EE2227 Control System Documentation.

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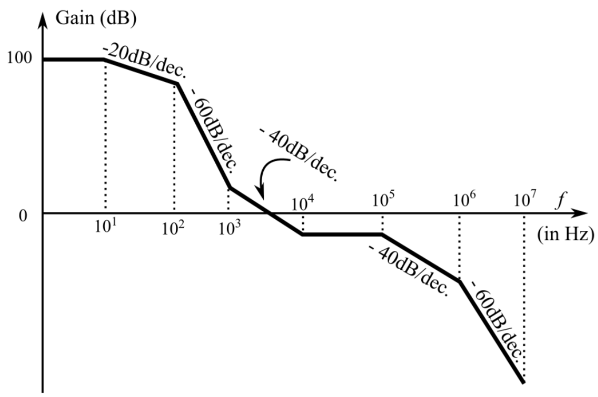
Abstract—**Gate Control System Question and Solution**

$\boxed{\text{Question-1}}$ For an LTI system, the Bode plot for its gain is as illustrated in the figure shown. The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz $\leq$ f $\leq$ 10^7 Hz is



$\boxed{\text{Solution:-}}$

Let us consider a generalized transfer gain

$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)}$

$Gain = 20log|H(s)| = 20log|k| + 20log|s - z_1| + 20log|s - z_2| + ... + 20log|s - z_m| - 20log|s - p_1| - 20log|s - p_2| - ..... - 20log|s - p_n|$

- When a pole is encountered the slope always decreases by -20 dB/decade
- When a zero is encountered the slope always increases by +20 dB/decade
- At f = 10 Hz , change in slope = -20dB/sec, Hence we have 1 pole here
- At f = 10^2 Hz, Change in slope = -40dB/sec, Hence we have 2 poles here
- At f = 10^3 Hz, Change in slope = +20dB/sec, Hence we have 1 zero here
- At f = 10^4 Hz, Change in slope = +40dB/sec, Hence we have 2 zeros here
- At f = 10^5 Hz, Change in slope = -40dB/sec, Hence we have 2 poles here
- At f = 10^6 Hz, Change in slope = -20dB/sec, Hence we have 1 pole here

$N_p = 6 \quad N_z = 3$

$\boxed{\text{Question-2}}$ Consider the following second order system with the transfer function:

$$G(s) = \frac{1}{1 + 2s + s^2}$$

with input unit step

$$R(s) = \frac{1}{s}$$

Let C(s) be the corresponding output. The time taken by the system output c(t) to reach 94% of its steady state value, rounded off to two decimal places is

(A)5.25 (B)4.50 (C)3.89 (D)2.81

$\boxed{\text{Solution:-}}$ The approach for finding the solution is as follows:

- finding C(s)
- finding c(t)
- finding the time at which c(t) attains 94% of its steady state value

We are given G(s) and R(s), to find C(s), we can simply multiply these two

$$C(s) = R(s).G(s) = (\frac{1}{s})(\frac{1}{1 + 2s + s^2})$$

$$C(s) = \frac{1}{s(1 + s)^2}$$

To find c(t), we have to do inverse Laplace transform on C(s)

$$c(t) \longleftrightarrow C(s)$$

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Inverse Laplace transform can be calculated by the formula:

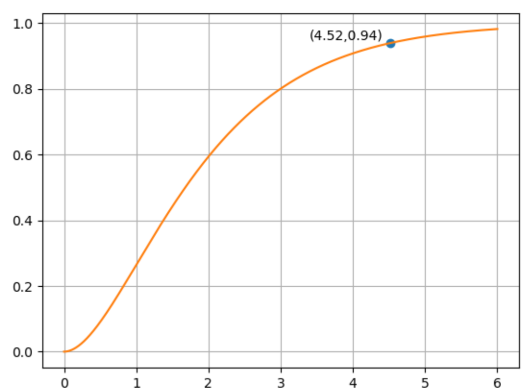
$$f(t) = \frac{1}{2\pi j}\int_{a-j\infty}^{a+j\infty} F(s)e^{st}ds$$

From the above formula, the inverse Laplace for some common expressions are:

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}$$

$$te^{-at}u(t) \longleftrightarrow \frac{1}{(s+a)^2}$$



We found C(s) as:

$$C(s) = \frac{1}{s(1+s)^2}$$

Now, we will use partial fractions to make applying Inverse Laplace easy.

$$C(s) = \frac{1}{s(1+s)^2} = \frac{A}{s} + \frac{B}{(1+s)} + \frac{C}{(1+s)^2}$$

We get,

$$A = 1 \qquad A + B = 0 \qquad 2A + B + C = 0$$
$$A = 1 \qquad B = -1 \qquad C = -1$$

Therefore,

$$C(s) = \frac{1}{s} - \frac{1}{(1+s)} - \frac{1}{(1+s)^2}$$

$$c(t) = L^{-1}(\frac{1}{s} - \frac{1}{(1+s)} - \frac{1}{(1+s)^2})$$

From the properties of inverse Laplace transform,

$$L^{-1}(F_1(s) + F_2(s) + F_3(s)) = L^{-1}(F_1(s))$$
$$+L^{-1}(F_2(s)) + L^{-1}(F_3(s))$$

Therefore;

$$c(t) = L^{-1}(\frac{1}{s}) - L^{-1}(\frac{1}{(1+s)}) - L^{-1}(\frac{1}{(1+s)^2})$$

Using the Known inverse transforms:

$$c(t) = (1 - e^{-t} - te^{-t}).u(t)$$

To know the steady state value of c(t), we calculate

$$\lim_{t\to\infty} c(t) = (1 + 0 + 0).(1) = 1$$

Now, 94% of 1 is 0.94, so we should now solve for a positive t such that

$$(1 - e^{-t} - te^{-t}) = 0.94$$

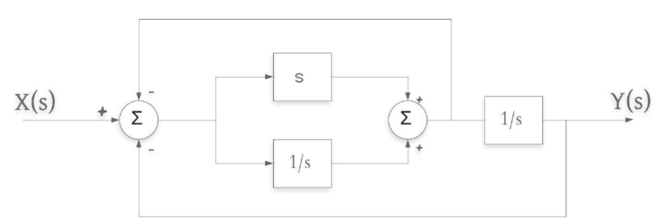
After calculation, t turns out to be

$$t = 4.5221$$

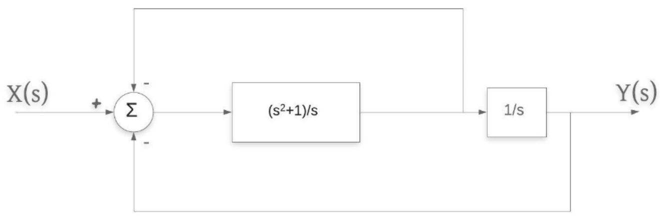
Therefore, answer is option (b) We can also find the solution by plotting c(t):

$\boxed{\text{Question-3}}$ The block diagram of a system is illustrated in the figure shown, where X(s) is the input and Y(s) is the output. The transfer function H(s)=$\frac{Y(s)}{X(s)}$ is

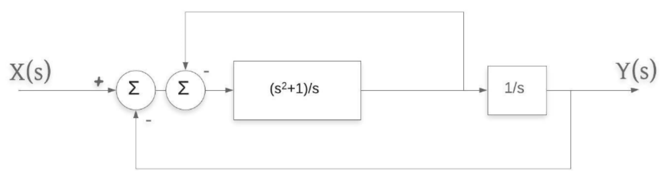
(A) H(s)=$\frac{s^2+1}{s^3+s^2+s+1}$
(B) H(s)=$\frac{s^2+1}{s^3+2s^2+s+1}$
(C) H(s)=$\frac{s^2+1}{s^2+s+1}$
(D) H(s)=$\frac{s^2+1}{2s^2+1}$



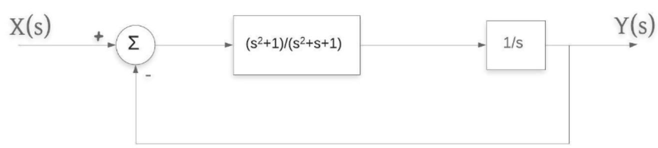
$\boxed{\text{Solution:-}}$ Here we have two transfer function s and $\frac{1}{s}$ in parallel with a adder as shown in figure. After solving these two parallel transfer function by just adding both of them we will get



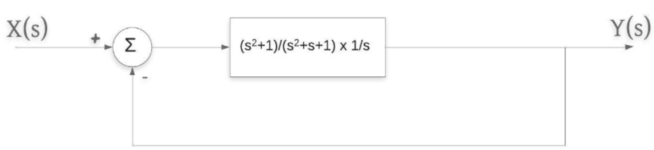
Now we will convert three input adder into two input adder as shown in figure given below.



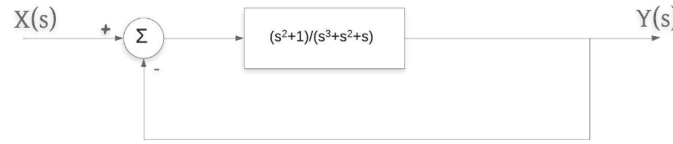
Now we have Negative Unity Feedback System(NUFS) in closed loop transfer function.
Let's say we have transfer function G(s) with Negative Unity Feedback System in closed loop then we will solve this by
$\frac{G(s)}{1+G(s)}$.
Here we have



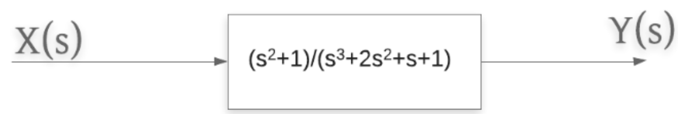
Here we have two transfer function in series



Now we have one more transfer function with negative unity feedback.



Again we will solve this then we will get



Now

X(s)($\frac{s^2+1}{s^3+2s^2+s+1}$)=Y(s)

$\frac{Y(s)}{X(s)}=\frac{s^2+1}{s^3+2s^2+s+1}$

The correct option is (B)

Question-4 Let the state-space representation of an LTI system be.

$\dot{x(t)} = Ax(t) + Bu(t)$
y(t)=Cx(t)+Du(t)
A,B,C are matrices, D is scalar, u(t) is input to the system and y(t) is output to the system. let

$$b1 = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix}$$

$b1^T = B$
and D=0. Find A and C.

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$$

Solution:- STATE MODEL
Let U1(t) and U2(t) are the inputs of the MIMO system and y1(t),y2(t) are the output of the system and x1(t) and x2(t) are the state variables.
so output equation is,

$$y1(t) = C_{11}\times x1(t)+C_{12}\times x2(t)+d_{11}\times U1(t)+d_{12}\times U2(t) \quad (1)$$

$$y2(t) = C_{21}\times x1(t)+C_{22}\times x2(t)+d_{21}\times U1(t)+d_{22}\times U2(t) \quad (2)$$

$$\begin{bmatrix} y1(t) \\ y2(t) \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{11} & C_{12} \end{bmatrix} \times \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{11} & d_{12} \end{bmatrix} \times \begin{bmatrix} U1(t) \\ U2(t) \end{bmatrix}$$

therefore Y(t)=C.X(t)+D.U(t)

$$x\dot{1}(t) = a_{11}\times x1(t)+a_{12}\times x2(t)+b_{11}\times U1(t)+b_{12}\times U2(tx \quad (3)$$

$$x\dot{2}(t) = a_{21}\times x1(t)+a_{22}\times x2(t)+b_{21}\times U1(t)+b_{22}\times U2(t) \quad (4)$$

$$\begin{bmatrix} x\dot{1}(t) \\ x\dot{2}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{bmatrix} \times \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{11} & b_{12} \end{bmatrix} \times \begin{bmatrix} U1(t) \\ U2(t) \end{bmatrix}$$

$therefore\ X\dot{(}t)$=A.X(t)+B.U(t)

FINDING TRANSFER FUNCTION

$So,\ X\dot{(}t)$=A.X(t)+B.U(t) *be equation* 1

and Y(t)=C.X(t)+D.U(t) *be equation* 2

by applying laplace transforms on both sides of equation 1

we get

S.X(S)-X(0)=A.X(S)+B.U(S)

S.X(S)-A.X(S)=B.U(S)+X(0)

(SI-A)X(S)=X(0)+B.U(S)

X(S)=X(0)([SI-A])$^{-}1 + B.([SI - A])^{-}1.U(S)$

Laplace transform of equation 2 and sub X(s)

$$Y(S) = C.X(S) + D.U(S)$$

$$Y(S) = C.[X(0)([SI-A])^{-}1+B.([SI-A])^{-}1.U(S)]+D.U(S)$$

$$If X(0) = 0$$

$$thenY(S) = C.[B.([SI - A])^{-}1.U(S)] + D.U(S)$$

$$\frac{Y(S)}{U(S)} = C.[B.([SI - A])^{-}1] + D = H(S)$$

As we know that

$$Y(s) = H(s) \times U(s) = (\frac{1}{s^3 + 3s^2 + 2s + 1}) \times U(s)$$

let X(S)= $\frac{U(S)}{denominator}$

$$Y(S) = X(S) \times numerotor$$

$$s^3X(s) + 3s^2X(s) + 2sX(s) + X(s) = U(S)$$

$$\begin{bmatrix} sx(s) \\ s^2x(s) \\ s^3x(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x(s) \\ sx(s) \\ s^2x(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U$$

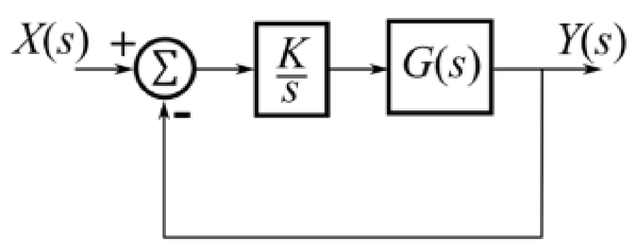
therfore A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$

Since Y(S)=X(S)×*numerator*
$therefore Y(S) = X(S);$

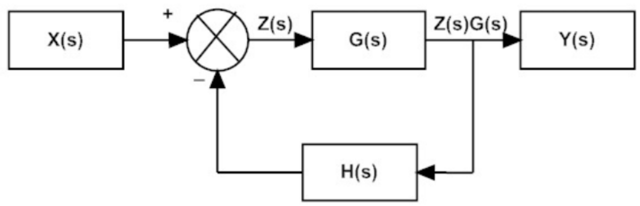
$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x(s) \\ sx(s) \\ s^2x(s) \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Question-5}}$ Consider a unity feedback system as shown in the figure,shown with an integral compensator k/s and open-loop transfer function G(s) = $\frac{1}{s^2+3s+2}$
where k>0. The positive value of k for which there are two poles of unity feedback system on jω axis is equal to—(rounded off to two decimal places)



$\boxed{\text{Solution:-}}$ A transfer function is the relative function between input and output.
In a negative feedback system an intermediate signal is defined as Z. .



Y(s) = Z(s).G(s)
Z(s) = X(s) - Y(s).H(s) ⇒ X(s) = Z(s)+Y(s).H(s)

X(s) = Z(s)+Z(s).G(s).H(s)

$\frac{Y(s)}{X(s)} = \frac{Z(s).G(s)}{Z(s)+Z(s).G(s).H(s)}$

So,the transfer function of negative feedback is $\frac{G(s)}{1+G(s).H(s)}$
Since unit feedback H(s) = 1
Now the transfer function of unity negative feedback is $\frac{G(s)}{1+G(s)}$
The net transfer function in the given question is.....

$\frac{Y(s)}{X(s)} = \frac{G(s)*k/s}{1+G(s)*k/s}$

The characteristic equation is 1 + (G(s)x$\frac{k}{s}$) = 0 that is..,

C.E = 1 + $\frac{k}{s(s^2+3s+2)}$ = 0 ⇒ $s(s^2+3s+2)+k=0$

⇒ $s^3+3s^2+2s+k=0$
Routh − HurwitzCriterion : −
. This criterion is based on arranging the coefficients of characteristic equation intoan array called Routh array.
q(s) = $a_0s^n + a_1s^{n-1} + a_2s^{n-2} + ... + a_{n-1}s + a_n = 0$

$$\begin{vmatrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \end{vmatrix} \begin{vmatrix} a_0 & a_2 & a_4 & \cdots & \\ a_1 & a_3 & a_5 & \cdots & \\ b_1 & b_2 & b_3 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots\cdots \end{vmatrix}$$

where $b_1 = \frac{a_1a_2-a_0a_3}{a_1}$ $b_2 = \frac{a_1a_4-a_0a_5}{a_1}$

$c_1 = \frac{b_1a_3-a_1b_2}{b_1}$ $c_2 = \frac{b_1a_5-a_1b_3}{b_1}$
. For poles to lie on imaginary axis any one entire row of hurwitz matrix should be zero.

For the given characteristic equation = $s^3+3s^2+2s+k=0$

$$\begin{vmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & k \\ \frac{6-k}{3} & 0 \\ k & 0 \end{vmatrix}$$

For poles on $j\omega$ axis any one of the row should be zero
⇒ $\frac{6-k}{3}$ = 0 or k = 0
But given k>0 ...
therefore, 6-k=0 ⇒ k = 6
To find the location of poles on jω axis

Auxillary equation of the given CE is $3s^2+k=0$
⇒ $3s^2+6=0$
⇒ s = $\pm j2$

$\boxed{\text{Question-6}}$ The output response of a system is denoted as y(t), and its Laplace transform is given by Y(s) = $\frac{10}{s(s^2+s+100(2)^{0.5})}$
. The steady state value of y(t) is

a) $100(2)^{0.5}$ $b)\frac{1}{10(2)^{0.5}}$

$c)10(2)^{0.5}$ $d)\frac{1}{100(2)^{0.5}}$

$\boxed{\text{Solution:-}}$

The final value theorem states that

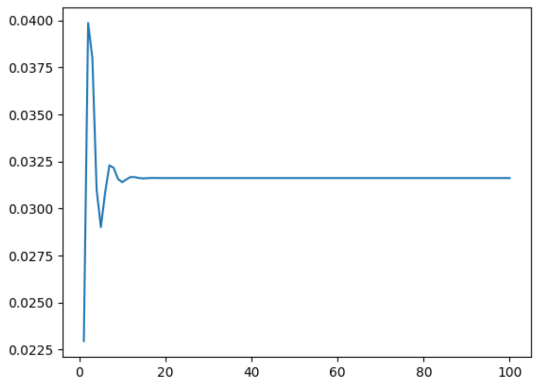
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

This is valid only when sY(s) has poles that lie in the negative half of the real side.

If the quadratic equation ax^2+bx+c has complex roots then the real part of those roots will be -b/2a Hence, verified that the roots of $s^2+s+100(2)^{0.5}$ have a negative real part which is -0.5. So, Final value theorem is applicable.
Steady state value of y(t) =

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0} \frac{10s}{s(s^2+s+100(2)^{0.5})}$$

$$= \frac{10}{100(2)^{0.5}} = \frac{1}{10(2)^{0.5}}.$$



We can see that y(t) is approaching a constant value 0.031 which is verifies our answer!

$\boxed{\text{Question-7}}$

- The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{\pi e^{-0.25s}}{s}$$

in G(s) plane,the Nyquist plot of G(s) passes through the negative real axis at the point (A)(-0.5,j0) (B)(-0.75,j0) (C)(-1.25,j0) (D)(-1.5,j0)

$\boxed{\text{Solution:-}}$

$$G(s) = \frac{\pi e^{-0.25s}}{s}$$

Nyquist plot cuts the negative real Axis at ω = phase cross over frequency
$G(j\omega) = \frac{\pi}{\omega}(-\sin 0.25\omega$-jcos $0.25\omega)$
$\angle G(j\omega)$=-90°-0.25ω×180°/π
$\angle G(j\omega)$—$_{\omega=\omega_{pc}}$=-180°
by solving for ω we get $\omega_{pc} = 2\pi$
magnitude at any point is X=|$G(j\omega)$|=$\frac{\pi}{\omega}$
substituting $\omega = 2\pi$ in magnitude we get X=0.5
hence it intersects at (-0.5,0j) so answer is A

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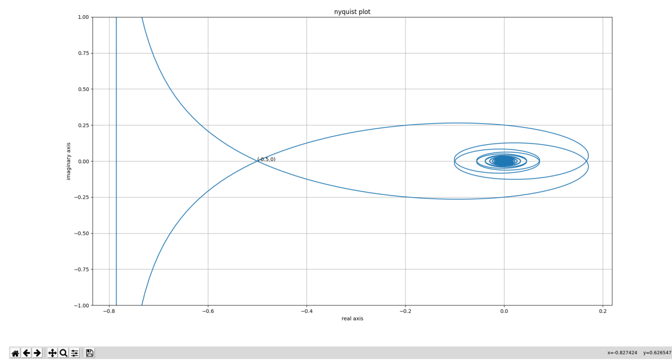
we can verify with the following plot that it intersects at (-0.5,0j)



$\boxed{\text{Question-8}}$ The characteristic equation of linear time invariant system is given by

$$\nabla(s) = s^4 + 3s^3 + 3s^2 + s + k = 0$$

The system is BIBO stable if

A.0<k< $\frac{12}{9}$

$B.k > 3$

$C.0 < k < \frac{8}{9}$

$D.k > 6$

$\boxed{\text{Solution:-}}$

Given data:

$$\nabla(s) = s^4 + 3s^3 + 3s^2 + s + k = 0$$

s^4	1	3	K
s^3	3	1	0
s^2	8/3	k	0
s	(8/3-3K)/(8/3)	0	0
s^0	k	0	0

$$\Rightarrow \frac{\frac{8}{3} - 3k}{\frac{8}{3}} > 0$$

$$\Rightarrow \frac{8}{3} - 3k > 0$$

$\Rightarrow 3k < \frac{8}{9}$

$\Rightarrow (0 < k < \frac{8}{9})$

.

for example the zeros of polynomial $s^4 + 3s^3 + 3s^2 + s + 0.5 = 0 are$

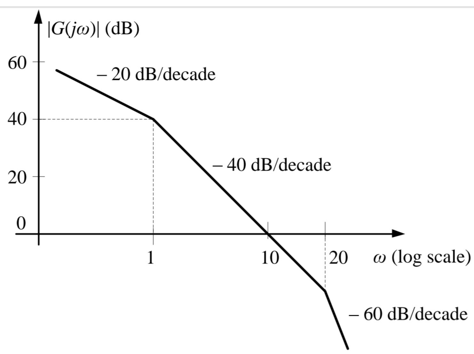
$s1 = 0.08373 + 0.45773i$

$$s2 = 0.083730.45773i$$

$$s3 = 1.41627 + 0.55075i$$

$$s4 = 1.416270.55075i$$

Question-9 The asymptotic Bode magnitude plot of minimum phase transfer function G(s) is show below.



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Consider the following two statements.
Statement 1: Transfer function G(s) has 3 poles and one zero
Statement 2: At very high frequency ($\omega \to \infty$), the phase
angle $\angle G(j\omega) = -3\pi/2$
Which of the following is correct ?
(A) Statement 1 is true and Statement 2 is false.
(B) Statement 1 is false and Statement 2 is true.
(C) Both the statements are true.
(D) Both the statements are false.

Solution:- Since, each pole corresponds to -20 dB/decade and each zero corresponds to +20 dB/decade.
Therefore, from the given Bode plot we can get the Transfer equation,

$$G(s) = \frac{k}{s(1 + s)(20 + s)}$$

Now, from the Transfer equation we can conclude that, there are three poles (0, -1 and -20) and no zeros.
$\therefore$ Statement 1 is false ....(1)

. Calculating phase
Since we know that,
phase ϕ is the sum of all the phases corresponding to each pole and zero.
phase corresponding to pole is =

$$-tan^{-1}(\frac{imaginary}{real})$$

phase corresponding to zero is =

$$tan^{-1}(\frac{imaginary}{real})$$

now take,

$$s = j\omega$$

$$\Rightarrow G(j\omega) = \frac{k}{j\omega(1 + j\omega)(20 + j\omega)}$$

Therefore,

$$\phi = -tan^{-1}(\frac{\omega}{0}) - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20})$$

$$\phi = -90^\circ - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20})$$

$$\because \omega \to \infty$$

.

$$\phi = -90^\circ - 90^\circ - 90^\circ$$

$$\phi = -270^\circ$$

$$\phi = -3\pi/2$$

$\therefore$ Statement 2 is true ....(2)
thus, from (1) and (2) option (B) is correct.

Question-10 The transfer function of phase lead compensator is given by

$$D(s) = \frac{3(s+\frac{1}{3T})}{(s+\frac{1}{T})}$$

The frequency (in rad/sec), at which $\angle D(j\omega)$ is maximum, is

(a) $\sqrt{\frac{1}{T^2}}$ **(b)** $\sqrt{\frac{1}{3T^2}}$ **(c)** $\sqrt{3T}$ **(d)** $\sqrt{3T^2}$

Solution:- The basic requirement of the phase lead network is that all poles and zeros of the transfer function of the network must lie on (-)ve real axis interlacing each other with a zero located as the nearest point to origin.

The given transfer fuction is

$$D(s) = \frac{3(s+\frac{1}{3T})}{(s+\frac{1}{T})}$$

. Now substituting $s = j\omega$ in D(s), we get

$$D(j\omega) = \frac{3(j\omega+\frac{1}{3T})}{(j\omega+\frac{1}{T})}$$

The phase of this transfer function $\phi(\omega)$ is given by

$$\phi(\omega) = \tan^{-1}(3\omega T) - \tan^{-1}(\omega T)$$

$\phi(\omega)$ has its maximum at ω_c such that $\phi'(\omega_c) = 0$,

$$\phi'(\omega_c) = \frac{3T}{1+(3\omega_c T)^2} - \frac{T}{1+(\omega_c T)^2}$$

. After simplification,

$$\omega_c^2 T^2 = \frac{1}{3}$$

$$\omega_c = \sqrt{\frac{1}{3T^2}}$$

Hence **(b)** is the correct option.

Question-11

Consider a state-variable model of a system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where y is the output, and r is the input. The damping ratio ζ and the undamped natural frequency ω_n (rad/sec) of the system are given by:

(A) $\zeta = \frac{\beta}{\sqrt{\alpha}}, \omega_n = \sqrt{\alpha}$

(B) $\zeta = \sqrt{\alpha}, \omega_n = \frac{\beta}{\sqrt{\alpha}}$

(C) $\zeta = \frac{\alpha}{\sqrt{\beta}}, \omega_n = \sqrt{\beta}$

(D) $\zeta = \sqrt{\beta}, \omega_n = \sqrt{\alpha}$

Solution:- Transformation of State Equations to a Single Differential Equation

The state equations $\dot{x}$ =Ax+Br for a linear second-order system with a single input are a pair of coupled first-order differential equations in the two state variables: $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} r.$

Or

$$\frac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_1 r.$$

$$\frac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_2 r.$$

The state-space system representation may be transformed into a single differential equation ineither of the two state-variables.

.Transformation of State Equations to a Single Differential Equation

Taking the Laplace transform of the state equations

$$(sI - A)X(s) = BR(s)$$

$$X(s) = (sI - A)^{-1}BR(s)$$

$$X(s) = \frac{1}{det[sI-A]} \begin{bmatrix} s-a_{22} & a_{12} \\ a_{21} & s-a_{11} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} R(s)$$

$$det[sI-A]X(s) = \begin{bmatrix} s-a_{22} & a_{12} \\ a_{21} & s-a_{11} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} R(s)$$

.Transformation of State Equations to a Single Differential Equation

from this we can write

$$\Rightarrow \frac{d^2x_1}{dt^2} - (a_{11}+a_{22})\frac{dx_1}{dt} + (a_{11}a_{22} - a_{12}a_{21})x_1$$
$$= b_1\frac{du}{dt} + (a_{12}b_2 - a_{22}b_1)r.$$

and

$$\Rightarrow \frac{d^2x_2}{dt^2} - (a_{11}+a_{22})\frac{dx_2}{dt} + (a_{11}a_{22} - a_{12}a_{21})x_2$$
$$= b_2\frac{du}{dt} + (a_{21}b_1 - a_{11}b_2)r.$$

which can be written in terms of the two parameters ω_n and ζ as follows:

.Transformation of State Equations to a Single

Differential Equation

$$\frac{d^2x_1}{dt^2}+2\zeta\omega_n\frac{dx_1}{dt}+\omega_n^2x_1 = b_1\frac{du}{dt}+(a_{12}b_2-a_{22}b_1)r.$$

and

$$\frac{d^2x_2}{dt^2}+2\zeta\omega_n\frac{dx_2}{dt}+\omega_n^2x_1 = b_2\frac{du}{dt}+(a_{21}b_1-a_{11}b_2)r.$$

where ζ is the system(dimensionless) damping ratio and the undamped natural frequency with units of radian/second is ω_n.

By comparing above equations we get that:

$$\omega_n = \sqrt{a_{11}a_{22} - a_{12}a_{21}}$$

and

$$\zeta = -\frac{(a_{11}+a_{22})}{\omega_n} = \frac{-(a_{11}+a_{22})}{2\sqrt{a_{11}a_{22}-a_{12}a_{21}}}$$

.Transformation of State Equations to a Single Differential Equation

Now putting the give values in the variables

$$\zeta = \frac{-(0-2\beta)}{2\sqrt{0(-2\beta)-1(-\alpha)}} = \frac{\beta}{\sqrt{\alpha}}$$

we get, $\omega_n = \sqrt{0(-2\beta)-1(-\alpha)} = \sqrt{\alpha}$

So the answer is Option(A).

Question-12 Match the transfer functions of the second-order systems with the nature of the systems given below

Transfer functions	Systems
P : $\frac{15}{s^2+5s+15}$	1:Overdamped
Q : $\frac{25}{s^2+10s+25}$	2:critically damped
R : $\frac{35}{s^2+18s+35}$	3:Underdamped

(A)P-1,Q-2,R-3
(B)P-2,Q-1,R-3
(C)P-3,Q-2,R-1
(D)P-3,Q-1,R-2

Solution:-

The standard transfer function H(s)=$\frac{\omega^2}{s^2+2\zeta\omega+\omega^2}$
where
"ω" is natural frequency
and"ζ" is damping factor

then compare the given functions with this we get

1. For Transfer function H(s)=$\frac{15}{s^2+5s+15}$,

$$\omega^2 = 15$$
$$2\zeta\omega = 5$$
$$\text{then we get } \zeta = \sqrt{\tfrac{5}{12}}<1$$ [

. 2. For Transfer function H(s)=$\frac{25}{s^2+10s+25}$,

$$\omega^2 = 25$$
$$2\zeta\omega = 10$$
$$\text{then we get } \zeta = \sqrt{\tfrac{5}{5}} = 1$$ [

3. For Transfer function H(s)=$\frac{35}{s^2+18s+35}$,

$$\omega^2 = 35$$
$$2\zeta\omega = 18$$
$$\text{then we get } \zeta = \sqrt{\tfrac{81}{35}}>1$$ [

.The damping of a system can be described as being one of the following:

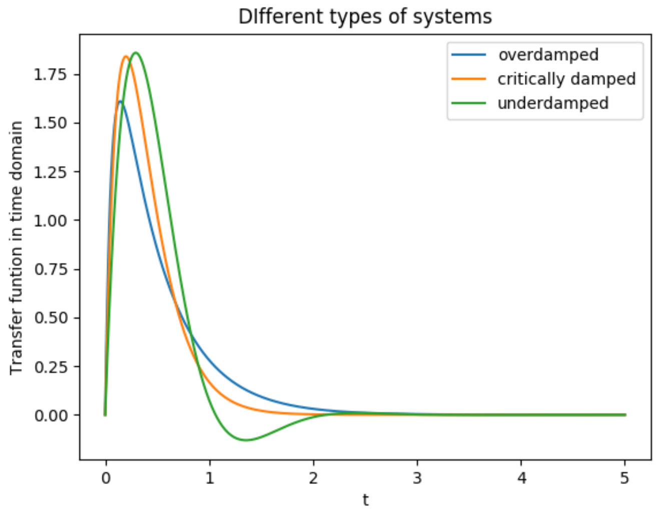
Overdamped The system returns to equilibrium without oscillating.For this ζ>1.

Critically damped The system returns to equilibrium as quickly as possible without oscillating.For this $\zeta = 1$

. Underdamped The system oscillates(at reduced frequency compared to the undamped case) with the amplitude gradually decreasing to zero.For this 0 <ζ<1

Undamped The system oscillates at its natural resonant frequency($\omega 0$).$Forthis\zeta = 0$

. As for P:ζ<1 It is Underdamped system
As for Q: $\zeta = 1$ It is critically damped system.
As for R: ζ>1 It is an overdamped system.
So,P-3,Q-2,R-1. Option (C) is correct. .



Question-13 The number of roots of the polynomial, $s^7+s^6+7s^5+14s^4+31s^3+73s^2+25s+200$, in the open left half of the complex plane is

(A) 3
(B) 4
(C) 5
(D) 6

Solution:-

We will be using the concept of Routh-Hurwitz Criterion.

Routh-Hurwitz Criterion: The number of roots of the polynomial that are in the right half-plane is equal to the number of sign changes in the first column.

- RouthHurwitz stability criterion is a mathematical test that is a necessary and sufficient condition for the stability of a linear time invariant control system.

.

Rules for generating Routh-Hurwitz Table.

1) Label the rows of Routh table from highest power to the lowest power.
2) List alternative coefficients starting with the highest order coefficients in the first row.
3) List alternative coefficients starting with the next highest order coefficients in the second row.
4) Each entry is the negative of determinant of the previous two entries in the previous two rows divided by the entry in the first column directly above the row.
5) The left hand column of the determinant is always the first column of the previous two rows.
6) The right hand column is the elements of the column above and to the right.
7) The table is complete when all of the rows are completed down to s^0.

. The Routh-Hurwitz Table for given equation $s^7+s^6+7s^5+14s^4+31s^3+73s^2+25s+$ 200, is calculated as follows

s^7	1	7	31	25
s^6	1	14	73	200

s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0

. .

s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0
s^4	8	48	200	0

s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0
s^4	8	48	200	0
s^3	0	0	0	

When such a case is encountered, we take the derivative of the expression formed the the coefficients above it i.e derivative of $8s^4+48s^2+200$.

$$\frac{d}{dx}(8s^4+48s^2+200)=32s^3+96s$$

The coefficients of obtained expression are placed in the table. . .

.

s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0
s^4	8	48	200	0
s^3	32	96	0	

s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0
s^4	8	48	200	0
s^3	32	96	0	
s^2	24	200	0	

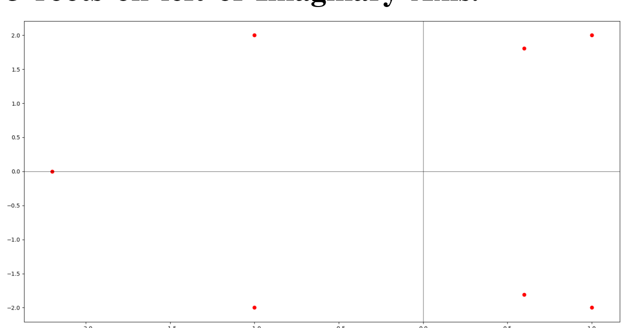
s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0
s^4	8	48	200	0
s^3	32	96	0	
s^2	24	200	0	
s^1	-170.67	0		

s^7	1	7	31	25
s^6	1	14	73	200
s^5	-7	-42	-175	0
s^4	8	48	200	0
s^3	32	96	0	
s^2	24	200	0	
s^1	-170.67	0		
s^0	200			

So, the above one is the Routh-Hurwitz Table. The no.of sign changes in first column of RouthHurwitz Table is the no.of roots on right side of imaginary axis.

So, for the given equation 4 roots lie on right-side of Imaginary Axis.

Given equation has a total of 7 roots in which 4 lie on right side of Imaginary Axis. **So there will be 3 roots on left of Imaginary Axis.**

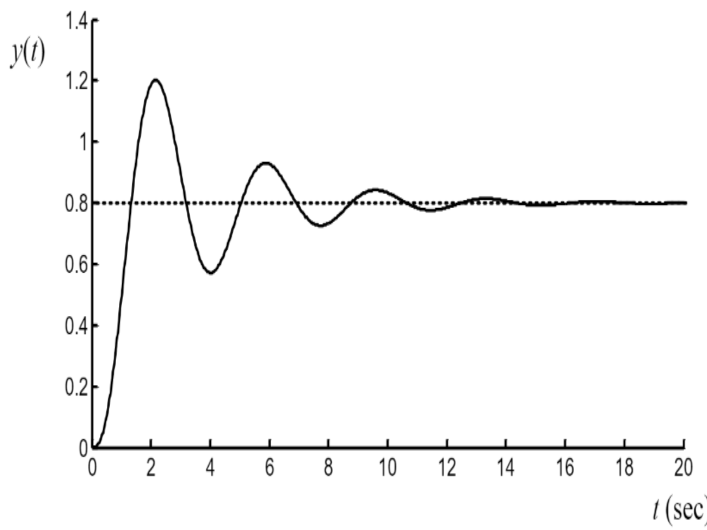


.

Question-14 The unit step response of y(t) of a unity feedback system with an open loop transfer function

$$G(s)H(S) = \frac{K}{(s+1)^2(s+2)}$$

is shown in figure. The value of K is(up to two decimal places). .



Solution:- Given,

$$G(s)H(s) = \frac{K}{(s+1)^2(s+2)}$$

We know that the input and output relation for open loop transfer function for a unity feedback system is given by,

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{G(s)}{1+G(s)}$$

Now, substituting the G(s)H(s) in the above equation we will get the below,

$$\frac{Y(s)}{X(s)} = \frac{\frac{K}{(s+1)^2(s+2)}}{1+\frac{K}{(s+1)^2(s+2)}}$$

.

$$\frac{Y(s)}{X(s)} = \frac{K}{K+(s+1)^2(s+2)}$$

According to the question,

$$X(s) = \frac{1}{s}$$

So,

$$Y(s) = \frac{1}{s}\ \frac{K}{K + (s+1)^2(s+2)}$$

From final value theorem,

$$y(\infty\ ) = \lim_{s\to 0} sY(s) = 0.8$$

[From the response shown in the figure steady state value in the time domain is 0.8] .

$$\frac{K}{K+2} = 0.8$$

$$K = 1.6 + 0.8K$$

$$K = 8$$

Hence,the value of K is 8

$\boxed{\text{Question-15}}$ An input p(t) = sin(t) is applied to the system $G(s) = \frac{s-1}{s+1}$. The corresponding steady state output is y(t) = sin(t + φ), where the phase φ (in degrees), when restricted to $0° \le \varphi \le 360°$, is ?

$\boxed{\text{Solution:-}}$

We have p(t) = sin(t)

We know that Laplace Transform of p(t) = $\mathcal{L}$(p(t)) = P(s)

So , P(s) = $\frac{1}{s^2+1}$

And we are given the steady state output $y_s(t)$ = sin(t+φ)

So , $y_s(t) = sin(t)cos(\varphi)$ + cos(t)sin(φ) ,

Hence Laplace transform of y(t) in steady state = $\mathcal{L}(y_s(t)) = Y_s(s) = \mathcal{L}(sin(t))cos(\varphi) + \mathcal{L}(\cos(t))\sin(\varphi)$

. Since , $\mathcal{L}$(sin(t)) = $\frac{1}{s^2+1}$ And $\mathcal{L}$(cos(t)) = $\frac{s}{s^2+1}$

So , $Y_s(s) = \frac{cos\varphi + s(sin\varphi)}{s^2+1}$

And G(s) = $\frac{s-1}{s+1}$

Hence , the output of the system in s-domain is Y(s) = P(s)G(s)

So , Y(s) = $\frac{1}{s^2+1}$. $\frac{s-1}{s+1}$, For solving this we can use the partial fractions :

.

Y(s) = $\frac{As+B}{s^2+1}$ + $\frac{C}{s+1}$ = $\frac{(A+C)s^2+(A+B)s+(B+C)}{(s^2+1)(s+1)}$

Hence , by comparing coefficients ,

we get A+C = 0 , A+B = 1 , B+C = -1

After solving these above equations , we get A = 1 , B = 0 , C = -1.

So , Y(s) = $\frac{s}{s^2+1}$ - $\frac{1}{s+1}$

Now we know that Laplace transform of $e^{-t}u(t)$ = $\mathcal{L}(e^{-t}u(t)) = \frac{1}{s+1}$

. i.e $\mathcal{L}^{-1}(\frac{1}{s+1}) = e^{-1}u(t)$

As for steady state analysis , we put t $\to \infty$,therefore $e^{-1}u(t)$ will disappear while taking inverse laplace transform . Hence in steady state , only $\frac{s}{s^2+1}$ term will appear in laplace transform of y(t) as t $\to \infty$

Hence , in steady state , Y(s) = $\frac{s}{s^2+1}$ = $Y_s(s)$ given previously.

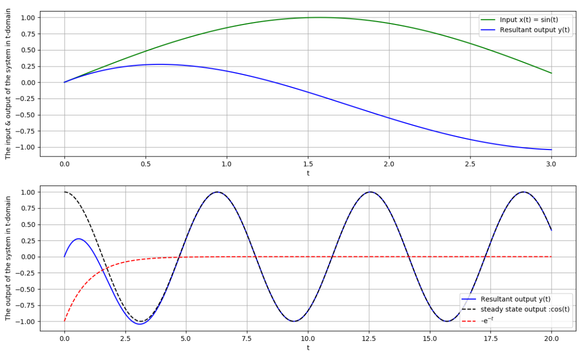
So , $\frac{s}{s^2+1}$ = $\frac{cos(\varphi)+s(cos(\varphi))}{s^2+1}$

By comparing coefficients of s and constants , we get $cos(\varphi)$ = 0 and $sin(\varphi)$ = 1

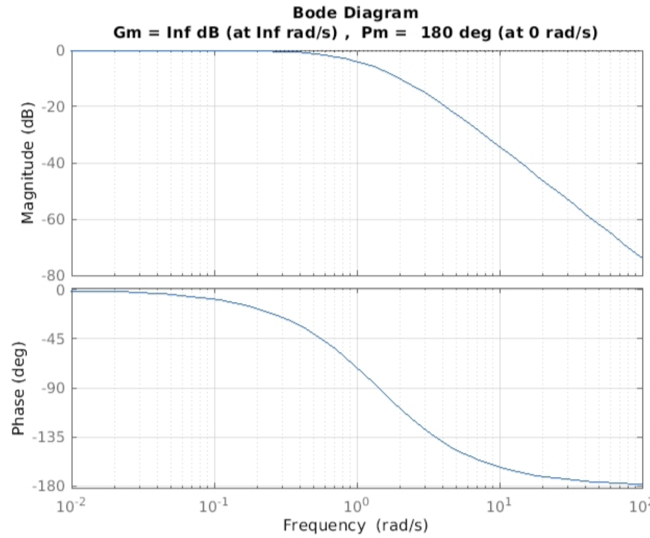
So, because $0° \le \varphi \le 360°$, therefore $\varphi = 90°$

.

Plot obtained for verification in python :



. In above plot , black color plot is of cos(t).And blue color plot is the plot of resultant y(t) .So , we can see from above plots that black and blue color plots are coinciding after t = 3 . Hence y(t) = sin(t + 90°) in steady state.



Question-16 Consider the transfer function $G(s) = \frac{2}{(s+1)(s+2)}$. The Phase Margin of $G(s)$ in degrees is ________

Solution:- **Gain Margin:**The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.
Gain Margin = $\frac{1}{|G(j\omega)|}$ at $\omega = \omega_{pc}$
ω_{pc} =phase crossover frequency (The frequency at which at which phase becomes -180°
Phase Margin: Phase margin refers to the amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees. .
Phase margin =$\phi - \angle(G(j\omega)|_{\omega=\omega_{pc}}$ =180°+ϕ
Where , $\phi = \angle G(j\omega)_{\omega=\omega_{gc}}$
ω_{gc} = The Gain crossover frequency (frequency where Gain becomes 0)
Given, $G(s) = \frac{2}{(s+1)(s+2)}$

$\implies G(j\omega) = \frac{2}{(j\omega+1)(j\omega+2)}$

$\implies |G(j\omega)| = \frac{2}{(\sqrt{\omega^2+1})(\sqrt{\omega^2+4})}$

$\implies \angle G(j\omega) = -tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{2})$.

To find find gain margin we need find $\angle G(j\omega)$

$\angle(G(j\omega)|_{\omega=\omega_{pc}} = -180°$=-tan$^{-1}(\omega) - tan^{-1}(\frac{\omega}{2})$

$\implies \omega_{pc} = \infty \implies$ Gain margin = ∞
We have to find phase margin, which is calculated over the gain cross over frequency (ω_{gc})
To find ω_{gc},
We know , Gain=0 at $\omega = \omega_{gc}$

$\implies log_{10}|G(j\omega)| = 0$ at $\omega = \omega_{gc}$

$\implies |G(j\omega_{gc})| = 1$.
So, $\frac{2}{(\sqrt{\omega_{gc}^2+1})(\sqrt{\omega_{gc}^2+4})} = 1$

$\implies (\omega_{gc}^2 + 1)(\omega_{gc}^2 + 4) = 4$

$\implies \omega_{gc}^4 + 5\omega_{gc}^2 + 4 = 4$

$\implies \omega_{gc}^2(\omega_{gc}^2 + 5) = 0$

$\therefore \omega_{gc} = 0, +j\sqrt{5}, -j\sqrt{5}$.

As frequency is a real quantity
Hence, $\omega_{gc} \neq$ Imaginary
So, $\omega_{gc} = 0$

$\therefore \angle G(j\omega_{gc}) = -tan^{-1}(0) - tan^{-1}(0) = 0$

$\implies \phi = 0°$
$\therefore PhaseMargin = 180°+0°$

$\therefore$ **Phase Margin = 180°** . we can verify the phase margin by bode plot

$\boxed{\text{Question-17}}$ Consider the linear system $\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$x , with initial condition x(0) = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The solution x(t) for this system is:

(A) x(t) = $\begin{bmatrix} e^{-t} & te^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(B) x(t) = $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(C) x(t) = $\begin{bmatrix} e^{-t} & t^2e^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(D) x(t) = $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\boxed{\text{Solution:-}}$ It is of the form $\dot{x} = Ax$. Therefore its solution is

$$x(t) = e^{At}x(0)$$

e^{At} is my state transition matrix and is equal to $\mathcal{L}^{-1}$[sI - A]$^{-1}$ (derivation in last slide)

A = $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ ∴ [sI - A] = $\begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$

$$\rightarrow Adj(sI - A) = \begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}$$

$\rightarrow det(sI - A) = (s+1)(s+2)$. ∴ [sI - A]$^{-1}$ = $\frac{Adj(A)}{det(A)} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$

$$e^{At} = \begin{bmatrix} \mathcal{L}^{-1}[\frac{1}{s+1}] & 0 \\ 0 & \mathcal{L}^{-1}[\frac{1}{s+2}] \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$$

$$\therefore x(t) = e^{At}x(0) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\rightarrow$ *OPTION (D)* . Derivation of state transition matrix $\rightarrow \dot{x} = Ax$

$$\xrightarrow{TakingLaplacian}$$

$$\rightarrow S.X(s) - x(0) = AX(s)$$

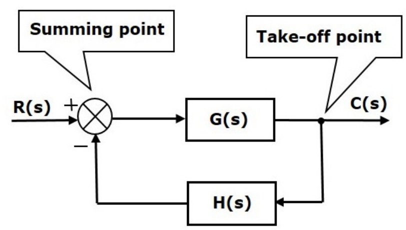
$$\therefore X(s) = [SI - A]^{-1}x(0)$$

$$X(t) = L^{-1}[SI - A]^{-1}x(0)$$

$\rightarrow$ *Here*, $\mathcal{L}^{-1}$[SI -A]$^{-1}$ is called the state transition matrix

$\boxed{\text{Question.18}}$ Consider a standard negative feedback configuration with G(s)=$\frac{1}{(s+1)(s+2)}$ and H(s)=$\frac{s+\alpha}{s}$ the closed loop system to have poles on the imaginary axis, the value of α should be equal to (up to one decimal place

$\boxed{\text{Solution:-}}$



. Negative feedback system with G(s)=$\frac{1}{(s+1)(s+2)}$,H(s)=$\frac{s+\alpha}{s}$

transfer function for negative feedback system $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

$$\frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{(s+1)(s+2)}}{1+\frac{1}{(s+1)(s+2)}\frac{s+\alpha}{s}}$$

transfer function = $\frac{s}{(s^3+3s^2+3s+\alpha)}$

. we have to find value of α for which the poles of the system will be lying on imaginary axis. for determining the location of closed loop poles

$s^3 + 3s^2 + 3s + \alpha = 0$

.

Hence we can form the Rouths array using characteristic polynomial

P(s)=$s^3 + 3s^2 + 3s + \alpha$

s^3	1	3	0
s^2	3	α	0

$$\begin{array}{llll} s^1 & \frac{(9-\alpha)}{3} & 0 & 0 \\ s^0 & \alpha & 0 & 0 \end{array}$$

. If closed loop poles will be lying on imaginary axis, then the system will be marginally stable and the conditions is satisfied by Rouths array, if any complete row (except last) become zero. Having seen the first column of Rouths array, poles will on imaginary axis, when

$$\frac{(9-\alpha)}{3} = 0$$

Hence, $\alpha = 9$

now put α=9 in characteristic equation now characteristic equation is P(s)=$s^3 + 3s^2 + 3s + 9$

poles are s= -3,j $\sqrt{3}$,-j $\sqrt{3}$

$\boxed{\text{Question-19}}$ Unit Step response of a linear time invariant (LTI) system is given by $y(t) = (1 - e^{-2t})u(t)$. Assuming zero initial condition, the transfer function of the system is

- (A) $\frac{1}{s+1}$
- (B) $\frac{2}{(s+1)(s+2)}$
- (C) $\frac{1}{s+2}$
- (D) $\frac{2}{s+2}$

$\boxed{\text{Solution:-}}$ Unit step response in time domain is

$$y(t) = (1 - e^{-2t})u(t)$$

We can convert this step response into s-domain using the Laplace Transform

$$Y(s) = \mathcal{L}(y(t))$$

$$where \quad \mathcal{L}(y(t)) = \int_{-\infty}^{\infty} y(t)e^{-st}dt$$

. From the properties of Laplace Transform we know that:

- $\mathcal{L}(ax(t) + by(t)) = a\mathcal{L}(x(t)) + b\mathcal{L}(y(t))$
- $\mathcal{L}(u(t)) = \frac{1}{s}$

where

$$u(t) = \begin{cases} 1 & \text{if t} \geq 0 \\ 0 & \text{if t ¡ 0} \end{cases}$$

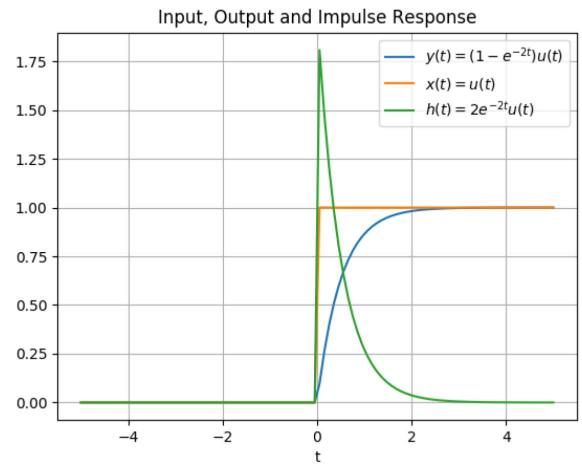


Fig. 7: All Functions Plot

- $\mathcal{L}(e^{-at}u(t)) = \frac{1}{s+a}$

.

$Y(s) = \mathcal{L}(y(t))$

$= \mathcal{L}(u(t) - e^{-2t}u(t))$

$= \mathcal{L}(u(t)) - \mathcal{L}(e^{-2t}u(t))$

$= \frac{1}{s} - \frac{1}{s+2}$

$= \frac{2}{s(s+2)}$

$x(t) = u(t)$ $\because$Input is the Unit step function

$\implies X(s) = \frac{1}{s}$

.

The Transfer Function $H(s)$ is given by

$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{2}{s(s+2)}}{\frac{1}{s}} = \frac{2}{s+2}$

Hence the Transfer Function $H(s)$ is $\frac{2}{s+2}$ Option(D)

. $\boxed{\text{Question-20}}$ The Nyquist stability criterion and the Routh criterion both are powerful analysis

tools for determining the stability of feedback controllers. Identify which of the following statements is FALSE:
(A) Both the criteria provide information relative to the stable gain range of the system.
(B) The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum-phase systems.
(C) The Routh criterion is not applicable in the condition of transport lag, which can be readily handled by the Nyquist criterion.
(D) The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

Solution:-

The answer is option:(D)
The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

Option(A) Both the criteria provide information relative to the stable gain range of the system is true.
Option(B) It's true because as in a minimum-phase system, Bode magnitude plot is enough to obtain a general approximation of its Nyquist plot.

Option(C) Routh criterion can be applied to any system to check the stability of a system but a transport lag controller can only by explained using Nyquist Criterion.
Option(D) We can obtain closed-loop frequency response for Unity Feedback system easily by substituting s = j, and draw the plot for different values of . Usually this is not done as it is not necessary as OLTF is enough to comment on the stability. Thus, (D) is false.

Question-21 The state equation and the output equation of a control system are given below :

$$\dot{X} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} X + \begin{bmatrix} 4 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} X$$

Then transfer function representation of the system is

(A) $\frac{(3s+5)}{(s^2+4s+6)}$ (B) $\frac{(3s-1.875)}{(s^2+4s+6)}$

(C) $\frac{(4s+1.5)}{(s^2+4s+6)}$ (D) $\frac{(6s+5)}{(s^2+4s+6)}$

Solution:- From the given state space representation of the system, we can find matrices as

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix}$$

when

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

where A, B, C, D are matrices
Then the transfer function can be find using

$$T(s) = C[(sI - A)^{-1}].B + D$$

We can find the transfer function using

$$T(s) = C[(sI - A)^{-1}].B \tag{1}$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s+4 & -1.5 \\ -4 & s \end{bmatrix} \tag{2}$$

$$|sI - A| = s(s+4) - (-4) \times (-1.5)$$

$$= s^2 + 4s + 6 \tag{3}$$

$$Adj[sI - A] = \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

Hence

$$[sI - A]^{-1} = \frac{Adj[sI - A]}{|sI - A|} = \begin{bmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{(s+4)}{(s^2+4s+6)} \end{bmatrix} \tag{4}$$

$$[sI-A]^{-1}.B = \begin{bmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{(s+4)}{(s^2+4s+6)} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix} \quad (5)$$

.

$$\therefore \quad [sI-A]^{-1}.B = \begin{bmatrix} \frac{2s}{(s^2+4s+6)} \\ \frac{8}{(s^2+4s+6)} \end{bmatrix} \quad (6)$$

Substituting values of [sI - A]$^{-1}$.B *and* C *in equation* (1)

$$T(s) = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} \begin{bmatrix} \frac{2s}{(s^2+4s+6)} \\ \frac{8}{(s^2+4s+6)} \end{bmatrix} \quad (7)$$

.

$$T(s) = \begin{bmatrix} \frac{3s}{(s^2+4s+6)} + \frac{5}{(s^2+4s+6)} \end{bmatrix}$$

the transfer function representation of the system is

$$T(s) = \begin{bmatrix} \frac{3s+5}{(s^2+4s+6)} \end{bmatrix}$$

$\boxed{\text{Question-22}}$ For a unity feedback control system with the forward path transfer function

$$G(s) = \frac{k}{s(s+2)}$$

The peak resonant magnitude M_r of the closed loop frequency is 2.The corresponding value of the gain K is

$\boxed{\text{Solution:-}}$

Given:

For a unity feedback control system $G(s) = \frac{k}{s(s+2)}$ and resonant peak M_r=2

1.

we can find its closed loop transfer function as,

$$C(s) = [R(S) - H(S)C(S)]G(S) \quad (5)$$

$$C(s) = R(S)G(S) - H(S)C(S)G(S) \quad (6)$$

$$C(S) + H(S)C(S)G(S) = R(S)G(S) \quad (7)$$

$$C(S)[1 + G(S)H(S)] = R(S)G(S \quad (8)$$

$$\frac{C(s)}{R(s)} = T(s) = \frac{G(s)}{1+G(s)H(s)} \quad (9)$$

$$T(s) = \frac{\frac{K}{s(s+2)}}{1+\frac{K}{s(s+2)*1}} = \frac{K}{s^2+2s+K} \quad (10)$$

.

standard equation of T(s) = $\dfrac{\omega_n^2}{s^2 + 2S\xi_n^{\omega} + \omega_n^2}$

formula for resonant peak as $M_r = \dfrac{1}{2\xi(1-(\xi)^2)^{\frac{1}{2}}}$

given resonant peak $M_r = 2$

$\dfrac{DCGain}{2\xi(1-(\xi)^2)^{\frac{1}{2}}} = 2(11)$

here DC gain is 1

squaring on both sides

$16\xi^2(\xi^2 - 1) = 1$

putting$\xi^2 = x$

$16x^2 - 16x + 1 = 0(12)$

$$x = \xi^2 = \frac{2-\sqrt{3}}{4} \quad (13)$$

$$x = \xi^2 = \frac{2+\sqrt{3}}{4} \quad (14)$$

characteristic equation is

$$s^2 + 2s + k$$

comparing it with standard equation we get ${\omega_n}^2 = k$

$2\xi\omega_n$=2

$$\xi = \frac{1}{\omega_n} = \frac{1}{k^{\frac{1}{2}}}$$

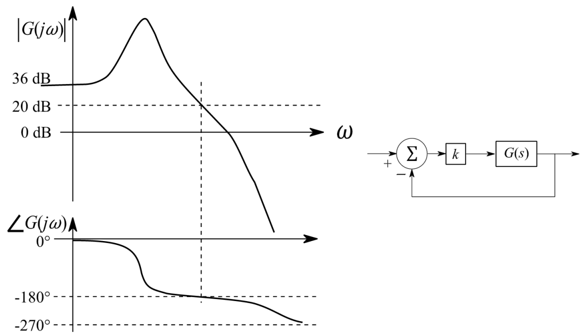
$$k = \frac{1}{\xi^2} = \frac{4}{2-\sqrt{3}} = 14.92$$

$\boxed{\text{Question-23}}$ The figure below shows the Bode magnitude and phase plots of a stable transfer function

$$G(s) = \frac{n_0}{s^3 + d_2 s^2 + d_1 s + d} \quad (15)$$

Consider the negative unity feedback configuration with gain k in the feedforward path. The closed loop is stable for $k < k_o$.

$The maximum value of k_o \ is$



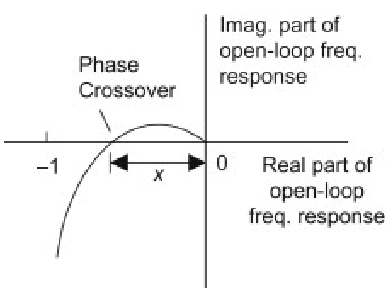
Solution:-

For a stable system, Gain margin at the phase cross-over frequency $> 0dB$.

Phase crossover frequency (ω_{pc}) The phase crossover frequency is the frequency at which the phase angle first reaches -180 °.

This is the factor by which the gain must be multiplied at the phase crossover to have the value 1.

The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable.



- The above shows nyquist plot of a stable transfer function.
- The phase crossover frequency is the frequency at which the phase angle first reaches -180°and thus is the point where the Nyquist plot crosses the real axis.

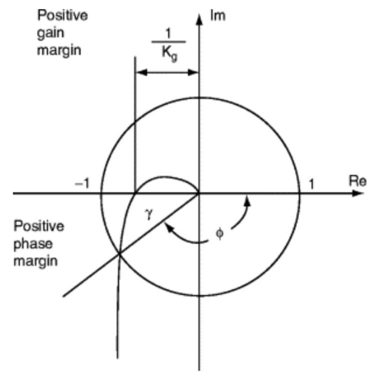
The gain margin is defined as

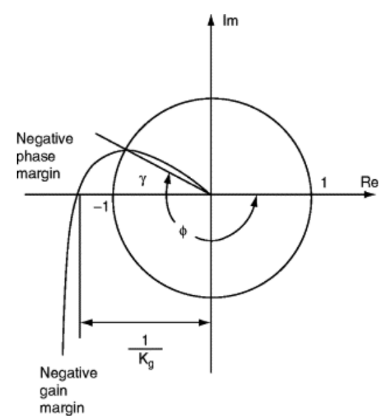
$$K_g = \frac{1}{|G(j\omega)|}$$

at the frequency at which the phase angle is -180°.

In terms of decibels:

$$K_g dB = -20log(|G(j\omega)|)dB$$





For a stable system, Gain margin at the phase cross-over frequency > 1.

G(s) is cascaded with k, so,

$$G_1(s) = kG(s)$$

$$K_g = \frac{1}{|G_1(j\omega_{pc})|} > 1$$

$$\implies K_{g(dB)} = -20log(|G_1(j\omega_{pc})|) > 0dB$$

$$\implies -20log(|G(j\omega_{pc})k|) > 0dB$$

$$\implies -20 - 20log(|k|) > 0dB$$

$$\implies 20log(k) < -20$$

$$\implies k < 10^{-1}$$

$$\implies k_{max} = 0.1$$

$\therefore k_o = 0.1$

$\boxed{\text{Question-24}}$. Consider the state space realization :

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

with the initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

, where u(t) denotes unit step function

The value of $\lim_{t\to\infty} |\sqrt{x_1^2(t) + x_2^2(t)}|$ **is ?**

$\boxed{\text{Solution:-}}$ **State Space Reprensentation** : A state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations.

For Linear systems :

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t)$$

where $\dot{\mathbf{x}}(t) := \frac{d}{dt}\mathbf{x}(t)$

$$\mathscr{L}\{x(t)\} = X(s)$$

$$\mathscr{L}\{\dot{x(t)}\} = sX(s) - x(0)$$

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

By applying Laplace transform on both sides, we get

$sX_1(s) - x_1(0) = 0X_1(s) = \frac{x_1(0)}{s} = 0 \qquad \because x_1(0) = 0$

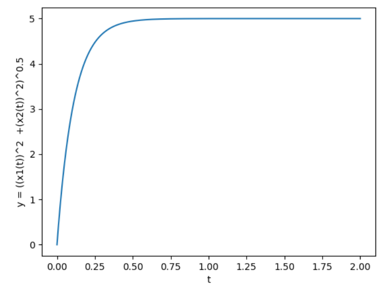
So, $\quad x_1(t) = 0$

and $\quad sX_2(s) - x_2(0) = -9X_1(s) + \frac{45}{s}$

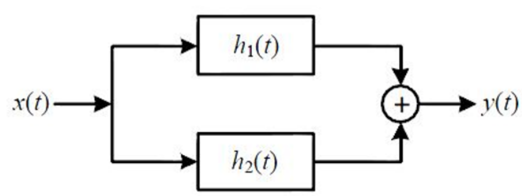
$$\text{Required value} = \lim_{t\to\infty} |\sqrt{x_1^2(t) + x_2^2(t)}| = \left|\lim_{t\to\infty} x_2(t)\right|$$

$\lim_{t\to\infty} x_2(t) = \lim_{s\to 0} sX_2(s) = \frac{45}{9} = 5$

So Required value = |5| = 5

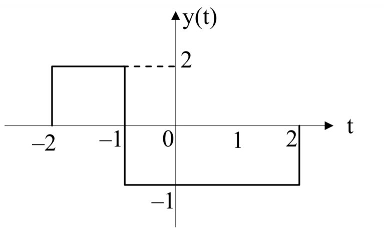


$\boxed{\text{Question-25}}$ Consider the parallel combination of two LTI systems shown in the figure. The impulse



response of the system are:
$h_1(t) = 2\delta(t+2) - 3\delta(t+1)$
$h_2(t) = \delta(t-2)$
If the input x(t) is a unit step signal,then the energy of y(t) is

$\boxed{\text{Solution:-}}$
Output of the system is found by convolving input x(t) with system's response h(t).
y(t)=x(t)*h(t)
Since $h_1(t)$ and $h_2(t)$ are connected in parallel the resultant system will be:
h(t)=[$h_1(t)+h_2(t)$] .
Given that x(t) is a unit step signal
$h_1(t)+h_2(t)=[2\delta(t+2) - 3\delta(t+1) + \delta(t-2)]$
y(t)=2u(t+2)-3u(t+1)+u(t-2)
Energy of the given is



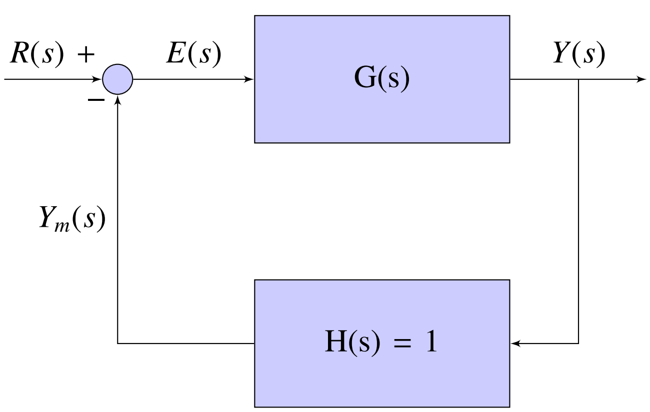
y(t)=$\int_{-2}^{-1}(2)^2dx + \int_{-1}^{2}(-1)^2dx$
=4[1]+1[3]
=7

Question-26 A unity feedback control system is characterised by the open-loop transfer function

$$G(S) = \frac{2(s+1)}{s^3 + ks^2 + 2s + 1}$$

the value of the k for which the system oscillates at 2 rad/s

.



Solution:-

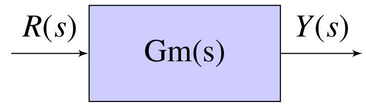
$$E(s) = R(S) - Y_m(s)$$

$$Y_m(s) = H(s)Y(s)$$

$$G(S) = \frac{Y(s)}{E(s)}$$

$$G(S) = \frac{Y(s)}{R(s) - H(s)Y(s)}$$

$$Gm(S) = \frac{Y(S)}{R(s)} = \frac{G(s)}{1 + H(s)G(s)}$$



.

Routh's Stability Condition
If the closed-loop transfer function has all poles in the left half of the s-plane,the system is stable.Thus,a system is stable if there are no sign changes in the first column of the Routh table. The Routh - Hurwitz criterion declares that the number of roots of the polynomial that are lies in the right half-plane is equal to the number of sign changes in the first column of Routh's array .Hence the system is unstable if the poles lies on the right hand side of the s-plane.

. Characteristic equation :

$$1 + G(s)H(s) = 0$$

$$H(s) = 1$$

$$1 + G(s) = 0$$

$$1 + \frac{2(s+1)}{s^3 + ks^2 + 2s + 1} = 0$$

$$\frac{s^3 + ks^2 + 4s + 3}{s^3 + ks^2 + 2s + 1} = 0$$

$$s^3 + ks^2 + 4s + 3 = 0$$

. Characteristic Equation :

$$s^3 + ks^2 + 4s + 3 = 0$$

Develop Routh's Array :

$$\begin{array}{c|cc} S^3 & 1 & 4 \\ S^2 & k & 3 \\ S & \frac{\begin{vmatrix} 1 & 4 \\ k & 3 \end{vmatrix}}{k} = \frac{3-4k}{k} & 0 \end{array}$$

. Given that ,System oscillates at a frequency 2 rad/s

- If all the coefficients in a row are zero, then auxiliary polynomial has pair of roots of equal magnitude and opposite sign is indicated. These could be two real roots with equal magnitudes and opposite signs or two conjugate imaginary roots.

- The auxiliary polynomial, is obtained from the values in the row above the zero row.
- Auxiliary polynomial is always even degree.

.

$$\frac{3-4k}{k} = 0$$

$$k = \frac{3}{4}$$

Auxilary polynomial

$$ks^2 + 3 = 0$$

$$\frac{3}{4}s^2 + 3 = 0$$

$$s^2 + 4 = 0$$

$$s = \pm 2j$$

Magnitude is 2 rad/sec

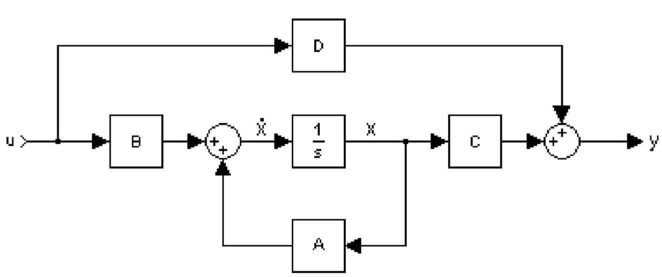
$\boxed{\text{Question-27}}$ A second-order LTI system is described by the following state equations:

- $\frac{\partial x_1(t)}{\partial t} - x_2(t) = 0$
- $\frac{\partial x_2(t)}{\partial t} + 2x_1(t) + 3x_2(t) = r(t)$

where $x_1(t)$ and $x_2(t)$ are the two state variables and $r(t)$ denotes the input. The output $c(t) = x_1(t)$. Identify the type of system.

$\boxed{\text{Solution:-}}$

- The corresponding state equations:
  1) $\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}r$
  2) $[c] = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$
- The state space model of a LTI system is:
  1) State equation: $\dot{X} = AX + BU$
  2) Output equation: $Y = CX + DU$



.

- Transfer Function from State Space model: $TF : H(s) = C[sI-A]^{-1}B+D = C\frac{Adj[sI-A]}{|sI-A|}B+D$
- $H(s) = \dfrac{\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}s+3 & 1\\ -2 & s\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}}{s(s+3)+2} = \frac{1}{s^2+3s+2}$
- Therefore the poles of the transfer function are: s = -1 and s = -2

.

- From first equation:

$$\frac{\partial x_1(t)}{\partial t} = x_2(t)$$

- Substitution in second equation results into the equation: $\frac{\partial^2 x_1}{\partial t^2} + 3\frac{\partial x_1(t)}{\partial t} + 2x_1(t) = r(t)$
- Taking Laplace transform on both sides: $s^2X_1(s) + 3sX_1(s) + 2X_1(s) - sx_1(0) - x_1'(0) - 3x_1(0) = R(s)$
- $H(s) = \frac{X_1(s)}{R(s)} = \frac{1}{s^2+3s+2}$
- Therefore the poles of the transfer function are: s = -1 and s = -2

.

- Since the poles of the transfer function are real and distinct, the system is **OVERDAMPED**.
- Solution: $h(t) = L^{-1}(H(s)) = e^{-t} - e^{-2t}$

$\boxed{\text{Question-28}}$ The number of directions and encirclements around the point -1+j0 in the complex plane by the Nyquist plot of

$$G(s) = \frac{1-s}{4+2s}$$

- Zero
- One, Anti-Clock wise
- One, Clock wise
- Two, Clock wise

$\boxed{\text{Solution:-}}$

First,we need to draw the polar plot of given G(S). In the polar plot,substitute s = jω
$G(j\omega) = \frac{1-j\omega}{4+2j\omega}$
$\lim_{\omega\to\infty} G(j\omega) = \frac{1-j\omega}{4+2j\omega}$
$\lim_{\omega\to\infty} G(j\omega) = \frac{j\omega(\frac{1}{j\omega}-1)}{j\omega(\frac{4}{j\omega}+2)}$
$\lim_{\omega\to\infty} G(j\omega) = \frac{-1}{2}\angle 0$

which is equal to $\frac{1}{2}\angle -180$
. As the Magnitude is taken positive in Nyquist Plot.

Now substitute $\omega = 0$
$\lim_{\omega \to 0} G(j\omega) = \frac{1-j\omega}{4+2j\omega} = \frac{1}{4}\angle 0$

$$\angle Num(G(j\omega)) = \frac{-\omega}{1} = -\frac{\omega}{1}$$

$$\angle Den(G(j\omega)) = \frac{\omega}{2} = \frac{\omega}{2}$$

$\angle$G(jω) = $\angle Num(G(j\omega)) - \angle Den(G(j\omega))$

so from this at $\omega = 0$ $\angle$G(jω) = 0

and so from this at ω =∞ $\angle$G(jω) =-π

.

$$|(G(j\omega))| = \frac{\sqrt{1+\omega^2}}{\sqrt{16+4\omega^2}}$$

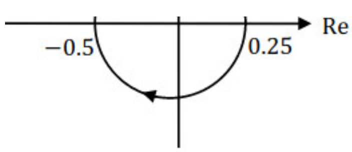
when $\omega = 0$ $|(G(j\omega))| = \frac{1}{4}$

when $\omega = \infty$ $|(G(j\omega))| = \frac{1}{2}$

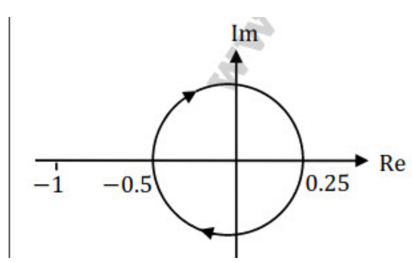
Hence,magnitude should be every time positive. So,we have to plot first 0.25 then we have to turn -180 degrees
to that point i.e 180 degrees clockwise(in this case)

.

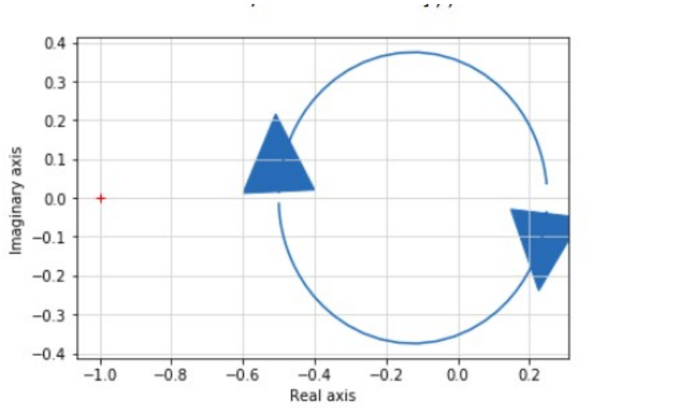
- Now Plot the Polar Plot 1 from ω=0 to ∞



- Draw the Mirror image of the Polar Plot 1.



- Find the points where $G(j\omega)$ intersects the real and imaginary axes(if needed) and then locate the given co-ordinate



.

.

$$Put\ s = Re^{j\theta}$$

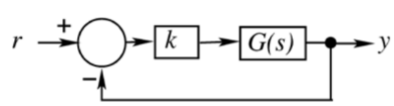
$$\lim_{R\to\infty} G(Re^{j\theta}) = \frac{1 - Re^{j\theta}}{4 + 2Re^{j\theta}} = \frac{-1}{2}$$

As there are no $e^{j\theta}$ terms,

There there will be no enclosed Nyquist path here. So, for this Transfer function $G(s)$,the Nyquist plot is same as the Polar plot.
. As from the observed plot the co-ordinate -1 + j0 is outside the contour

Hence,the number of encirclements around the the given co-ordinate is zero.

$\boxed{\text{Question-29}}$ In the feedback system given below G(s)= $\frac{1}{s^2+2s}$.
The step response of the closed-loop system should have minimum settling time and have no overshoot.



The required value of gain k to achieve this is __________

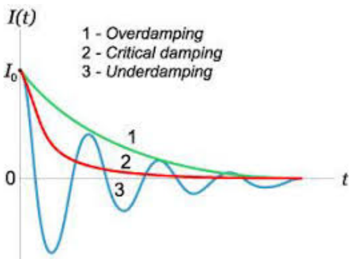
$\boxed{\text{Solution:-}}$ *SettlingTime*: The time required for the transient's damped oscillations to reach and stay within 2% of the steady-state value.

Overshoot:The amount that the waveform overshoots the steady state, or final, value at the peak time, expressed as a percentage of the steady-state value.

The Transfer function of the negative unity feedback system is given by $\frac{H(s)}{1+H(s)}$ (Where H(s) is the open-loop gain of the system)

.

In the given Question H(s) = k x G(s).So, Transfer function of the whole feedback system is $\frac{kG(s)}{1+kG(s)}$ By Substituting G(s) function we get $\frac{k}{s^2+2s+K}$



By observing the above figure,minimum settling time is obtained for Critical Damped System.

. Also, Critically Damped System doesn't have overshoot.
Transfer function of the Critical Damped System is given by $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ (Where, $\zeta = 1$)
$By comparing Obtained Transfer function \frac{k}{s^2+2s+K}$ and the general transfer function of Critical Damped System $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

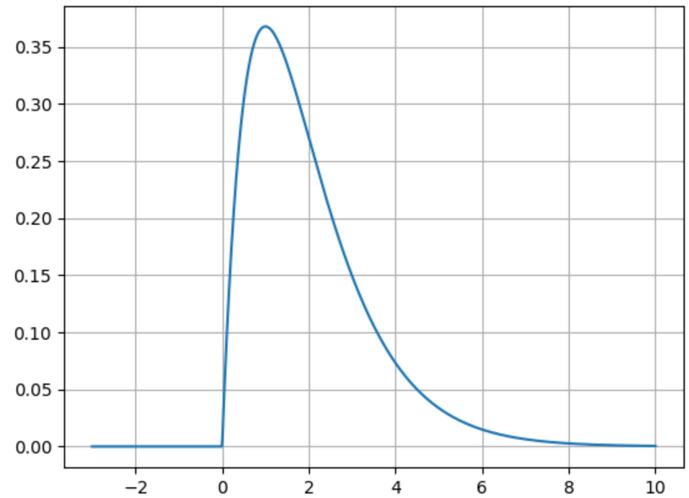
We get $\zeta = \frac{1}{\sqrt{K}}$
As $\zeta = 1$

$\frac{1}{\sqrt{K}}$ =1

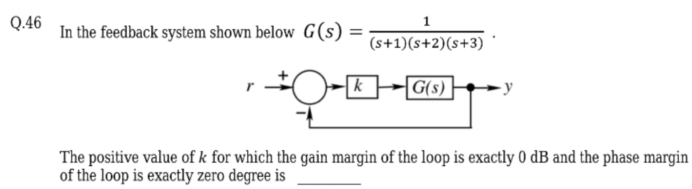
K=1

Therefore, The value of K is 1.

.



Question-30



Solution:-

The gain margin is defined as the change in open-loop gain required to make the closed-loop system unstable. Systems with greater gain margins can withstand greater changes in system parameters before becoming unstable in closed-loop.

The phase margin is defined as the change in open-loop phase shift required to make the closed-loop system unstable. The phase margin also measures the system's tolerance to time delay

.

As given in question we see that gain margin is 0 dB and phase margin is 0 degrees. This implies that system is just enough stable and will become destabilized on just small increase in gain. Hence the system is marginbally stable.

The stability of the system can be checked by Routh-Hurwitz Stability Criterion

.

s^n	a_0	a_2
$s^(n-1)$	a_1	a_3
$s^(n-2)$	2	0
$s^(n-3)$	0	0

s^3	1	11
s^2	6	(6+k)
s^1	$\frac{66-(6+K)}{6}$	0
s^0	$(6+k)$	0

Routh-Hurwitz Stability Criterion:-

Necessary condition for Routh-Hurwitz Stability Criterion: The necessary condition is that the coefficients of the characteristic polynomial should be positive. This implies that all the roots of the characteristic equation should have negative real parts.

Sufficient condition for Routh-Hurwitz Stability Criterion:The sufficient condition is that all the elements of the first column of the Routh array should have the same sign. This means that all the elements of the first column of the Routh array should be either positive or negative.

. The Routh array for characteristic equation $a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3}..... + a_n$

. The Routh array for equation $s^3 + 6s^2 + 11s^1 + (6+k)$ Now since the system is marginally stable therefore s^1 row is ≥ 0

Hence $\frac{66-(6+K)}{6} \geq 0$ Hence k=60