

## Lesson 1.4

# SHORTEST PATHS IN GRAPHS

## Do Less and Accomplish More

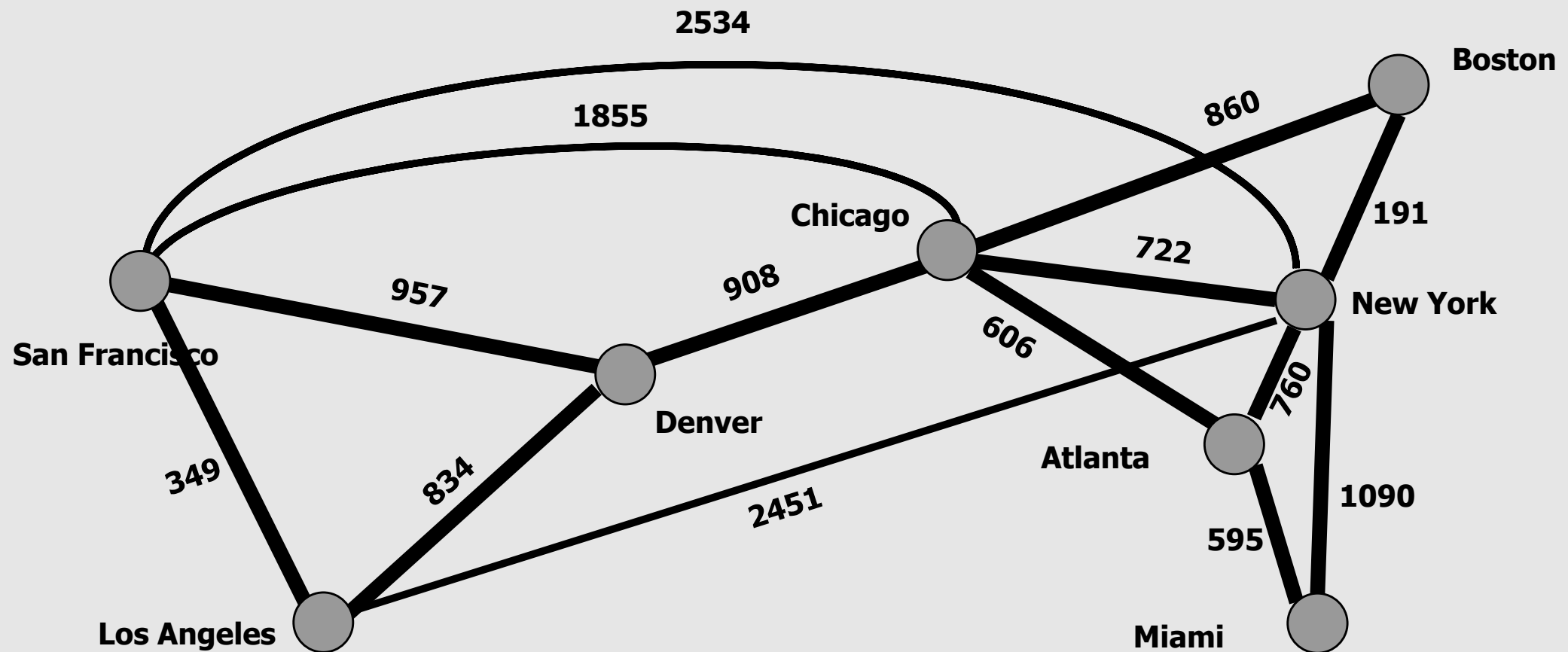
**Wholeness of the lecture:** A graph can have many paths, each with different lengths or different weights. We use algorithms to find the best—or optimal—path for a specific application. *Science of Consciousness:* In Transcendental Meditation, the mantra gives us the fastest, most comfortable ride to the transcendental field of our own consciousness.

### Main Points

1. A *weighted graph* has numbers or *weights* assigned to each path. The sum of the weights on a given path tell us how efficient or costly the path is. *Science of Consciousness:* Regular practice of the Transcendental Meditation program increases our efficiency in activity.<sup>3</sup>
2. Dijkstra's algorithm is a greedy iterative algorithm that finds the shortest path from a given vertex to every other vertex in a weighted graph. *Science of Consciousness:* Nature functions according to the path of least action. With regular transcending, we develop the ability to act in accord with natural law, following the path of least action.
3. The traveling salesperson problem, which asks for the shortest Hamilton circuit in a weighted graph, is very hard; the best approaches can only give approximations to the shortest circuit. *Science of Consciousness:* Spontaneous right action is possible only when established in Cosmic Consciousness, when we are functioning from the field of all possibilities.

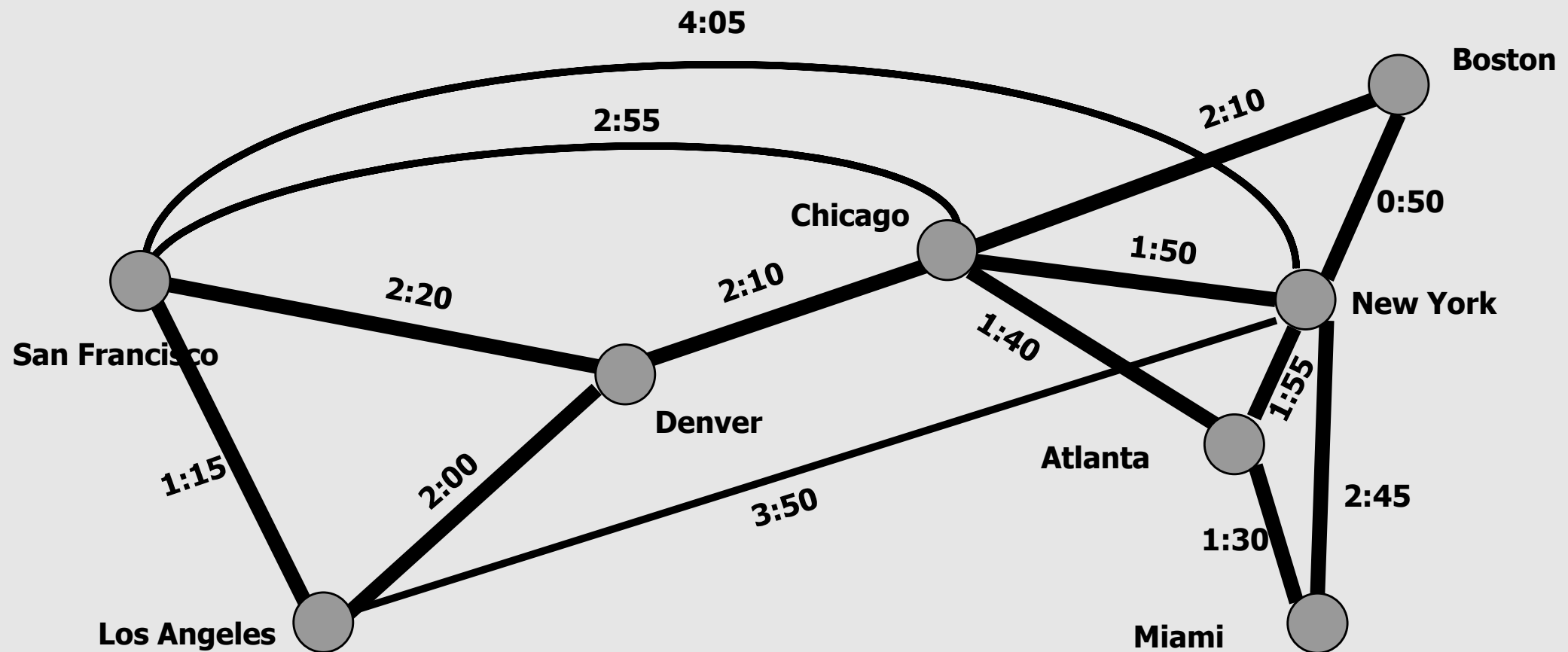
# Shortest Path

## Mileage based paths



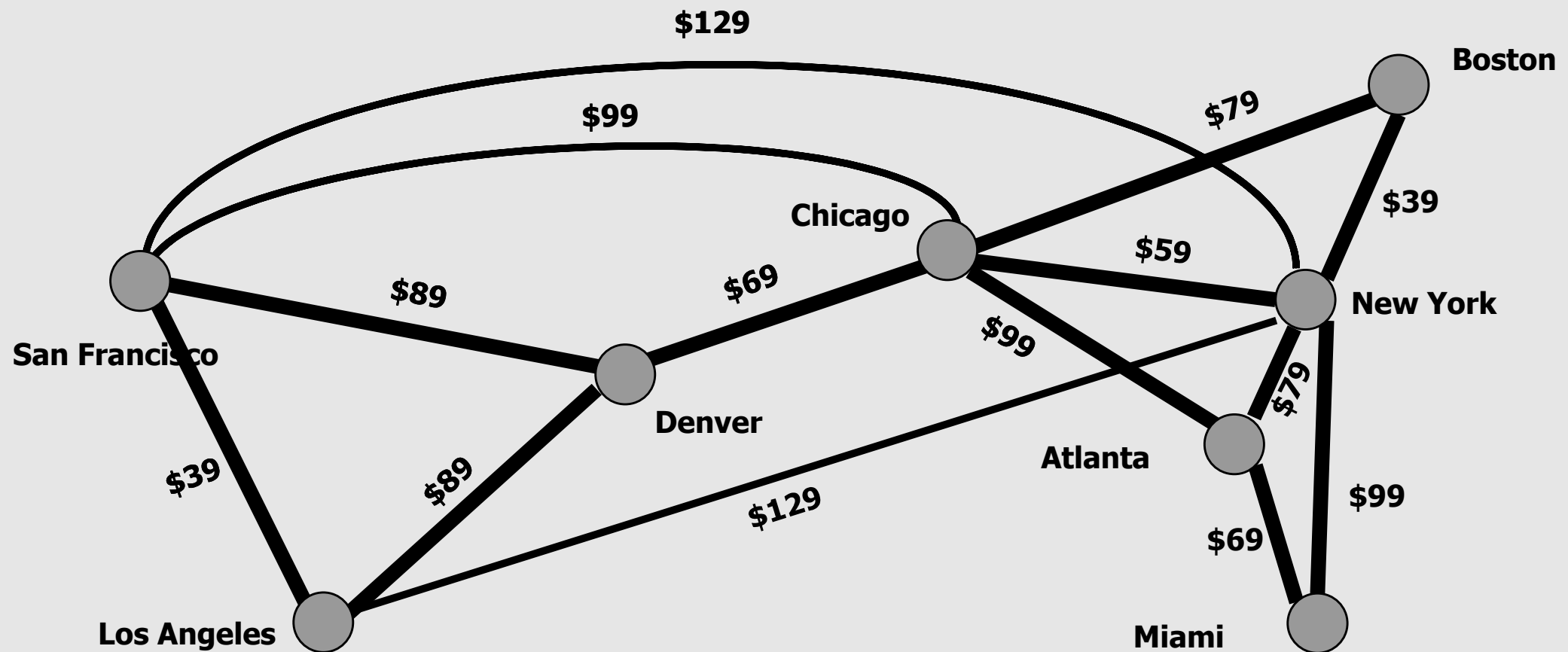
# Shortest Path

## Flight Times



# Shortest Path

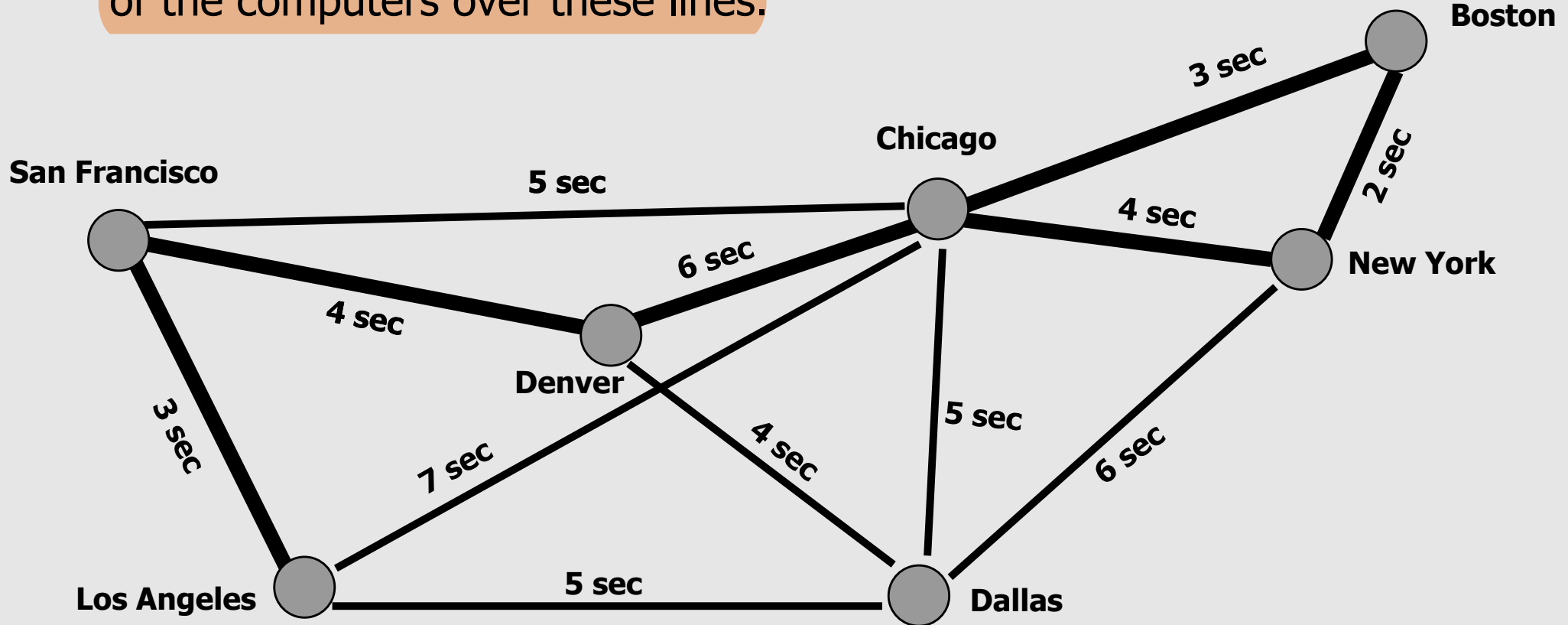
## Fares



# Shortest Path

## Response Time

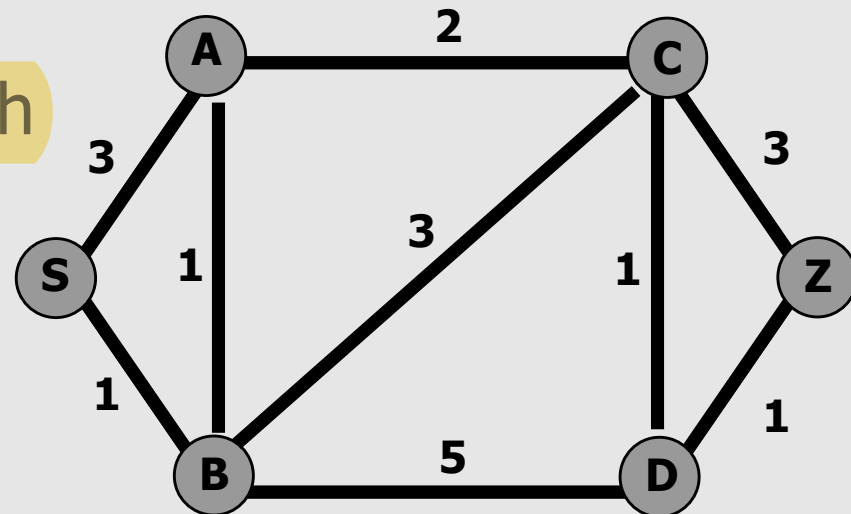
- Weighted graphs are used to model computer networks, the response times of the computers over these lines.



# Shortest Path

## Weighted Graphs

- A graph in which a number called weight is assigned to each *edge*, then the graph is said to be a weighted graph. In a weighted graph, the weight of the path is the sum of the weights of the *edges* in the path.
- In a weighted graph, one common problem is to find the shortest path between given vertices.



# Dijkstra's Algorithm: A Shortest-Path Algorithm

## EDSGER WYBE DIJKSTRA (1930-2002)



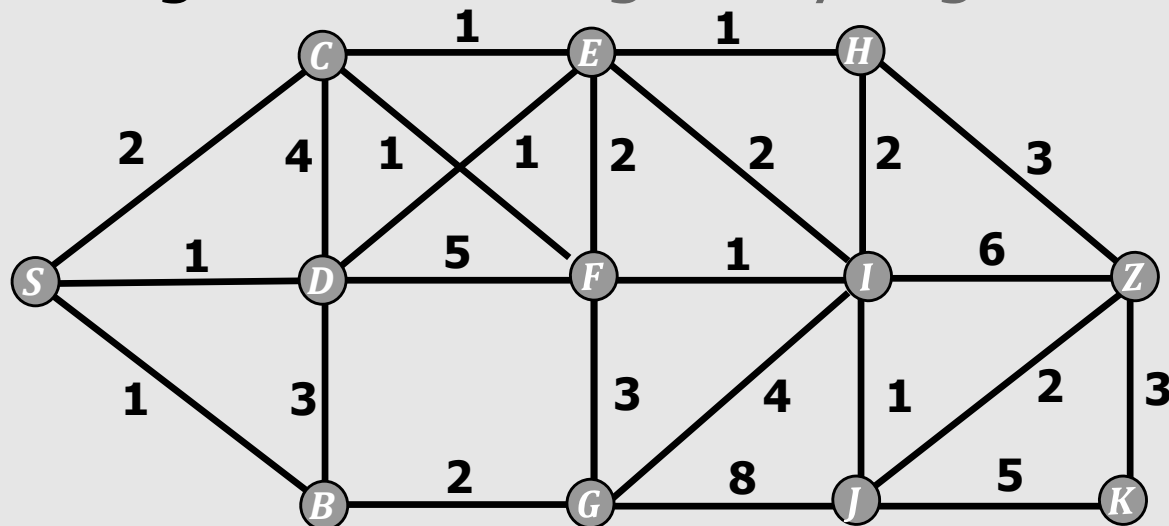
- In 1972, Edsger Dijkstra, a Dutch Computer Scientist received one of the most prestigious **Turning Award for contribution to developing computer programming languages.**
- In 1984 he was appointed to a chair in Computer Science at the University of Texas, Austin.
- Dijkstra contributed to **SHORTEST PATH ALGORITHM**, also known as **Dijkstra's Algorithm.**
- **Fundamental contributions to the areas of operating systems, including deadlock avoidance, programming languages, including the motion of structured programming and algorithms.**

*The Shortest-Path algorithm has been used in GPS navigating systems.*

# Shortest Path

## Dijkstra's Algorithm

- Dijkstra's Algorithm can be applied on directed and undirected graphs when all the weights are positive.
- Dijkstra's Algorithm provides the shortest path from source vertex to any other vertices in the graph.
- Dijkstra's algorithm is a greedy algorithm.

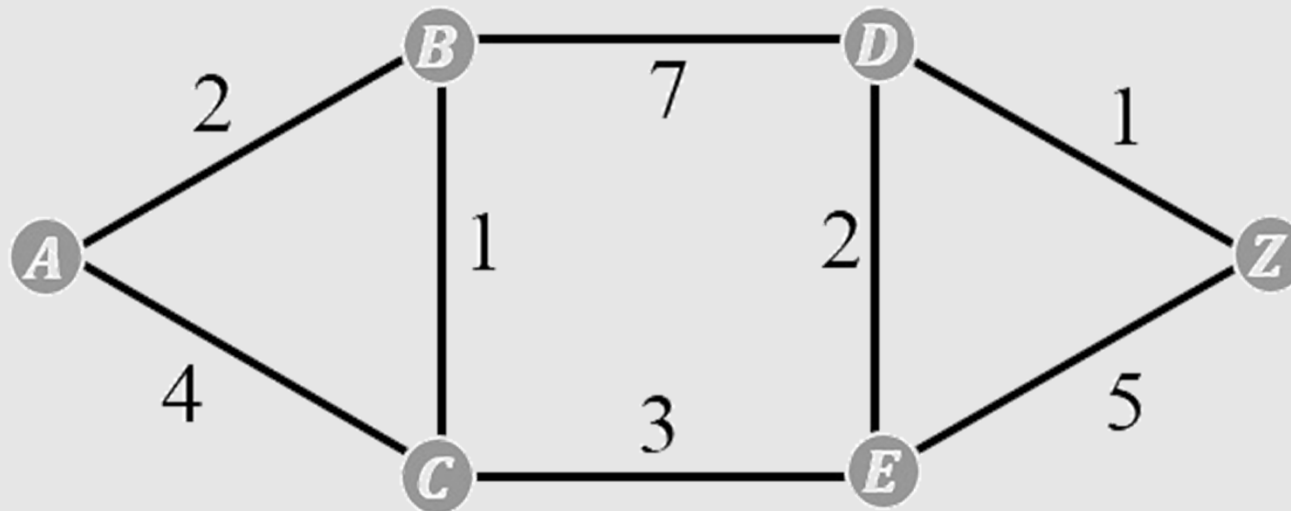




# Dijkstra's Algorithm

## Example

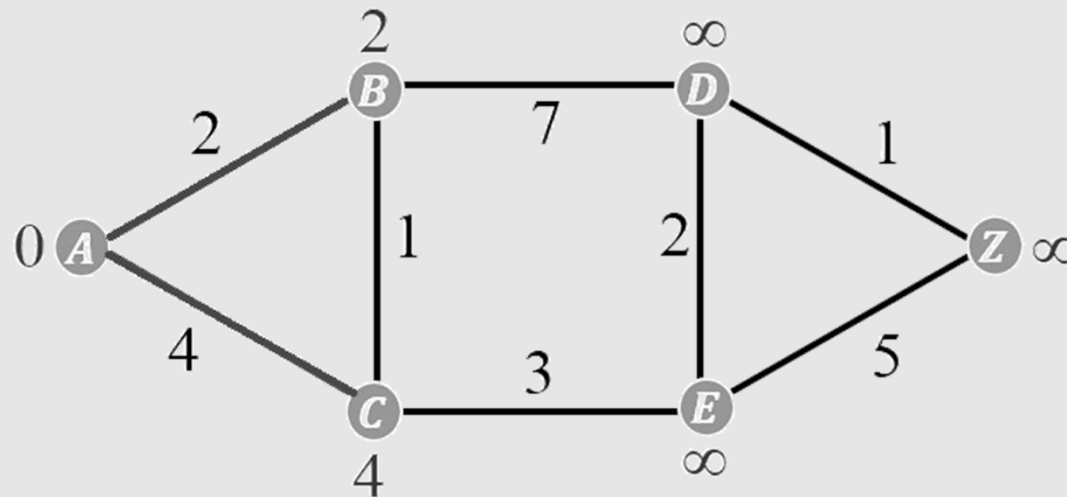
- Find the Shortest-Path from A vertex to another Z by using Dijkstra's Algorithm.
- Consider the below graph with positive weights.



# Dijkstra's Algorithm

## Examples:

- Start from the vertex **A** and update the direct connecting vertices, marking it as visited.
- The shortest-paths **B** and **C** are updated by using the concept of relaxation  **$\text{dist}[v] > \text{dist}[u] + w(u, v)$** .



# Dijkstra's Algorithm

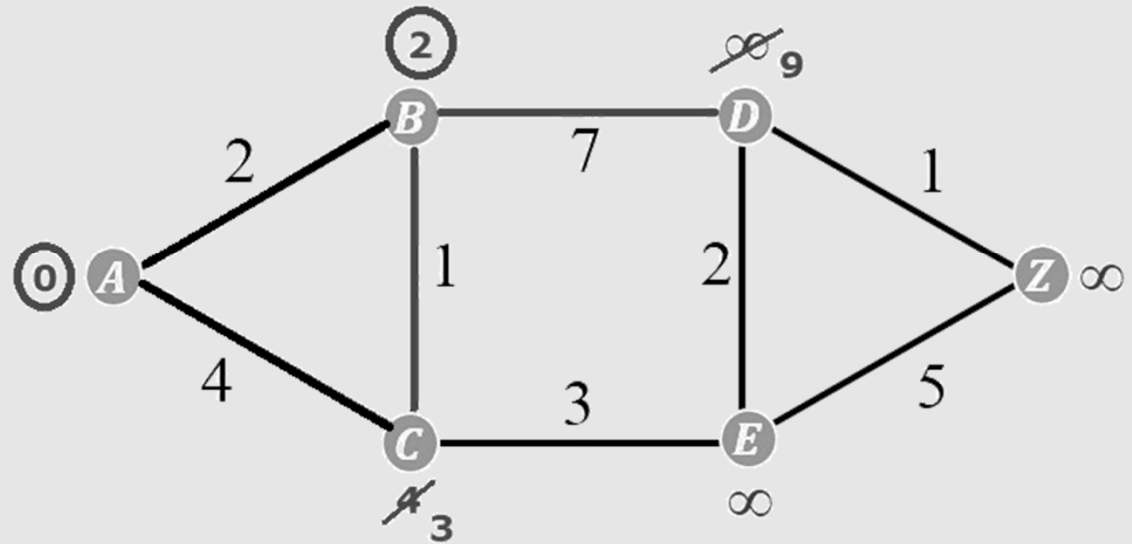
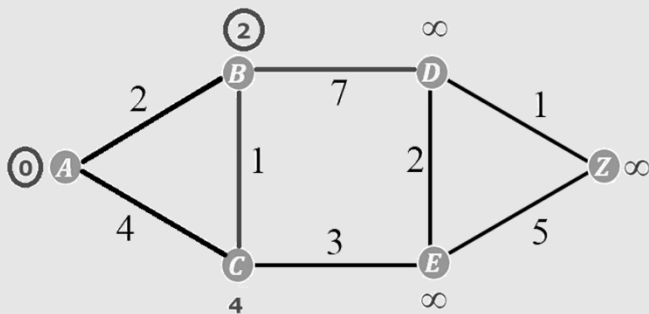
- The next vertex we visit is vertex **B** and start relaxation.

$$\diamond 1 + 2 < 4$$

$$\diamond 2 + 7 < \infty$$

Relax vertex **C**

Relax vertex **D**



# Dijkstra's Algorithm

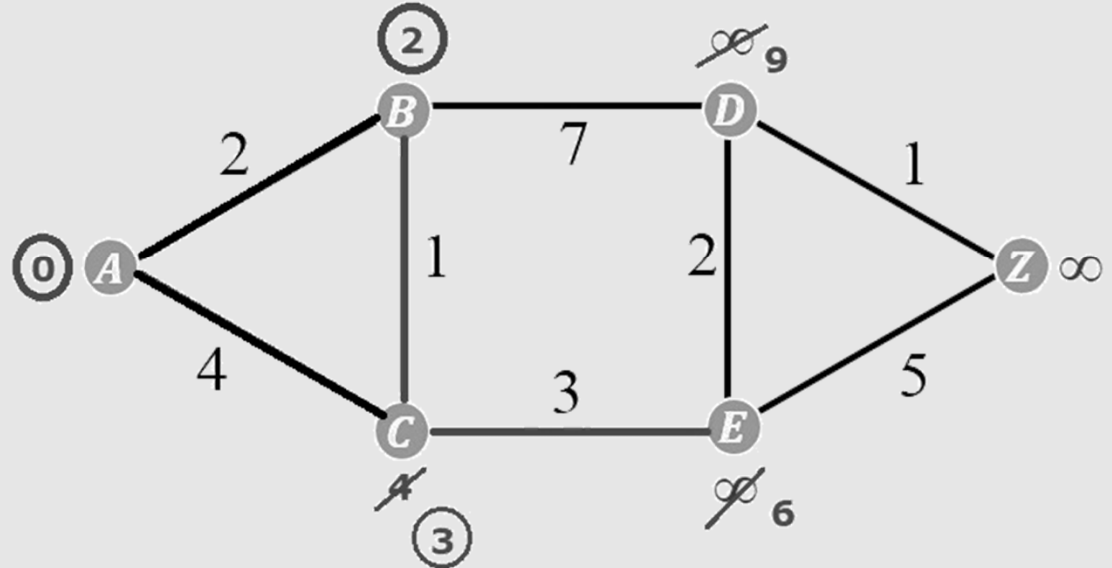
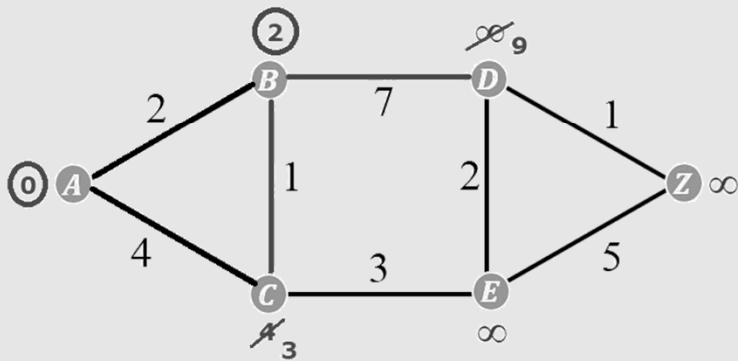
- The next vertex we visit is vertex **C** and start relaxation.

❖  $3 + 3 < \infty$

❖  $3 + 1 > 2$

Relax vertex **E**

Don't Relax vertex **B** or B is already selected



# Dijkstra's Algorithm

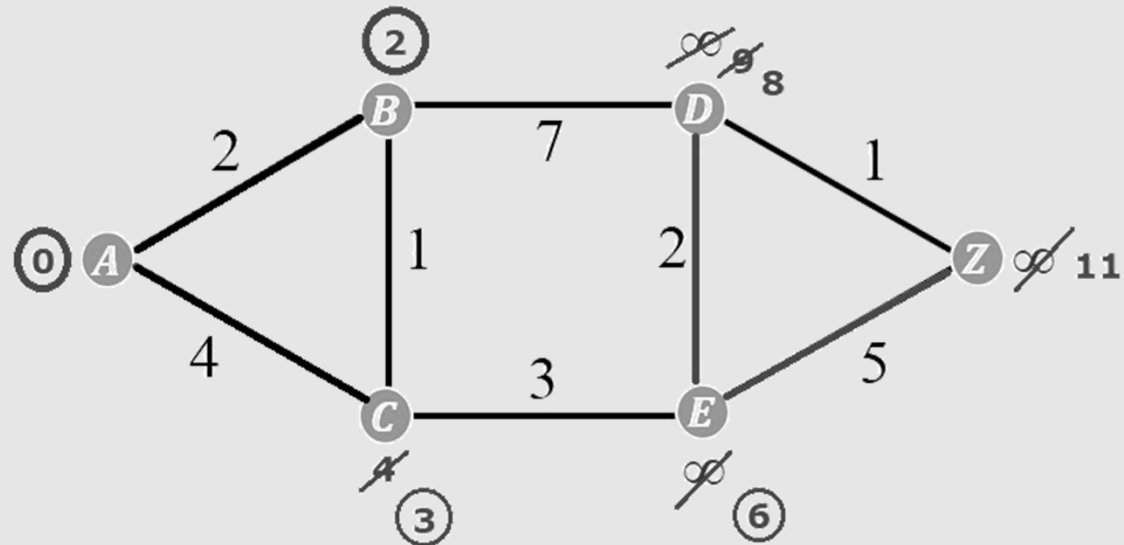
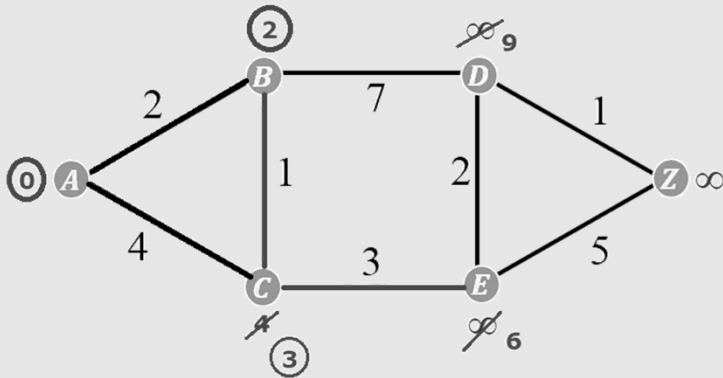
- The next vertex we visit is vertex **E** and start relaxation.

$$\diamond 6 + 2 < 9$$

$$\diamond 6 + 5 < \infty$$

Relax vertex **D**

Relax vertex **Z**



# Dijkstra's Algorithm

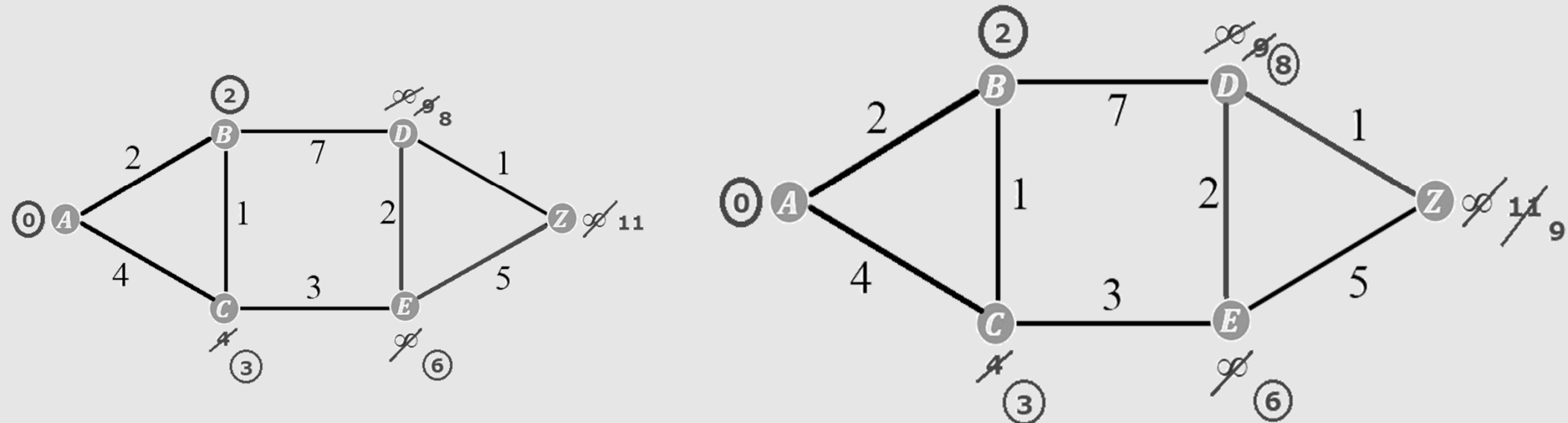
- The next vertex we visit is vertex **D** and start relaxation.

$$\diamond 8 + 2 > 4$$

$$\diamond 8 + 1 < 11$$

Don't Relax vertex **C**

Relax vertex **Z**

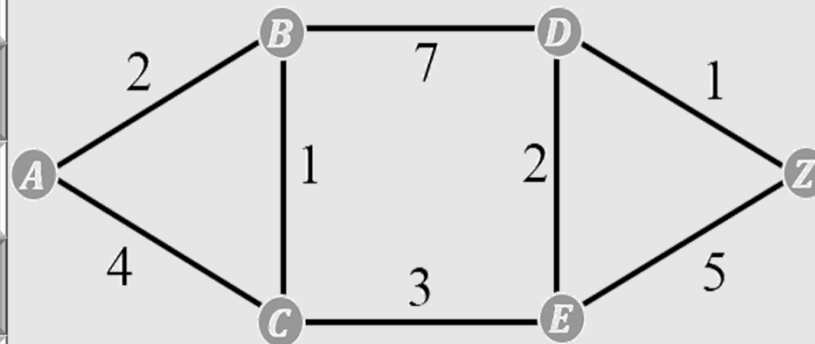


# Dijkstra's Algorithm

- Find the Shortest-Path from Vertex **A** to Vertex **Z**.
- The adjoining table represents the shortest path of each vertex from the source vertex.

Another way to find the Shortest-Path

	A	B	C	D	E	Z
A	0	2	4	$\infty$	$\infty$	$\infty$
B	0	2	3	9	$\infty$	$\infty$
C	0	2	3	9	6	$\infty$
E	0	2	3	8	6	11
D	0	2	3	8	6	9



Vertex	Shortest-Distance
A	0
B	2
C	3
D	8
E	6
Z	9

# Shortest Path

## Examples

- We can find the length of shortest path from vertex  $s$  to all other vertices  $V$  of the graph  $G$  if we continue this procedure until all vertices are added to the distinguished set.

```
dist[s] ← 0                                (distance to source vertex is zero)
for all  $v \in V - \{s\}$ 
    do  $\text{dist}[v] \leftarrow \infty$           (set all other distances to infinity)
 $S \leftarrow \emptyset$                       (S, set of visited vertices is initially empty)
 $Q \leftarrow V$                             (Q, queue initially contains all vertices)
while  $Q \neq \emptyset$                      (while the queue is not empty)
do  $u \leftarrow \text{mindistance}(Q, \text{dist})$    (select the element of Q with the min. distance)
     $S \leftarrow S \cup \{u\}$                (add u to list of visited vertices)
    for all  $v \in \text{neighbors}[u]$ 
        do if  $\text{dist}[v] > \text{dist}[u] + w(u, v)$  (if new shortest path found)
            then  $d[v] \leftarrow d[u] + w(u, v)$  (set new value of shortest path)
            (if desired, add traceback code)
return dist
```



# Shortest Path

## UNITY CHART

Connecting The Parts Of Knowledge With The Wholeness Of Knowledge

### Finding an Optimal Path

1. There are many paths in a weighted graph, each having a different length or weight.
  2. Using algorithms, we can find the best path for a given situation.
- 
3. Transcendental Consciousness is a field of all possibilities.
  4. Impulses within the transcendental field give a direction to manifestation.
  5. Wholeness moving within itself: In Unity Consciousness, functioning from the field of all possibilities (the Self), we spontaneously perform right action.

