***Abstract*—These problems have been selected from GATE question papers and can be used for conducting tutorials in courses related to the course Digital Signal Processing in practice.**

1) The inverse Laplace trasform of H(s)=$\frac{s+3}{s^2+2s+1}$ for t≥0
   (A) $3te^{-t}+e^{-t}$
   (B) $3e^{-t}$
   (C) $2te^{-t}+e^{-t}$
   (D) $4te^{-t}+e^{-t}$

2) A system Transfer Function is H(s)=$\frac{a_1s^2+b_1s+c_1}{a_2s^2+b_2s+c_2}$. If $a_1 = b_1 = 0$ and all other coefficients are positive, the transfer function represents a-
   (A) $lowpassfilter$
   (B) $HighPassfilter$
   (C) $bandpassfilter$
   (D) $notchfilter$

3) The symbols, a and T, represent positive quantities, and u(t) is the unit step function. Which one of the following impulse responses is NOT the output of a causal linear time-invariant system-
   (A) $e^{at}u(t)$
   (B) $e^{-a(t+T)}u(t)$
   (C) $1+e^{-at}u(t)$
   (D) $e^{-a(t-T)}u(t)$

4) A periodic function f(t) with a period of 2π is represented as its Fourier series,

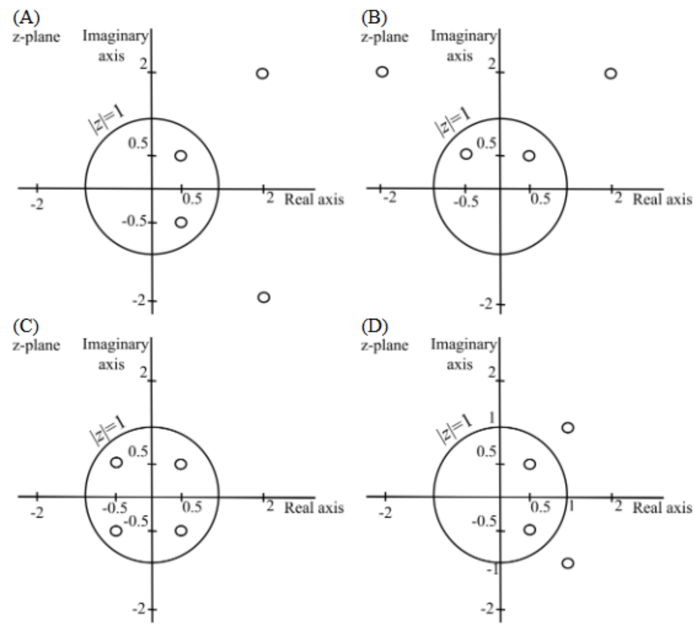
$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} a_n \sin nt \quad (1)$$

   if

$$f(t) = \begin{cases} A\sin t, & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi, \end{cases}$$

   the Fourier series coefficients a_1 and b_1 of f(t) are-
   (A) $a_1 = \frac{A}{\pi}; b_1 = 0$
   (B) $a_1 = \frac{A}{2}; b_1 = 0$
   (C) $a_1 = 0; b_1 = \frac{A}{\pi}$
   (D) $a_1 = 0; b_1 = \frac{A}{2}$

5) Let H(z) be the z-transform of a real-valued discrete-time signal h[n]. If $P(z) = H(z)H(\frac{1}{z})$ has a zero at $z = \frac{1}{2} + \frac{1}{2}j$, and P(z) has a total of four zeros, which one of the following plots represents all the zeros correctly?



6) Consider the signal f(t)=1 + $2\cos \pi t$ + $3\sin(\frac{2\pi}{3})t + 4\cos(\frac{\pi}{2}t + \frac{\pi}{4})$ where t in seconds. Its Fundamental time period, in second is. ________

7) It is desired to find a three-tap causal filter which gives zero signal as an output to an input of the form

$$x[n] = c_1 e^{\frac{-j\pi n}{2}} + c_2 e^{\frac{j\pi n}{2}} \quad (2)$$

   where c_1 and c_2 are arbitrary real numbers. The desired three-tap filter is given by h[0]=1, h[1]=a, h[2]=b and h[n]=0 otherwise
   What are the values of the filter taps a and b if the output is y[n]=0 for all n, when x[n] is as given above?
   (A) $a_1 = 1; b_1 = 1$
   (B) $a_1 = 0; b_1 = -1$

(C) $a_1 = -1; b_1 = 1$

(D) $a_1 = 0; b_1 = 1$

8) Let h[n] be a length-7 discrete-time finite impulse response filter, given by:
h[0]=4, h[1]=3, h[2]=2, h[3]=1
h[-1]=-3, h[-2]=-2, h[-3]=-1
and h[n]=0 for $|n| \geq 4$. A length-3 finite impulse response approximation g[n] of h[n] has to be obtained such that-
E(h,g)= $\int_{-\pi}^{\pi} (|H(e^{j\omega})| - |G(e^{j\omega})|)^2 d\omega$
is minimized, where $H(e^{j\omega})$ and $G(e^{j\omega})$ are the discrete-time Fourier transforms of h[n] and g[n],respectively. For the filter that minimizes E(h,g), the value of 10g[-1]+g[1], rounded off to 2 decimal places, is __________

9) A discrete-time all-pass system has two of its poles at $0.25\angle 0°$ and $2\angle 30°$. Which one of the following statements about the system is TRUE?

(A) It has two more poles at $0.5\angle 30°$ and $4\angle 0°$

(B) It is stable only when the impulse response is two-sided.

(C) It has constant phase response over all frequencies.

(D) It has constant phase response over the entire z-plane.

10) Let x(t) be a periodic function with period T=10. The Fourier series coefficients for this series are denoted by a_k, that is-
$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{\frac{jk2\pi t}{T}}$
The same function x(t) can also be considered as a periodic function with period T'=40. Let b_k be the Fourier series coefficients when period is taken as T'. If $\sum_{k=-\infty}^{\infty} |a_k|$ =16. Then $\sum_{k=-\infty}^{\infty} |b_k|$ is equal to-

(A) 256

(B) 64

(C) 16

(D) 4

11) Let X[k]=k+1, $0 \leq k \leq 7$ be a 8-point DFT of a sequence x[n], where
X[k]=$\sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi nk}{N}}$ The value (correct to two decimal places) of $\sum_{n=0}^{3} x[2n]$ is __________

12) Let the input be u and the output be y of a system and the other parameters are real constants. Identify which among the following system is not a linear system

(A) $\frac{\mathrm{d}^3 y}{\mathrm{d}t^3} + a_1 \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + a_2 \frac{\mathrm{d}y}{\mathrm{d}t} + a_3 y = b_3 u + b_2 \frac{\mathrm{d}u}{\mathrm{d}t} + b_1 \frac{\mathrm{d}^2 u}{\mathrm{d}t^2}$ (with initial rest conditions)

(B) $y(t) = \int_0^t e^{\alpha(t-\tau)} \beta u(\tau) d\tau$

(C) $y = au + b$

(D) $y = au$

13) Let f be the real valued function of a real variable defined $f(x) = x - [x]$, where [x] denotes the largest integer less than or equal to x. The value of $\int_{0.25}^{1.25} f(x) dx$ is - __________

14) Consider the two continuous-time signals defined below:

$$x_1(t) = \begin{cases} |t|, & -1 \leq t \leq 1 \\ 0, & otherwise, \end{cases}$$

$$x_2(t) = \begin{cases} 1 - |t|, & -1 \leq t \leq 1 \\ 0, & otherwise, \end{cases}$$

These signals are sampled with a sampling period of T=0.25 seconds to obtain discrete-time signals $x_1[n]$ and $x_2[n]$, respectively.Which one of the following statements is true ?

(A) The energy of $x_1[n]$ is greater than the energy of $x_2[n]$

(B) The energy of $x_2[n]$ is greater than the energy of $x_1[n]$

(C) $x_![n]$ and $x_2[n]$ have equal energies.

(D) Neither $x_1[n]$ nor $x_2[n]$ is a finite-energy signal.

15) The signal energy of the continuous time signal $x(t) = [(t-1)u(t-1)] - [(t-2)u(t-2)] - [(t-3)u(t-3)] + [(t-4)u(t-4)]$ is

(A) $\frac{11}{3}$

(B) $\frac{7}{3}$

(C) $\frac{1}{3}$

(D) $\frac{5}{3}$

16) The input x[n] and output y[n] of a discrete-time system are related as $y[n] = \alpha[n-1] + x[n]$. The condition on α for which the system is Bounded-Input Bounded-Output(BIBO) stable is-

(A) $|\alpha| < 1$

(B) $|\alpha| = 1$

(C) $|\alpha| > 1$

(D) $|\alpha| < \frac{3}{2}$

17) The output of a continuous-time system y(t) is related to input x(t) as $y(t) = x(t) + \frac{1}{2}x(t-1)$. If the Fourier transform of x(t) and y(t) are $X(\omega)$ and $Y(\omega)$ respectively, and $|X(0)|^2 = 4$, the value of $|Y(0)|^2$ is __________

18) A discrete-time signal $x[n] = e^{\frac{j5\pi n}{8}} + e^{\frac{j\pi n}{4}}$ is down-sampled to the signal $x_d[n]$ such that $x_d[n] = x[4n]$. The fundamental period of the down-sampled signal $x_d[n]$ is __________

19) Two periodic signals $x(t)$ and $y(t)$ have the same fundamental period of 3 seconds. Consider the signal $z(t) = x(-t) + y(2t+1)$. The fundamental period of $z(t)$ in seconds is-

(A) 1

(B) 1.5

(C) 2

(D) 3

20) Consider signal -

$$x(t) = \begin{cases} 1, & -2 \le t \le 2 \\ 0, & otherwise, \end{cases}$$

Let $\delta(t)$ denote the unit impulse (Dirac-delta) function. The value of integral $\int_0^5 2x(t-3)\delta(t-4)dt$ is-

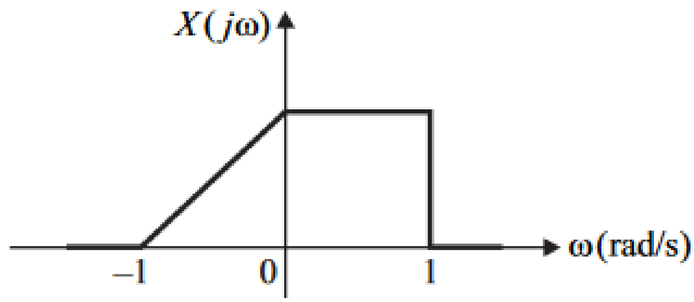
(A) 2

(B) 1

(C) 0

(D) 3

21) The Fourier transform of a signal $x(t)$, denoted by $X(j\omega)$, is shown in the figure. Let $y(t) =$



$x(t) + e^{jt}x(t)$. The value of Fourier transformof $y(t)$ evaluated at the angular frequency ω= 0.5 rad/s is-

(A) 0.5

(B) 1

(C) 1.5

(D) 2.5

22) Let $y[n] = x[n] * h[n]$, where $*$ denotes convolution and x[n] and h[n] are two discrete time sequences. Given that the z-transform of $y[n]$ is $Y(z) = 2 + 3z^{-1} + z^{-2}$, the z-transform of $p[n] = x[n] * h[n-2]$ is

(A) $2 + 3z + z^{-2}$

(B) $3z + z^{-2}$

(C) $2z^2 + 3z + 1$

(D) $2z^{-2} + 3z^{-3} + z^{-4}$

23) For the sequence $x[n]$= 1,-1,1,-1 with n=0,1,2,3 the DFT is computed as $X(k) = \sum_{k=0}^{3} x[n]e^{\frac{-j2\pi nk}{4}}$, for k=0,1,2,3. The value of k for which X(k) is not zero is-

(A) 0

(B) 1

(C) 2

(D) 3

24) Unit step response of a linear time invariant (LTI) system is given by $y(t) = (1 - e^{-2t})u(t)$. Assuming zero initial condition, the transfer function of the system is-

(A) $\frac{1}{s+1}$

(B) $\frac{2}{(s+1)(s+2)}$

(C) $\frac{1}{s+2}$

(D) $\frac{2}{s+2}$