NIRMA UNIVERSITY
Institute of Technology, Ahmedabad
2CS305- Discrete Mathematics
Academic Year: 2022-2023

Tutorial 4
Propositional and Predicate Logic

Section 1- Propositional Logic

1.Let p and q be the propositions
p :You drive over 65 miles per hour.   q :You get a speeding ticket.
Write these propositions using p and q and logical connectives (including negations).
a) You drive over 65 miles per hour, but you do not get a speeding ticket.
c) You will get a speeding ticket only if you drive over 65 miles per hour.
c) Driving over 65 miles per hour is necessary and sufficient for getting a speeding ticket.

2. Construct a truth table for each of these compound propositions.
a) $(p \lor q) \to (p \oplus q)$ **b)** $(p \oplus q) \to (p \land q)$

3. Explain, without using a truth table, why $(p \lor q \lor r) \land (\neg p \lor \neg q \lor \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

4. Determine whether each of these compound propositions is satisfiable.
$(p \lor \neg q) \land (\neg p \lor q) \land (\neg p \lor \neg q)$

5. Show that $\neg (p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

6. Show that $(p \to q) \land (q \to r) \to (p \to r)$ is a tautology.

7. Show that the argument form with premises $(p \land t) \to (r \lor s)$, $q \to (u \land t)$, $u \to p$, and $\neg s$ and conclusion $q \to r$ is valid

8. What rule of inference is used in each of these arguments?
a) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

b) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

Topic-First Order Logic

Q.1 What is the first order predicate calculus statement equivalent to the following?

Every teacher is liked by some student

(A) ∀(x) [teacher (x) → ∃ (y) [student (y) → likes (y, x)]]

(B) ∀ (x) [teacher (x) → ∃ (y) [student (y) ^ likes (y, x)]]

(C) ∃ (y) ∀ (x) [teacher (x) → [student (y) ^ likes (y, x)]]

(D) ∀ (x) [teacher (x) ^ ∃ (y) [student (y) → likes (y, x)]]

Q.2 Some oranges in the box are more ripen than apples.
Write down the above statement by using Quantifiers.

Q.3 Consider the following:

I. $\neg\forall x(P(x))$ II. $\neg\exists x(P(x))$

III. $\neg\exists x(\neg P(x))$ IV. $\exists x(\neg P(x))$

Which of the above two are equivalent?

A. I and III

B. I and IV

C. II and III

D. II and IV

Q.4 Let Graph(x) be a predicate which denotes that x is a graph. Let Connected(x) be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement: "Not every graph is connected"?

(A) $\neg\forall x(Graph(x) \Rightarrow Connected(x))$ (B) $\exists x(Graph(x) \wedge \neg Connected(x))$

(C) $\neg\forall x(\neg Graph(x) \vee Connected(x))$ (D) $\forall x(Graph(x) \Rightarrow \neg Connected(x))$

Q.5 Which one of the first order predicate calculus statements given below correctly express the following English statement?

Tigers and lions attack if they are hungry or threatened

(A) $\forall x\left[(\text{tiger}(x) \wedge \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)\}\right]$

(B) $\forall x\left[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \wedge \text{attacks}(x)\}\right]$

(C) $\forall x\left[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{\text{attacks}(x) \rightarrow (\text{hungry}(x) \vee \text{threatened}(x))\}\right]$

(D) $\forall x\left[(\text{tiger}(x) \vee \text{lion}(x)) \rightarrow \{(\text{hungry}(x) \vee \text{threatened}(x)) \rightarrow \text{attacks}(x)\}\right]$

Q. 6 Which one of the following is the most appropriate logical formula to represent the statement? "Gold and silver ornaments are precious". The following notations are used: G(x): x is a gold ornament S(x): x is a silver ornament P(x): x is precious

A. $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$
B. $\forall x((G(x) \wedge S(x)) \rightarrow P(x))$
C. $\exists x((G(x) \wedge S(x)) \rightarrow P(x)$
D. $\forall x((G(x) \vee S(x)) \rightarrow P(x))$

Q.7 Suppose U is the power set of the set S = {1,2,3,4,5,6}. For any T ∈ U, let |T| denote the number of elements in T and T′ denote the complement of T.
For any T, R ∈ U, let T\R be the set of all elements in T which are not in R. Which one of the following is true?

(A) $\forall X \in U(|X| = |X'|)$

(B) $\exists X \in U \exists Y \in U(|X| = 5, |Y| = 5 \text{ and } X \cap Y = \phi)$

(C) $\forall X \in U \forall Y \in U(|X| = 2, |Y| = 3 \text{ and } X \setminus Y = \phi)$

(D) $\forall X \in U \forall Y \in U(X \setminus Y = Y' \setminus X')$

Q. 8 **.**Let P(x), Q(x), and R(x) be the statements “x is a professor,” “x is ignorant,” and “x is vain,” respectively.
Express each of these statements using quantifiers; logical connectives; and P(x), Q(x), and R(x), where the domain consists of all people.

a) No professors are ignorant. **b)** All ignorant people are vain.
c) No professors are vain.
d) Does (c) follow from (a) and (b)?