Inverse Kinematics of Sirius 2

To find inverse kinematics solution means to calculate angles of robot arm's joints given target position of the end effector. In the case of the manipulator of the Sirius 2 rover, five joint angles must be solved for, each corresponding to one of the joints. The structure of the robot allows for a relatively simple analytical solution. The purpose of this document is to present such solution currently used in *ik.py* module.

Target roll and yaw

The robot arm has five degrees of freedom, meaning there are physical constraints that disallow the end effector from reaching all positions with all possible orientations. Presented solution assumes we have no control over the yaw and doesn't take it into consideration. Following that, the 4th joint contributes only to the roll of the end effector. Thus, denoting the target roll by θ:

$$s_4 = \theta$$

First joint - base rotation

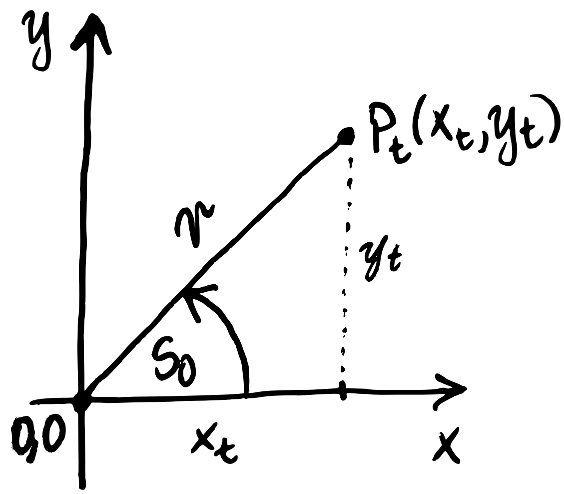

Figure 1: Top view of the robot arm

The rotation of base allows for movement in the y axis. The angle can be simply calculated using basic trigonometry. The solution can be easily seen on figure 2. P_t denotes target point.

$$s_0 = atan2(y_t, x_t)$$

The difficult part

For further calculations, a new reference frame has to be defined. It is placed at the beginning of link 0 and lays on the plane of the arm. The z' axis is parallel to z. Simply put, this is the side view of the manpulator.

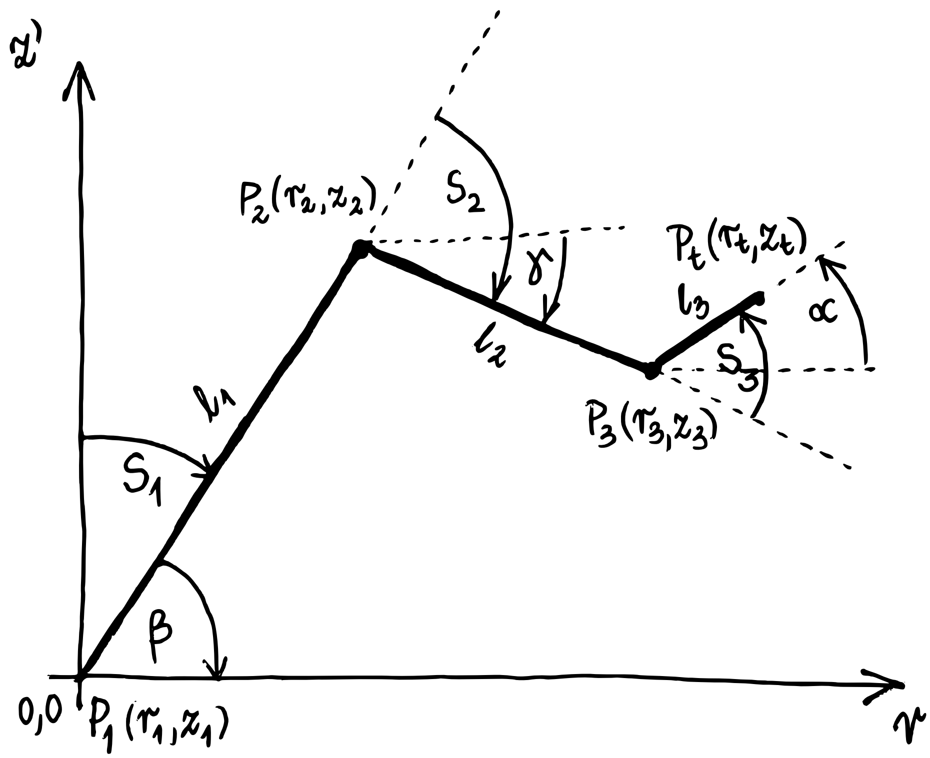

Figure 2: Side view of the robot arm

In order to solve the task, points P_2 and P_3 must be found. P_3 turns out quite straightforward as P_t and α are given (That's target position and target pitch) – again simple trigonometry:

$$r_3 = r_t - l_3 \cdot cos(\alpha)$$

$$z_3 = z_t + l_3 \cdot sin(\alpha)$$

The distance between P_2 and P_3 equals l_2. This can be stated as following:

$$\sqrt{(r_3 - r_2)^2 + (z_3 - z_2)^2} = l_2$$

Additionaly, the distance between P_2 ans P_1 is also known:

$$\sqrt{r_2^2 + z_2^2} = l_1$$

These equations together form a system, from which r_2 and z_2 can be calculated. There are two solutions of this system, but only one of them is possible for the arm (the other would force the manipulator to bend its elbow backwards.

When both P_2 and P_3 are found, joint angles can be calculated.

$$\beta = atan2(z_2, r_2)$$

$$\gamma = atan2(z2 - z_3, r_3 - r_2)$$

$$s_1 = \frac{\pi}{2} - \beta$$

$$s_2 = \beta + \gamma$$

$$s_3 = \alpha - \gamma$$