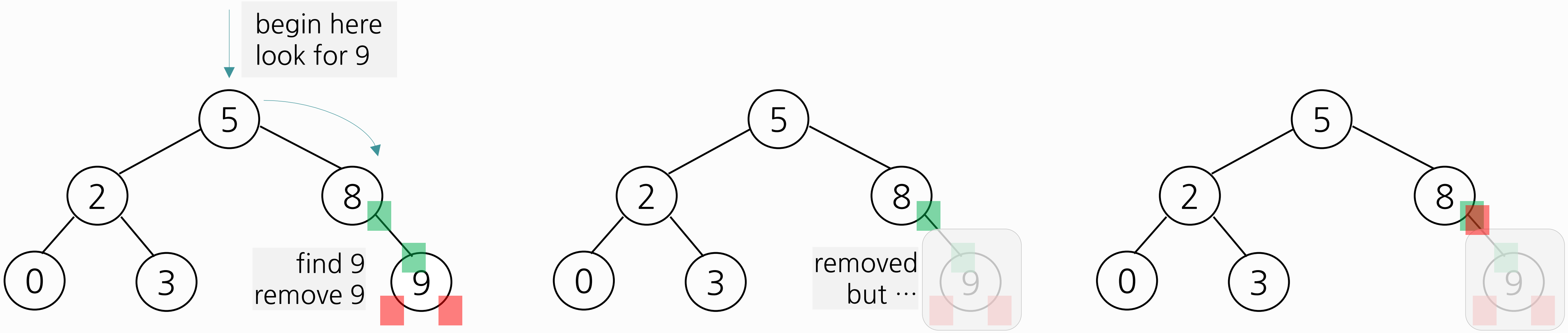
find

```
def _delete(self, node, key):
    ...
    elif key > node.key:
        node.right = self._delete(node.right, key)
    ...
    ...
    ...
    return node
```

```
def _delete(self, node, key):
    ...
    else:  # key == node.key:
        if node.left and node.right:
          two children case
        elif node.left or node.right:
          one child case
        else:              # no child
            node = None
    return node
```

delete() Example

- Case 1: No child - Simply return **None**.



It sets 8's right to None.

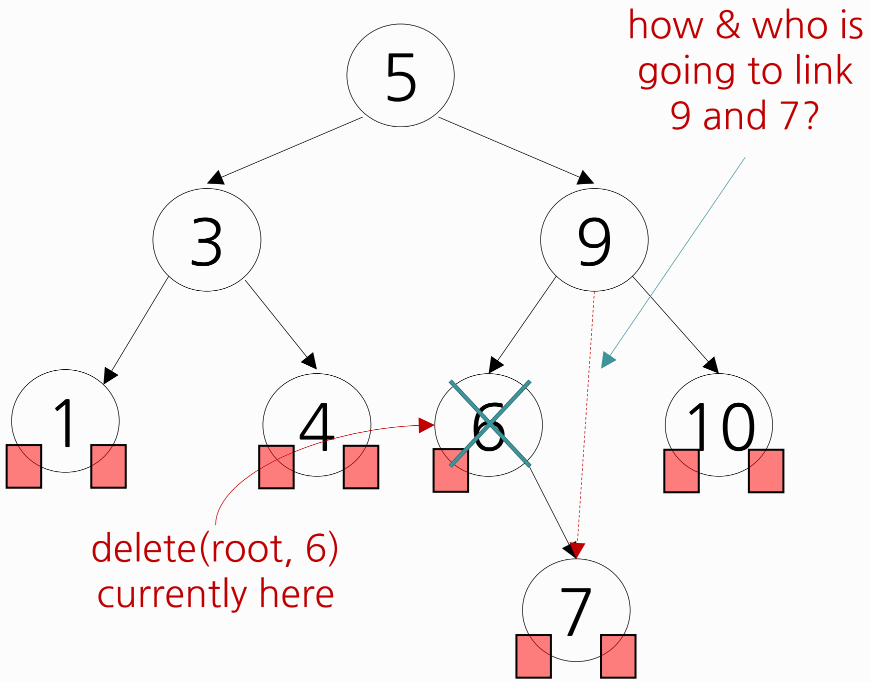
find

```
def _delete(self, node, key):
    ...
    elif key > node.key:
        node.right = self._delete(node.right, key)
    ...
    ...
    ...
    return node
```

```
def _delete(self, node, key):
    ...
    else:  # key == node.key:
        if node.left and node.right:
          two children case
        elif node.left or node.right:
          one child case
        else:                  # no child
            node = None
    return node
```

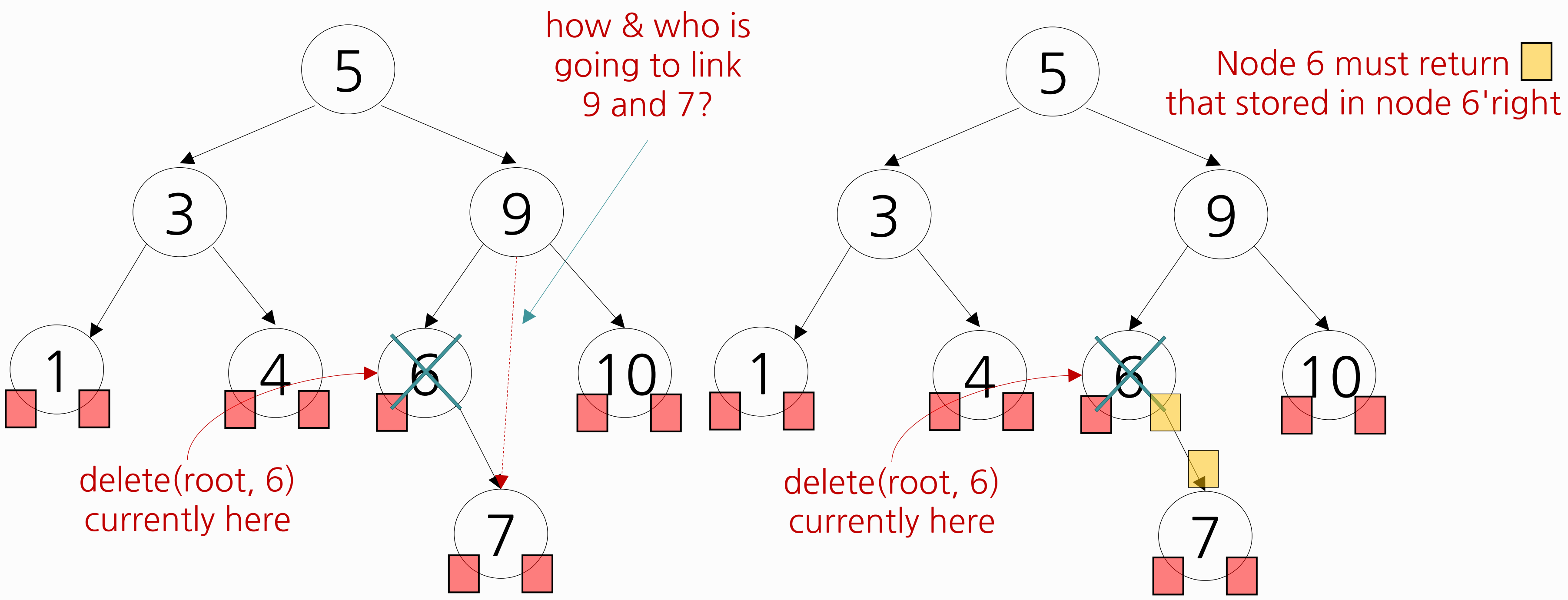
delete()

- **Case 2:** One child



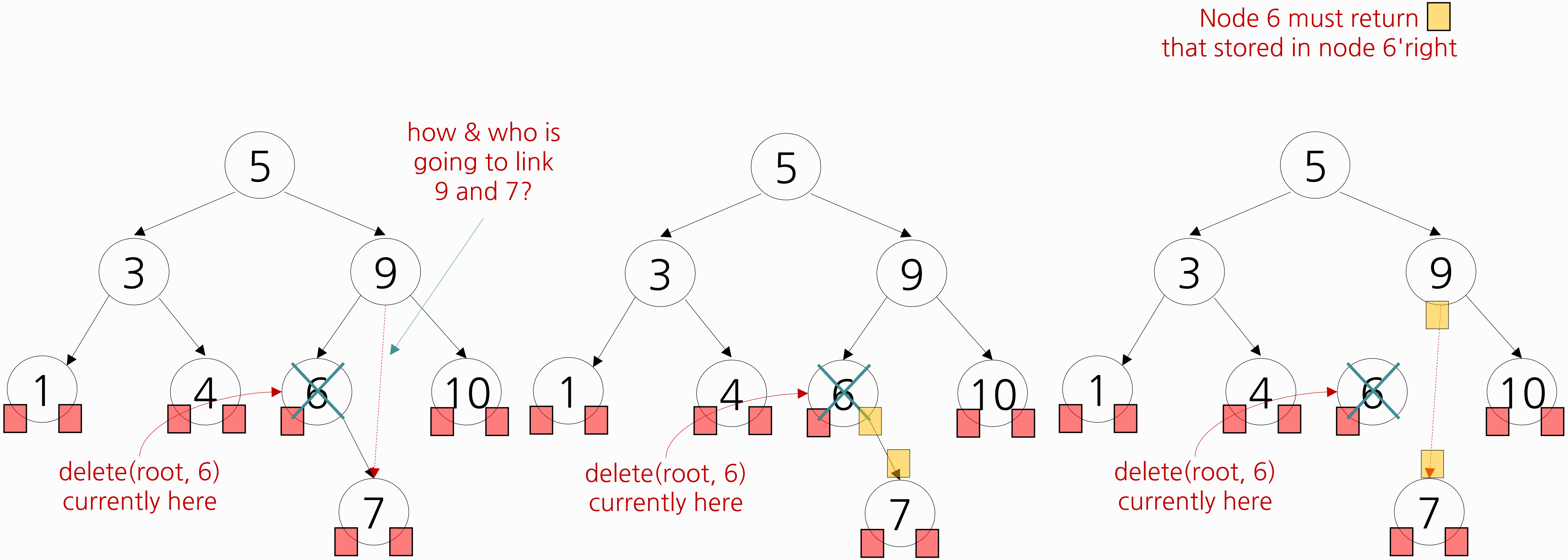
delete()

- **Case 2:** One child



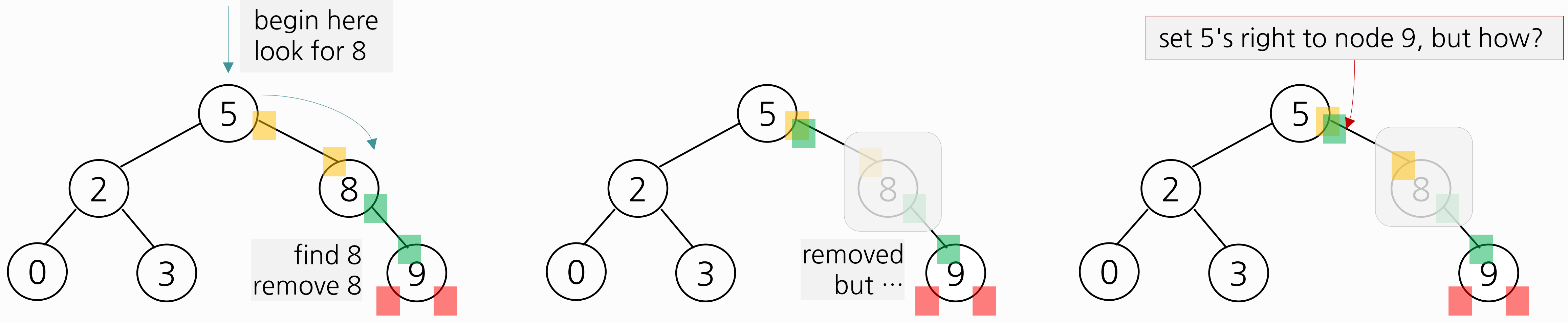
delete()

- **Case 2:** One child



delete() Example

- Case 2: One child

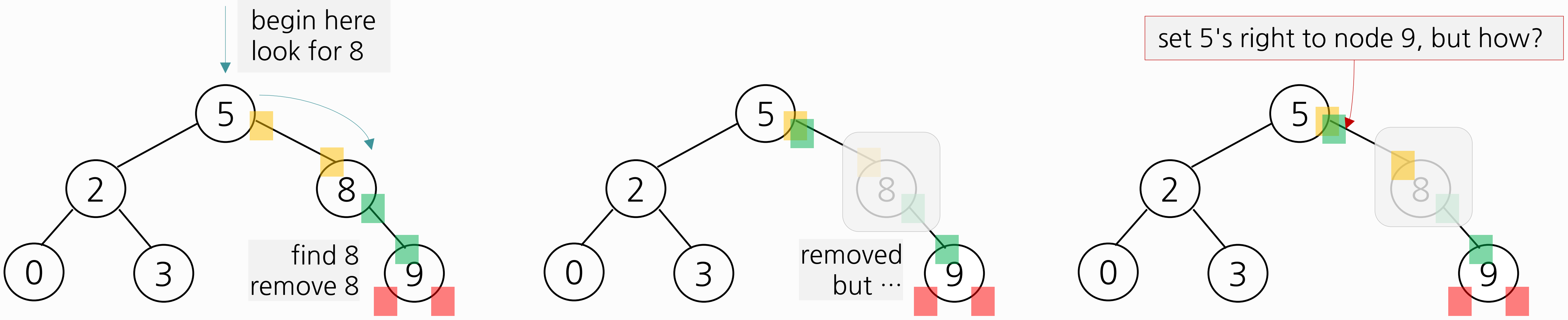


find

```
def _delete(self, node, key):
    ...
    elif key > node.key:
        node.right = self._delete(node.right, key)
    ...
    ...
    ...
    return node
```

delete() Example

- Case 2: One child



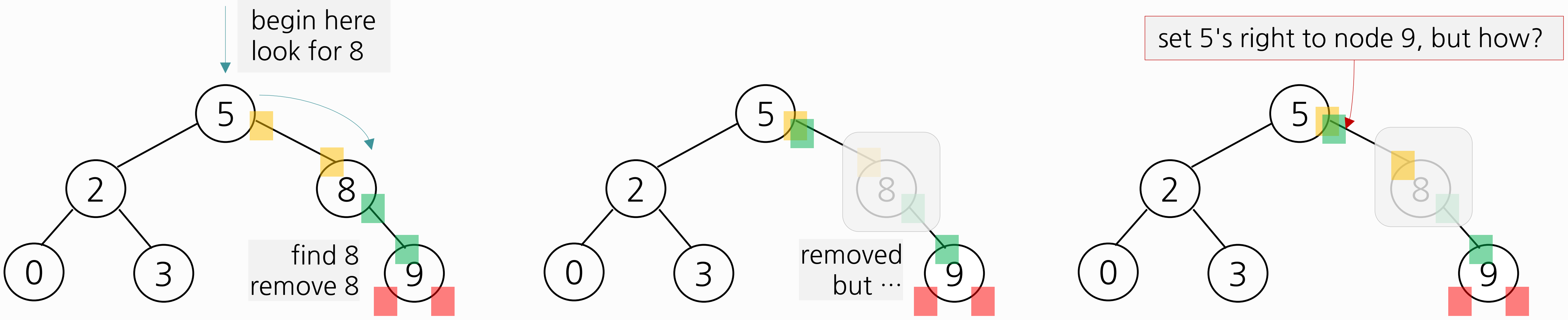
find

```
def _delete(self, node, key):
    ...
    elif key > node.key:
        node.right = self._delete(node.right, key)
    ...
    ...
    ...
    return node
```

```
def _delete(self, node, key):
        ...
        elif node.left or node.right: # one child
            if node.left:
                node = node.left
            else:
                node = node.right
        else:                          # no child
            node = None
    return node
```

delete() Example

- Case 2: One child



It sets 5's right to node 9.

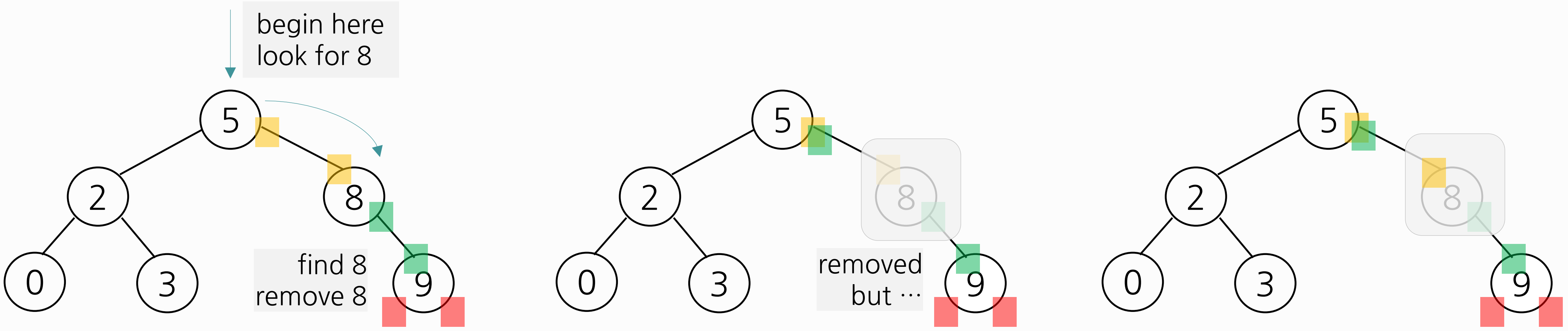
find

```
def _delete(self, node, key):
    ...
    elif key > node.key:
        node.right = self._delete(node.right, key)
    ...
    ...
    ...
    return node
```

```
def _delete(self, node, key):
        ...
        elif node.left or node.right: # one child
            if node.left:
                node = node.left
            else:
                node = node.right
        else:                          # no child
            node = None
    return node
```

delete() Example

- Case 2: One child



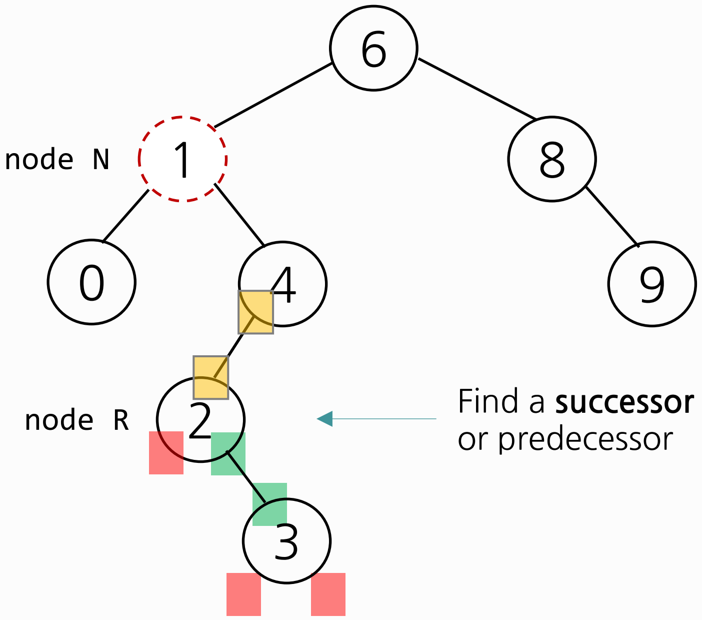

```
def _delete(self, node, key):
        ...
        elif node.left or node.right: # one child
            node = node.left or node.right
        else:                         # no child
            node = None
    return node
```

can be simplified

```
def _delete(self, node, key):
        ...
        elif node.left or node.right: # one child
            if node.left:
                node = node.left
            else:
                node = node.right
        else:                         # no child
            node = None
    return node
```

delete() Example

- Case 3: Two children
  1. Find the node **N** to delete, but do not delete it.
  2. Choose either its **successor** or its **predecessor** node, **R**.
  3. Simply, replace N's key with R's key
  4. Then, recursively call to delete on node R in the subtree. The node R must be in either Case 1 or Case 2.



find

```
def _delete(self, node, key):
    if node is None: return node

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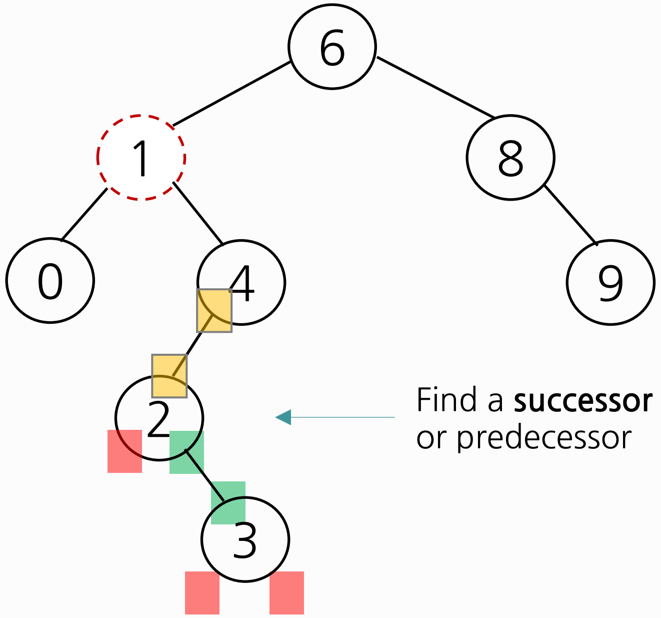
    else:    # key == node.key:
        if node.left and node.right: # two children

                  two children case

        elif node.left or node.right: # one child
                  one child case
        else:                         # no child
                  no child case
    return node
```

delete() Example

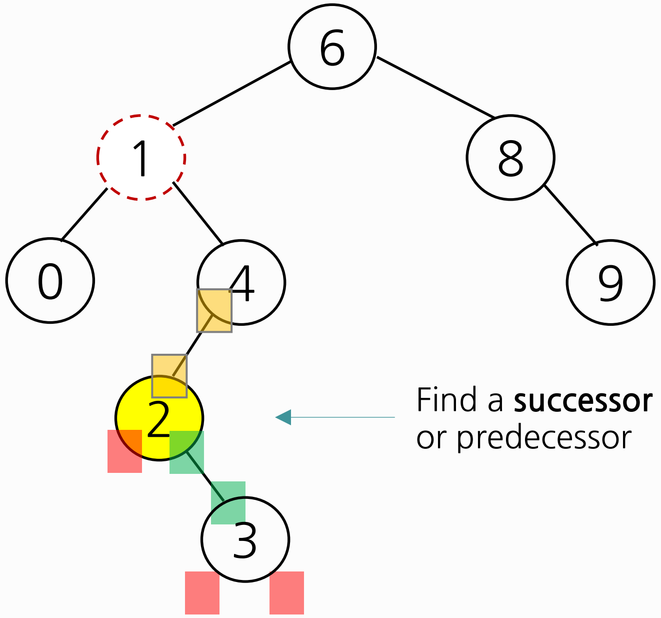
- Case 3: Two children




```
1. find the node 1 to delete
```

delete() Example

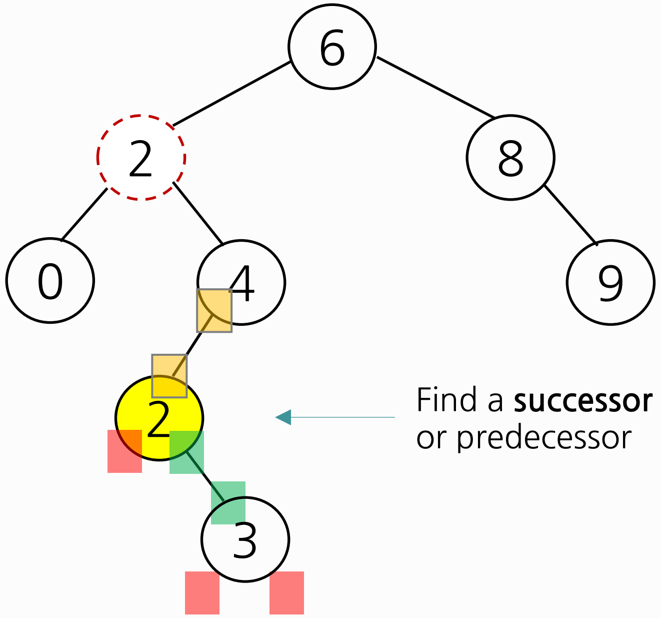
- Case 3: Two children




```
1. find the node 1 to delete
2. if (two children case),
   find 1's successor's key = 2
```

delete() Example

- Case 3: Two children




```
1. find the node 1 to delete
2. if (two children case),
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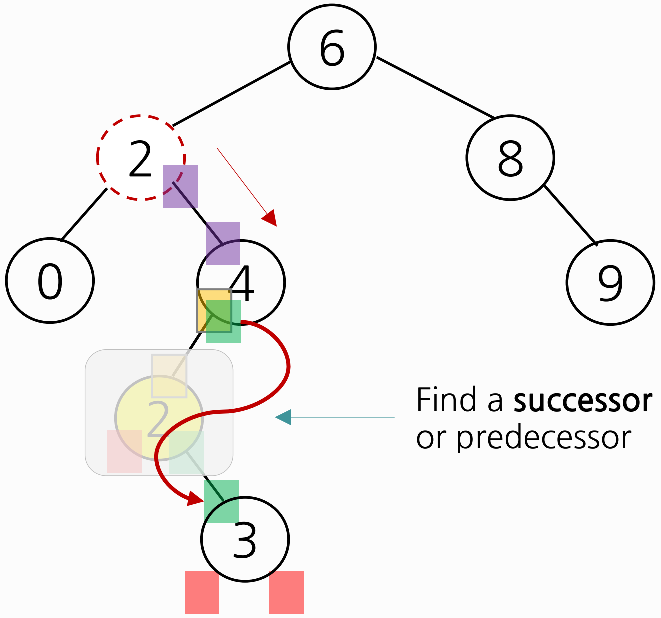
3. replace the node 1 with 2
```

delete() Example

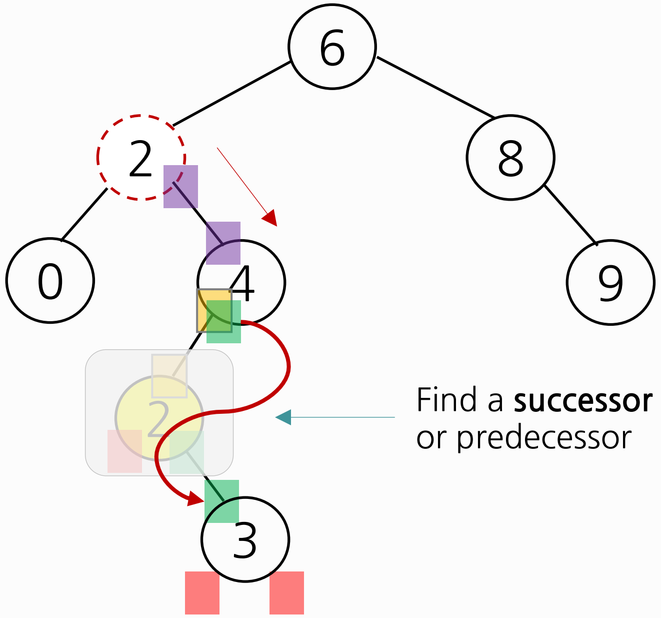
- Case 3: Two children




```
1. find the node 1 to delete
2. if (two children case),
   find 1's successor's key = 2
3. replace the node 1 with 2
4. invoke
   node.right = self._delete(node.right, 2)
```

delete() Example

- Case 3: Two children

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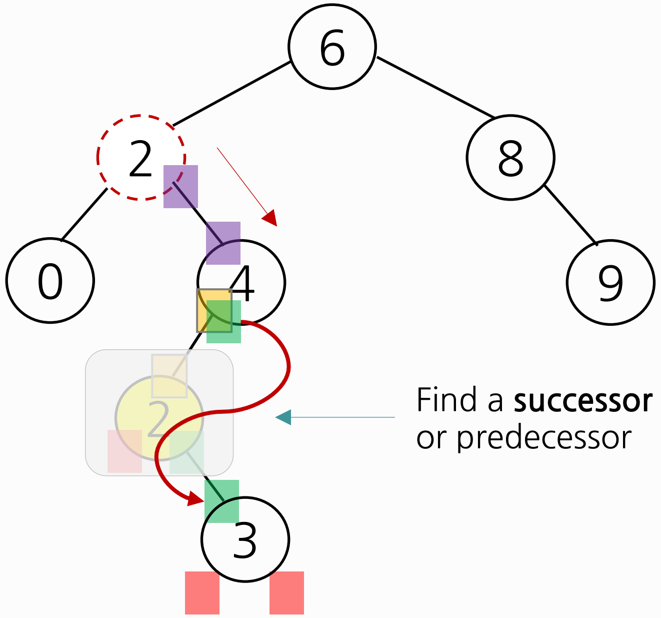
Some thoughts:

- Step 2 Get the heights of two subtree first.
  - If right subtree height is larger, then use the successor. Otherwise use the predecessor to shorten the tree height.
- Step 4 simply uses the code for one-child case deletion, recusively.

delete() Example

- Case 3: Two children

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Some thoughts:

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- Step 4 simply uses the code for one-child case deletion, recursively.

Some questions:

- What if successor has **two** children?
  - **Not possible !**
  - Because if it has two nodes, at least one of them is less than it, then in the process of finding successor, we won't pick it !

delete(): Exercise

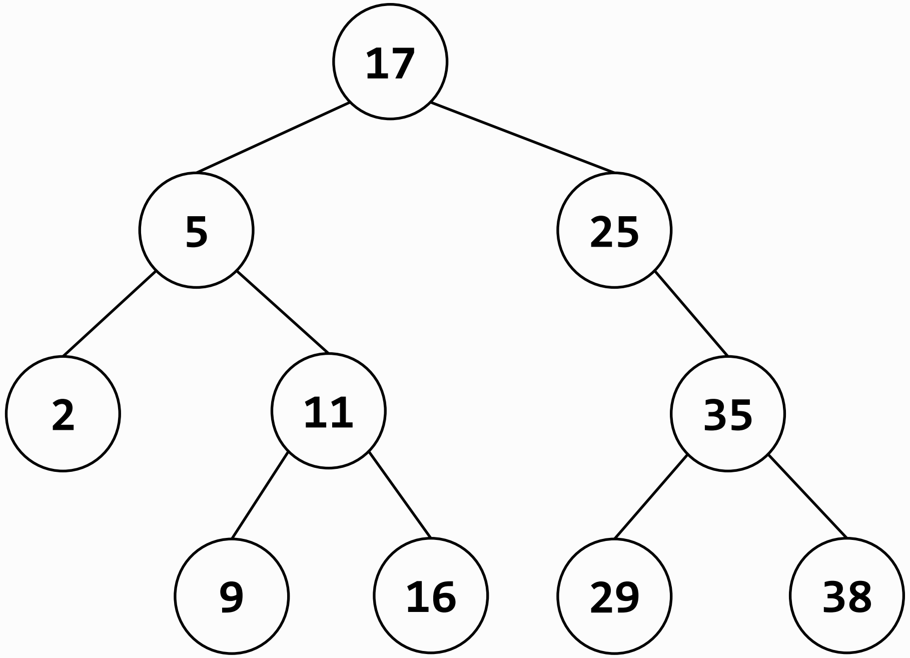
- Delete the root 5 times consecutively. Delete the node from the higher subtree if necessary.

```
       ____________17
      /              \
     5______          25___
    /       \              \
   2         11            _35
            /  \          /   \
          __9   16       29    39
         /
        7
         \
          8
```

isBST() - Validate Binary Search Tree

- A BST is defined as follows:
  - The left subtree of a node contains only nodes with **keys less than** the node's key.
  - The right subtree of a node contains only nodes with **keys greater than** the node's key.
  - Both the left and right subtrees must also be binary search trees.

```
_isBST(self, node, min, max)
```



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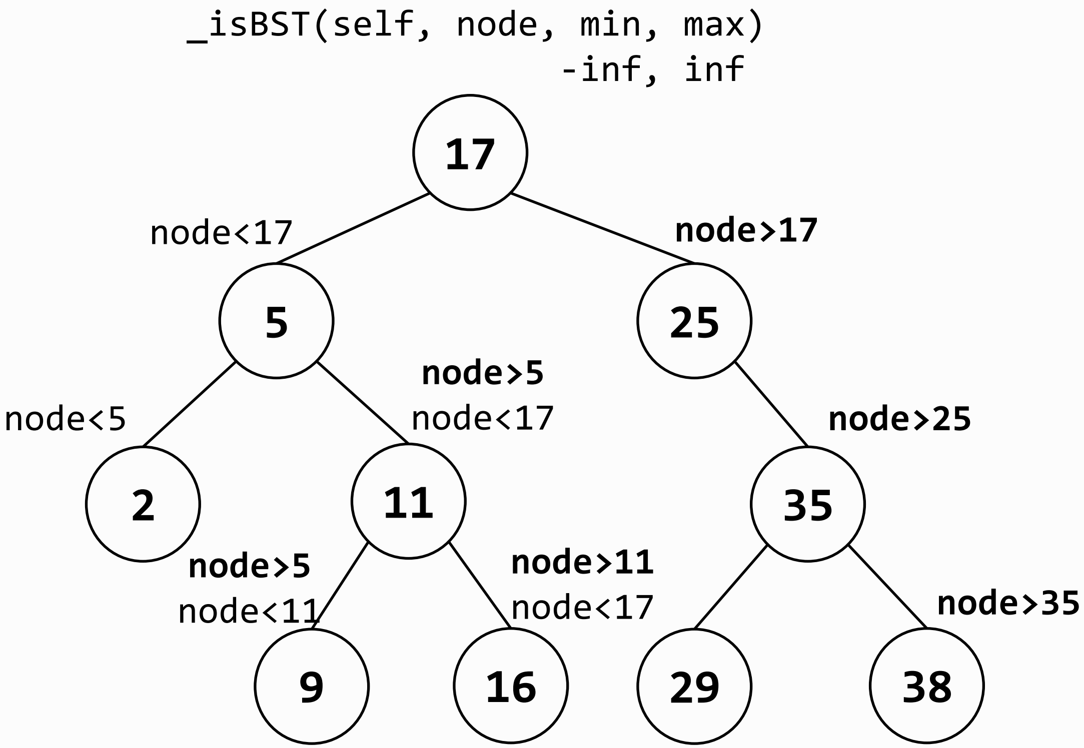


```
def isBST(self, node = None):
    if node is None: node = self.root
    return self._isBST(node, float('-inf'), float('inf'))

def _isBST(self, root, min, max):
```

isBST() - Validate Binary Search Tree

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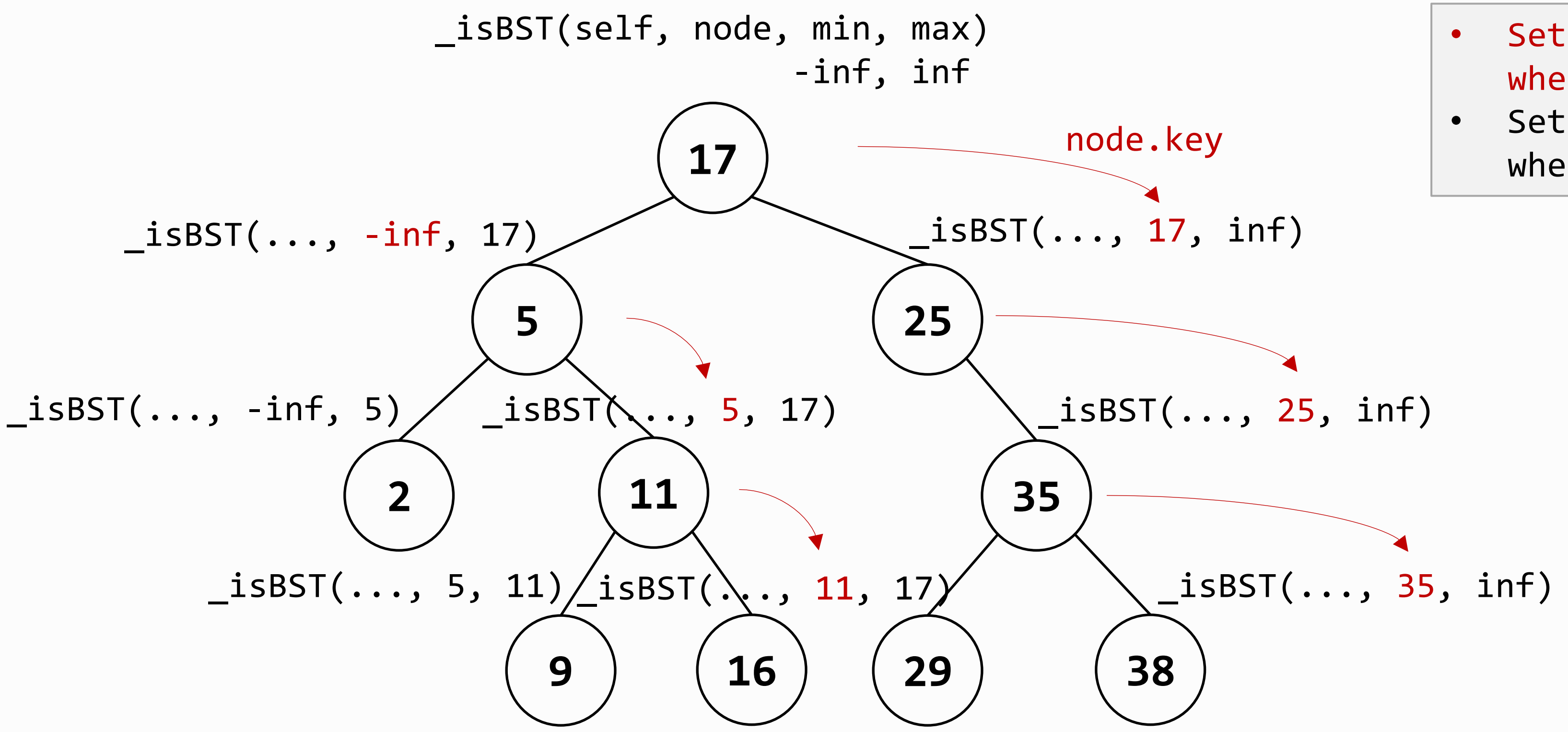

- Set 'min' with node.key when move to the right.
- Set 'max' with node.key when move to the left.

Summary

- To delete a node in a BST, we must consider three different cases.
  - no child
  - one child
  - two children
- Both predecessor and successor at a node in a BST always has only one child or none, never two children.

학습 정리

1) Predecessor, successor는 트리의 root를 삭제할 경우, 대체할 값을 찾기 위해 사용된다

2) 노드를 삭제할 때는 no child, one child, two children의 세가지 경우로 나누어 생각한다

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수고했습니다
곧 다음 시간에
다시 뵙겠습니다