Tutorial 2
Topic: Relations

1. For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is anti-symmetric, and whether it is transitive.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}

c) {(2, 4), (4, 2)}

d) {(1, 2), (2, 3), (3, 4)}

e) {(1, 1), (2, 2), (3, 3), (4, 4)}

f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

2. Give an example of a relation on a set that is

a) both symmetric and antisymmetric.
b) neither symmetric nor antisymmetric

3. Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

a) a is taller than b.
b) a and b were born on the same day.
c) a has the same first name as b.
d) a and b have a common grandparent.

4. Let S be the set of all strings of English letters. Determine whether these relations are reflexive, irreflexive, symmetric, anti-symmetric, and/or transitive.

a) R1 = {(a,b) | a and b have no letters in common}
b) R2 = {(a,b) | a and b are not the same length}
c) R3 = {(a,b) | a is longer than b}

5. Determine whether the relation R on the set of all Web pages is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if

a) everyone who has visited Web page a has also visited Web page b.
b) there are no common links found on both Web page a and Web page b.
c) there is at least one common link on Web page a and Web page b.
d) there is a Web page that includes links to both Web page a and Web page b.

6. Let R be the relation on the set of all URLs (or Web addresses) such that xRy if and only if the Web page at *x* is the same as the Web page at *y*. Show that R is an equivalence relation.

7. How many reflexive, symmetric, and antisymmetric relations exist on a set of n elements.

8. Let R be an reflexive relation on a set A. Show that R is an equivalence relation if and only if (a,b) and (a,c) are in R implies that (b,c) is in R.