Developing a Bayesian method for locating breakpoints in time series data.

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Key words: keyword a; keyword b; keyword c; keyword d; keyword e

1 Introduction

2 Math

Here is the math for our paper

2.1 Derivations of Ratio

To start we need to find the ratio

$$
\begin{aligned}
ratio &= \frac{g(\tau_n K_n | x_1, \ldots, x_n)}{g(\tau_o K_o | x_1, \ldots, x_n)} \times \frac{q(\tau_o K_o | \tau_n K_n)}{q(\tau_n K_n | \tau_o K_o)} \\
&= \frac{\left[\frac{f(x_1,\ldots,x_n|\tau_n K_n)\pi(\tau_n K_n)}{\int f(x_1,\ldots,x_n|\tau_n K_n)\pi(\tau_n K_n) d\tau_n K_n}\right] q(\tau_o K_o | \tau_n K_n)}{\left[\frac{f(x_1,\ldots,x_n|\tau_o K_o)\pi(\tau_o K_o)}{\int f(x_1,\ldots,x_n|\tau_o K_o)\pi(\tau_o K_o) d\tau_o K_o}\right] q(\tau_n K_n | \tau_o K_o)}
\end{aligned}
$$

Then we have,

$$
ratio = \frac{\left[\frac{\int f(x_1,\ldots,x_n|\tau_n K_n)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{new} d\tau dK}{\int f(x_1,\ldots,x_n|\tau_n K_n)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{new} d\tau dK d\beta d\sigma}\right] q(\tau_o K_o | \tau_n K_n)}{\left[\frac{\int f(x_1,\ldots,x_n|\tau_o K_o)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{old} d\tau dK}{\int f(x_1,\ldots,x_n|\tau_o K_o)\left(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\right)_{old} d\tau dK d\beta d\sigma}\right] q(\tau_n K_n | \tau_o K_o)}
$$

Basing priors of the BARS paper (Kass & Wasserman, 1995) we have that $\pi(\theta) = \pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)$ for both the θ_n and the θ_o.

$\pi(\beta)$ is an unit information prior, multivariate normal

$\pi(\sigma)$ is an inverse gamma

$\pi(\tau|K)$ might be uniform

$\pi(K)$ might be a Poisson or uniform

2.1.1 BIC

Applying this ratio into our own model we need to first take the log such that,

$$
\begin{aligned}
ratio &= \frac{\Big[f(x_1,\ldots,x_n|\tau_n K_n)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)_{new}\Big]q(\tau_o K_o|\tau_n K_n)}{\Big[f(x_1,\ldots,x_n|\tau_o K_o)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)_{old}\Big]q(\tau_n K_n|\tau_o K_o)} \\
log(ratio) &= log\Big[\frac{\Big[f(x_1,\ldots,x_n|\tau_n K_n)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)\Big]q(\tau_o K_o|\tau_n K_n)}{\Big[f(x_1,\ldots,x_n|\tau_o K_o)\big(\pi(\tau|K)\pi(K)\pi(\beta)\pi(\sigma)\big)\Big]q(\tau_n K_n|\tau_o K_o)}\Big] \\
&= \Big[log\big[f(x_1,\ldots,x_n|\tau_n K_n)\big] - log\big[f(x_1,\ldots,x_n|\tau_o K_o)\big]\Big] \\
&\qquad + \Big[log\big[q(\tau_o K_o|\tau_n K_n)\big] - log\big[q(\tau_n K_n|\tau_o K_o)\big]\Big]
\end{aligned}
$$

From the knowledge gained by the (Kass & Wasserman, 1995) paper we have that $\Big[log\big[f(x_1,\ldots,x_n|\tau_n K_n)\big] - log\big[f(x_1,\ldots,x_n|\tau_o K_o)\big]\Big]$ approximates BIC. Thus,

$$
log(ratio) = \frac{-\Delta BIC}{2} + \Big[log\big[q(\tau_o K_o|\tau_n K_n)\big] - log\big[q(\tau_n K_n|\tau_o K_o)\big]\Big]
$$

The q values depend on whether the step is addition or subtraction.

Addition: $q(\tau_o K_o|\tau_n K_n) = c \cdot d \cdot dpois(K_{old}, \lambda)$, $q(\tau_n K_n|\tau_o K_o) = c \cdot b \cdot dpois(K_{old}, \lambda)$

Subtraction: $q(\tau_o K_o|\tau_n K_n) = c \cdot b \cdot dpois(K_{new}, \lambda)$, $q(\tau_n K_n|\tau_o K_o) = c \cdot d \cdot dpois(K_{old}, \lambda)$

2.2 Derivations of β and σ draws

The posterior for the β coefficient is laid out in detail in the *Forecasting time series* (Pesaran Paper, 2006) such that,

$$y \sim N(x\beta, \sigma) \longrightarrow \beta|\sigma^2, b_0, B_0, V_0, d_0, p, S_\gamma, Y_\gamma \sim N(\overline{\beta_j}, \overline{V_j})$$

Where $\overline{V}_j = (\sigma^{-2}x^T x + B_0^{-1})^{-1}$, and $\overline{\beta}_j = \overline{V}_j(\sigma^{-2}x^T y + B_0^{-1}b_0)$. Thus we have that $\beta \sim N(\overline{\beta}_j, \overline{V}_j)$.

The *Forecasting time series* (Pesaran Paper, 2006) also lays out the σ posterior. $y \sim N(x\beta, \sigma)$ and

$$\sigma_j^{-2} \sim \Gamma(v_0, d_0) \longrightarrow \sigma_j^{-2}|\beta, b_0, B_0, v_0, d_0, p, S_\gamma, Y_\gamma \sim \Gamma(d = \overline{v}_0, \beta = \overline{d}_0)$$

Where $\overline{v}_0 = v_0 + \frac{n_j}{2}$, and $\overline{d}_0 = d_0 + \frac{1}{2}(y - x\beta)^T(y - x\beta)$

3 Methods

4 Results

5 Discussion

6 Appendix

Acknowledgements

We want to thank...