# 파이썬으로 배우는 데이터 구조

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# 학습 목표

AVL 트리의 균형을 유지하게하는
Rebalance 알고리즘을 학습하고 구현할 수 있다

**Data Structures in Python**
**Chapter 7 - 2**

- Binary Search Tree(BST)
- BST Algorithms
- AVL Tree
- **AVL Algorithms**

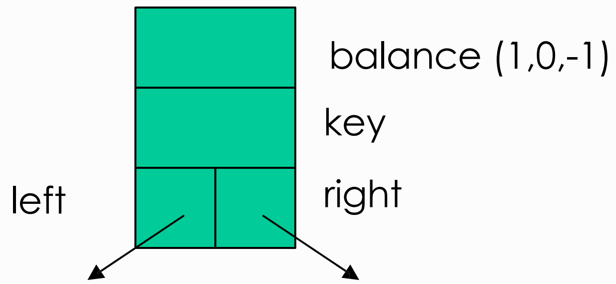
# Agenda & Readings

- AVL Tree
  - Balance Factor
  - Rotations:
    - Single Rotation - LL and RR cases (Outside case)
    - Double Rotation - LR and RL cases (Inside case)
  - Define AVL Class
  - Coding:
    - height(), balanceFactor(), _add() & _delete(), rebalance()

- Reference:
  - Problem Solving with Algorithms and Data Structures
    Chapter 6 - Tree
  - Wikipedia: AVL tree

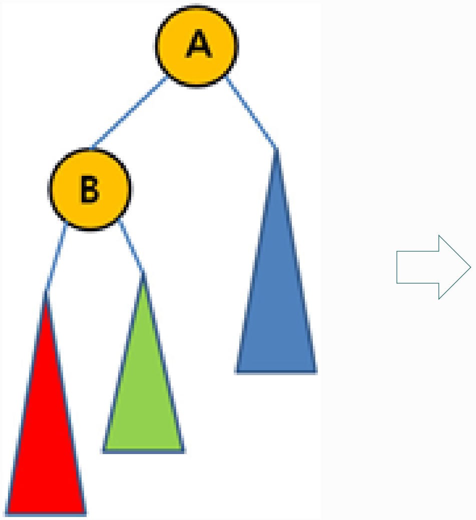
# Balance Factor

- You can either keep the height or just the difference in height, i.e., the balance factor; this must be modified on the path of insertion or deletion even if you do not perform rotations.
  - Once you have performed a rotation (single or double) you won't need to go back up the tree for the computation.
- You may compute the balance factor **on the fly** after the insert is done during the recursion.



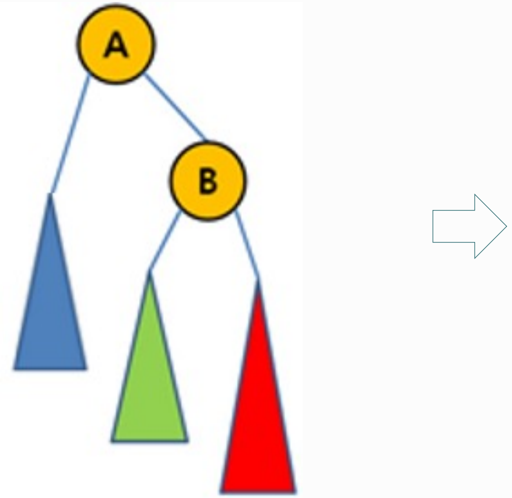
# Single Rotation(Outside Case)

## LL Case




```
def LLcase(A):



    return
```

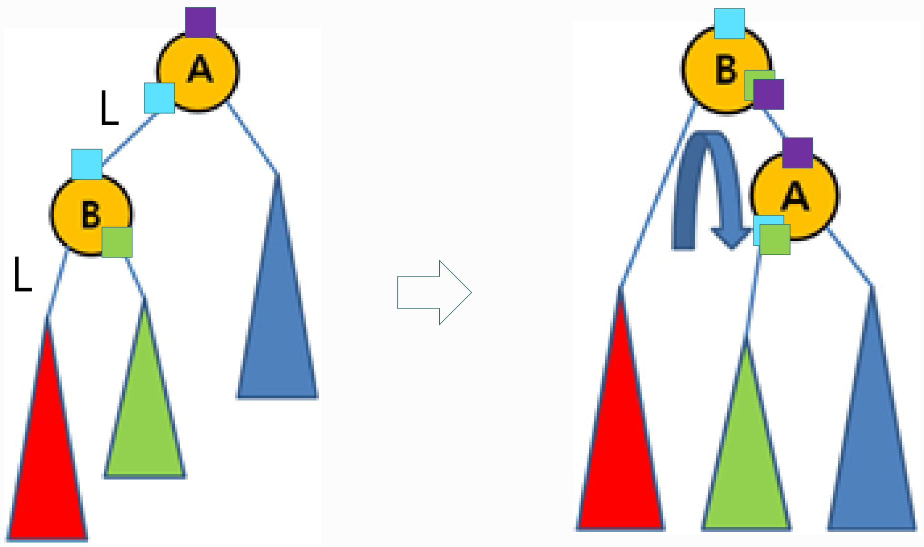
## RR Case



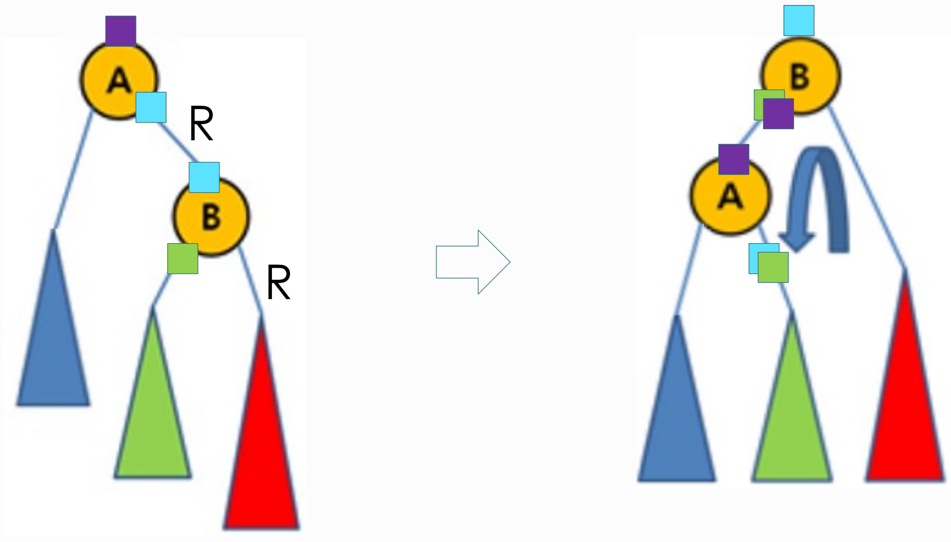

```
def RRcase(A):



    return
```

# Single Rotation(Outside Case)

## LL Case




## RR Case

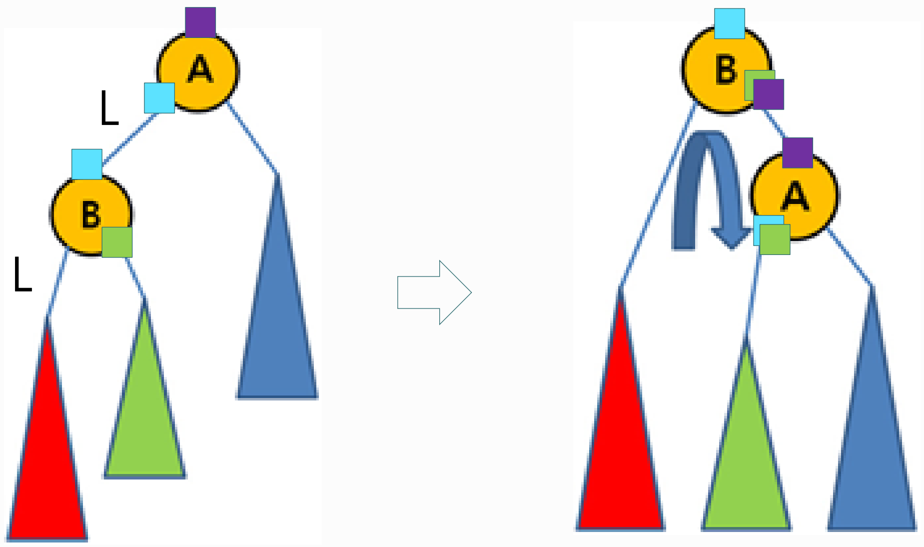



```
def LLcase(A):



    return
```

```
def RRcase(A):



    return
```

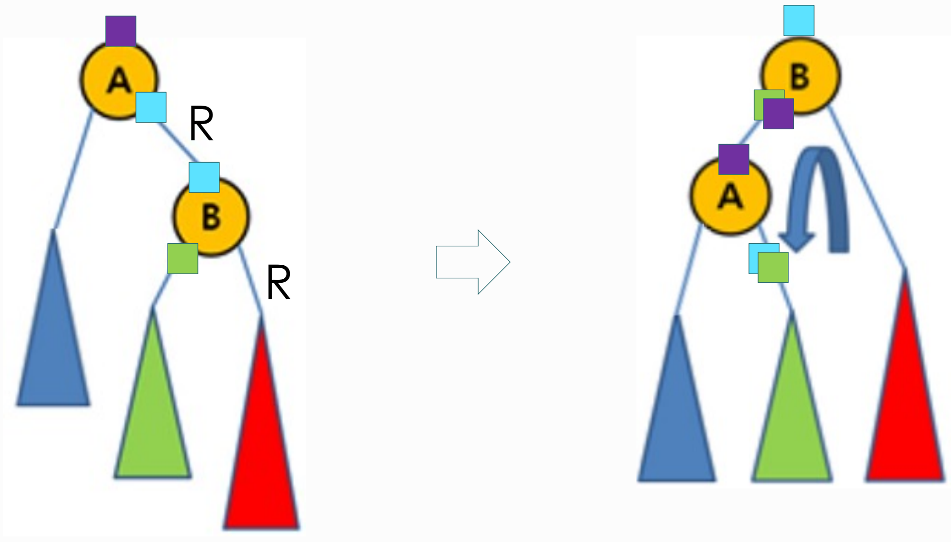
# Single Rotation(Outside Case)

## LL Case




```
def LLcase(A):
    B = A.left


    return
```

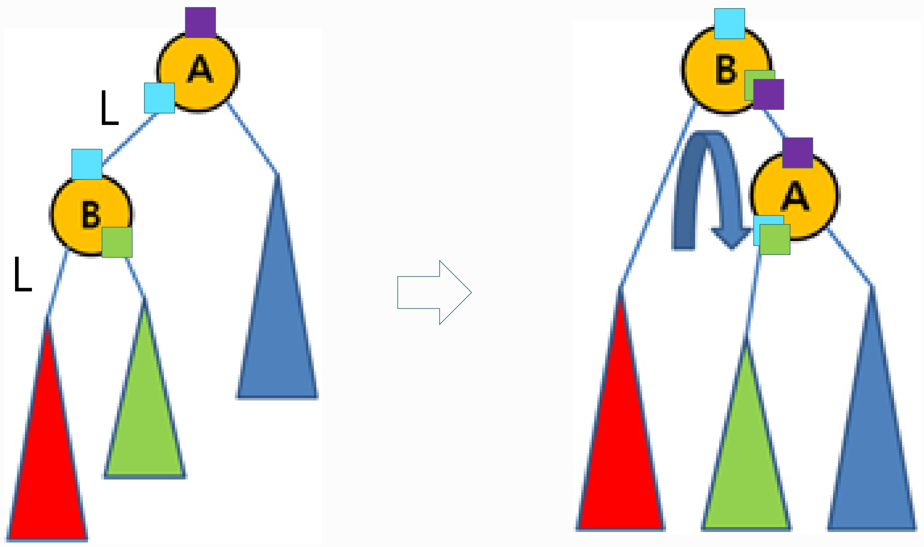
## RR Case



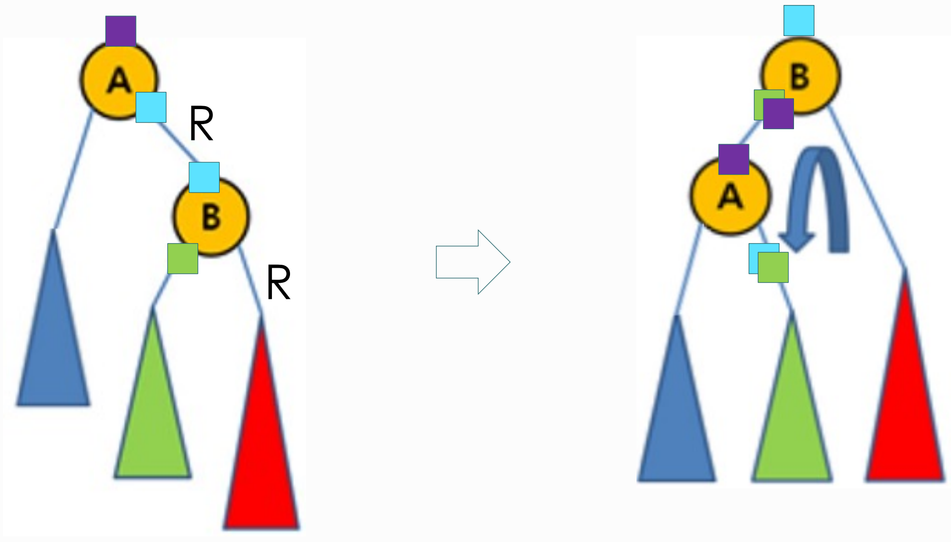

```
def RRcase(A):



    return
```

# Single Rotation(Outside Case)

## LL Case




## RR Case

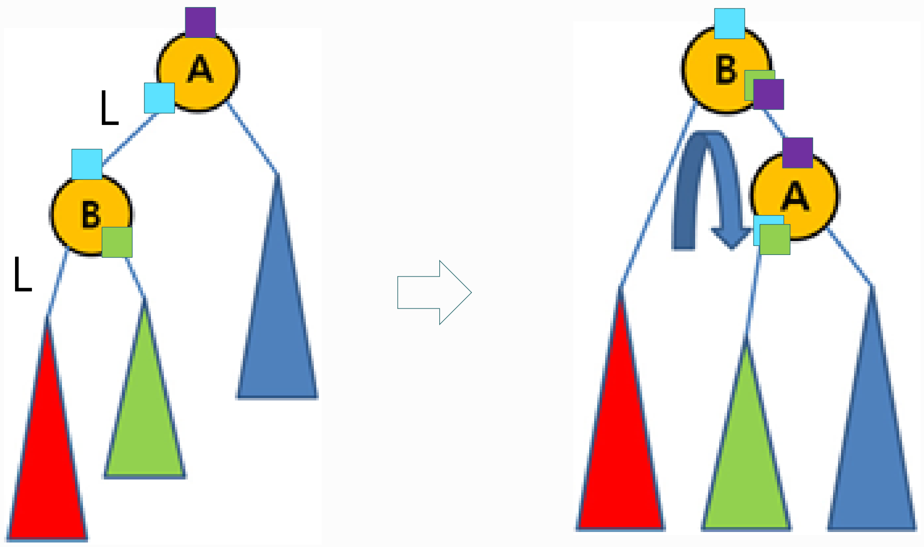



```
def LLcase(A):
    B = A.left
    A.left  = B.right

    return
```

```
def RRcase(A):



    return
```

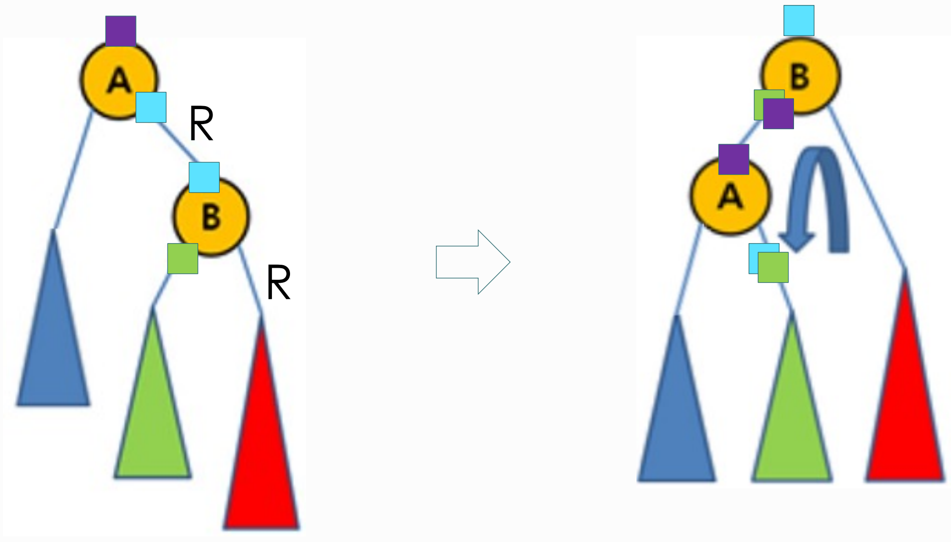
# Single Rotation(Outside Case)

## LL Case




```
def LLcase(A):
    B = A.left
    A.left  = B.right
    B.right = A
    return
```

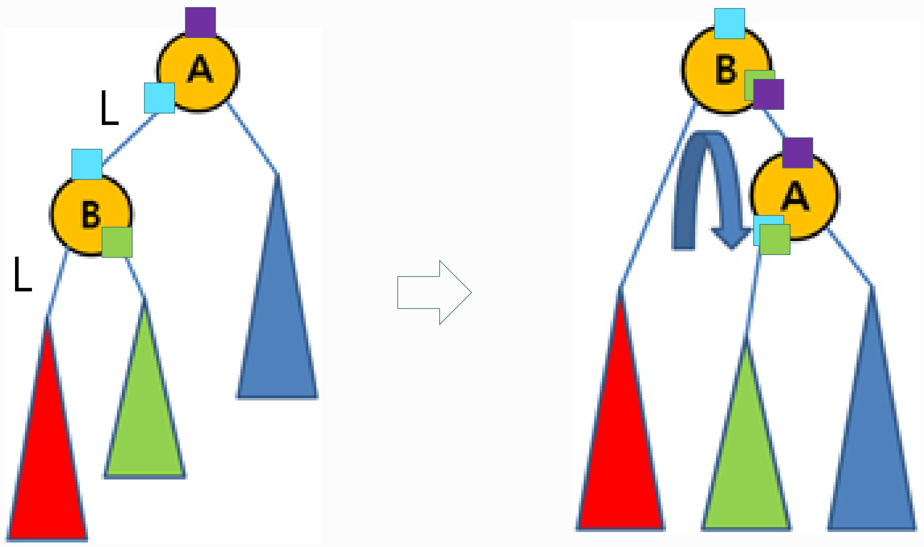
## RR Case




```
def RRcase(A):



    return
```

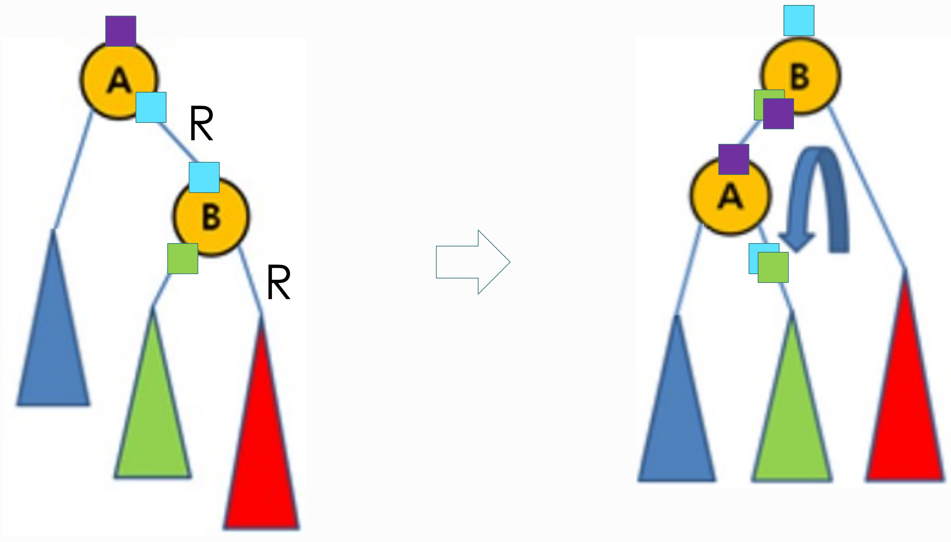
# Single Rotation(Outside Case)

## LL Case




```
def LLcase(A):
    B = A.left
    A.left  = B.right
    B.right = A
    return B
```

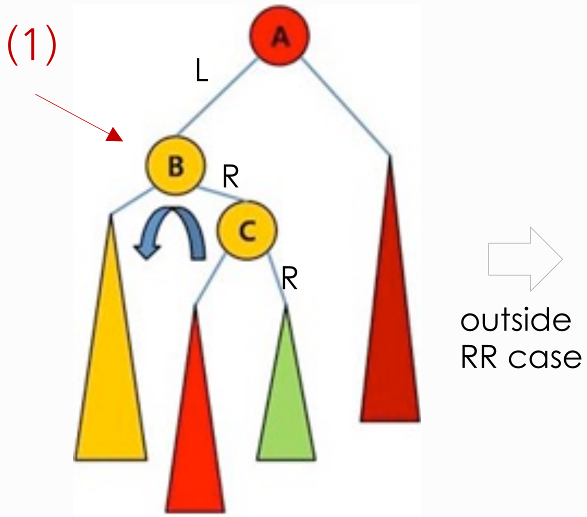
## RR Case




```
def RRcase(A):



    return
```

# Double Rotation(Inside Case)

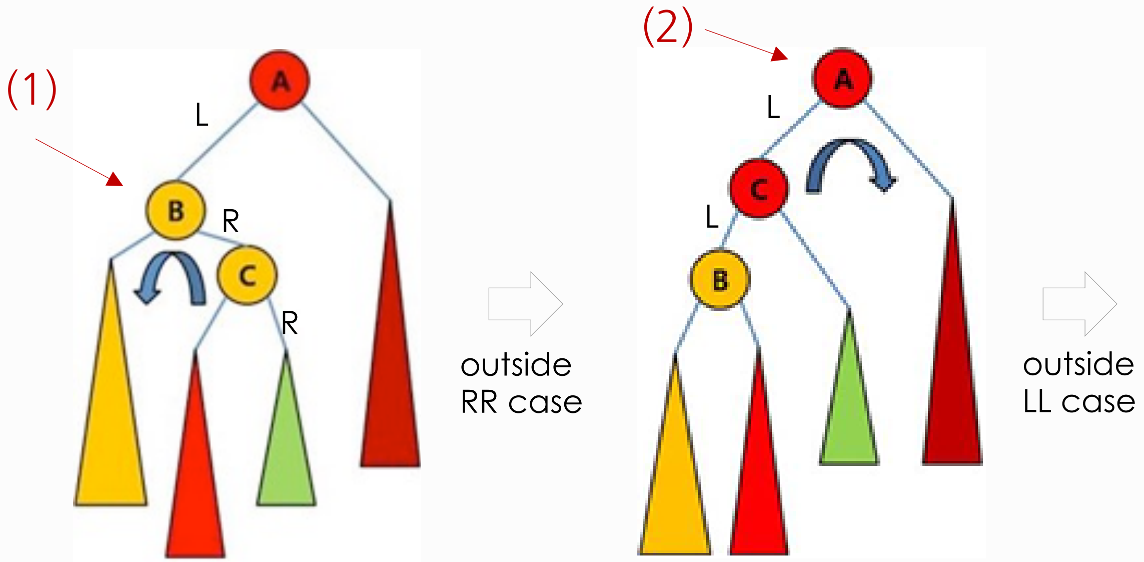
## LR case




```
def LRcase(A):
```

# Double Rotation(Inside Case)

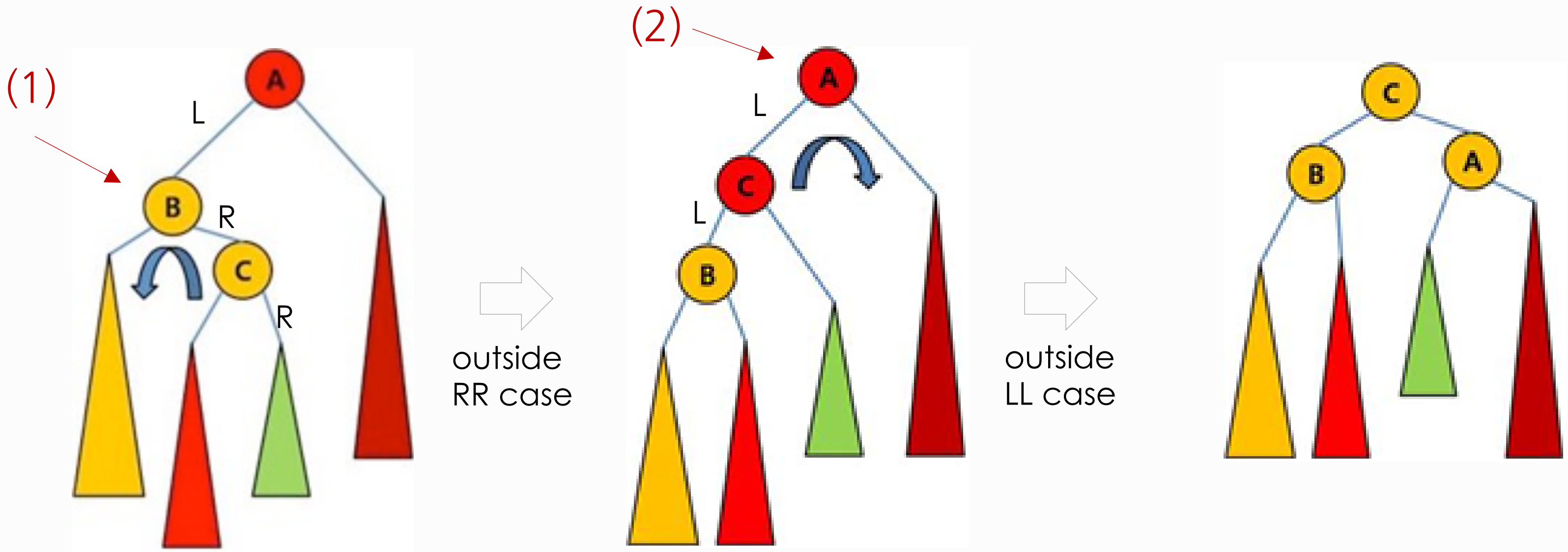
## LR case




```
def LRcase(A):
```

# Double Rotation(Inside Case)

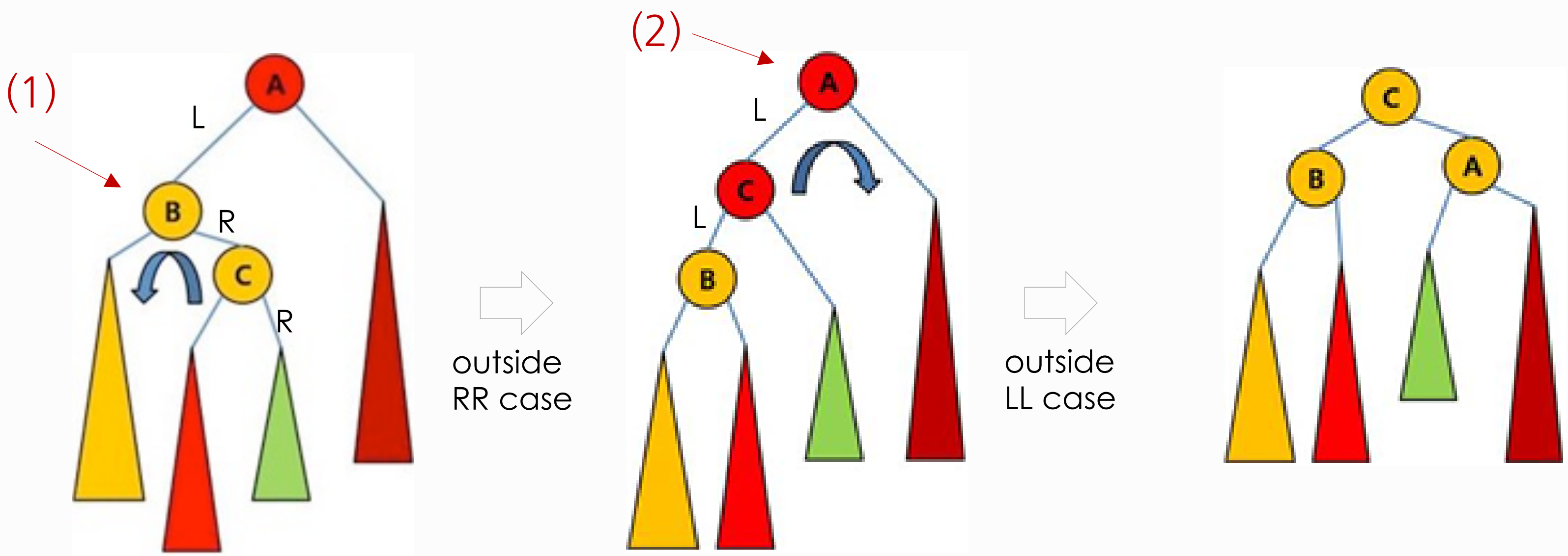
## LR case




```
def LRcase(A):
```

# Double Rotation(Inside Case)

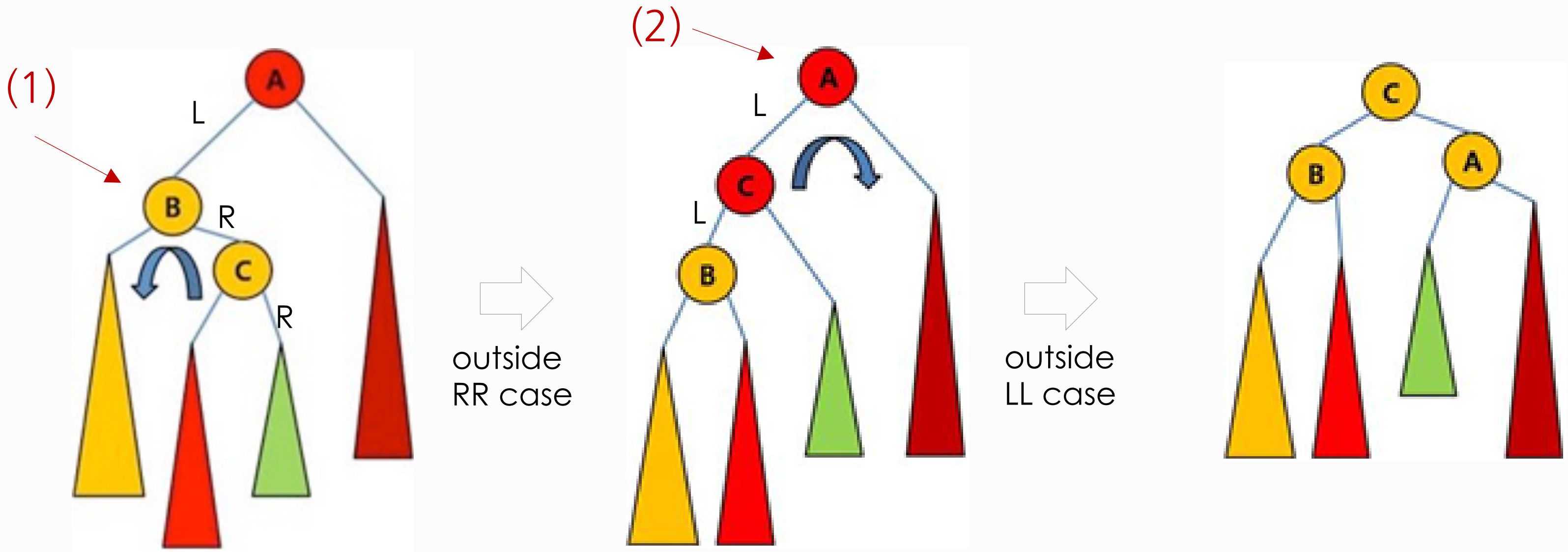
## LR case




```
def LRcase(A):
    B = A.left
```

# Double Rotation(Inside Case)

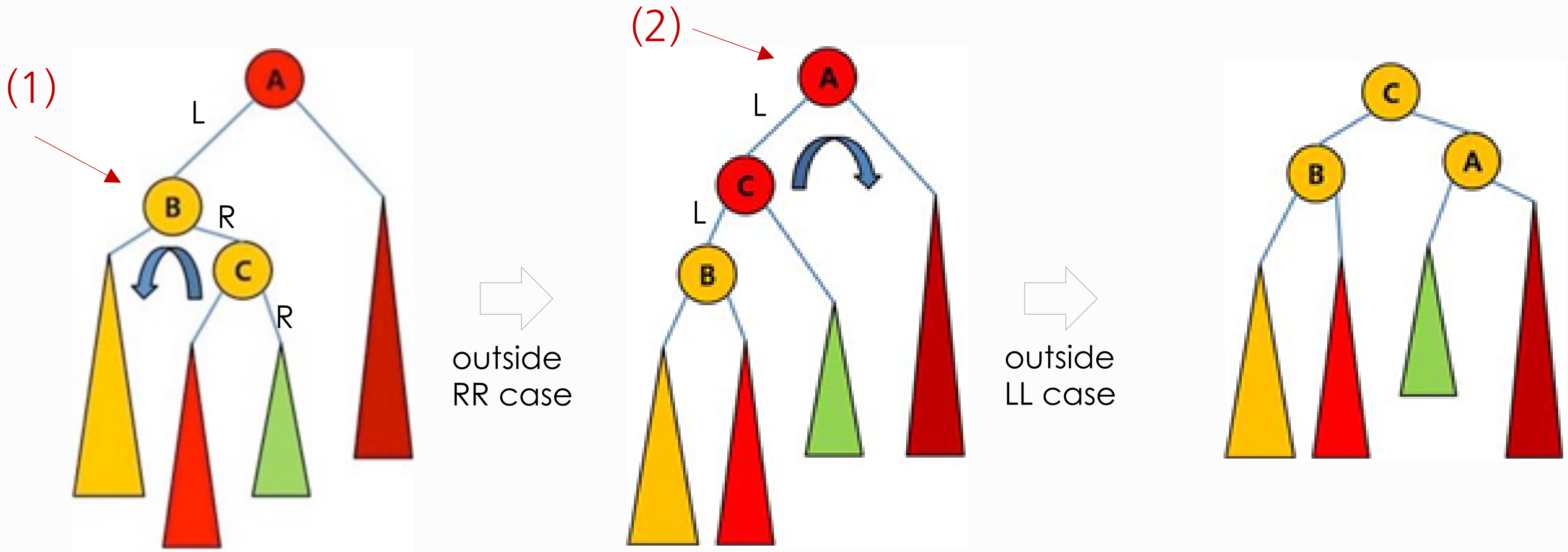
## LR case




```
def LRcase(A):
    B = A.left
    A.left = RRcase(B)
```

# Double Rotation(Inside Case)

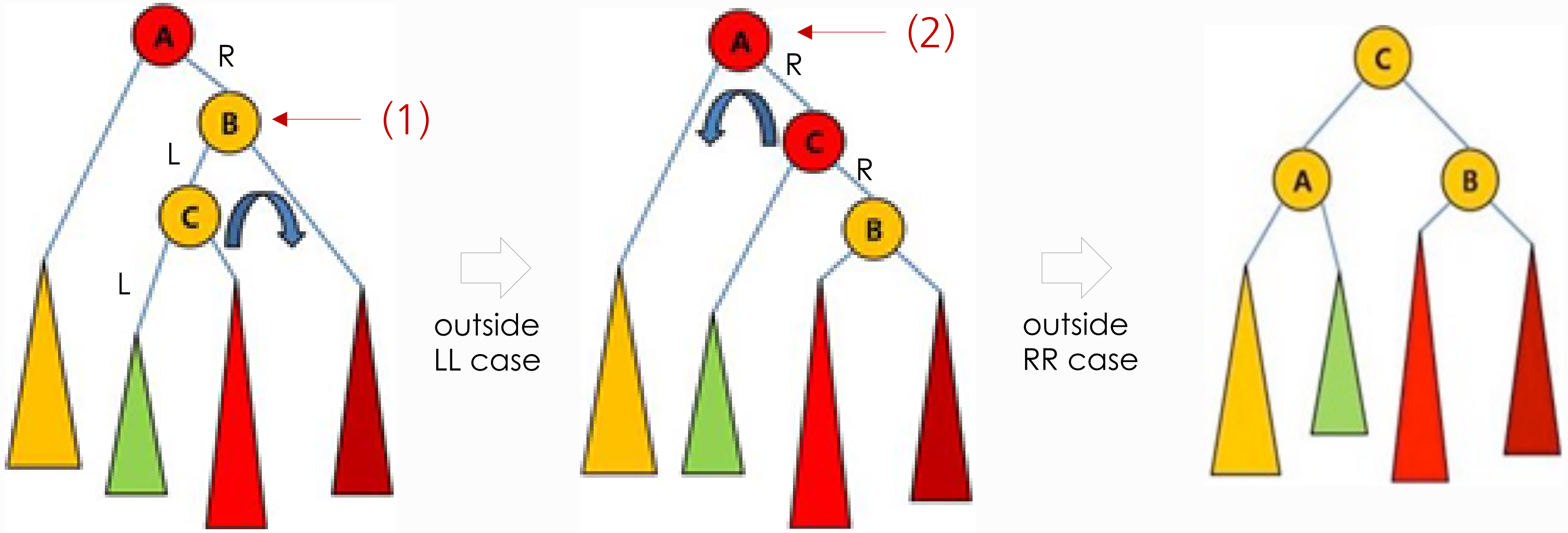
## LR case




```
def LRcase(A):
    B = A.left
    A.left = RRcase(B)
    return LLcase(A)
```

# Double Rotation(Inside Case)

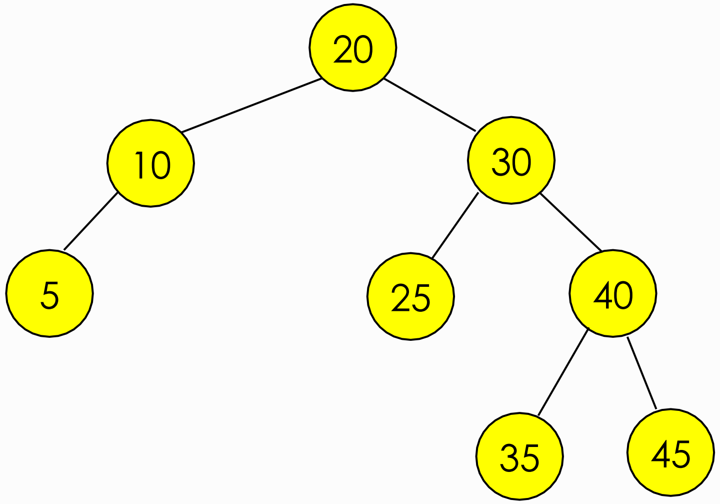
## RL case




```
def RLcase(A):
```

# Double Rotation - ??case

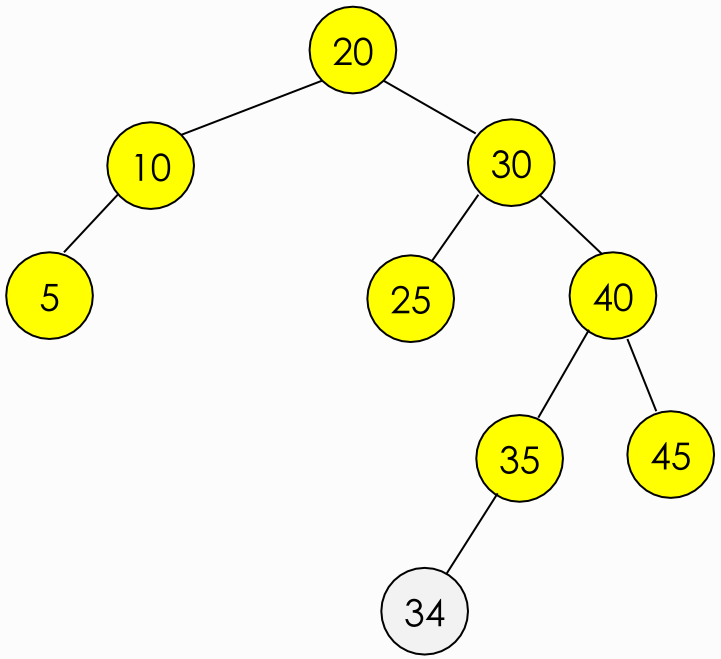
- **Insertion of 34**
- Imbalance at ?
- Balance factor ??
- Rotation ___??___ case




AVL balanced tree

# Double Rotation - ??case

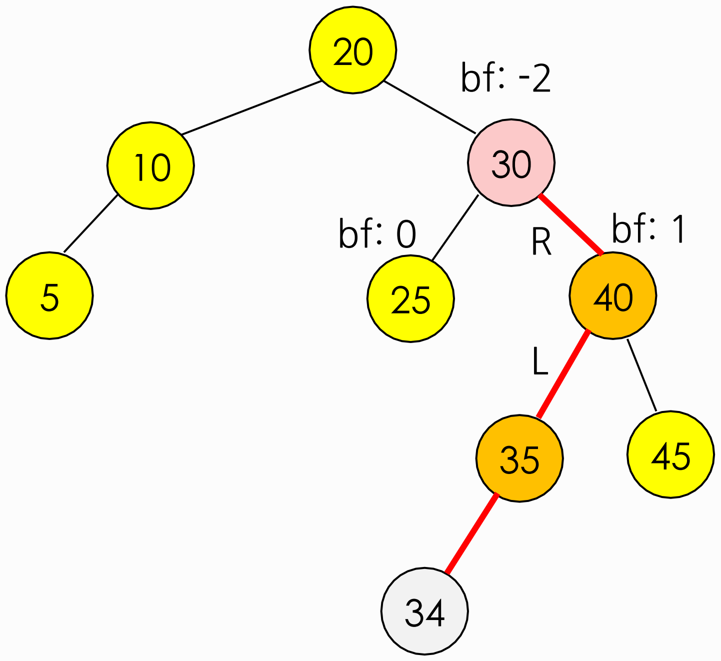
- Insertion of 34
- Imbalance at ?
- Balance factor ??
- Rotation ___??___ case




After insertion, it is AVL imbalanced at ?

# Double Rotation - ??case

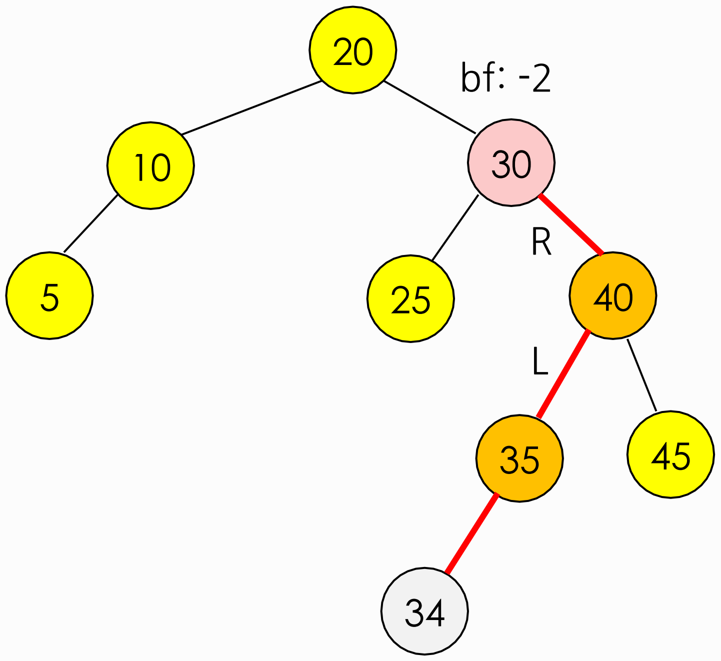
- Insertion of 34
- Imbalance at 30
- Balance factor **-2**
- Rotation ___??___ case



# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

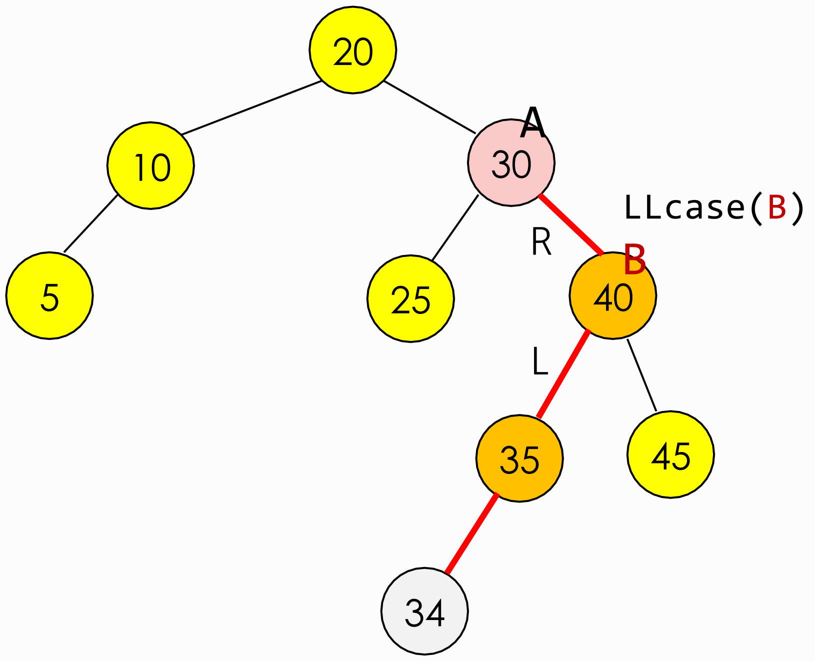
```
def RLcase(A):
  B = A.right
  A.right = LLcase(B)
  return RRcase(A)
```



# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

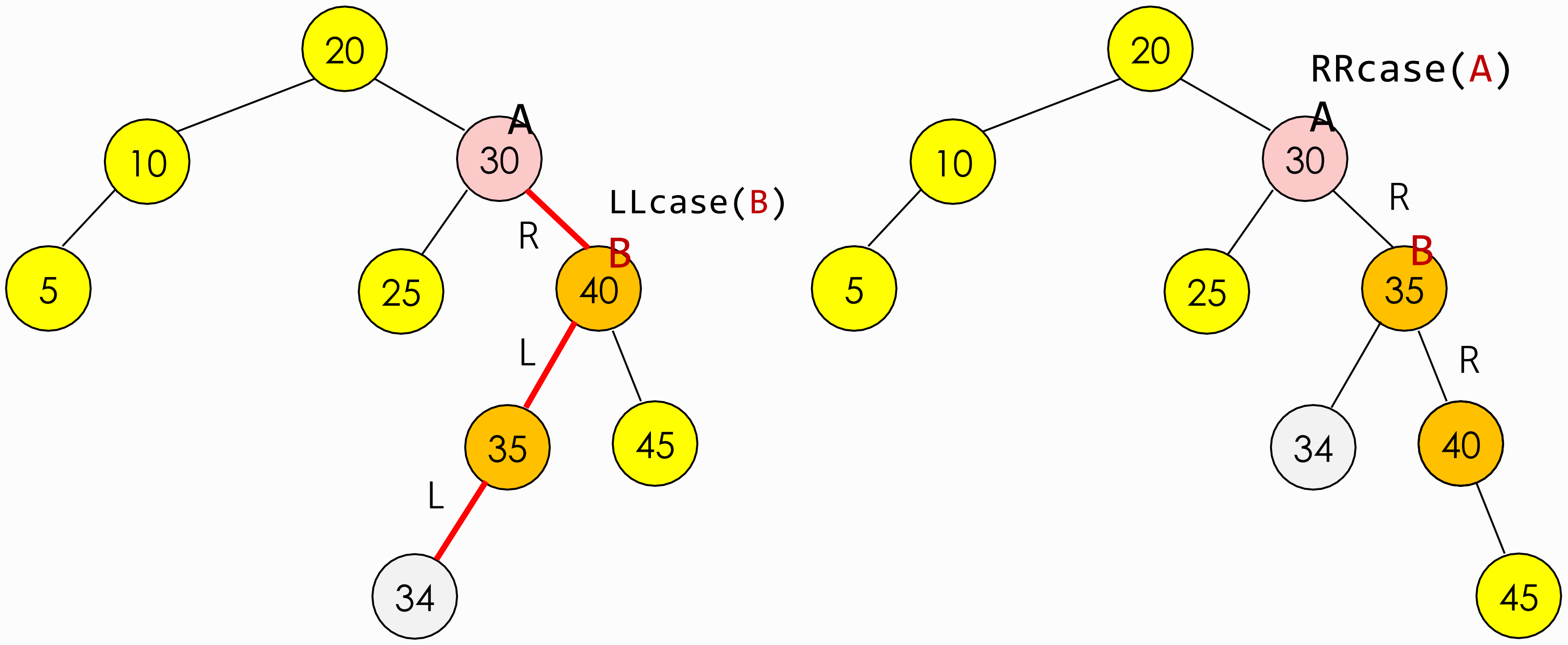
```
def RLcase(A):
  B = A.right
  A.right = LLcase(B)
  return RRcase(A)
```



# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

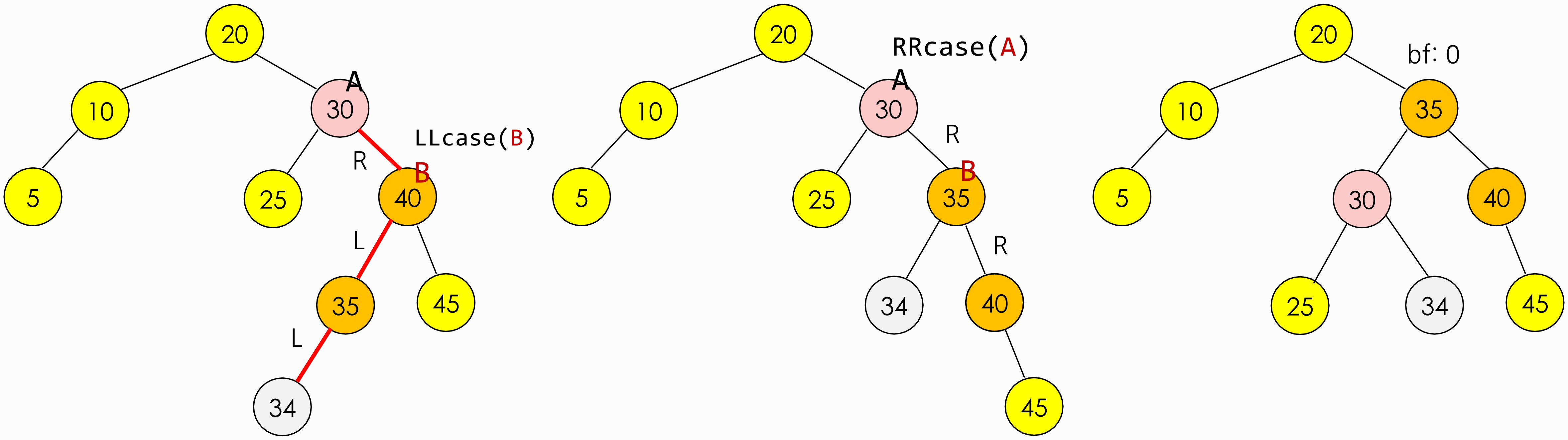
```
def RLcase(A):
  B = A.right
  A.right = LLcase(B)
  return RRcase(A)
```



# Double Rotation - RL case

- Insertion of 34
- Imbalance at 30
- Balance factor -2
- Rotation ___RL___ case

```
def RLcase(A):
  B = A.right
  A.right = LLcase(B)
  return RRcase(A)
```




After insertion, AVL imbalanced tree

After insertion, AVL balanced tree

# height() and _height()

```
def height(self):
    return self._height(self.root)
```

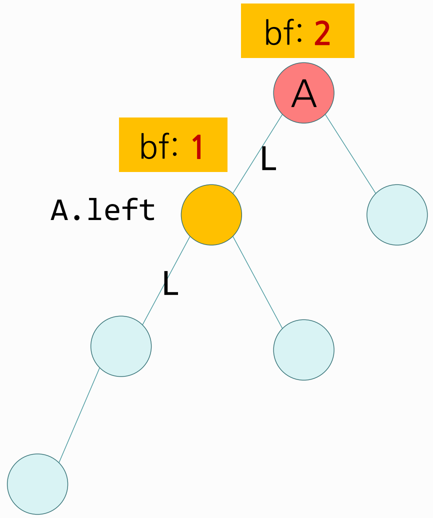
```
def _height(self, node):
    if node is None: return -1  # if empty

    l = self._height(node.left)
    r = self._height(node.right)
    return max(l, r) + 1
```

# _add() & _delete()

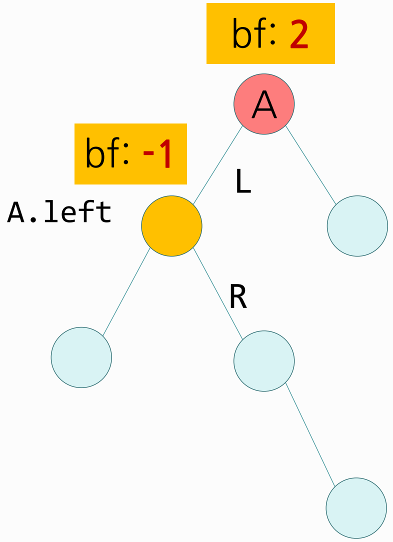
```
class AVL(BST):
    def _add(self, node, key):
        curr = super()._add(node, key)
        return self.rebalance(curr)

    def _delete(self, node, key):
        curr = super()._delete(node, key)
        return self.rebalance(curr)

    def rebalance(self, A):
        def balanceFactor(A):
            if A == None: return
            l = self._height(A.left)
            r = self._height(A.right)
            return l - r
        # your code here: define LLcase(), RRcase(), LRcase(), RLcase()
        # your code here: invoke rotate functions depending on balance factor.
        pass
```

# rebalance()

LL Case




LR case




```
def rebalance(A):
    ...
    bf = balanceFactor(A)
    if bf == 2:
        bf:1, 0
        bf: -1

    elif bf == -2:




    return A          # no rebalanced needed
```

checking single or double rotation

LL Case

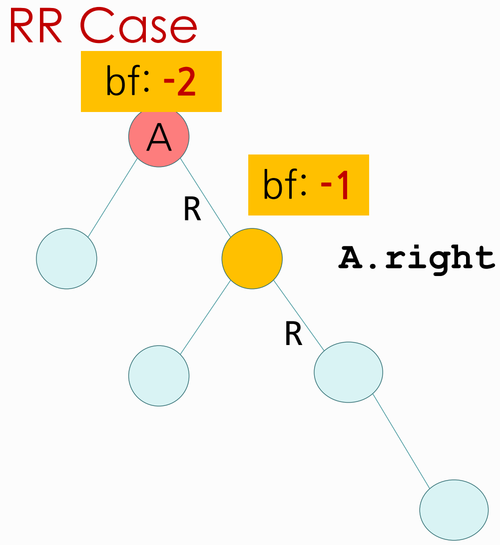
LR case

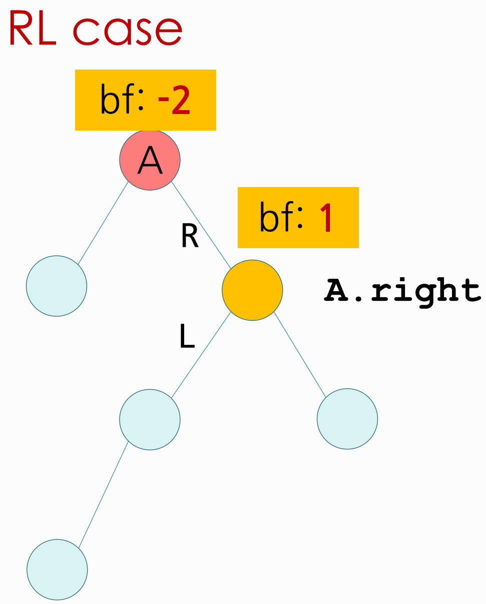
**Observation:** If A and its child have the same sign in bf's, a single rotation is needed, a double rotation otherwise.
If the bf of A's child is 0, treat it like the same sign of A.

# rebalance()

```
def rebalance(A):
    ...
    bf = balanceFactor(A)
    if bf == 2:



                                      checking single or
                                      double rotation
    elif bf == -2:

        bf:-1, 0                     RR Case

        bf: 1                        RL case

    return A         # no rebalanced needed
```

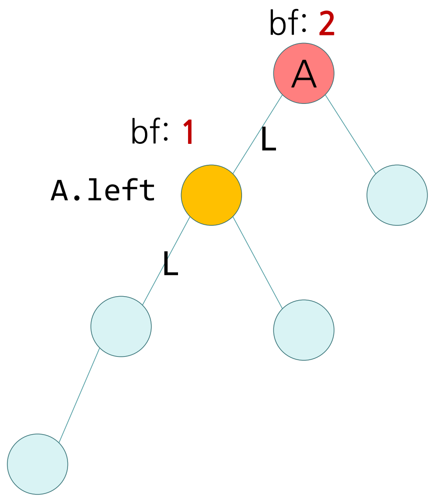
**Observation:** If A and its child have the same sign in bf's, a single rotation is needed, a double rotation otherwise.
If the bf of A's child is 0, treat it like the same sign of A.



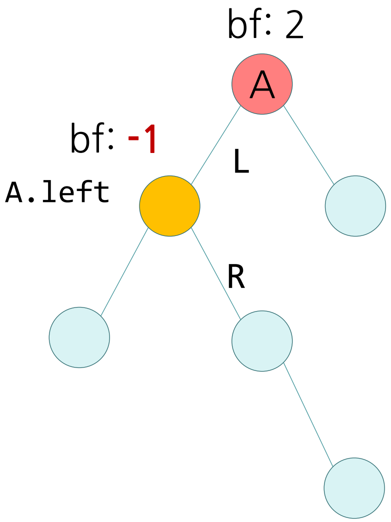



# rebalance()

## LL Case



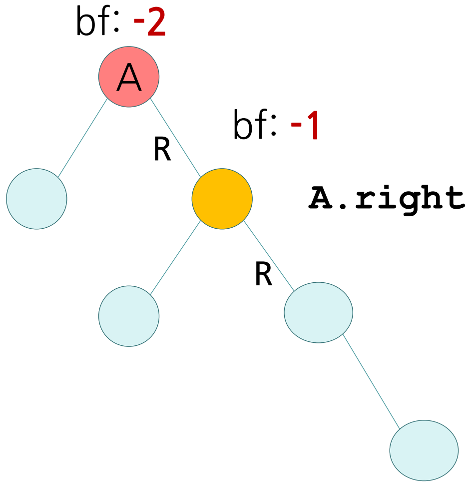

## LR case



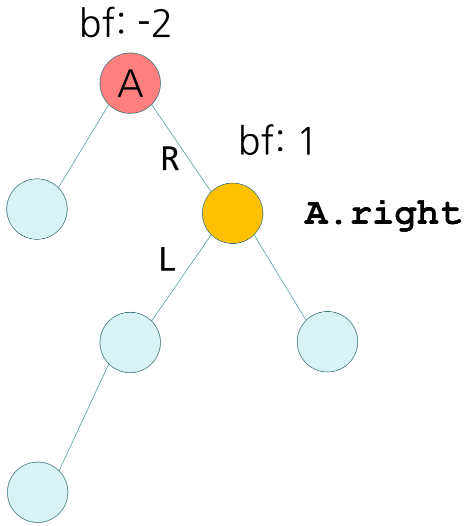

```
def rebalance(A):
    ...
    bf = balanceFactor(A)
    if bf == 2:

                                      LL Case

                                      LR case
    elif bf == -2:

                                      RR Case

                                      RL case
    return A          # no rebalance needed
```

**Observation:** If A and its child have the same sign in bf's, a single rotation is needed, a double rotation otherwise.
If the bf of A's child is 0, treat it like the same sign of A.

## RR Case

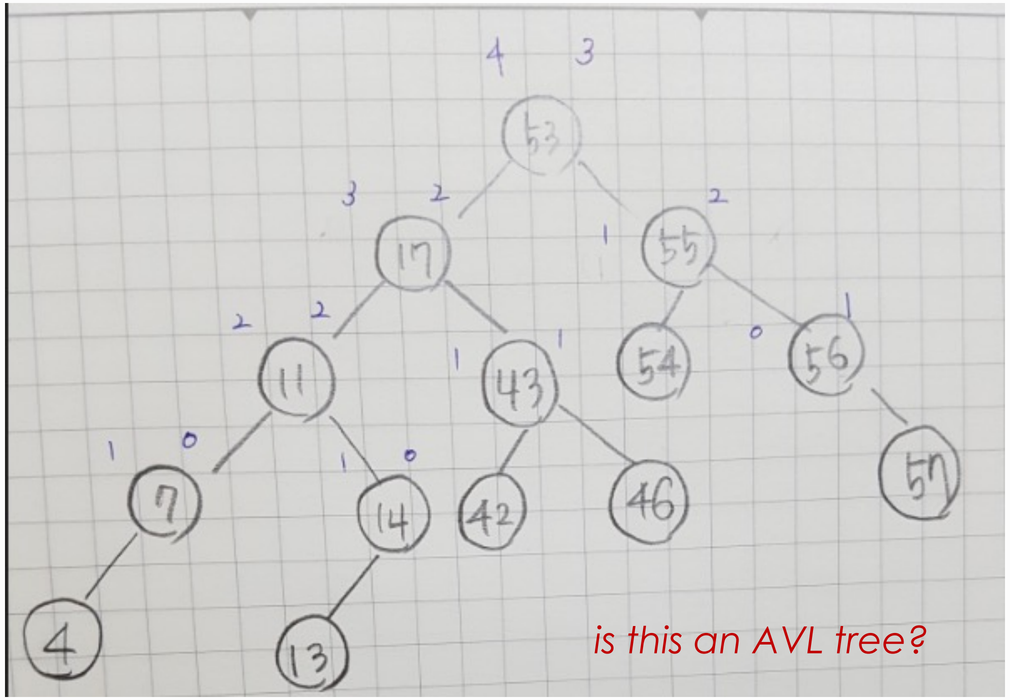



## RL case



# Summary

- AVL tree is binary search tree(BST) that balances every time a node is **inserted or deleted**.
- **Each node** of an AVL tree has the property that the heights of the sub-tree rooted at its children differ **by at most one**.
- The maximum height of an AVL tree having exactly n nodes is $\mathbf{h} \le \mathbf{1.44} * \boldsymbol{log_2 n}$.
  For proof, refer to Wikipedia [AVL tree](AVL tree).




*is this an AVL tree?*

# 학습 정리

1) AVL 트리는 노드가 삽입 혹은 삭제될 때마다 균형을 맞춘다

2) AVL 트리의 각 노드의 두 sub-tree 높이 차이는 최대 1 이다

3) 노드 n개를 가지고 있는 AVL 트리 최대 높이를 h라 하면,
$h \leq 1.44 * \log n$이다

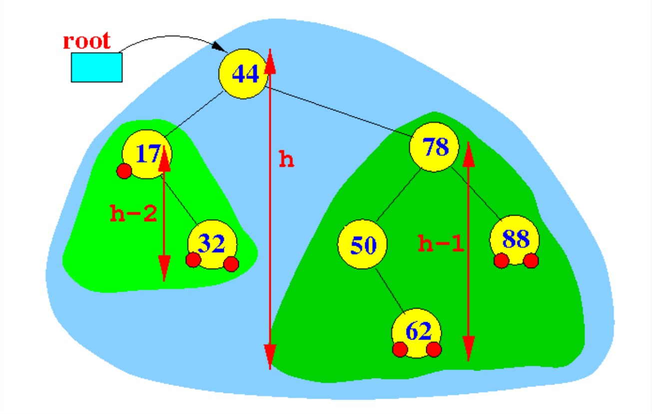
# 파이썬으로 배우는 데이터 구조

수고했습니다
곧 다음 시간에
다시 뵙겠습니다

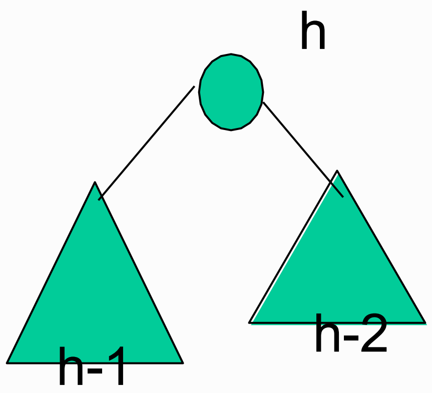
# Height of an AVL Tree

- What is the maximum height of an AVL tree having exactly n nodes?
  - To answer this question, we must ask this question first:
    What is the minimum number of nodes (sparsest possible AVL tree) an AVL tree of height h?

- Consider **the minimum number of nodes** in an AVL tree of height h:
  We can get the recurrence relationship:
  $n(0) = 1, n(1) = 2, n(2) = 4, \cdots$
  $n(h) = n(h-1) + n(h-2) + 1$, where $h > 1$
  This approximate solution of the recurrence is known as $\mathrm{n(h)} \cong 1.618^{h}$




- **Solve the equation** above for h to get **the max height of an AVL tree** with n
  $\log_2 n \geq \mathrm{h} * \log_2 1.62$
  $\mathrm{h} \leq 1/\log_2 1.618 * \log_2 n$
  $\mathbf{h} \leq \mathbf{1.44} * \boldsymbol{log_2 n}$



# Height of an AVL Tree

- AVL trees are binary search trees that balances itself every time an element is **inserted or deleted**. **Each node** of an AVL tree has the property that the heights of the sub-tree rooted at its children differ **by at most one**.

- If there are n nodes in AVL tree, minimum height of AVL tree is floor($\log_2 n$).
- If there are n nodes in AVL tree, maximum height can't exceed $1.44 * \log_2 n$.
- If height of AVL tree is h, maximum number of nodes can be $2^{h+1} - 1$.
- Minimum number of nodes in a tree with height h can be represented as:
  N(h) = N(h-1) + N(h-2) + 1, where N(0) = 1 and N(1) = 2.
- The complexity of searching, inserting and deletion in AVL tree is O($\log_2 n$).
- The cost of balancing AVL tree is O(1).
  What is the time complexity of adding N elements to an empty AVL tree?
  Time complexity: log(1) + log(2) + …. + log(n) <= log(n) + log(n) + … + log(n) = n log (n)