Developing Bayesian method for locating breakpoints in time series data.

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Abstract: An abstract of up to 200 words should precede the text together with 5 or 6 keywords in alphabetical order to describe the content of the paper. Authors should take great care in preparing the abstract and not simply lift it from the main text. The abstract should describe the background and contribution of the manuscript and give a clear verbal description of the results and examples, and avoid citations

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Key words: keyword a; keyword b; keyword c; keyword d; keyword e

1 Math

Here is the math for our paper

1.1 Derivations of Ratio

To start we need to find the ratio

$$
\begin{aligned}
ratio &= \frac{g(\tau_n K_n | x_1, \ldots, x_n)}{g(\tau_o K_o | x_1, \ldots, x_n)} \times \frac{q(\tau_o K_o | \tau_n K_n)}{q(\tau_n K_n | \tau_o K_o)} \\
&= \frac{\left[\frac{f(x_1,\ldots,x_n|\tau_n K_n)\pi(\tau_n K_n)}{\int f(x_1,\ldots,x_n|\tau_n K_n)\pi(\tau_n K_n) d\tau_n K_n}\right] q(\tau_o K_o | \tau_n K_n)}{\left[\frac{f(x_1,\ldots,x_n|\tau_o K_o)\pi(\tau_o K_o)}{\int f(x_1,\ldots,x_n|\tau_o K_o)\pi(\tau_o K_o) d\tau_o K_o}\right] q(\tau_n K_n | \tau_o K_o)}
\end{aligned}
$$

Then we have,

$$
ratio = \frac{\left[\frac{\int f(x_1,\ldots,x_n|\tau_n K_n)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{new} d\tau dK}{\int f(x_1,\ldots,x_n|\tau_n K_n)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{new} d\tau dK d\beta d\sigma}\right] q(\tau_o K_o | \tau_n K_n)}{\left[\frac{\int f(x_1,\ldots,x_n|\tau_o K_o)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{old} d\tau dK}{\int f(x_1,\ldots,x_n|\tau_o K_o)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{old} d\tau dK d\beta d\sigma}\right] q(\tau_n K_n | \tau_o K_o)}
$$

Basing priors of the BARS paper (Kass & Wasserman, 1995) we have that

$\pi(\theta) = \pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)$ for both the θ_n and the θ_o.

$\pi(\beta)$ is an unit information prior, multivariate normal

$\pi(\sigma)$ is an inverse gamma

$\pi(\tau|K)$ might be uniform

$\pi(K)$ might be a Poisson or uniform

1.1.1 BIC

Applying this ratio into our own model we need to first take the log such that,

$$\begin{aligned} ratio &= \frac{\Big[f(x_1,\ldots,x_n|\tau_n K_n)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)_{new}\Big]q(\tau_o K_o|\tau_n K_n)}{\Big[f(x_1,\ldots,x_n|\tau_o K_o)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)_{old}\Big]q(\tau_n K_n|\tau_o K_o)} \\ log(ratio) &= log\Big[\frac{\Big[f(x_1,\ldots,x_n|\tau_n K_n)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)\Big]q(\tau_o K_o|\tau_n K_n)}{\Big[f(x_1,\ldots,x_n|\tau_o K_o)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)\Big]q(\tau_n K_n|\tau_o K_o)}\Big] \\ &= \Big[log\big[f(x_1,\ldots,x_n|\tau_n K_n)\big] - log\big[f(x_1,\ldots,x_n|\tau_o K_o)\big]\Big] \\ &\qquad + \Big[log\big[q(\tau_o K_o|\tau_n K_n)\big] - log\big[q(\tau_n K_n|\tau_o K_o)\big]\Big] \end{aligned}$$

From the knowledge gained by the (Kass & Wasserman, 1995) paper we have that $\Big[log\big[f(x_1,\ldots,x_n|\tau_n K_n)\big] - log\big[f(x_1,\ldots,x_n|\tau_o K_o)\big]\Big]$ approximates BIC. Thus,

$$log(ratio) = \frac{-\Delta BIC}{2} + \Big[log\big[q(\tau_o K_o|\tau_n K_n)\big] - log\big[q(\tau_n K_n|\tau_o K_o)\big]\Big]$$

The q values depend on whether the step is addition or subtraction.

Addition: $q(\tau_o K_o|\tau_n K_n) = c \cdot d \cdot dpois(K_{old}, \lambda)$, $q(\tau_n K_n|\tau_o K_o) = c \cdot b \cdot dpois(K_{old}, \lambda)$

Subtraction: $q(\tau_o K_o|\tau_n K_n) = c{\cdot}b{\cdot}dpois(K_{new}, \lambda)$, $q(\tau_n K_n|\tau_o K_o) = c{\cdot}d{\cdot}dpois(K_{old}, \lambda)$

1.2 Derivations of β and σ draws

The posterior for the β coefficient is laid out in detail in the *Forecasting time series* (Pesaran Paper, 2006) such that,

$$y \sim N(x\beta, \sigma) \longrightarrow \beta|\sigma^2, b_0, B_0, V_0, d_0, p, S_\gamma, Y_\gamma \sim N(\overline{\beta_j}, \overline{V_j})$$

Where $\overline{V}_j = (\sigma^{-2}x^T x + B_0^{-1})^{-1}$, and $\overline{\beta}_j = \overline{V}_j(\sigma^{-2}x^T y + B_0^{-1}b_0)$. Thus we have that $\beta \sim N(\overline{\beta}_j, \overline{V}_j)$.

The *Forecasting time series* (Pesaran Paper, 2006) also lays out the σ posterior. $y \sim N(x\beta, \sigma)$ and

$$\sigma_j^{-2} \sim \Gamma(v_0, d_0) \longrightarrow \sigma_j^{-2}|\beta, b_0, B_0, v_0, d_0, p, S_\gamma, Y_\gamma \sim \Gamma(d = \overline{v}_0, \beta = \overline{d}_0)$$

Where $\overline{v}_0 = v_0 + \frac{n_j}{2}$, and $\overline{d}_0 = d_0 + \frac{1}{2}(y - x\beta)^T(y - x\beta)$

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