Physical Layer Implementation for Digital Video Broadcast

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CONTENTS

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***Abstract*—An end to end implementation of the physical layer for the Digital Video Broadcasting - Satellite - Second Generation (DVB-S2) standard is provided in this**

paper. This work builds upon various receiver design techniques available in the literature for carrier and frame synchronization and channel coding. Numerical results are provided based on the actual parameters given in the standard.

1. Introduction

2. Modulation and Demodulation

The baseband signal representaion for PSK in [1] can be expressed as

$$Y_k = X_k + V_k, \quad k = 1, \dots, N \tag{2.1}$$

The respective mapping and demapping expressions for upto 8-PSK are available in Table I

Constellation	N	Mapping	Demapping
BPSK	2	$X_k \in \left\{e^{\jmath \frac{2\pi n}{N}}\right\}$	$\frac{2\pi}{N} i < \angle Y_k < \frac{2\pi(i+1)}{N}$
QPSK	4		
8-PSK	8		

TABLE I: PSK for $N = 2, 4, 8$. $n = 0, 1, \dots, N-1$.

A. 16-APSK

Fig. 1 shows the constellation mapping for 16-APSK symbols, that are generated by

$$X_k \in \{X\} = \begin{cases} r_1 e^{j(\phi_1 + \frac{2\pi}{4} n)} & n = 0, \dots, 3 \\ r_2 e^{j(\phi_2 + \frac{2\pi}{12} n)} & n = 0, 1, \dots, 11 \end{cases} \tag{2.2}$$

where $\frac{r_2}{r_1} = 2.6$, $\phi_1 = 45^\circ, \phi_2 = 15^\circ$. Demapping can be done as,

$$|Y_k| < \frac{r_1 + r_2}{2}, \quad \frac{2\pi}{4} i < \angle Y_k < \frac{2\pi}{4}(i+1) \tag{2.3}$$

$$i = 0, \dots, 3 \tag{2.4}$$

$$|Y_k| > \frac{r_1 + r_2}{2}, \quad \frac{2\pi}{12} i < \angle Y_k < \frac{2\pi}{12}(i+1) \tag{2.5}$$

$$i = 4, \dots, 15 \tag{2.6}$$

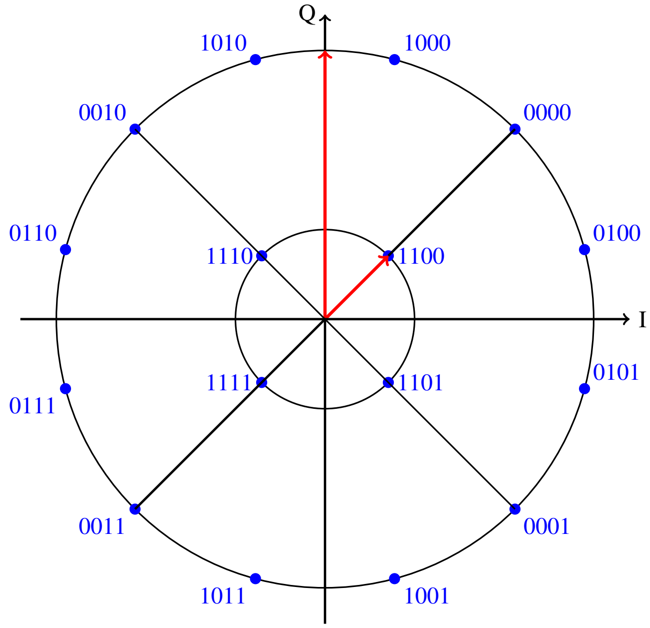

Fig. 1: Constellation diagram of 16APSK

B. 32-APSK

Fig. 2 shows the constellation mapping for 32-APSK symbols generated by

$$X_k \in \{X\} = \begin{cases} r_1 e^{j(\phi_1 + \frac{2\pi}{4} n)} & n = 0, \dots, 3 \\ r_2 e^{j(\phi_2 + \frac{2\pi}{12} n)} & n = 0, 1, \dots, 11 \\ r_3 e^{j(\phi_3 + \frac{2\pi}{16} n)} & n = 0, 1, \dots, 16 \end{cases} \tag{2.7}$$

where $\frac{r_2}{r_1} = 2.54$, $\frac{r_3}{r_2} = 4.33$, and $\phi_1 = 45^\circ, \phi_2 = 15^\circ, \phi_3 = 0^\circ$. Demapping can be done as,

$$|Y_k| < \frac{r_1 + r_2}{2}, \quad \frac{2\pi}{4} i < \angle Y_k < \frac{2\pi}{4}(i+1) \tag{2.8}$$

$$i = 0, \dots, 3 \tag{2.9}$$

$$\frac{r_1 + r_2}{2} < |Y_k| < \frac{r_2 + r_3}{2}, \quad \frac{2\pi}{12} i < \angle Y_k < \frac{2\pi}{12}(i+1) \tag{2.10}$$

$$i = 4, \dots, 15 \tag{2.11}$$

$$|Y_k| > \frac{r_2 + r_3}{2}, \quad \frac{2\pi}{12} i < \angle Y_k < \frac{2\pi}{12}(i+1) \tag{2.12}$$

$$i = 16, \dots, 31 \tag{2.13}$$

3. Carrier Synchronization

A. Time Offset: Gardner TED

Let the mth sample in the rth received symbol time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1, \dots, M. \tag{3.1}$$

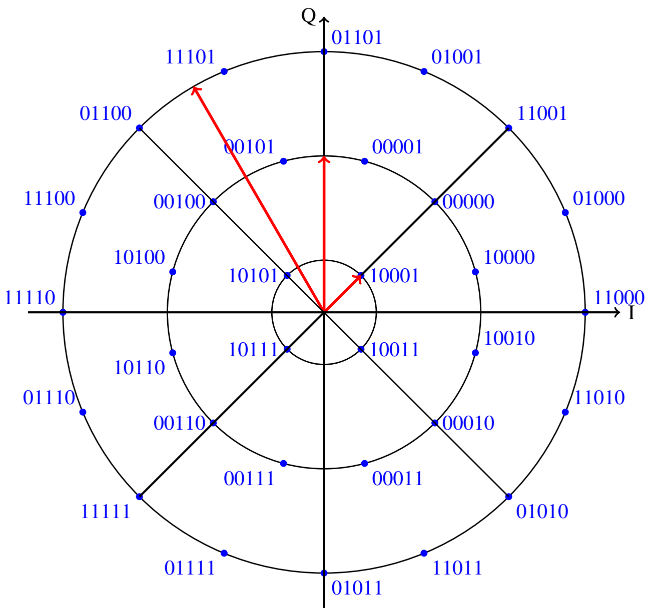

Fig. 2: Constellation diagram of 32APSK

where X_k is the transmitted symbol in the kth time slot and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$. The decision variable for the kth symbol is [6]

$$U_k = \frac{1}{N} \sum_{i=1}^{N} Y_{k-i} \left(\frac{M}{2}\right) \left[Y_{k-i+1}(M) - Y_{k-i}(M)\right] \quad (3.2)$$

B. Frequency Offset: LR Technique

Let the frequency offset be Δf [7] . Then

$$Y_k = X_k e^{j2\pi\Delta f k M} + V_k, \quad k = 1, \ldots, N \qquad (3.3)$$

From (3.3),

$$Y_k X_k^* = |X_k|^2 \, e^{j2\pi\Delta f k M} + X_k^* V_k \qquad (3.4)$$

$$\implies r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \qquad (3.5)$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \qquad (3.6)$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \leq k \leq N-1 \qquad (3.7)$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta\hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \operatorname{Im}(R(k))}{\sum_{k=1}^{P} k \operatorname{Re}(R(k))}, \quad P\Delta f M << 1 \quad (3.8)$$

where P is the number of pilot symbols.

C. Phase Offset: Feed Forward Maximum Likelihood (FF-ML)technique

Let the phase offset be $\Delta\phi$ [8] . Then for the kth pilot,

$$Y_k = X_k e^{j\Delta\phi_k} + V_k, \quad k = 1, \ldots, P \qquad (3.9)$$

From (3.9),

$$Y_k X_k^* = |X_k|^2 \, e^{j\Delta\phi_k} + X_k^* V_k \qquad (3.10)$$

$$\implies r_k = e^{j\Delta\phi_k} + \bar{V}_k \qquad (3.11)$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \qquad (3.12)$$

From (3.11), the estimate for the kth pilot is obtained as

$$\Delta\hat{\phi}_k = \arg(r_k) \qquad (3.13)$$

The phase estimate is then obtained using $\Delta\hat{\phi}_k$ in the following update equation as

$$\Delta\theta_k = \Delta\theta_{k-1} + \alpha SAW\left[\Delta\hat{\phi}_k - \Delta\theta_{k-1}\right] \qquad (3.14)$$

Where SAW is sawtooth non-linearity

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \qquad (3.15)$$

and $\alpha \leq 1$. The estimate is then obtained as $\Delta\theta_P$.

D. Automatic Gain Controller (AGC): Data-Aided Vector-Tracker (DA-VT)

Let the random AGC offset α, then the received symbol equation with amplitude offset as,

$$Y_k = \alpha X_k + V_k \quad k = 1, \ldots, P \qquad (3.16)$$

where $\alpha = \alpha_I + j\alpha_Q$ is the gain parameter. According to [9], the $\hat{\alpha}_k$ estimate for the kth pilot is

$$\alpha_{k+1} = \alpha_k - \gamma\left[\alpha_k Y_k^p - X_k^p\right]\left[X_k^p\right]^*, \qquad (3.17)$$

where γ is the AGC step size.

4. Frame Synchronization : Global Summation of SOF/PLSC Detectors

Let the frequency offset be Δf and phase offset be $\Delta\phi$.Then,

$$Y_k = X_k e^{j(2\pi\Delta f k M + \phi_k)} + V_k, \quad k = 1, \ldots, N \qquad (4.1)$$

assuming that no pilot symbols are trasmitted. Let the phase information be θ_k, and defined as

$$\theta(k) = \frac{Y_k}{|Y_k|} \tag{4.2}$$

At the receiver, the header information is available in the form of

$$g_i(l) = x_s(l)x_s(l-i), l = 0, \ldots, SOF-1 \tag{4.3}$$
$$h_i(l) = x_p(l)x_p(l-i), l = 0, \ldots, PLSC-1 \tag{4.4}$$

where x_s are the mapped SOF symbols, x_p are the scrambled PLSC symbols, both modulated using $\pi/2$ BPSK for $i = 1, 2, 4, 8, 16, 32$. A special kind of correlation is performed to obtain

$$m_i(k) = \sum_{l=0}^{PLSC-1} e^{j(\theta(k-l)-\theta(k-l-i))}h_i(l), \tag{4.5}$$
$$n_i(k) = \sum_{l=0}^{SOF-1} e^{j(\theta(k-l)-\theta(k-l-i))}g_i(l), \tag{4.6}$$
$$k = 1, \ldots, N \tag{4.7}$$

Compute

$$p_i(k) = \begin{cases} \max\left(|n_i(k-PLSC) + m_i(k)|\right., & \\ \left.|n_i(k-PLSC) - m_i(k)|\right) & k > PLSC \\ \max|m_i(k)| & k < 64 \end{cases} \tag{4.8}$$

GLOBAL variable $G_{R,T}(k)$ [10] defined as,

$$G_{R,T}(k) = \sum_{i \geq 1} p_i(k), \quad i = 1, 2, 4, 8, 16, 32 \tag{4.9}$$

At the receiver, let us consider we have sent two types of transmission. One is PLHEADER+DATA (Y_{k1}) and another is only DATA (Y_{k2}) and the GLOBAL variables for (Y_{k1}) and (Y_{k2}) from (4.9) are $G1_{R,T}(k)$, $G2_{R,T}(k)$ respectively.

A. Global Threshold Calculation

The Global Threshold variable is defined as

$$T = \max\left(\max\left(G1_{R,T}(k)\right), \max\left(G2_{R,T}(k)\right)\right) \tag{4.10}$$

The probability of false detection of plheader when only DATA frame (Y_{k2}) has been sent is defined as

$$P_{FA} = \frac{\sum \frac{sign(|Y_{k2}-T|)+1}{2}}{N} \tag{4.11}$$

The probability of missed detection of plheader when PLHEADER+DATA (Y_{k1}) has been sent is defined as

$$P_{MD} = \frac{\sum \frac{sign(T-|Y_{k1}|)+1}{2}}{N + PLSC + SOF} \tag{4.12}$$

5. BCH: Generator Polynomial

For a BCH code, the minimal polynomials are given by

$$g_1(x) = 1 + x + x^3 + x^5 + x^{14} \tag{5.1}$$
$$g_2(x) = 1 + x^6 + x^8 + x^{11} + x^{14} \tag{5.2}$$
$$g_3(x) = 1 + x + x^2 + x^6 + x^9 + x^{10} + x^{14} \tag{5.3}$$
$$g_4(x) = 1 + x^4 + x^7 + x^8 + x^10 + x^{12} + x^{14} \tag{5.4}$$
$$g_5(x) = 1 + x^2 + x^4 + x^6 + x^8 + x^9 + x^{11} + x^{13} + x^{14} \tag{5.5}$$
$$g_6(x) = 1 + x^3 + x^7 + x^8 + x^9 + x^{13} + x^{14} \tag{5.6}$$
$$g_7(x) = 1 + x^2 + x^5 + x^6 + x^7 + x^{10} + x^{11} + x^{13} + x^{14} \tag{5.7}$$
$$g_8(x) = 1 + x^5 + x^8 + x^9 + x^{10} + x^{11} + x^{14} \tag{5.8}$$
$$g_9(x) = 1 + x + x^2 + x^3 + x^9 + x^{10} + x^{14} \tag{5.9}$$
$$g_{10}(x) = 1 + x^3 + x^6 + x^9 + x^{11} + x^{12} + x^{14} \tag{5.10}$$
$$g_{11}(x) = 1 + x^4 + x^{11} + x^{12} + x^{14} \tag{5.11}$$
$$g_{12}(x) = 1 + x + x^2 + x^3 + x^5 + x^6 + x^7 + x^8 + x^{10} + x^{13} + x^{14} \tag{5.12}$$

The generator polynomial is obtained as

$$g(x) = \prod_{i=1}^{m} g_i(x) \tag{5.13}$$

6. BCH: Encoding

Let $\vec{m}$ be a $k \times 1$ message vector and

$$m(x) = m_{k-1}x^{k-1} + m_{k-2}x^{k-2} + \cdots + m_1 x + m_0 \tag{6.1}$$

be the corresponding Message polynomial. Let

$$m(x)x^{n-k} = q(x)g(x) + d(x) \tag{6.2}$$

and

$$c(x) = m(x)x^{n-k} + d(x) \tag{6.3}$$

7. Berlekamp's Decoding Algorithm

Let $\vec{\alpha}_i, 2 \leq i \leq 2m+1$ be the ith row of $\vec{A}$ and $\alpha_i(x)$ be the corresponding polynomial. Let $\vec{r}$ be the received codeword (noisy). $r(x)$ is then defined to be the received polynomial.the correspondig syndromes.

$$S_i(x) = r(\alpha_i(x)) \tag{7.1}$$

Intialization : $k = 0, \Lambda^{(0)}(x) = 1, T^{(0)} = 1$ Let $\Delta^{(2k)}$ be the coefficient of x^{2k+1} in $\Lambda^{(2k)}[1+S(x)]$ Compute

$$\Delta^{(2k+2)}(x) = \Lambda^{(2k)}(x) + \Delta^{(2k)}[x.T^{(2k)}(x)] \tag{7.2}$$

Compute

$$T^{(2k+2)}(x) = \begin{cases} x^2 T^{(2k)}(x) & \text{if } \Delta^{(2k)} = 0 \text{ or } \deg\left[\Lambda^{(2k)}(x)\right] : \\ \frac{x\Lambda^{(2k)}(x)}{\Delta^{(2k)}} & \text{if } \Delta^{(2k)} \neq 0 \text{ or } \deg\left[\Lambda^{(2k)}(x)\right] : \end{cases} \tag{7.3}$$

Set $k = k + 1$.If $k < t$ then go to step3. Return the Error Locator polynomial $\Lambda(x) = \Lambda^{(2k)}(x)$

8. BCH Decoding: The Chien's Search Algorithm

8.1 Take α^j as test root . $0 \leq j \leq n-1$.
8.2 if Λ_i test every root and if its equals to zero. Then that is root.
8.3 Flip the bit values at root positions.
8.4 Let the Received polynomial be $r(x)$ i.e which contains both transmitted codeword polynomial $c(x)$ and the error polynomial $e(x)$

$$e(x) = e_0 + e_1 x^1 + ... + e_{n-1} x^{n-1} \tag{8.1}$$

Where e_i represents the value of the error at the location. For binary BCH codes e_i is either 0 or 1.

$$r(x) = c(x)+e(x) = r_{n-1}x^{n-1}+r_{n-2}x^{n-2}+\cdots+r_1 x+r_0 \tag{8.2}$$

Definie, Syndrome

$$S_i = r(\alpha^i) = c(\alpha^i) + e(\alpha^i) = e(\alpha^i) \tag{8.3}$$

Where α^i is a root of the codeword. Suppose that v errors occurred, and $0 \leq v \leq t$. Let the error occurs at $i_1, i_2, \ldots, i_v$. The Decoding Process, for a t-error correcting code will follows the basic steps,

9. LDPC: Introduction

Let the Channel model be,

$$Y_k = X_k + V_k, \quad k = 0, \ldots, 6 \tag{9.1}$$

where X_k is the transmitted symbol in the kth time slot using the BPSK modulation and $V_k \sim \mathcal{N}\left(0, \sigma^2\right)$.

10. LDPC: Encoding

LDPC codes are popular linear block codes with closest Shannon limit channel capacity [2]. As an example, lets take (7,4) Hamming parity check matrix.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{10.1}$$

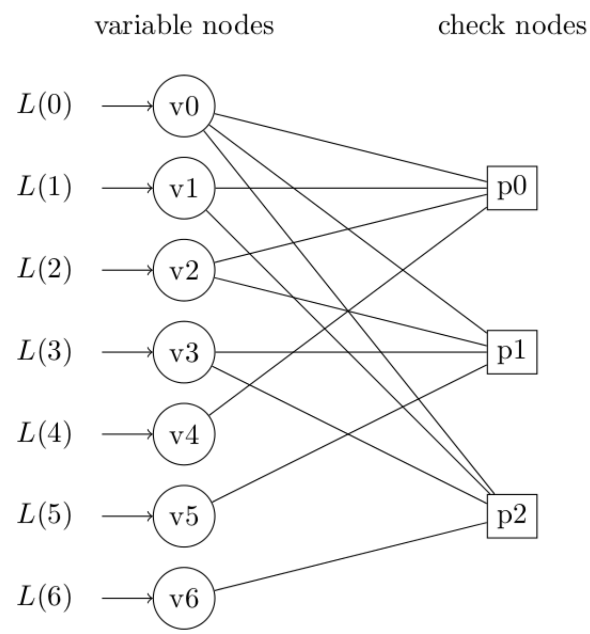

Fig. 3: Tanner Graph Representation for (7,4) Hamming parity check matrix

Encoding can be carried out by using

$$H \times c^T = 0 \tag{10.2}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = 0 \tag{10.3}$$

solving we get

$$p_0 = m_0 \oplus m_1 \oplus m_2 \tag{10.4}$$

$$p_1 = m_0 \oplus m_2 \oplus m_3 \tag{10.5}$$

$$p_2 = m_0 \oplus m_1 \oplus m_3 \tag{10.6}$$

This is called Systematic Encoding.i.e Encoder will ensures information bits followed by parity bits.

11. LDPC: Decoding

A. Useful Calculations for proceeding LDPC Decoding

1) Calculation of Input Channel Log Likelihood Ratio LLR

$$L(x_j) = \log\left(\frac{Pr(x_j = 1|y)}{Pr(x_j = -1|y)}\right) \quad X = 1 - 2c \tag{11.1}$$

$$= \log\left(\frac{f(y|x_j = 1)Pr(x_j = 1)}{f(y|x_j = -1)Pr(x_j = -1)}\right) \tag{11.2}$$

$$= \log\left(\frac{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_j-1)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_j+1)^2}{2\sigma^2}}}\right) \tag{11.3}$$

$$= \log\left(e^{\frac{2y_j}{\sigma^2}}\right) \tag{11.4}$$

$$L(x_j) = \frac{2y_j}{\sigma^2} \tag{11.5}$$

2) Check Node Operation :
Lets assume that we have initilized all LLR values to variable nodes and we sent to check nodes. V_j represents all the variable nodes which are connected to j^{th} check node. Using the min-sum approximation [3], the message from j^{th} check node to i^{th} variable node given by, since parity node equation for the first

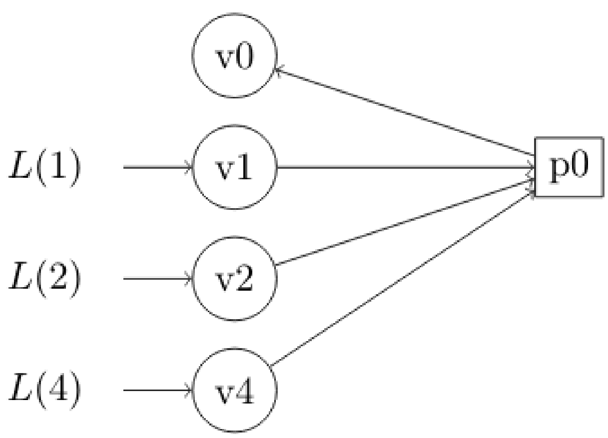

Fig. 4: Check node operation

check node is $p_0 = m_0 + m_1 + m_2 + m_4$. we need to calculate

$$L_{ext0,0} = \log\left(\frac{Pr(x_0 = 0|y_1, y_2, y_4)}{Pr(x_0 = 1|y_1, y_2, y_4)}\right) \tag{11.6}$$

Defining,

$$L_1 = \log\left(\frac{Pr(x_1 = 0|y_1)}{Pr(x_1 = 1|y_1)}\right) = \log\left(\frac{p_1}{1 - p_1}\right) \tag{11.7}$$

$$L_2 = \log\left(\frac{Pr(x_2 = 0|y_2)}{Pr(x_2 = 1|y_2)}\right) = \log\left(\frac{p_2}{1 - p_2}\right) \tag{11.8}$$

$$L_4 = \log\left(\frac{Pr(x_4 = 0|y_4)}{Pr(x_4 = 1|y_4)}\right) = \log\left(\frac{p_4}{1 - p_4}\right) \tag{11.9}$$

(11.10)

Using Table. III we can find the

c_0	c_1	c_2	c_4
0	0	0	0
1	0	0	1
1	0	1	0
0	0	1	1
1	1	0	0
0	1	0	1
0	1	1	0
1	1	1	1

TABLE II: Probability of a varibale node from other check nodes

$$p_0 = Pr(c_0 = 0|c_1, c_2, c_4) \tag{11.11}$$

$$\begin{aligned} p_0 &= p_1p_2p_4 + p_1(1 - p_2)(1 - p_4) \\ &\quad + (1 - p_1)p_2(1 - p_4) + (1 - p_1)(1 - p_2)p_4 \\ 1 - p_0 &= p_1p_2(1 - p_4) + p_1(1 - p_2)p_4 \\ &\quad + (1 - p_1)p_2p_4 + (1 - p_1)(1 - p_2)(1 - p_4) \end{aligned}$$

by rearranging above equations,

$$p_0-(1-p_0) = p_1-(1-p_1)+p_2-(1-p_2)+p_4-(1-p_4) \tag{11.12}$$

Where p_i is the probability. getting message from check to variable node by taking all variable node informations.

$$\frac{1 - \frac{1-p_0}{p_0}}{1 + \frac{1-p_0}{p_0}} = \frac{1 - \frac{1-p_1}{p_1}}{1 + \frac{1-p_1}{p_1}} \times \frac{1 - \frac{1-p_2}{p_2}}{1 + \frac{1-p_2}{p_2}} \times \frac{1 - \frac{1-p_4}{p_4}}{1 + \frac{1-p_4}{p_4}} \tag{11.13}$$

$$\frac{1 - e^{-L_{ext0,0}}}{1 + e^{-L_{ext0,0}}} = \frac{1 - e^{-L_1}}{1 + e^{-L_1}} \times \frac{1 - e^{-L_2}}{1 + e^{-L_2}} \times \frac{1 - e^{-L_4}}{1 + e^{-L_4}} \tag{11.14}$$

$$-\tanh\left(\frac{L_{ext0,0}}{2}\right) = \left(-\tanh\left(\frac{L_1}{2}\right)\right)\left(-\tanh\left(\frac{L_2}{2}\right)\right)$$

$$\left(-\tanh\left(\frac{L_4}{2}\right)\right) \tag{11.15}$$

$$L_{ext0,0} = \left(\prod_{k\in V_j\setminus i} \alpha_{k,0}\right) f\left(\sum_{k\in V_j\setminus i} f\left(\beta_{k,0}\right)\right) \tag{11.16}$$

Where,

$$\alpha_{k,j} = sign(L_{k,j}) \tag{11.17}$$

$$\beta_{k,j} = \left|L_{k,j}\right| \tag{11.18}$$

$$f(x) = -\log\left(\tanh\frac{x}{2}\right) \tag{11.19}$$

Using the Fig 5 and using its 45° symmetricity,

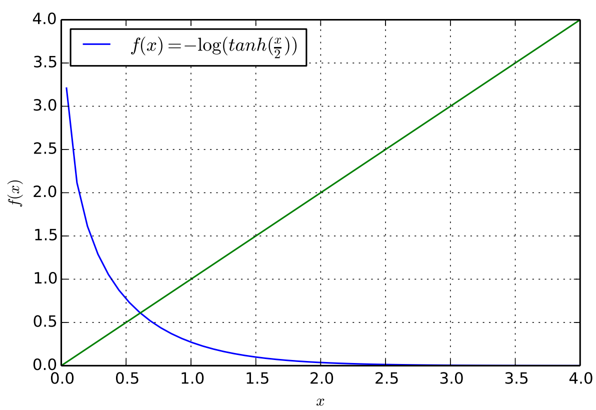

Fig. 5: Plot of function $f(x)$

we can approximate the above equation as, given by minimum sum approximation [3]

$$f\left(\sum_{k\in V_j\setminus i} f\left(\beta_{k,0}\right)\right) \approx f\left(f\left(\min_{k\in V_j\setminus i}\left(\beta_{k,0}\right)\right)\right) \tag{11.20}$$

$$= \min_{k\in V_j\setminus i}\left(\beta_{k,0}\right) \tag{11.21}$$

Combining (11.21) in (11.16),

$$L(r_{j=0,i=0}) = \left(\prod_{k\in V_j\setminus i} \alpha_{k,0}\right)\left(\min_{k\in V_j\setminus i}\left(\beta_{k,0}\right)\right) \tag{11.22}$$

3) Variable Node Operation :
Let C_i denotes all the check nodes connected to i^{th} variable node. The message from i^{th} variable node to j^{th} check node given by,

$$L(q_{i=0,j=0}) = \log\left(\frac{Pr(x_j = 1|y_0, y_1, y_2)}{Pr(x_j = -1|y_0, y_1, y_2)}\right) \quad X = 1 - 2c \tag{11.23}$$

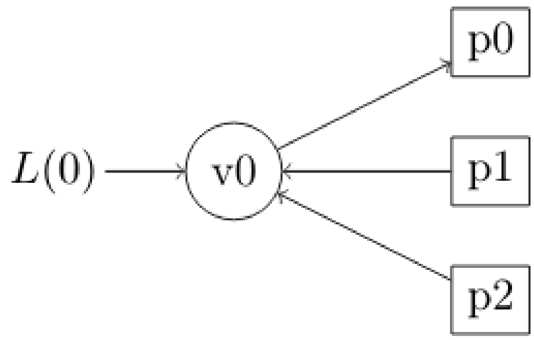

Fig. 6: Variable node operation

$$= \log\left(\frac{f(y_0, y1, y_2|x_j = 1)Pr(x_j = 1)}{f(y_0, y_1, y_2|x_j = -1)Pr(x_j = -1)}\right) \tag{11.24}$$

$$= \log\left(\frac{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^3 e^{\frac{-(y_0-1)^2}{2\sigma^2}} e^{\frac{-(y_1-1)^2}{2\sigma^2}} e^{\frac{-(y_2-1)^2}{2\sigma^2}}}{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^3 e^{\frac{-(y_0+1)^2}{2\sigma^2}} e^{\frac{-(y_1+1)^2}{2\sigma^2}} e^{\frac{-(y_2+1)^2}{2\sigma^2}}}\right) \tag{11.25}$$

$$= \log\left(e^{\frac{2(y_0+y_1+y_2)}{\sigma^2}}\right) \tag{11.26}$$

$$L(q_{i=0,j=0}) = \frac{2(y_0 + y_1 + y_2)}{\sigma^2} = L(x_i) + \sum_{k\in C_i\setminus j} L(r_{ki}) \tag{11.27}$$

B. Message Passing Algorithm using min-sum Approximation

Transmitted frames = N, Total number of bits = N × 7 and Total number of information bits = N × 4. For Each Frame,

1) Initialize $L(q_{ij})$ using (11.5) for all i, j for which $h_{ij} = 1$ with channel LLR's.
2) Update $\left\{L(r_{ji})\right\}$ using (11.22)
3) Update $\left\{L(q_{ji})\right\}$ using (11.27).
4) Update $\{L(V_i)\}$ using,

$$L(V_i) = L(x_i) + \sum_{k\in C_i} L(r_{ki}) \quad i = 0, \ldots, 6. \tag{11.28}$$

5) Proceed to step 2.
After maximum specified iterations,

Decoding can be done using,

$$\hat{c}_i = \begin{cases} 1 & L(V_i) < 0 \\ 0 & else \end{cases} \tag{11.29}$$

C. Simulation Results

For frames N=10000. Fig 7 Shows the Comparison of Probability error with channel coding and without channel coding. Since the parity check matrix taken was not much sparse, we are not getting near shannon limit performance. (Good sparse matrix i.e number of entries in H $<< m \times n$)

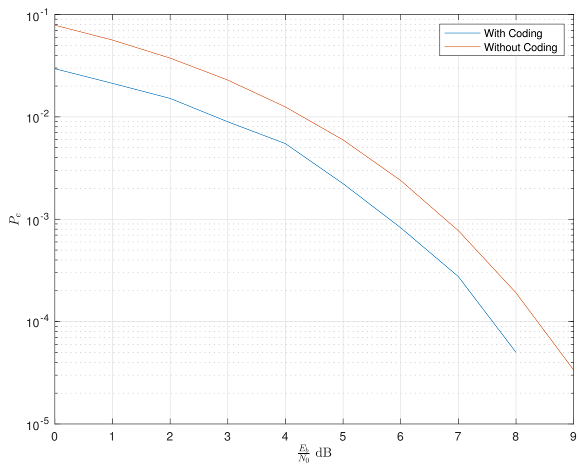

Fig. 7: SNR vs BER curves using LDPC channel coding and no channel coding

12. LDPC Decoding for Higher Order Mapping Schemes

In the case of higher order mapping schemes, calculation of Log Likelihood Ratio's are crutial. A generalized and approximated Log Likelihood Ratio given by [4]

A. General Expression for Calculation of Log Likelihood Ratio(LLR)

$$LLR(b_j) = \log\left(\frac{Pr(b_j = 0|y)}{Pr(b_j = 1|y)}\right) \tag{12.1}$$

$$LLR(b_j) = \log\left(\frac{\sum_{i\in\{K_{b_j=0}\}} \frac{1}{2\sigma^2} e^{\frac{-|y-x_i|^2}{2\sigma^2}}}{\sum_{i\in\{K_{b_j=1}\}} \frac{1}{2\sigma^2} e^{\frac{-|y-x_i|^2}{2\sigma^2}}}\right) \tag{12.2}$$

Where the alphabet $\{K_{b_j=b}\}$ is the set of all symbols representing the j^{th} bit equals to $b = 0$ or $b = 1$.

$$\log\left(e^a + e^b\right) \approx \max(a, b) \tag{12.3}$$

Shows the Jacobian-Logarithmic approximation using [5]. By Using the Above equation, The LLR expression can be written as,

$$LLR(b_j) \approx \max_{i\in\{K_{b_j=0}\}}\left(e^{\frac{-|y-x_i|^2}{2\sigma^2}}\right) - \max_{i\in\{K_{b_j=1}\}}\left(e^{\frac{-|y-x_i|^2}{2\sigma^2}}\right) \tag{12.4}$$

B. Approximated LLR's for QPSK Mapping Scheme

For the QPSK mapping scheme showed in Fig. 8 is grey code constellation and b_1, b_0 are the MSB and LSB of the mapped symbols.

$$LLR(b_1) \approx \max(L_0, L_1) - \max(L_3, L_2) \tag{12.5}$$

$$LLR(b_0) \approx \max(L_0, L_2) - \max(L_1, L_3) \tag{12.6}$$

Where,

$$L_i = e^{-\frac{|y-x_i|^2}{2\sigma^2}} \quad i = 0, 1, 2, 3 \tag{12.7}$$

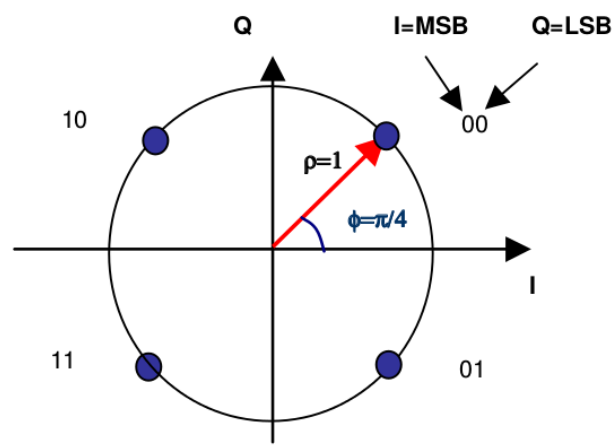

Fig. 8: Constellation Diagram of QPSK

C. Approximated LLR's for 8-PSK Mapping Scheme

For the 8-PSK mapping scheme showed in Fig. 9 is grey code constellation and b_2, b_0 are the MSB and LSB of the mapped symbols.

$$LLR(b_2) \approx \max(L_0, L_1, L_3, L_2) - \max(L_6, L_7, L_5, L_4) \quad (12.8)$$

$$LLR(b_1) \approx \max(L_0, L_1, L_5, L_4) - \max(L_3, L_2, L_6, L_7) \quad (12.9)$$

$$LLR(b_0) \approx \max(L_0, L_2, L_6, L_4) - \max(L_1, L_3, L_7, L_5) \quad (12.10)$$

Where,

$$L_i = e^{-\frac{|y-x_i|^2}{2\sigma^2}} \qquad i = 0, 1, \ldots, 7 \quad (12.11)$$

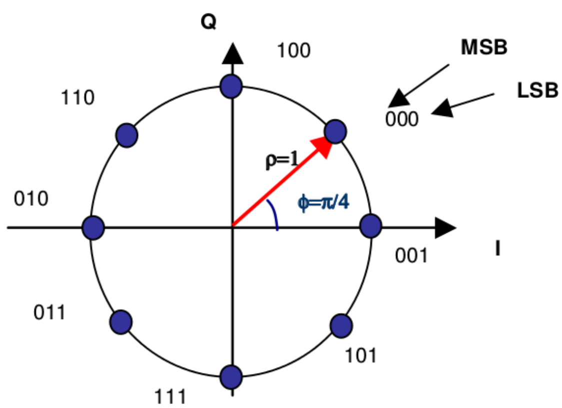

Fig. 9: Constellation Diagram of 8-PSK

13. Interleaver/Deinterleaver

For 8PSK, 16APSK, and 32APSK mapping schemes, a block interleaver [?] is used to mitigate the effects of bursty channel. For Concatenated Channel coding schemes bit interleaving is necessary. The mapped data is serially put as column wise and searially read out row wise. Fig. 10 shows bit interleaving scheme for 8PSK.

Fig. 11 shows the BER comparison of 8PSK mapping scheme with and without interleaver.

14. Physical Layer Framing(PLFRAMING)

PLFRAMING is useful for specifying the modulation scheme, code rate and frame characteristics, useful for Frame synchronization at the receiver. Fig. 12 shows the Typical Structure of PLFRAME according to [?]. SOF=Starting Of Frame and $PLSC$=Physical Layer Signalling Code.

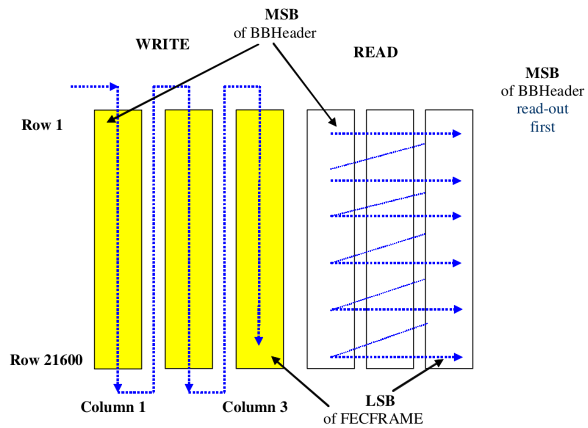

Fig. 10: Bit Interleaver Structure for 8PSK mapping scheme

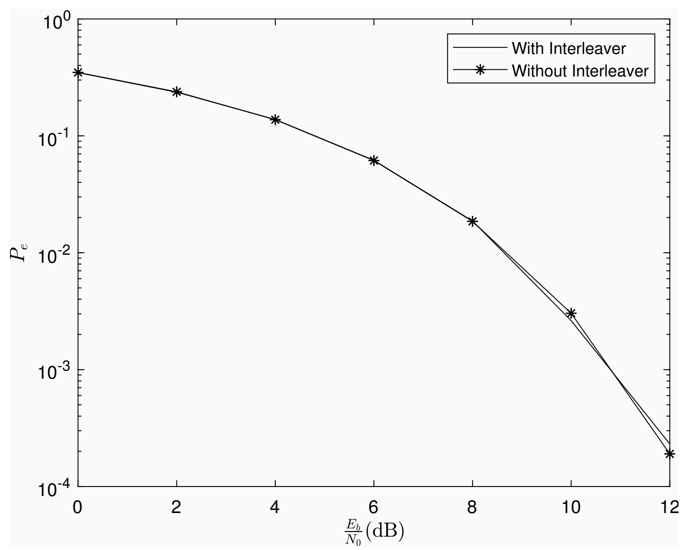

Fig. 11: Bit interleaver for 8PSK

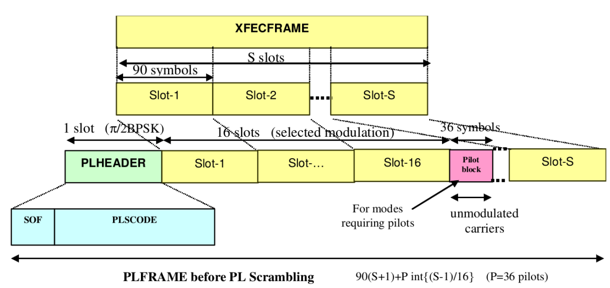

Fig. 12: Structure of PLFRAME.

A. *Generation of SOF*

SOF consistutes a fixed sequence $18D2E82_{HEX}$ (length=26 bits) in binary format which is from right to left.

B. Generation of PLSC

- Generation of PLSC involes definining 7 symbols and multiplying first 6 symbols with the defined G matrix in [**?**]. First 5 symbols called as MODCOD filed and next 2 symbols as TYPE field.
- First 5 symbols represents MODCOD which specifies Frame's mapping scheme and code rate. Fig. 13 shows the MODCOD coding for various mapping schemes.
- Next 2 symbols represents TYPE filed which specifies Frame length and presence and absesnce of pilot fileds. This is shown in Table III.
- Similarly, Fig. 14 shows the generation of 64 bit using the MODCOD and TYPE bits.
- After the generation of PLS code, we will again scramble the PLS Code with the fixed SCR sequence which is defined in [**?**].

$$PLSC = PLSC \oplus SCR \tag{14.1}$$

Bit-6	Bit-7	Frame Type	Pilots
0	0	Normal	No
0	1	Normal	Yes
1	0	Short	No
1	1	Short	Yes

TABLE III: Frame Type

Mode	MOD COD	Mode	MOD COD	Mode	MOD COD	Mode	MOD COD
QPSK 1/4	1_D	QPSK 5/6	9_D	8PSK 9/10	17_D	32APSK 4/5	25_D
QPSK 1/3	2_D	QPSK 8/9	10_D	16APSK 2/3	18_D	32APSK 5/6	26_D
QPSK 2/5	3_D	QPSK 9/10	11_D	16APSK 3/4	19_D	32APSK 8/9	27_D
QPSK 1/2	4_D	8PSK 3/5	12_D	16APSK 4/5	20_D	32APSK 9/10	28_D
QPSK 3/5	5_D	8PSK 2/3	13_D	16APSK 5/6	21_D	Reserved	29_D
QPSK 2/3	6_D	8PSK 3/4	14_D	16APSK 8/9	22_D	Reserved	30_D
QPSK 3/4	7_D	8PSK 5/6	15_D	16APSK 9/10	23_D	Reserved	31_D
QPSK 4/5	8_D	8PSK 8/9	16_D	32APSK 3/4	24_D	DUMMY PLFRAME	0_D

Fig. 13: MODCOD coding for various mapping schemes. Subscript D denotes decimal e.g. 1_D = 00001 in binary

C. PLHEADER

The PLHEADER has SOF (26 symbols) and PLSC (64 symbols) sequences, that are modulated

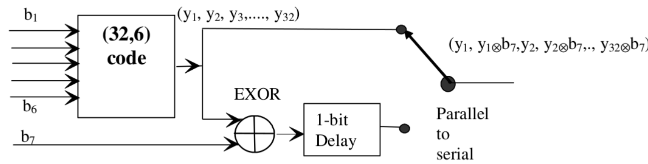

Fig. 14: Physical Layer Signalling generation

by rotating QPSK by $\frac{\pi}{4}$. This is also known as $\frac{\pi}{2}$-BPSK.

$$I_{2i-1} = Q_{2i-1} = \frac{1}{\sqrt{2}}(1 - 2y_{2i-1}) \quad i = 1, 2, \ldots, 45 \tag{14.2}$$

$$I_{2i} = -Q_{2i} = -\frac{1}{\sqrt{2}}(1 - 2y_{2i-1}) \quad 1, 2 \ldots, 45 \tag{14.3}$$

where $y_i \in \{0, 1\}, i = 1, \ldots, 90$.

D. Generation of Pilots

Pilot block consists of $P = 36$ symbols. Each pilot is composed of un-modulated complex symbol.Where, $I = Q = \frac{1}{\sqrt{2}}$

E. PLFRAME Calculations

PLFRAME calculations are available in Tables IV and V. The PLFRAME length is calculated as

$$L = SOF + PLSC + (90 * S_{SLOTS}) + (36) * P_{SLOTS} \tag{14.4}$$

where,

$$S_{SLOTS} = \frac{N}{90 \times \log_2 M}, P_{SLOTS} = \left\lfloor \frac{(S_{SLOTS} - 1)}{16} \right\rfloor$$

N = Frame Type(64800/16200 bits),M = Mapping order(4/8/16/32), depending upon the modulation scheme.

	SHORT FRAME (16200 bits)								
PLFRAME	SOF	PLSC	Slot-i	S_{SLOTS}	Pilot Symbols	P_{SLOTS}	#Pilots	Without Pilots	With Pilots
QPSK	26	64	90	90	36	5	180	8190	8370
8PSK	26	64	90	60	36	3	108	5490	5598
16APSK	26	64	90	45	36	2	72	4140	4212
32APSK	26	64	90	36	36	2	72	3330	3402

TABLE IV: Short frame details.

PLFRAME	SOF	PLSC	Slot-i	S_{SLOTS}	Pilot Symbols	P_{SLOTS}	#Pilots	Without Pilots	With Pilots
	NORMAL FRAME (64800 bits)								
QPSK	26	64	90	360	36	22	792	32490	33282
8PSK	26	64	90	240	36	14	504	21690	22194
16APSK	26	64	90	180	36	11	396	16290	16686
32APSK	26	64	90	144	36	8	288	13050	13338

TABLE V: Long frame details.

15. Pulse Shaping

Let X_k be the modulated symbol in the kth time slot. Then the mth sample of the transmitted symbol in the kth time slot is

$$S_k(m) = h_k(m) * X_k \tag{15.1}$$

where $h_k(m), m = 1, \ldots, M; k = 1, \ldots, N$ are samples of the pulse shaping filter [**?**] obtained from

$$H(f) = \begin{cases} 1 & |f| < f_N(1-\alpha) \\ \left\{\frac{1}{2} + \frac{1}{2}\sin\frac{\pi}{2}\left[\frac{f_N - |f|}{\alpha}\right]\right\}^{\frac{1}{2}} & |f| = f_N(1-\alpha) \\ 0 & |f| > f_N(1+\alpha) \end{cases} \tag{15.2}$$

where f_N is the Nyquist frequency and $\alpha = 0.35, 0.25$ or 0.2. Let the mth received sample in the kth time slot be $Y_k(m)$. At the Receiver, the symbol to be demodulated is then obtained as

$$Y_k(m) * h_k(M - m) \tag{15.3}$$

The following code plots Fig. 15 using pulse shaping.

```
wget https://github.com/gadepall/EE5837/raw/master/ETD/codes/pulse_shaping_qpsk_final.py
```

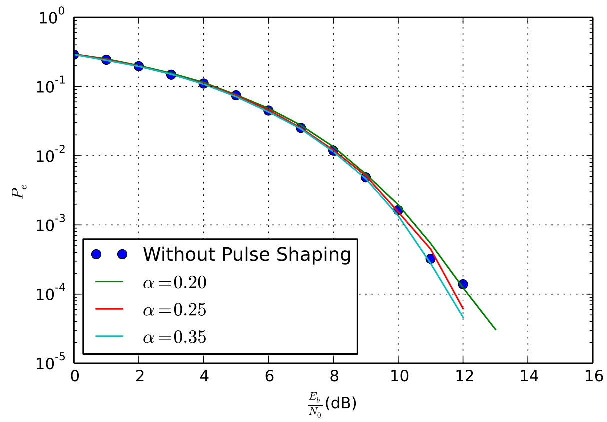

Fig. 15: SER comparison for QPSK with and without the pulse in (15.2).

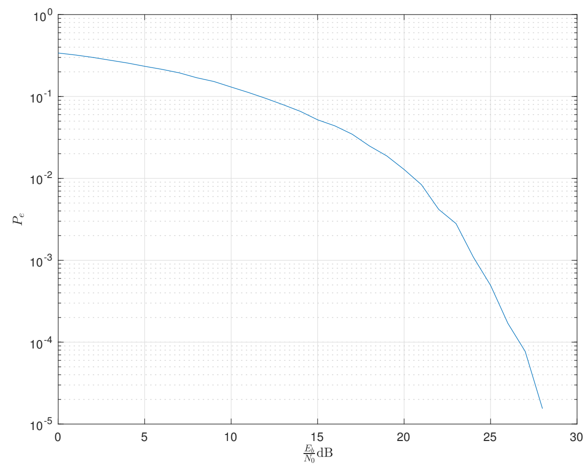

Fig. 16: SNR vs BER for 32-APSK

16. Results and Discussion

and Fig. 16 shows the Simulation diagram. and Fig. 17 Shows the Simulation diagram.

A. Time Offset

Fig. 19 Shows the Simulation diagram. Fig. 20 Shows the Simulation diagram.

Fig. 21 is generated by the following code

```
https://github.com/gadepall/EE5837/raw/master/synctech/codes/time_sync_offsets.py
```

and shows the variation of the BER with respect to the SNR with different timing offsets τ for $N = 6$.

B. Frequency

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 22 shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. 23 shows the variation of the error with respect to the SNR for Δf = 5MHz.

C. Phase

Fig. 24 is generated using

```
https://github.com/gadepall/EE5837/raw/master/synctech/codes/Error_vs_lp.py
```

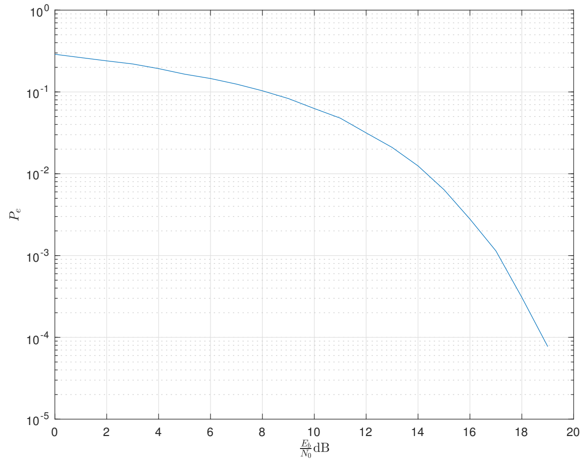

Fig. 17: SNR vs BER for 16-APSK

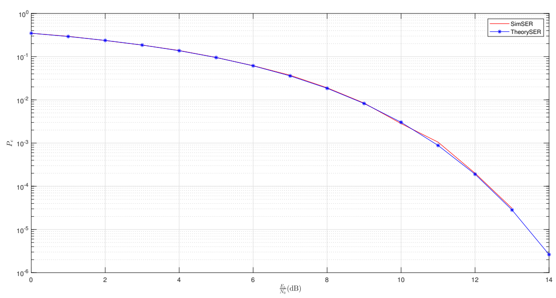

Fig. 18: SNR vs BER for 8-PSK

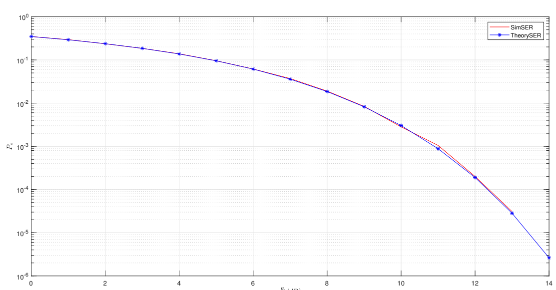

Fig. 19: SNR vs BER for 8-PSK

and shows the variation of the phase error in the offset estimate with respect to the pilot symbols when the SNR = 10 dB and $\alpha = 0.5$.

Similarly Fig. 25 generated by

```
https://github.com/gadepall/EE5837/blob/master/
    synctech/codes/Error_vs_snr.py
```

shows the variation of the error with respect to the SNR for pilot symbols $P = 18$ and $\alpha = 1$.

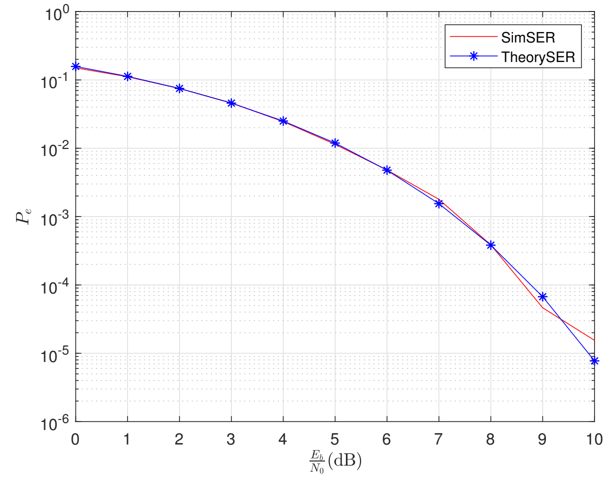

Fig. 20: SNR vs BER for QPSK

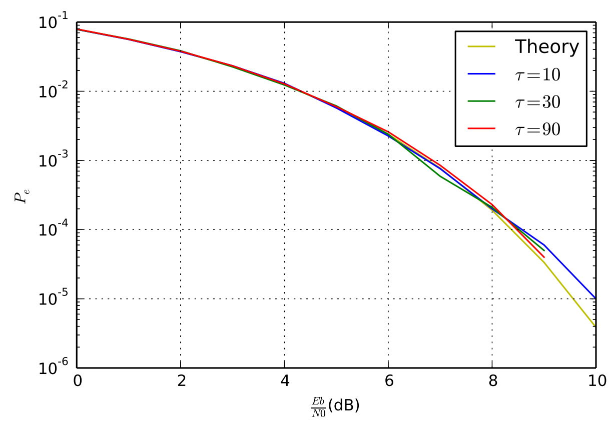

Fig. 21: SNR vs BER for varying τ.

D. Automatic Gain Control

The following code plots the real and imaginary parts of the gain parameter α with respect to the number of pilot symbols P. in Fig. 26. $\gamma = 10^{-3}, SNR = 10dB$.

```
https://github.com/gadepall/EE5837/raw/master/
    synctech/codes/
    Digital_AGC_with_fixed_SNR.py
```

E. Frame

Fig.27 shows the ROC curve ($P_{FA} vs P_{MD}$) at the receiver for frame synchronization at $\frac{E_b}{N_0} = -2$ dB.

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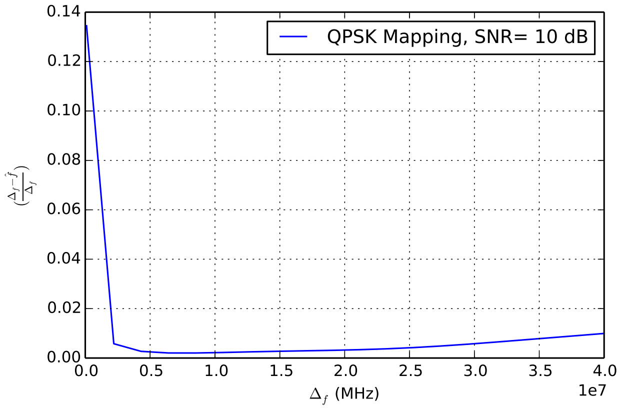

Fig. 22: Error variation with respect to frequency offset.

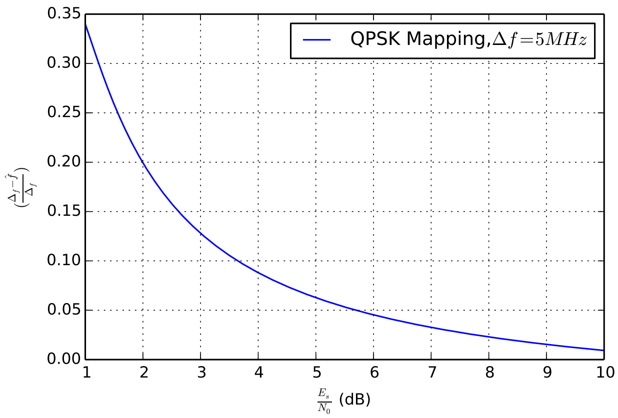

Fig. 23: Error variation with respect to the SNR. Δf = 5 MHz, Center frequency f_c = 25 GHz

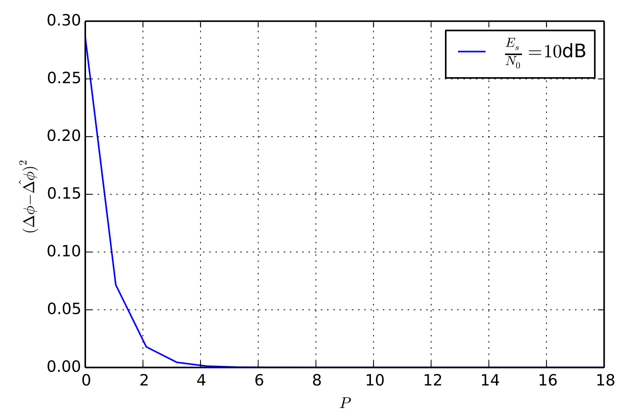

Fig. 24: Phase error variation with respect to pilot symbols

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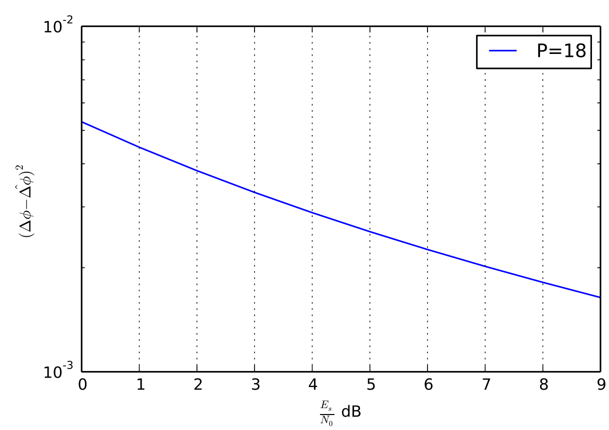

Fig. 25: Δf = 5 MHz

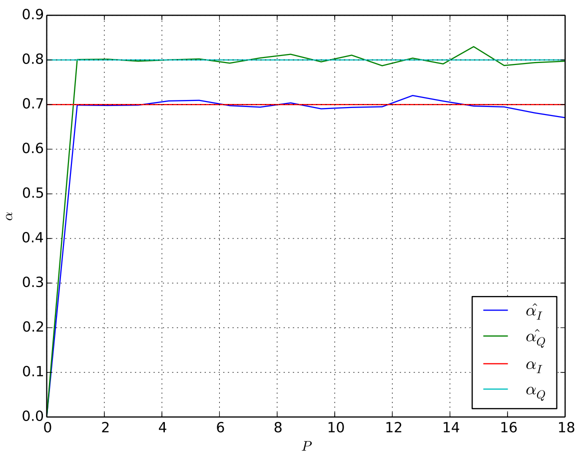

Fig. 26: Convergence of Digital AGC with resepct to P.

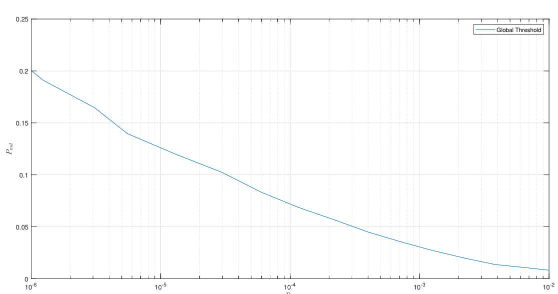

Fig. 27: Frame Synchronization Receiver Operating Characteristcs (ROC)