파이썬으로 배우는
데이터 구조
한동대학교
전산전자공학부
김영섭 교수

학습 목표

해시 테이블을 구현할 때 피할 수 없는 충돌을
해결하는 방법들을 이해하고 적용할 수 있다

Data Structures in Python
Chapter 6

- Hash Table
- **Collision Resolutions**
- Double Hashing & Rehashing
- Hash Implementation

Agenda & Readings

- Collision Resolution
  - Separate chaining
  - Open addressing
    - Linear Probing
    - Quadratic Probing
    - Double Hashing

- Reference:
  - Problem Solving with Algorithms and Data Structures
  - Chapter 5 - Hashing

Collision Resolution

- **Perfect hash functions** are hard to come by, especially if you do not know the input keys beforehand.

- If multiple keys map to the same hash value this is called **collision**.
  - For non-perfect hash functions we need systematic way to handle collisions.

- Handling collisions systematically is required - collision resolution.

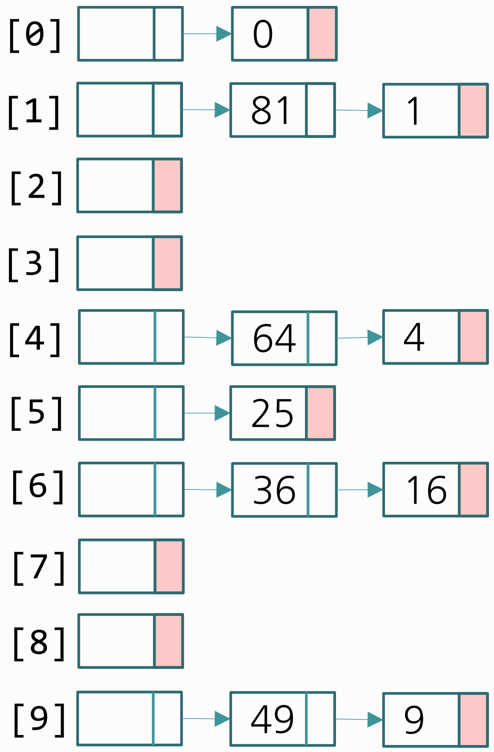
Collision Resolution

- Collision resolution methods
  - **Chaining** - Store colliding keys in a linked list at the same hash table index
  - **Open addressing** - Store colliding keys elsewhere in the table
    - Linear Probing
    - Quadratic Probing
    - Double hashing.

Collision Resolution by Chaining

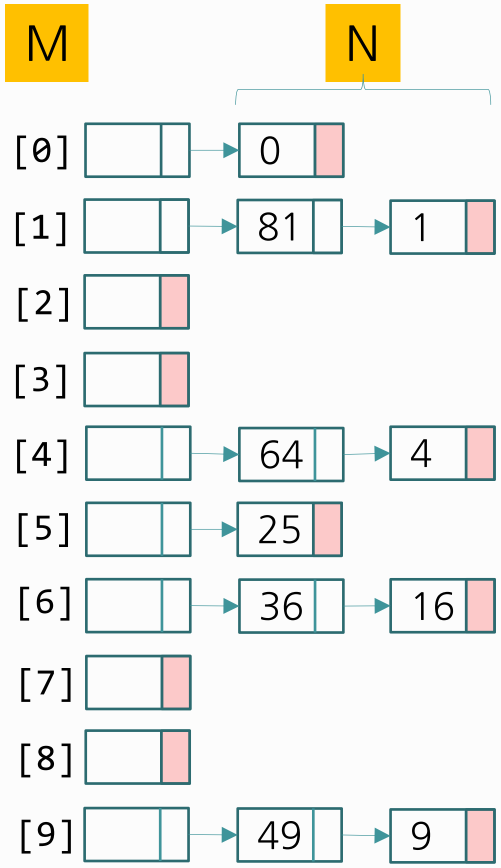
- Maintains a linked list at every hash index for collided elements
  - Hash table T is a vector of linked lists.
    - Insert element at the head (as shown here) or at the tail.
  - Key k is stored in list a HashTable[h(k)]
  - For example,
    - TableSize = 10
    - h(k) = k % 10
    - Insert first 10 perfect squares

```
Insertion sequence:
{ 0 1 4 9 16 25 36 49 64 81 }
```



Collision Resolution by Chaining

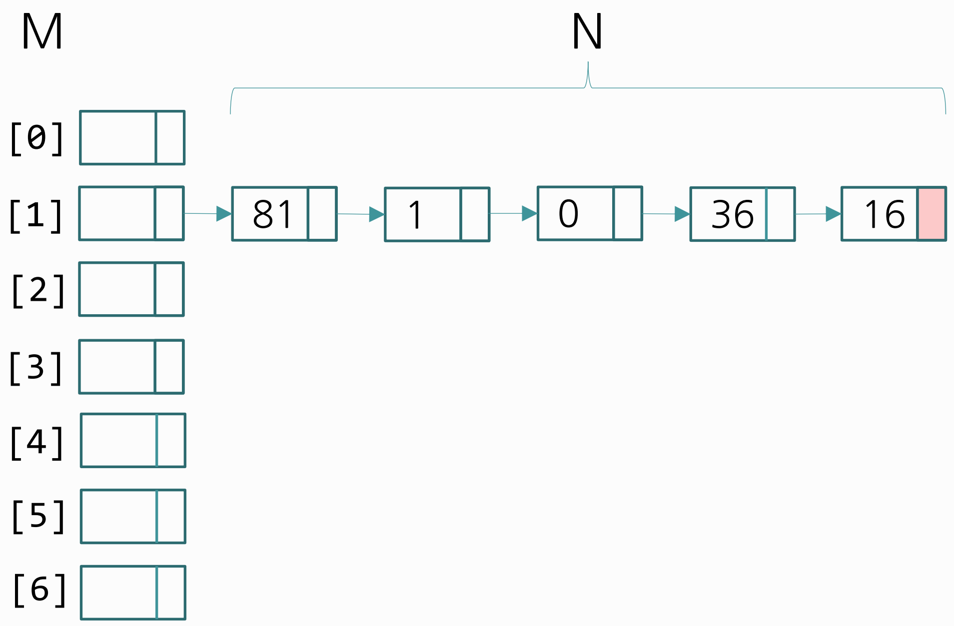
- Load factor λ of a hash table T is defined as follows:
  - Element size: N = number of elements in T
  - Table size: M = size of T
  - Load factor: $\lambda = N/M$ (적재율)
    - i.e., λ is the average length of a chain



- Unsuccessful search time: $O(\lambda)$
  - Same for insert time
- Successful search time average: $O(\lambda/2)$
- Ideally, want $\lambda \leq 1$ (then, not a function of N)

Collision Resolution by Chaining

- Potential disadvantages of Chaining
  - Linked lists could get long
    - Especially when N >> M
    - Longer linked lists could negatively impact performance
  - More memory because of pointers
  - Absolute worst-case (even if N << M):
    - All N elements in one linked list!
      Typically the result of a bad hash function



Collision Resolution by Open Addressing

1. Linear Probing (선형조사법)
2. Quadratic Probing (이차조사법)
3. Double Hashing (이중해싱법)

Collision Resolution by Open Addressing

- When a collision occurs, look elsewhere in the table for an empty slot.

- Advantages over chaining
  - No need for list structures
  - No need to allocate/deallocate memory during insertion/deletion (slow)

- Disadvantages
  - Slower insertion - May need several attempts to find an empty slot.
  - Table needs to be bigger (than chaining-based table) to achieve average-case constant-time performance.
    - Load factor $\lambda \approx 0.5$

Collision Resolution by Open Addressing

- A "**Probe sequence**" is a sequence of slots in hash table while searching for an element k
  - $h_0(k)$, $h_1(k)$, $h_2(k)$, …
  - Needs to visit each slot exactly once
  - Needs to be repeatable (so we can find/delete what we've inserted before)

- Hash function
  - $h_i(k) = (h(k) + f(i)) \% TableSize$
  - $f(0) = 0$ → position for the 0^{th} probe
  - $f(i)$ is "the distance to be traveled relative to the 0^{th} probe position, during the i^{th} probe". It can be linear, quadratic etc.

Collision Resolution by Open Addressing - Linear Probing

- $f(i)$ = is a **linear** function of i, e.g., $f(i) = i$

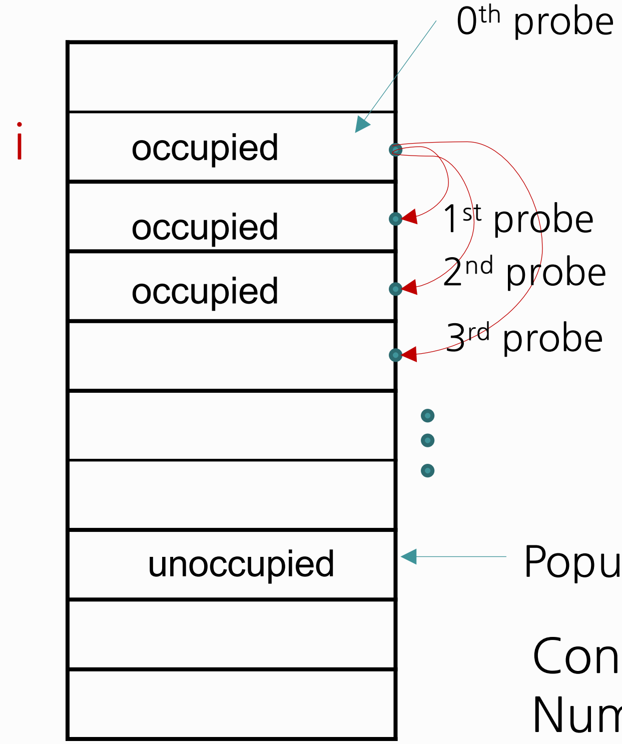
$$h_i(k) = (\ h(k) + i\ )\ \%\ TableSize$$

i^{th} probe index 0th probe index $f(i)$

Probe sequence: +0, +1, +2, +3, +4, …

linear

Linear probing

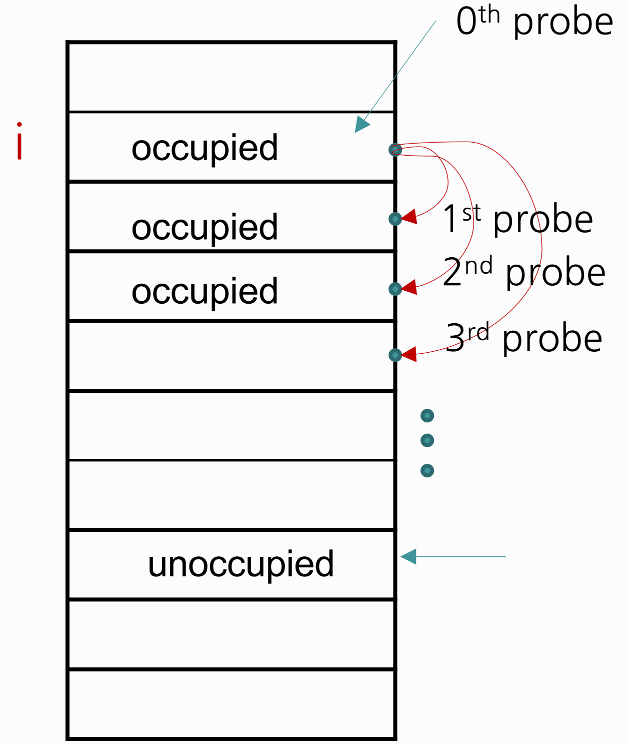


Populate x here

Continue until an empty slot is found
Number of failed probes is a measure of performance

Collision Resolution Example - Linear Probing

Linear probing



- $f(i)$ = is a **linear** function of i, e.g., $f(i) = i$

$$h_i(k) = (\ h(k) + i\ )\ \%\ TableSize$$

i^{th} probe index 0th probe index $f(i)$

Probe sequence: +0, +1, +2, +3, +4, …

linear

- Example: h(k) = k % TableSize
  - $h_0(89) = (h(89) + 0)\ \%\ 10 = 9$
  - $h_0(18) = (h(18) + 0)\ \%\ 10 = 8$
  - $h_0(49) = (h(49) + 0)\ \%\ 10 = 9$ (collision)
    $h_1(49) = (h(49) + 1)\ \%\ 10 = (h(49) + 1)\ \%\ 10 = 0$

Collision Resolution Example - Linear Probing

Insert sequence: **8, 1, 9, 6, 15**

$h(k) = k \% 7$

	Empty Table	After 8	After 1	After 9	After 6	After 15
0						
1		8	8			
2			1			
3						
4						
5						
6						

$h_0(8) = 8 \% 7 = 1$

$h_0(1) = 1 \% 7 = 1$
$h_1(1) = (h(1)+1) \% 7 = 2$

Collision Resolution Example - Linear Probing

Insert sequence: **8, 1, 9, 6, 15**

$h(k) = k \% 7$

	Empty Table	After 8	After 1	After 9	After 6	After 15
0						
1		8	8	8	8	
2			1	1	1	
3				9	9	
4						
5						
6					6	

$h_0(8) = 8 \% 7 = 1$

$h_0(1) = 1 \% 7 = 1$
$h_1(1) = (h(1)+1) \% 7 = 2$

$h_0(9) = 9 \% 7 = 2$
$h_1(9) = (h(9)+1) \% 7 = 3$

$h_0(6) = 6 \% 7 = 6$

Collision Resolution Example - Linear Probing

Insert sequence: **8, 1, 9, 6, 15**

$h(k) = k \% 7$

	Empty Table	After 8	After 1	After 9	After 6	After 15
0						
1		8	8	8	8	8
2			1	1	1	1
3				9	9	9
4						15
5						
6					6	6

$h_0(8) = 8 \% 7 = 1$

$h_0(1) = 1 \% 7 = 1$
$h_1(1) = (h(1)+1) \% 7 = 2$

$h_0(9) = 9 \% 7 = 2$
$h_1(9) = (h(9)+1) \% 7 = 3$

$h_0(6) = 6 \% 7 = 6$

$h_0(15) = (h(15) + 0) \% 7 = 1$ (collision)
$h_1(15) = (h(15) + 1) \% 7 = 2$ (collision)
$h_2(15) = (h(15) + 2) \% 7 = 3$ (collision)
$h_3(15) = (h(15) + 3) \% 7 = 4$

probe sequence
and it is linear

Collision Resolution Exercise - Linear Probing

Insert sequence: **89, 18, 49, 58, 69**

$h(k) = k \ \% \ 10$

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	
1						
2						
3						
4						
5						
6						
7						
8			18	18	18	
9		89	89	89	89	
Unsucessful no. of probes		0	0	1	3	3

probe sequence

For example, linear probing for 58

$h_0(58) = (h(58)+f(0)) \ \% \ 10$
$= (\quad 8 + 0) \ \% \ 10 = 8$ (collision)

$h_1(58) = (h(58)+ 1) \ \% \ 10 = 9$ (collision)

$h_2(58) = (h(58)+ 2) \ \% \ 10 = 0$ (collision)

$h_3(58) = (h(58)+ 3) \ \% \ 10 = 1$

Collision Resolution Exercise - Linear Probing

Insert sequence: **89, 18, 49, 58, 69**

$h(k) = k \% 10$

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	49
1					58	58
2						
3						
4						
5						
6						
7						
8			18	18	18	18
9		89	89	89	89	89
Unsucessful no. of probes		**0**	**0**	**1**	**3**	**3**

probe sequence

For example, linear probing for 58

$h_0(58) = (h(58)+f(0)) \% 10$
$= (\quad 8 + 0) \% 10 = 8$ (collision)

$h_1(58) = (h(58)+1) \% 10 = 9$ (collision)

$h_2(58) = (h(58)+2) \% 10 = 0$ (collision)

$h_3(58) = (h(58)+3) \% 10 = 1$

Complete the linear probing for 69

$h_0(69) =$

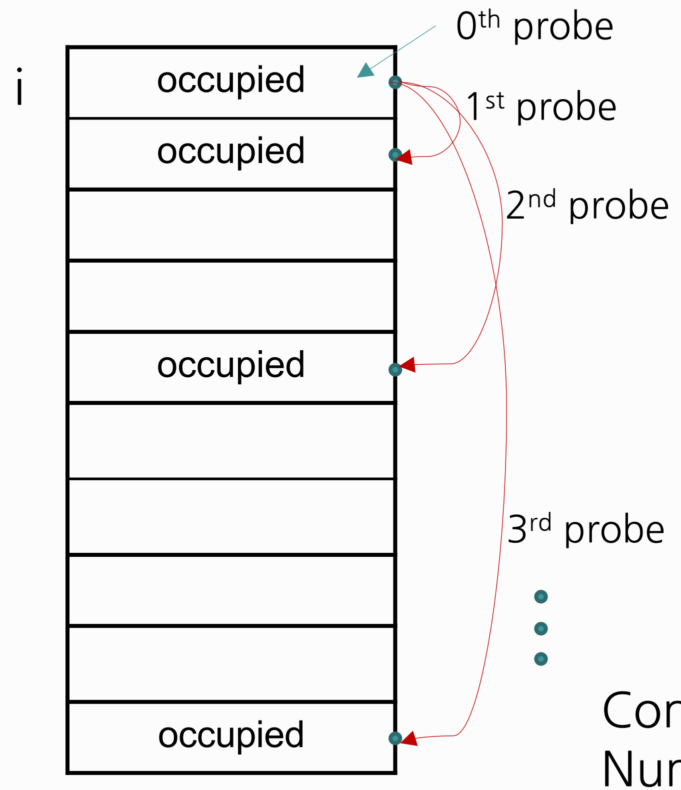
Collision Resolution - Linear Probing Issues

- Probe sequences can get longer with time.
- Primary clustering
  - Keys tend to cluster in one part of table.
  - Keys that hash into cluster will be added to the end of the cluster (making it even bigger).
  - Side effect:
    Other keys could also get affected if mapping to a crowded neighborhood.

Collision Resolution - Quadratic Probing이차조사법

- Avoids primary clustering
- f(i) is quadratic in i, e.g., **$f(i) = i^2$**

Linear probing



$$h_i(k) = (h(k) + i^2\ )\ \%\ \text{TableSize}$$

ith probe index   0th probe index   f(i)

Probe sequence: +0, +1, +4, +9, +16, …

Continue until an empty slot is found
Number of failed probes is a measure of performance

Collision Resolution - Quadratic Probing 이차조사법

- Avoids primary clustering
- $f(i)$ is quadratic in i, e.g., $f(i) = i^2$

$$h_i(k) = (h(k) + i^2) \ \% \ TableSize$$

i^{th} probe index $\quad$ 0^{th} probe index $\quad$ $f(i)$

Probe sequence: +0, +1, +4, +9, +16, … $\quad$ quadratic

- Example:
  - $h_0(58) = (h(58) + 0^2) \ \% \ 10 = 8$ (collision)
  - $h_1(58) = (h(58) + 1^2) \ \% \ 10 = 9$ (collision)
  - $h_2(58) = (h(58) + 2^2) \ \% \ 10 = 2$

Collision Resolution Exercise - Quadratic Probing이차조사법

Insert sequence: **89, 18, 49, 58, 69**

$h(k) = k \% 10$

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49 $+1^2$	49	49
1						
2					58 $+2^2$	58
3						
4						
5						
6						
7						
8			18 $+0^2$	18	18 $+0^2$	18
9		89 $+0^2$	89	89 $+0^2$	89 $+1^2$	89
Unsucessful no. of probes		0	0	1	2	2

probe sequence

For example, quadratic probing for 58

$h_0(58) = (h(58)+f(0)) \% 10$
$= (\ 8 + 0) \% 10 = \boxed{8}$ (collision)

$h_1(58) = (h(58)+ 1) \% 10 = \boxed{9}$ (collision)

$h_2(58) = (h(58)+ 4) \% 10 = \boxed{2}$

Collision Resolution Exercise - Quadratic Probing이차조사법

Insert sequence: **89, 18, 49, 58, 69**

h(k) = k % 10

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49 $+1^2$	49	49
1						
2					58 $+2^2$	58
3						
4						
5						
6						
7						
8			18 $+0^2$	18	18 $+0^2$	18
9		89 $+0^2$	89	89 $+0^2$	89 $+1^2$	89
Unsucessful no. of probes		0	0	1	2	2

probe sequence

For example, quadratic probing for 58

$h_0(58)$ = (h(58)+f(0)) % 10
= (8 + 0) % 10 = 8 (collision)

$h_1(58)$ = (h(58)+ 1) % 10 = 9 (collision)

$h_2(58)$ = (h(58)+ 4) % 10 = 2

Complete quadratic probing for 69

$h_0(69)$ =

Collision Resolution - Quadratic Probing Analysis

- Difficult to analyze
- Theorem:
  - New element can always be inserted into a table that is at least half empty and TableSize is prime.
  - Otherwise, may never find an empty slot, even is one exists.
- Ensure table never gets half full.
  - If close, then expand (rehash) it.
- May cause "secondary clustering"

Summary

- Table size prime
- Table size larger than number of inputs (to maintain $\lambda \ll 1.0$)
- Two types of collision resolution
  - Separate chaining
  - Open addressing - probing
  - Tradeoffs between chaining vs. probing
- Collision chances decrease in this order:
  linear probing → quadratic probing → double hashing
- Rehashing recommended to resize hash table at a time when λ exceeds 0.5

학습 정리

1) 충돌 해결 방법은 크게 Separate chaining과 Open addressing 두 가지가 있다

2) 적재율(λ)이 0.5 보다 작아야 O(1)을 유지할 수 있다

3) Linear probing > Quadratic probing > Double hashing 순서로 충돌 횟수가 줄어든다

파이썬으로 배우는 데이터 구조

수고했습니다
곧 다음 시간에
다시 뵙겠습니다