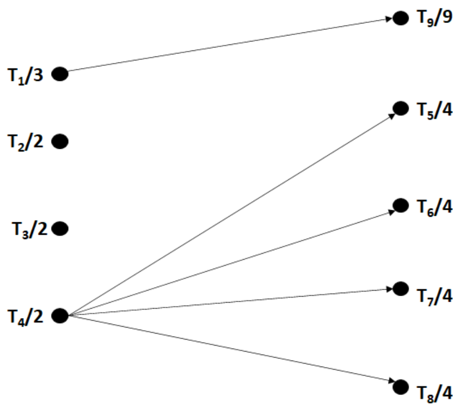
Tutorial 3
Nirma University
Institute of Technology
2CS305 – Discrete Mathematics
Tutorial 3

Topic: Job Scheduling problem

1. The procedure of scheduling a set of tasks according to the rule of never leaving a processor idle intentionally and assigning the priorities to the tasks and schedule a task that has the highest priority among all executable tasks at any time instant. The set of tasks is to be executed on a computing system with three processors. The priorities are assigned in the decreasing order of the tasks T1, T2, T3, T4, T5, T6, T7, T8, T9.

    a) Construct the corresponding schedule.
    b) Suppose we remove the arrows between T4 and T5 and between T4 and T6 in the Figure, construct the corresponding schedule.
    c) Suppose we reduce the execution time of each task by 1. Construct the corresponding schedule.
    d) Suppose we execute the set of tasks on a computing system with four processors. Construct the corresponding schedule.



Topic: Functions

2. Is relation a function or vice versa? Justify your answer. Let A = {a, b, c, d}, B = {p, q, r, s} denote sets. R: A –> B, R is a function from A to B. Then which of the following relations are not functions?
    a. {(a, p) (b, q) (c, r)}
    b. {(a, p) (b, q) (c, s) (d, r)}
    c. {(a, p) (b, s) (b, r) (c, q)}

3. Find the domain and range of these functions.
    a. the function that assigns to each pair of positive integers the first integer of the pair
    b. the function that assigns to each positive integer its largest decimal digit
    c. the function that assigns to a bit string the number of ones minus the number of zeros in the string
    d. the function that assigns to each positive integer the largest integer not exceeding the square root of the integer
    e. the function that assigns to a bit string the longest string of ones in the string.

4. Let f(x) = log|x| and g(x) = sin x. If A is the range of f(g(x)) and B is the range of g(f(x)). What is the value of A ∩ B?

5. Draw the graph of the function f (x) = $\lfloor x \rfloor + \lfloor x/2 \rfloor$ from R to R.

6. Find f ◦g and g ◦f , where f (x) = $x^2 + 1$ and g(x) =x + 2, are functions from R to R.

7. Determine if each function is one-to-one.
   a. To each person on the earth assign the number which corresponds to his age. n
   b. To each country in the world assign the latitude and longitude of its capital. y
   c. To each book written by only one author assign the author. n
   d. To each country in the world which has a prime minister assign its prime minister y

8. Determine whether each of these functions from Z to Z is one-to-one.
   a. f (n) = n –1 y
   b. f (n) = $n^2 + 1$ n
   c. f (n) = n^3 y
   d. f (n) = $\lfloor n/2 \rfloor$ n

9. Determine whether each of these functions is a bijection from R to R.
   a. f (x) = 2x + 1 y
   b. f (x) = $x^2 + 1$ n
   c. f (x) = x^3 y
   d. f (x) = $(x^2 + 1)/(x^2 + 2)$ n

10. If f_1: R →R; and f_2: R →R, where R is the set of real numbers. Compute $f_1 \circ f_2$ and $f_2 \circ f_1$ where $f_1(y)= y^2-2$ and $f_2(y)=y+4$. Determine whether these functions are one-to-one, onto or one-to-one onto? onto

11. If the function f and g are defined as f(x) = e^x and g(x)=3x-2, where f: R →R and g: R →R, then find the function f ˚g and g ˚ f. Also find $(f \circ g)^{-1}$ and $(g \circ f)^{-1}$.

12. Find the inverse of following functions
   a. F(x) = (x-3) x^2
   b. F(x) = -x-4 -y-4
   c. F(x) = ln(x+2)-3

13. Find the inverse of the quadratic functions
   a. F(x) = (x-3)2 +3, for x≥ 3
   b. F(x) = -x2 +4x -4, for x≤2

14. Let g(x) = 1+x-[x] and f(x) = $\{-1 \quad x < 0\ ;\ 0 \quad x = 0\ ;\ 1 \quad x > 0\ \}$. Then for all x find f(g(x)).

Topic: Pigeonhole Principle

15. Show that if 10 colors are used to paint 101 buildings, then at least 11 building have the same colour.
16. Show that if any seven numbers are chosen from 1 to 11, then two of them will add up to 12.
17. Show that if 11 numbers are chosen from the set {1, 2, ….20}, then one of them will be a multiple of another.

18. Show that if any 30 people are selected from any group, then a subset of 5 can be chosen such that all 5 were born on the same day of the week.
19. If n+1 numbers are chosen from {1, 2, …2n}, show that one must divide the other.
20. There are 35,000 students at the university. Each of them takes four (distinct) courses. The university offers 999 different courses. When a student who has taken a course in discrete mathematics learned that the largest classroom holds only 135 students, she realized that there is a problem. What is the problem?
21. From the integers 1-200, 101 of them are chosen arbitrarily. Show that, among the chosen numbers, there exist two such that one divides another.