MATH 257 Discrete Mathematics

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Kenneth Rosen, *Discrete Mathematics and its Applications*, 7th edition, McGraw Hill

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Lesson 1.1
DISCRETE MATHEMATICS
Knowledge has Organizing Power

Wholeness of the lecture: Knowledge of discrete structures like sets, graphs, matrices, trees, algorithms, sequences, functions, and relations gives us organizing power that can solve problems of sorting, searching, counting, and optimizing. Knowledge has organizing power; knowledge is for action, achievement, and fulfillment. *Science of Consciousness:* Our practice of the Transcendental Meditation technique enlivens the pure knowledge inherent within our consciousness.

Main Points

1. To solve a real-world problem, formulate it as a mathematical problem, solve the mathematical problem, and translate the solution into the terms of the real-world problem. Go from the surface level of the problem to a deeper, more abstract level of the problem where there is greater organizing power. *Science of Consciousness:* In the Transcendental Meditation technique, we experience the deepest level of our mind, our own pure consciousness, and enliven our own infinite organizing power.
2. Many problems involve objects and how they are related to each other. Graphs model such problems using vertices (objects) and edges (relationships). Since graphs are abstract, they help us model and solve many different types of problems. *Science of Consciousness:* Regular practice of the Transcendental Meditation technique develops problem-solving abilities.[1]

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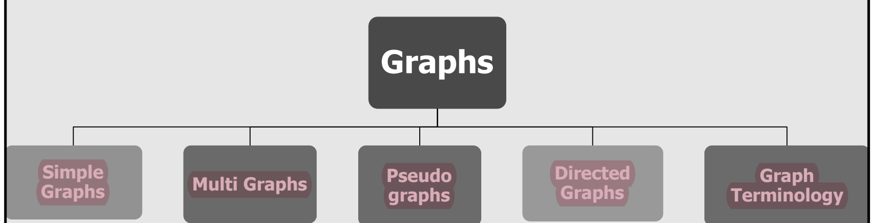

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Graphs

Discrete Mathematics

- What is Discrete Mathematics?
  - Discrete mathematics is the part of mathematics which devoted to the study of discrete objects.
  - The kind of problems solved by using Discrete mathematics are:
    - How many ways to choose a valid password on a computer system?
    - Is there a link between two computers in a network?
    - What is the shortest distance between two cities using a transport system?
    - How many valid internet addresses are there?
    - How can a circuit add two integers be designed?

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Graphs

Why we study Graphs?

- Graphs can be used to visualize the problem and easily provide the solution from a small scale to large scale.
- Graphs model a wide range of structures such as
  - Electric power lines
  - Fiber optic cables
  - Transportation system
  - Computers connected to data center to share the information from small level to the worldwide.
  - Data which is numerous or complicated to be represented in text or less space.

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Graphs

Definition

- "A graph is represented as ordered pairs $G = (V, E)$, where V is a non-empty **set of vertices** and E, a **set of edges**".
  - **Remarks**
    - "**Vertices** (or nodes) and **Edges** are typically both **finite.**
    - Each edge has either one or two vertices associated with it, known as its endpoints.
    - Edge connects the **endpoints.**
    - A graph with finite edge and vertex set is called a **finite graph** and in comparison, a graph with infinite vertex set or an infinite number of edges called an **infinite graph**.

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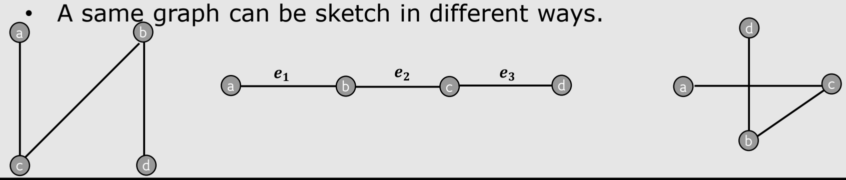
Graphs

Example

- Suppose we have four computers labeled a, b, c, and d, where is a flow of information between computers a and c, c and b, and b and d. This is usually referred to as a communication network.
  In set notation
  $$V = \{a, b, c, d\} \text{ and } E = \{(a, c), (c, b), (b, d)\}$$
- Graph is accomplished by moving from one vertex to another along a sequence of adjacent edges. This is represented by writing $ae_1be_2ce_3d$.
- A same graph can be sketch in different ways.



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Graphs

Examples

- Suppose that a network is made up of data centers and communication links between computers. We can represent the location of each data center by a point and each communications link by a line segment.

- This computer network can be modeled using a graph in which the vertices of the graphs represent the data centers, and the edges represent communication links.

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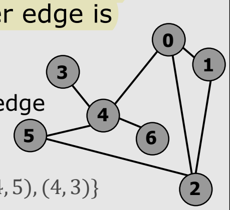
Graphs

Simple Graph

- "A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph."
  - **Note**
    - "In a simple graph each edge is associated to an unordered pair of vertices, and no other edge is associated to this same edge".
    - **Example**
      - Let V be the set of vertices, and no other edge
      - $V = \{0, 1, 2, 3, 4, 5, 6\}$
      - Let $E = \{(u, v) | u \text{ adjoins } v\}$
      - $= \{(0,1), (0,2), (0,4), (2,5), (1,2), (4,6), (4,5), (4,3)\}$



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Graphs

Multigraphs

- "Graphs that may have multiple edges connecting the same vertices are called multigraphs."

OR

- Like simple graphs, but there may be more than one edge connecting two given vertices.
  - **Examples**
    - "A computer network may contain multiple links between data centers. To model such networks, we need graphs that have more than one edge connecting the same pair of vertices.
    - Furthermore, as an example we consider vertices as cities and edges as major highways connecting the cities.

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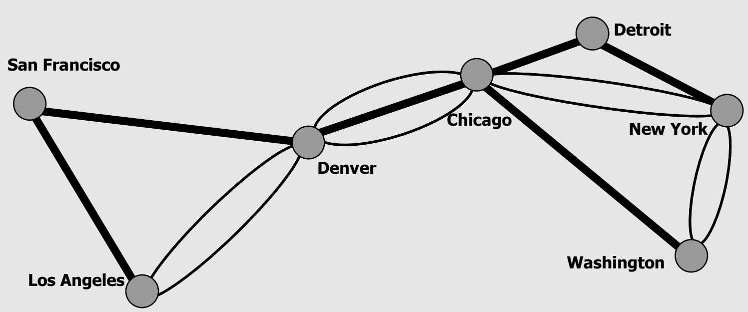
Graphs

Multigraph



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Graphs

Pseudographs

- "Like a multigraph, but edges connecting a vertex to itself are allowed."
  OR
- Graphs that may include loops, and possibly multiple edges connecting the same pair of vertices or a vertex to itself, are called Pseudographs.
  - **Note**
  - Some authors make a distinction between pseudographs (**with** loops) and multigraphs (without loops).
    - **Example**
      - A communication link connects a data center with itself, a feedback loop for diagnostic purpose.
      - To model this network, we need to include edges that connect a vertex to itself.

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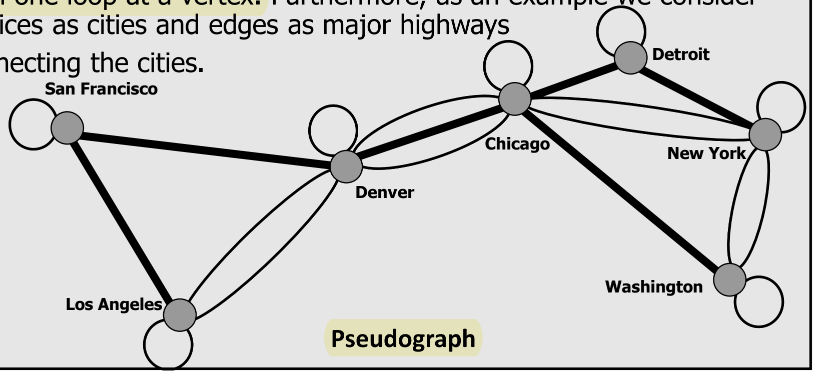
Graphs

Such edges are called loops, and sometimes we may even have more than one loop at a vertex. Furthermore, as an example we consider vertices as cities and edges as major highways connecting the cities.



Pseudograph

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Graphs

Directed Graphs

- "A directed graph $G = (V, E)$ *consists of* non-empty set of vertices V and E, a set of directed edges. Each directed edge is associated with an ordered pair of vertices as (u, v), which start at u and end at v. "

OR

- A directed graph (V, E) consists of a set of vertices V and a binary relation (need not to be symmetric) E on V.
- e.g.: $V = set\ of\ people$, $E = \{(u, v) | u\ looks\ directly\ v\}$

i. **Simple Directed graph:**
   - "A directed graph is said to be simple directed graph if graph has no loops and has no multiple directed edges."

ii. **Directed multigraph:**
   - "Like directed graphs, but there may be more than one edges from one vertex to another."

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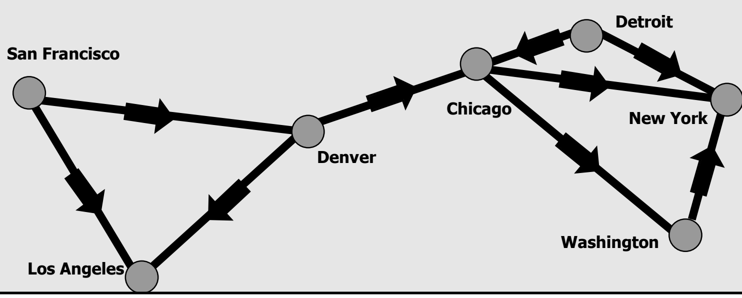
Graphs

Example

- In computer network some links may operate in only one direction. To model such a computer network we use a simple directed graphs.



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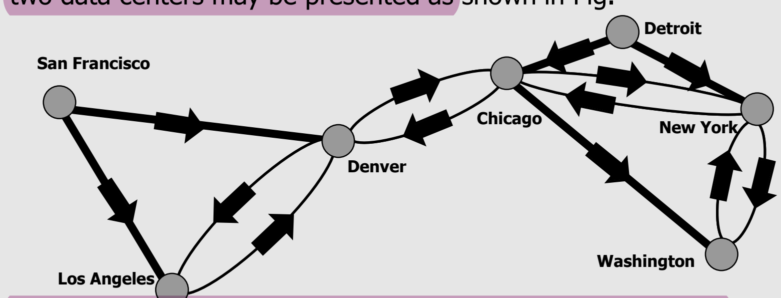
Graphs

Example

- In some computer networks, multiple communication links between two data centers may be presented as shown in Fig.



- Directed graphs that may have multiple directed edges from a vertex to a second vertex are used to model such networks.

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Graphs

Types of Graphs

- Keep in mind this terminology is not fully standardized across different authors.

Types of Graph	Types of Edge	Multiple Edges Permitted?	Self Loops Permitted?
Simple Graph	Undirected	NO	NO
Multigraph	Undirected	YES	NO
Pseudograph	Undirected	YES	YES
Simple Directed Graph	Directed	NO	NO
Directed Multigraph	Directed	YES	YES

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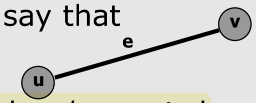
Graphs

Graph Terminology

i. Adjancy:

- Let us suppose that G be an undirected graph with E edge sets.
- If $\mathrm{E} = (u, v)$ is an edge, then we say that
- Edge e is incident with u.
- Edge e is incident with v.
- Vertices u and v are adjacent/neighbors/connected.
- Edge e connects u and v.
- Vertices u and v are endpoints of edge e.



ii. Neighborhood of a Vertex:

- The set of all neighbors of a vertex v of $G = (V, E)$, denoted by $N(v)$, is called *the neighborhood of* v.

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Graphs

iii. Degree of a Vertex

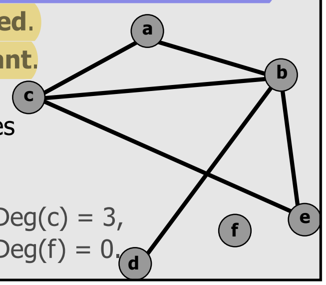
iii. Let $G = (V, E)$ be an undirected graph, $v \epsilon \mathrm{V}$ is a vertex.

iv. The degree of vertex v is denoted by $\deg(v)$ and defined as the number of edges incident to v. (Self-loops are counted twice)

v. A vertex of degree 0 is called **isolated**.

vi. A vertex of degree 1 is called **pendant**.

- **Example**
  - What are degrees of the vertices in the graph.
  - **Solution**
    - Deg(a) = 2, Deg(b) = 4, Deg(c) = 3,
    - Deg(e) = 2, Deg(d) = 1, Deg(f) = 0.



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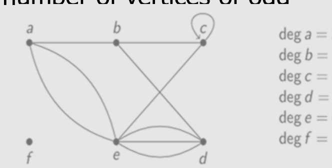
Graphs

Handshaking Theorem

- Let $G = (V, E)$ be an undirected graph, V is a vertex set and edge set E. Then

$$\sum_{v \in V} deg(v) = 2|E|$$

- **Note**
  - This theorem applied even if multiple edges and loops are present.
  - An undirected graph has an even number of vertices of odd degree.
  - **Example**
    - **Sum of degrees = 2 ×number of edges**



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Graphs

Definition

- When (u, v) is an edge of the graph G with directed edges, u is said to be adjacent to v and v is said to be adjacent from u. The vertex u is called the initial vertex of (u, v), and v is called the terminal or end vertex of (u, v).
  - **Note**
    - The initial vertex and terminal vertex of a loop are the same.

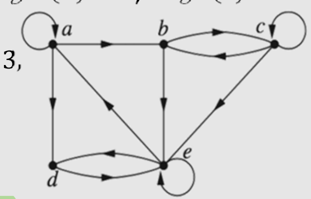
Definition

- In a graph with directed edges the in-degree of a vertex v, denoted by $deg^-(v)$, is the number of edges with v as their terminal vertex. The out degree of v, denoted by $deg^+(v)$, is the number of edges with v as their initial vertex.
  - **Note**
    - A loop at a vertex contributes 1 to both the in-degree and the out-degree of this vertex.

Graphs

Example

- Find the in-degree and out-degree of each of vertex in the graph G with directed edges.
  - **Solution**
    - The in-degree in G are $deg^-(a) = deg^-(b) = deg^-(c) = deg^-(d) = 2, deg^-(e) = 4$
    - The out-degree in G are $deg^+(a) = deg^+(c) = deg^+(e) = 3,$ $deg^+(b) = 2, deg^+(d) = 1$



Theorem

- Let $G = (V, E)$ be a graph with directed edges. Then
- Sum of $deg^-(v) =$ Sum of $deg^+(a) = |E|$

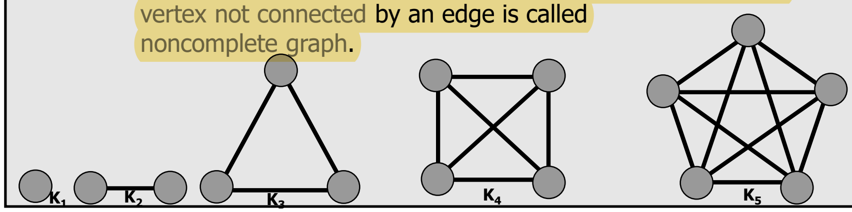
Graphs

Some Special Simple Graphs

- There is a class of simple graphs, often used as examples and arise in many applications.

i. **Complete Graphs**

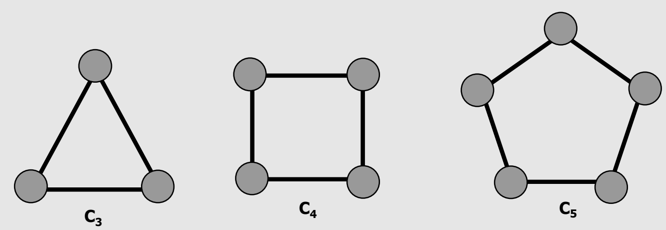
- A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices. The graphs K_n, for n = 1, 2, 3, 4, 5 are displayed in Fig.
- A simple graph for which there is at least one pair of distinct vertex not connected by an edge is called noncomplete graph.



Graphs

ii. **Cycles**

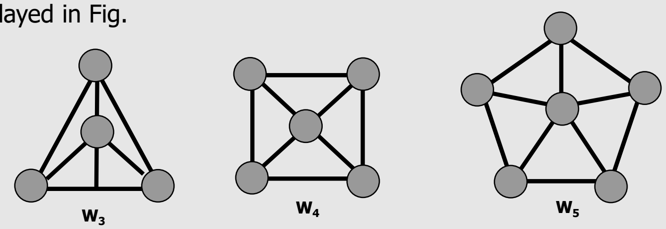
- A cycle C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$. The graphs of cycles C_n, for n = 1, 2, 3, 4, 5 are displayed in Fig.



Graphs

iii. Wheels

- We obtain a wheel W_n, when we add an additional vertex to a cycle C_n, $n \geq 3$, and connect this new vertex to each of the n vertices in C_n by new edges. The wheels W_3, W_4, and W_5 are displayed in Fig.



UNITY CHART

Connecting The Parts Of Knowledge With The Wholeness Of Knowledge

Solving Finite Problems

1. There are many real-world problems like sorting, searching, counting, and optimizing that are finite or discrete in their nature.
2. Discrete mathematics gives us universal mathematical techniques to solve discrete problems.

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3. **Transcendental Consciousness** is an infinite field of all possibilities.
4. **Impulses within the transcendental field** structure the finite isolated parts of creation.
5. **Wholeness moving within itself:** In Unity Consciousness, we find the solutions to all finite problems in the infinite unboundedness of our own Self.

[1]Dillbeck, M.C. Meditation and flexibility of visual perception and verbal problem-solving. *Memory & Cognition* 10(3): 207–215, 1982.