Welcome

To

MKS TUTORIALS

By

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Topics covered in playlist of PERT AND CPM: Network Techniques, Project, CPM, PERT, Common terms used in Arrow diagram: Activity, Event or Nodes, Path, Network; Network Construction, Dummy, Looping, Fulkerson's Rule, Problems on CPM, Optimistic Time Estimate, Most Likely Time Estimate, Pessimistic Time Estimate, Standard Deviation, Variance, Problems on PERT, Problem on Crashing of Network.

Complete playlist of PERT & CPM-
https://www.youtube.com/playlist?list=PLhSp9OSVmeyJKg_QaMV6qQdcGTFp2SbUH

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Network Analysis in Project Planning
(PERT and CPM)

Network Techniques:

- Symbolic representation [redacted] of a project.
- Most commonly used techniques → (1) PERT
  (2) CPM

Project:

- It is defined as the combination of interrelated activities which must be executed in a certain order before the entire task is completed.
  ↓
  (Non-repetitive task)

CPM (Critical Path Method)

- It uses activity oriented network which [redacted] tasks or consists of [redacted] activities.
- Each activity is represented by arrow and the activities are joined together by events.

$$(i) \longrightarrow (j)$$

- CPM is generally used for simple, repetitive types of projects for which activity times and costs are certainly and precisely known.
- It is a deterministic model.
- Eg. Construction of a bridge, road, building, etc.

PERT (Programme Evaluation & Review Technique)

- It uses event oriented network in which successive events are joined by arrows.
- It is preferred for projects that are non-repetitive and in which time for various activities cannot be precisely pre-determined.
- It is a probabilistic model.
- There are three time estimates associated with each and every activity:

1. Optimistic time (to)
2. Pessimistic time (tp)
3. the most likely time (tm)

- Eg: launching of a satellite, R and B of a new product.

Network (or) Arrow Diagram

Common terms used in network diagram :

1. **Activity** :

- It is the task that requires time and resources for its execution.
- An activity is represented by an arrow, the tail represents the start and the head represents the finish of the activity.

$$\xrightarrow[d_{ij}]{\text{Activity}} \qquad d_{ij} \rightarrow \text{activity duration}$$

- The length, shape, direction of arrows has no relation to the size of activity.

2. **Event or nodes** :

- It is the beginning and end points of an activity.
- It is a point in time.

$$(i) \xrightarrow{\text{Activity}} (j)$$

Starting event   Finishing event

- It doesnot consume any resources.
- Represented by a circle.
- Tail event → i^{th} event

  Head event → j^{th} event

  $(j > i)$

3. PATH :

- It is an unbroken chain of activity arrows connecting the initial event to some other event.

4. NETWORK :

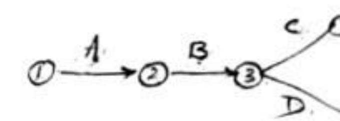
- Arrow diagram
- Graphical representation of sequentially connected arrows and nodes representing activities and events of a project.

NETWORK CONSTRUCTION

- Firstly split the project into activities.
- Start and finish events of projects are then decided.
- After deciding the precedence order, the activities are put in a logical sequence by using graphical notations.
- While constructing any network, following points must be followed:

(i) What activities must be completed before a particular activity starts?

(ii) What activities follow this?

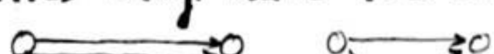


- Predecessor activities: Activities which must be completed before a particular activity starts.
- Successor activities: Activities which must follow a particular activity.

Following points should be kept in mind while drawing any network diagram:

1. Each activity is represented by only one arrow.
2. Arrow (Time) flows from left to right.

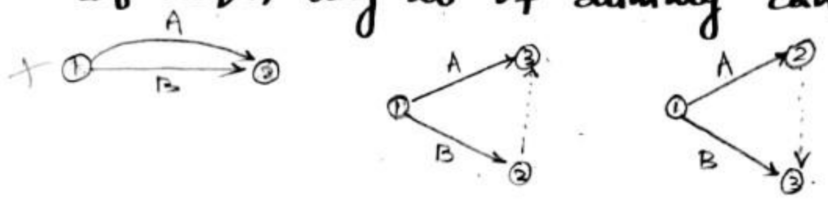
3. Each activity must have a tail and a head event. (No two or more activities may have the same tail and head events)



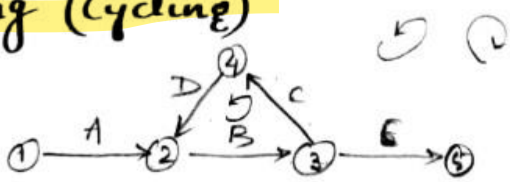
4. Arrows should not cross eachother. (Bridging should be done where crossing cannot be avoided.

Dummy :

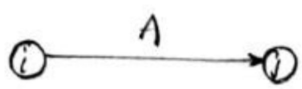
- Imaginary activity
- It only determines the dependency of one activity on other.
- It does not consume any time. (duration = zero)
- It is used to satisfy predecessor relationship
- If reqd, any no. of dummy can be used.



Looping (Cycling)



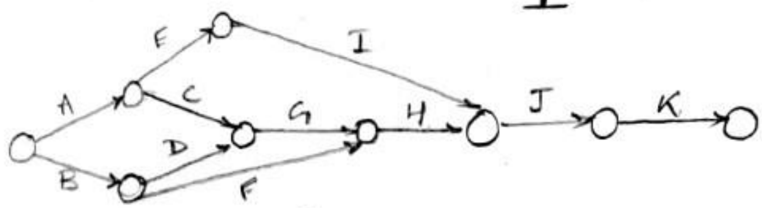
Fulkerson's Rule (Numbering the events)

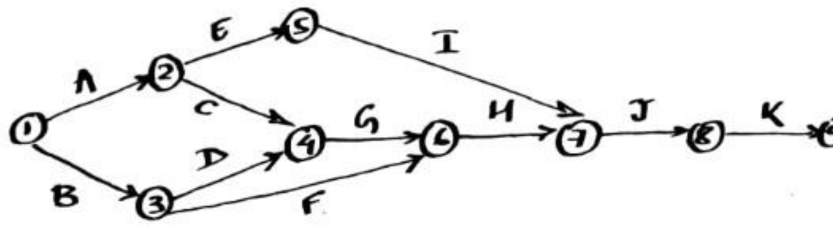


$i < j$

Que ① Draw a network :

Activity	Preceded by	Activity	Preceded by
A	—	F	B
B	—	G	C, D
C	A	H	G, F
D	B	I	E
E	A	J	H, I
		K	J





Que ② A project consists of a series of tasks labelled A, B, ···, H, I with the following relationships :

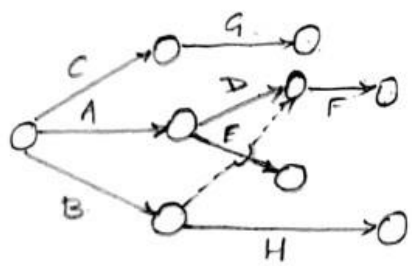
$W < X, Y$ means X and Y cannot start until W is completed.

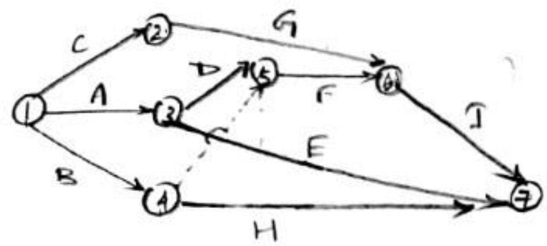
$X, Y < W$ means W cannot start until both X and Y are completed.

With this notation, construct the network diagram having the following constraints:

$A < D, E$; $B, D < F$; $C < G$; $B < H$; $F, G < I$.

A, B, C, ~~D~~, ~~E~~, ~~F~~, ~~G~~, ~~H~~, ~~I~~





Que (3) Draw a network for the following project and number the events according to Fulkerson's rule :

A is start event and K is end event,

A precedes event B,

J is successor event to F,

C and D are successor events to B,

D is the preceding event to G,

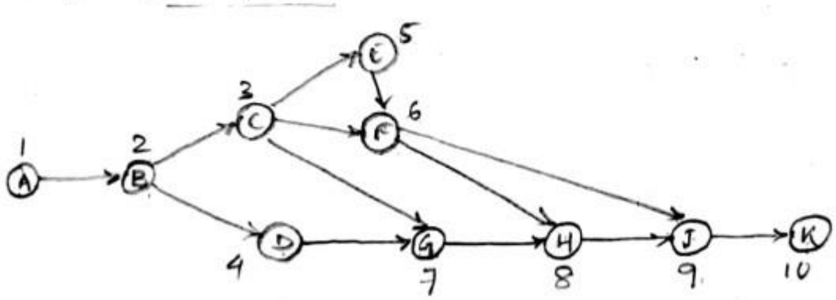
E and F occur after event C,

E precedes F,

C restraints the occurance of G and G precedes H,

H precedes J and K succeeds J,

F restraints the occurence of H.



(8)

CRITICAL PATH METHOD

Que (1) Task A, B, C, D, E, F, G, H, I constitute a project. The procedure relationships are

$A < D$; $A < E$; $B < F$; $D < F$; $C < G$; $C < H$; $F < I$; $G < I$.

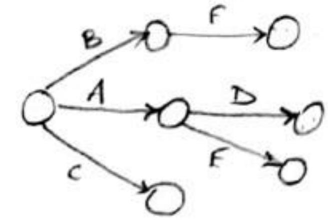
Draw a network to represent the project and find the min time of completion of project when time (in days) of each task is as follows:

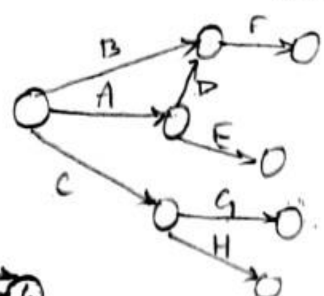
Task :	A	B	C	D	E	F	G	H	I
Time :	8	10	8	10	16	17	18	14	9

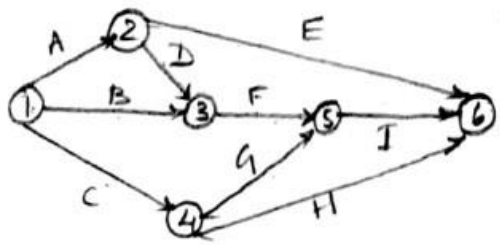
Also, identify the critical path.

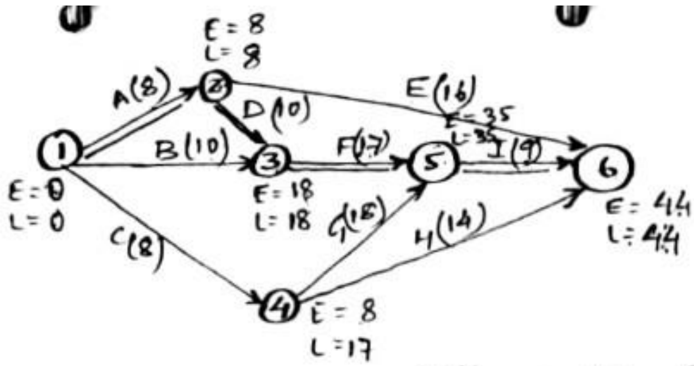
$A < D$; $A < E$; $B < F$; $D < F$; $C < G$; $C < H$; $F < I$; $G < I$

A, B, C, ~~D~~, ~~E~~, ~~F~~, ~~G~~, ~~H~~, ~~I~~









Project duration (or) longest path

Forward pass computations, (to find E) → Earliest starting time (occurrence)

$E_1 = 0$

$E_2 = 0+8 = 8$

$E_3 = \text{Max}(10, 18) = 18$

$E_4 = 0+8 = 8$

$E_5 = \text{Max}(35, 26) = 35$

$E_6 = \text{Max}(24, 44, 22) = 44$

Backward pass computations, (to find L) → latest finishing time

$L_6 = 44$

$L_5 = 44 - 9 = 35$

$L_4 = \text{Min}(17, 30) = 17$

$L_3 = 18 = 35 - 17$

$L_2 = \text{Min}(28, 8) = 8$

$L_1 = \text{Min}(0, 8, 9) = 0$

For critical path, following conditions must be followed:

(i) E = L for head event

(ii) E = L for tail event

(iii) $E_j - E_i = L_j - L_i$ = duration of activity

The activities 1-2, 2-3, 3-5, 5-6 follow above conditions.

Hence, the critical path is 1-2-3-5-6

(Trick)

Path	Duration
1-2-6	24
1-2-3-5-6	44 → Longest path (or) zero delay path
1-3-5-6	36
1-4-5-6	35
1-4-6	22

Que (2) A project schedule has following characteristics:

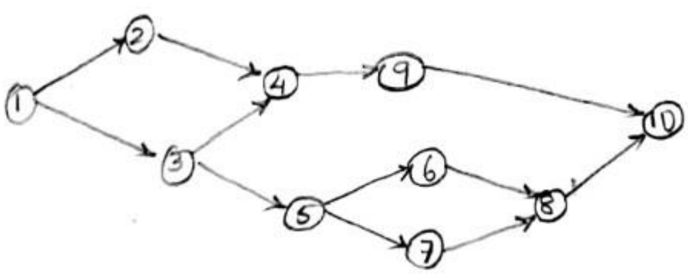
Activity	Time (weeks)	Activity	Time (weeks)
1-2	4	5-6	4
1-3	1	5-7	8
2-4	1	6-8	1
3-4	1	7-8	2
3-5	6	8-10	5
4-9	5	9-10	7

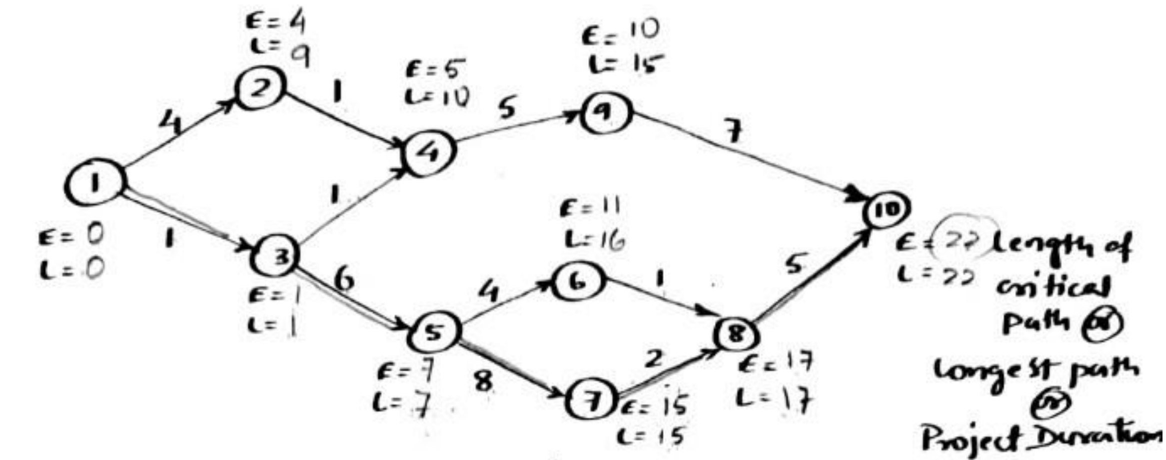
(i) Construct the network

(ii) Compute E and L for each event

(iii) Find the critical path.

1-2, 1-3, 2-4, 3-4, 3-5, 4-9, 5-6, 5-7, 6-8,
7-8, 8-10, 9-10





Forward pass computations,	Backward pass computations
$E_1 = 0$	$L_{10} = 22$
$E_2 = 0+4 = 4$	$L_9 = 22-7 = 15$
$E_3 = 0+1 = 1$	$L_8 = 22-5 = 17$
$E_4 = Max(5, 2) = 5$	$L_7 = 17-2 = 15$
$E_5 = 6+1 = 7$	$L_6 = 17-1 = 16$
$E_6 = 4+7 = 11$	$L_5 = Min.(12, 7) = 7$
$E_7 = 7+8 = 15$	$L_4 = 15-5 = 10$
$E_8 = Max(12, 17) = 17$	$L_3 = Min(9, 1) = 1$
$E_9 = 5+5 = 10$	$L_2 = 10-1 = 9$
$E_{10} = Max(17, 22) = 22$	$L_1 = Min(5, 0) = 0$

For critical path, the following conditions must be checked:

(i) $E = L$ for tail event

(ii) $E = L$ for head event

(iii) $E_j - E_i = L_j - L_i =$ duration of activity

The activities that follow above conditions are

1-3, 3-5, 5-7, 7-8, 8-10

Hence, the critical path is

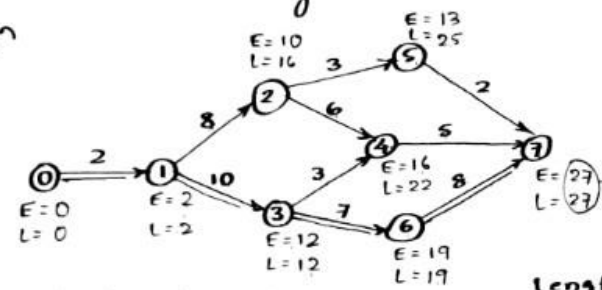
1-3-5-7-8-10.

Que (3) The utility data for a network are given below:

Activity: 0-1 1-2 1-3 2-4 2-5 3-4 3-6 4-7 5-7 6-7
Duration: 2 8 10 6 3 3 7 5 2 8

Determine total, free, independent and interfering floats. Also, identify the critical path.

Sol^n



Project duration (or) Longest path of Network (or) Length of Critical path.

Critical path: 0-1-3-6-7

Total Float = |LFT − EFT| = |LST − EST|
Free Float = TF − Slack of head event
Independent Float = FF − Slack of Tail event
Interfering float = TF − FF

EFT = EST + Duration
LST = LFT − Duration

Slack = L − E of the event

Activity	Duration	Start Time EST	Start Time LST	Finish Time EFT	Finish Time LFT	Slack of Head event	Slack of Tail event	Total Float (TF)	Free Float (FF)	Independent Float	Interfering Float
0-1	2	0	0	2	2	0	0	0	0	0	0
1-2	8	2	8	10	16	6	0	6	0	0	6
1-3	10	2	2	12	12	0	0	0	0	0	0
2-4	6	10	16	16	22	6	6	6	0	0	6
2-5	3	10	22	13	25	12	6	12	0	0	12
3-4	3	12	19	15	22	6	0	7	1	1	6
3-6	7	12	12	19	19	0	0	0	0	0	0
4-7	5	16	22	21	27	0	6	6	6	0	0
5-7	2	13	25	15	27	0	12	12	12	0	0
6-7	8	19	19	27	27	0	0	0	0	0	0

PERT (Programme Evaluation and Review Techniques)

1. New project
2. Activity duration is not fixed
   (It follows β- distribution)
3. Probabilistic model.
4. Three time estimates:
   (i) Optimistic time estimate (t_o)
   (ii) Most likely time estimate (t_m)
   (iii) Pessimistic time estimate (t_p)

Order: ($t_o - t_m - t_p$) $\quad$ ($t_p \geq t_m \geq t_o$)

Expected activity duration

$$T_e = \frac{t_o + 4t_m + t_p}{6}$$

$$① \xrightarrow[t_o - t_m - t_p]{A} ② \Rightarrow ① \xrightarrow[T_e]{A} ②$$

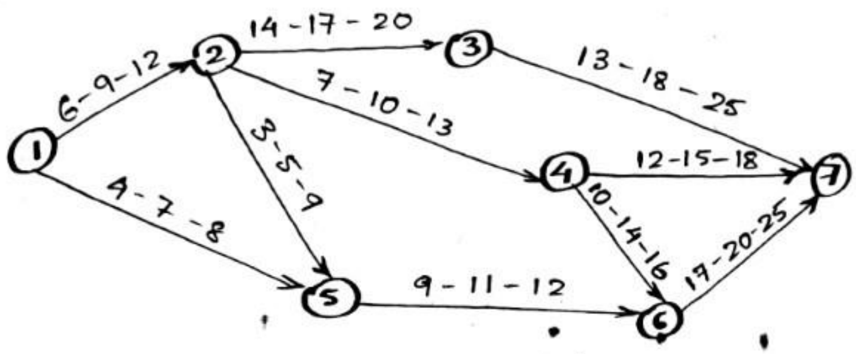
Standard deviation, $\sigma = \dfrac{t_p - t_o}{6}$

Variance, $\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$

$$\sigma_{CP} = \sqrt{\Sigma \sigma^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots}$$

(13)

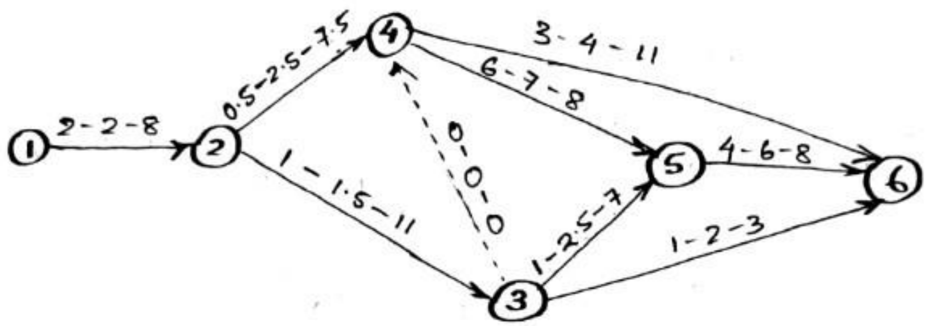
Que① Consider the network shown. For each activity, the three time estimates t_o, t_m and t_p are given along the arrows in t_o-t_m-t_p order. Determine the variance and expected time for each activity.



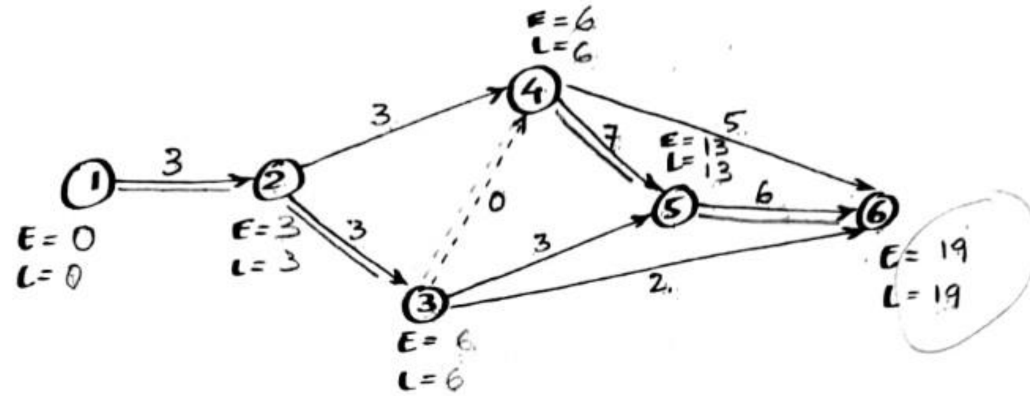
Activity	t_o	t_m	t_p	Variance $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$	$T_e = \left(\frac{t_o + 4t_m + t_p}{6}\right)$
1-2	6	9	12	1.	9
1-5	4	7	8	0.44	6.7
2-3	14	17	20	1	17
2-4	7	10	13	1	10
2-5	3	5	9	1	5.33
3-7	13	18	25	4	18.33
4-6	10	14	16	1	13.67
4-7	12	15	18	1	15
5-6	9	11	12	0.25	10.83
6-7	17	20	25	1.78	20.33.

Ans.

Que② Consider the network shown in fig. The three time estimates for the activities are given along the arrows. Determine the critical path. What is the probability that the project will be completed in 20 days?



Activity	t_o	t_m	t_p	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$	$T_e = \left(\frac{t_o + 4t_m + t_p}{6}\right)$
1-2	2	2	8	1	3
2-3	1	1.5	11	2.8	3
2-4	0.5	2.5	7.5	1.36	3
3-4	0	0	0	0	0
3-5	1	2.5	7	1	3
4-5	6	7	8	0.11	7
4-6	3	4	11	0.11	5
3-6	1	2	3	1.78	2
5-6	4	6	8	0.44	6



Expected project duration, $T_{cp} = 19$ days.

and $T = 20$ days (given)

Normal Deviate, $Z = \frac{T - T_{cp}}{\sigma_{cp}}$

$\sigma_{cp} = \sqrt{\Sigma \sigma^2} = \sqrt{1 + 2.8 + 0 + 0.11 + 0.44}$

$= 2.0857$

$\Rightarrow Z = \frac{20 - 19}{2.0857} = 0.4795$

$P(Z = 0.4795) = 68.42\%$

CRASHING the NETWORK

Que Activity A must precede all other while activity E must follow others.

Activity	Normal Duration (days)	Normal Cost (Rs.)	Crash Duration (days)	Crash Cost (Rs.)
A	5	3000	4	4000
B	6	1200	2	2000
C	4	1000	3	1800
D	5	1200	3	2000
E	3	1600	3	1600.
		8000		

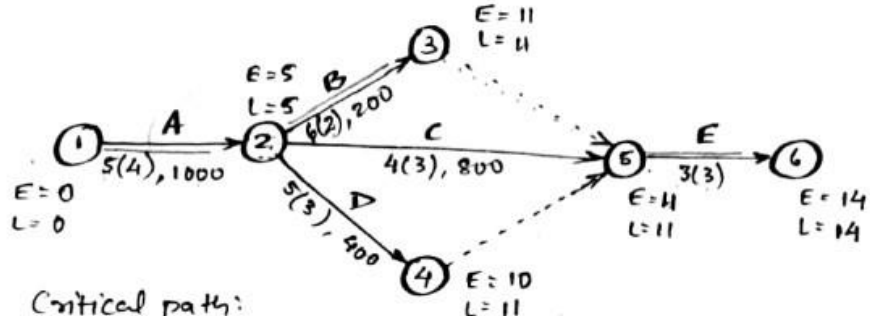
(i) Draw the network and identify the critical path.
(ii) Crash the network fully to find out min duration.
(iii) If indirect costs are Rs 300/day, determine the time cost trade off for the project.

Sol.^n First of all, the cost slope for each activity is calculated.

Use formula: $\text{Cost Slope} = \dfrac{C_c - C_n}{T_n - T_c}$

Activity:	A	B	C	D	E
Cost Slope: (Rs/day)	1000	200	800	400	—

Now, draw the network:



Critical path:
1 - 2 - 3 - 5 - 6

Normal duration = 14 days

Total cost = Normal Cost + ~~Crash Cost~~ 0 + Indirect cost (overhead)

$= 8000 + 0 + (300 \times 14) = 12,200/-$

Note: Project length can be reduced by crashing the critical activitio

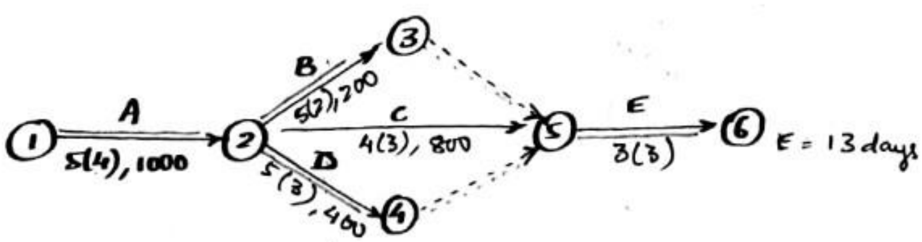
1 - 2 - 3 - 5 - 6 → 14 days → Critical path
1 - 2 - 5 - 6 → 12 days
1 - 2 - 4 - 5 - 6 → 13 days

Crash activity B by one day.

∴ Project duration = 14 - 1 = 13 days.

Total cost = Previous T.C. + Crash Cost - Indirect Cost

$= 12,200 + (200 \times 1) - (300 \times 1) = 12,100/-$



Options: A → 1000
B+D → 600

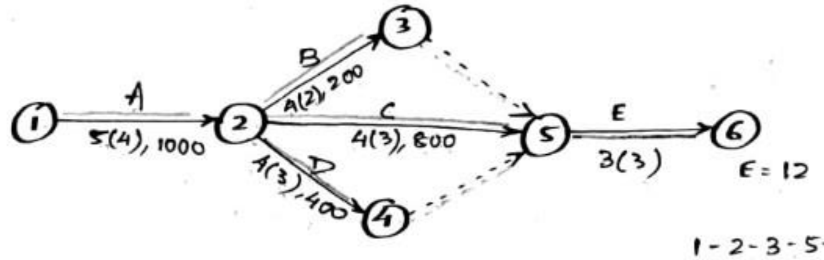
1 - 2 - 3 - 5 - 6 → 13
1 - 2 - 5 - 6 → 12
1 - 2 - 4 - 5 - 6 → 13
→ Critical path.

Crash activity (B+D) by one day.

Project duration = 13 − 1 = 12 days.

Total cost = Previous T.C. + Crash cost − Indirect cost

= 12,100 + (600 × 1) − (300 × 1) = 12,400/-



1-2-3-5-6
1-2-5-6
1-2-4-5-6
→ Critical path

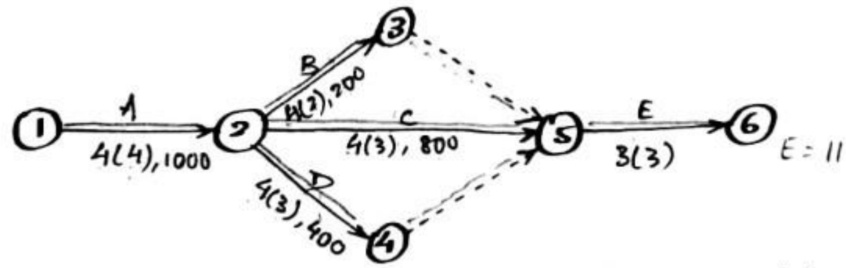
Option: A → 1000 ✓
B+C+D → 1400

Crash activity A by one day.

Project duration = 12 − 1 = 11 days.

Total cost = Previous T.C + Crash cost − Indirect cost

= 12,400 + (1000 × 1) − (300 × 1) = 13,100/-



All are critical activities.

Only one option: (B+C+D)

Crash activity (B+C+D) by 1 day

$\therefore$ Project duration = 11-1 = 10 days Ans.

Total cost = Previous T.C. + Crash cost - Indirect cost

= 13,100/- + (1400 x 1) -(300 x 1)

= 14,200/- Ans.

Duration (days)	Total cost (Rs.)
14	12,200
13	12,100
12	12,400
11	13,100
10	14,200

$\Rightarrow$ Time cost trade off exists at 13 days when B is crashed by 1 day and total min cost is 12,100/-. Ans.

THANK YOU SO MUCH