CHE212: Graded take-home simulation 4

No mobile phones allowed. No discussion allowed during the session, raise your hand if you need help on ode45/bvp4c algorithms.

Directions on report submission:

1. Save your files with question number as the prefix. Example: Q1_odefun.m, Q1_script.m
2. After finishing your each question in Matlab, add inferences as comments therein.
3. Upload the code at hello iitk.

There will be question in quiz/endsem on your coding experience of these problems :)

Best policy is to not copy. Copied assignments will be given zero across all simulations.

Consider the unsteady heat transfer problem governed by:

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial X^2}. \tag{1}$$

We scaled the temperature using $\Theta = (T - T_\infty)/(T_i - T_\infty)$. Length and time scales were chosen as L and L^2/α, respectively. The following boundary conditions are satisfied:

$$\text{BC}_1: \quad \frac{\partial \Theta}{\partial X} = 0 \text{ at } X = 0 \qquad \text{BC}_2: \quad \frac{\partial \Theta}{\partial X} = -Bi\Theta \text{ at } X = 1 \tag{2}$$

$$\text{Initial BC}: \quad \Theta = 1 \text{ at } \tau = 0$$

Question 1: plot and compare solutions [4 marks]

The series solution for the above system of equations is found using Separation of variables as:

$$\Theta(X,t) = \sum_{n=1}^{N} \frac{4\sin(\lambda_n X)}{2\lambda_n + \sin(2\lambda_n)} \cos(\lambda_n X) \exp\left[-\lambda_n \tau^2\right] \tag{3}$$

The eigenvalues λ_n in the above equation satisfy a transcendental equation: $\lambda_n \tan(\lambda_n) = Bi$ that has infinite roots. n=0 in the above equation is also a solution, but a trivial one ($\lambda = 0$), hence it is neglected.

1. Consider a low conductive material in a highly convective environment ($Bi = 10$). Plot the above solution Θ vs X for N=1 (First-term approximation) at $\tau = 0.05$. Compare it with another (more accurate solution) N=3 in the same plot.

2. Continue with same Biot number and plot at times $\tau = 0.1$ and $\tau = 1$. Do spatial temperature profiles merge for higher times?

3. Keep N=3 and plot Θ vs τ at X=0 for Bi = 10. Plot from $\tau = [0, 2]$.
Change the $Bi = 1$ and do the same. Compare these profiles for two Biot numbers in the same plot. Which Biot case decays faster?

4. Again plot the temporal profile (at $X = 0$) for Bi=0.1 and compare it with the lumped solution:

$$\Theta_{\text{lumped}} = \exp\left[-Bi\,\tau\right].$$

Question 2: pdepe [4 marks]

1. Use the PDEPE command to write a code to evaluate the spatio-temporal temperature profiles for the system described by Eq.[1-2].

2. Match the spatial profile with that obtained in eq. [3] (choose N=5) for times $\tau = 0.001$, $\tau = 0.01$ and $\tau = 0.1$.

Hint: Utilize "Practice coding session – 2" in the helloiitk portal for help.