REAL NUMBERS

Contents

In this chapter we will discuss following topics
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(6) Unit digit of any power
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Euclid's division Lemma

Given two positive integers a and b, there exist unique integers q and r satisfying a = bq + r where $0 \le r < b$.
For instance,
$11 = 2 \times 5 + 1$
Here a = 11, b = 2, q = 5, r = 1
where $0 \le r < 2$
Here, a is the dividend, q is the quotient, b is the divisor and r is the remainder. Here q or r can be zero also.
For instance
$15 = 17 \times 0 + 15$ (here q is 0)
$10 = 2 \times 5 + 0$ (here r is zero)

Euclid's division algorithm

Euclid's division algorithm is a technique to find HCF (highest common factor) of 2 positive integers.
Euclid's division algorithm can be stated as follows:
HCF of 2 positive integers c and d (c > d) can be obtained by following these steps:

Steps 1: Apply Euclid's division lemma to c and d. So, we find q and r such that
$c = qd + r \qquad 0 \le r < d.$

Steps 2: If r = 0, then d is the HCF of c and d. If $r \neq 0$, then apply division lemma to d and r.

Steps 3: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.
We illustrate the above algorithm with the help of an example:

Illustration 1:

Obtain HCF of 12056 and 420.

Solution: Step1 : Since 12056 > 420, we apply division lemma to 12056 and 420 to get $12056 = 420 \times 28 + 296$.
Since the remainder $296 \neq 0$, we apply the division lemma to 420 and 296.
Step 2: $420 = 296 \times 1 + 124$.
Again, remainder is not zero.
$\therefore$ division lemma is again applied to 296 and 124.
Step 3: $296 = 124 \times 2 + 48$
Now, 48 becomes the new divisor and 124 the new dividend.

Step 4: $124 = 48 \times 2 + 28$
Step 5: $48 = 28 \times 1 + 20$
Step 6: $28 = 20 \times 1 + 8$
Step 7: $20 = 8 \times 2 + 4$
Step 8: $8 = 4 \times 2 + 0$
Here, the remainder becomes zero, so we stop the algorithm here. Since, the divisor at this stage is 4, HCF of 12056 and 420 is 4.
Notice that 4 = HCF (8, 4) = HCF (20, 8) = HCF (28, 20)
= HCF (48, 28) = HCF (124, 48)
= HCF (296, 124) = HCF(420, 296) = HCF (12056, 420).

☞ ***Illustration 2:***

Show that any positive odd integer is of the form of $8q + 1$ or $8q + 3$ or $8q + 5$ or $8q + 7$, where q is some integer.

Solution: Let 'a' be any positive odd integer and b = 8. If we apply Euclid's division lemma, r can be 0, 1, 2, 3, 4, 5, 6 or 7.
i.e., a can be $8q$, $8q + 1$, $8q + 2$, $8q + 3$, $8q + 4$, $8q + 5$, $8q + 6$, $8q + 7$.
Since a is odd, it cannot be of the form $8q$, $8q + 2$, $8q + 4$, $8q + 6$.
$\therefore$ any positive odd integer will be of the form of $8q + 1$, $8q + 3$, $8q + 5$ or $8q + 7$.

Exercise 1:

(i) Show that cube of any positive integer is of the form of $9m$, $9m + 1$ or $9m + 8$.
(ii) Use Euclid's division algorithm to find HCF of 460 and 20468.
(iii) Show that $n^2 - 1$ is divisible by 8, if n is an odd positive integer.
(iv) Prove that product of every three consecutive integers is always divisible by 3 as well as 2.

The Fundamental Theorem of Arithmetic

The theorem states that every composite number can be expressed as a product of prime numbers, and their factorization is unique, apart from the order in which the prime factors occur.
i.e., given any composite number, there is one and only one way to write it as a product of primes.

☞ ***Illustration 3:***

Consider the numbers of the form 3^n(where n is a natural number). Check whether there is any value of n for which 3^n ends with 2.

Solution: If a number of the form 3^n ends with 2, then it will be divisible by 2. i.e., the prime number 2. This is not possible because prime factorization of 3^n contains only 3. The uniqueness property of the fundamental theorem of Arithmetic guarantees that there are no other primes in the factorization of 3^n.
Hence, there is no natural number n for which 3^n ends with 2.

HCF and LCM of two positive integers (Using Fundamental Theorem of Arithmetic):

We can find HCF and LCM of two positive integers using the prime factorization method. Now, the prime factorization of a positive integer is nothing but its all about using the Fundamental Theorem of Arithmetic.

☞ *Illustration 4:*

Find the HCF and LCM of 35 and 115 by prime factorization method.

Solution: $35 = 5 \times 7$

$115 = 5 \times 23$
As studied in earlier classes
HCF (35, 115) = 5
LCM (35, 115) = $5 \times 7 \times 23 = 805$.
Also, for two positive numbers a and b,
HCF (a, b) × LCM (a, b) = a × b.
We can use the above result to find one unknown quantity out of four if three of them are known.

☞ *Illustration 5:*

Find HCF of 315 and 462 and hence find their LCM.

Solution: $315 = 3^2 \times 7 \times 5$

$462 = 2 \times 3 \times 7 \times 11$
HCF (315, 462) = $3 \times 7 = 21$.
Now, {product of two numbers = their HCF × their LCM}
HCF × LCM = 315×462

$$\text{LCM} = \frac{315 \times 462}{21} = 6930$$

∴ LCM (315, 462) = 6930.

Exercise 2:

(i) *Find HCF and LCM of the following pairs of numbers:*
(a) *98 and 70;*
(b) *112 and 64;*

(ii) *There are 3 alarms A, B, and C. A rings after every 3 hrs, B rings after every 4 hrs and C rings after every 6 hrs. Find the time interval after which the 3 alarms ring together after ringing once together.*

(iii) *Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.*

Decimal Representation of Rational Numbers

Any rational number can be represented either in terminating decimal or non-terminating recurring decimals. In case of terminating decimal representation, the remainder becomes zero.

It can be seen that any rational number, whose decimal expansion terminates, can be expressed as a rational number whose denominator is a multiple of 10. For e.g.,

$$0.78 = \frac{78}{10^2} \qquad 0.3027 = \frac{3027}{10^4} \text{ and so on.}$$

The only prime factorization of 10 is 2 × 5 i.e., the denominator is always of the form 2^a5^b where a and b are non negative integers. From the above discussion, we arrive at the following result:
Theorem: If 'a' is a rational number whose decimal expansion terminates. Then 'a' can be expressed in the form p/q (form of a rational number) where q is always of the form 2^m5^n (m and n are non negative integers).

The converse of this theorem is also true i.e., if $x = \frac{p}{q}$ is a rational number such that q is of the form 2^m5^n (m and n are non negative integers), then x has a terminating decimal representation. The decimal representation of the remaining rational numbers of the form $\frac{p}{q}$, (where q cannot be expressed as $2^m \times 5^n$) is non-terminating repeating.

Decimal Representation of an irrational number

Any number is said to be irrational if its decimal representation is non-terminating and non-recurring. We can get so many decimal numbers which represent some or other irrational numbers e.g.

(a) 0.323323332 ... (b) 0.0200200020 ...
(c) 0.50000500005000005 ... (d) 0.234917643....
(e) 0.1332148631142 .... (f) 3.141761453...

All the above six decimal numbers represent a particular irrational number.

Illustration 6:

Which of the following rational numbers can be represented as terminating decimals?

(a) 3/5 *(b) 2/13* *(c) 7/20*

Solution: We know that a rational number can be represented in the form of p/q, where p and q do not have any common divisor. Then p/q can be represented as terminating decimal if q has only divisor 2 or 5 or both.

(a). $\frac{3}{5} = \frac{p}{q}$ $\because$ $q = 5 = 5^1$

$\therefore$ 3/5 can be represented as terminating decimals.

(b). $\frac{2}{13} = \frac{p}{q}$ here q = 13.

Since divisor of q is only 13, hence $\frac{2}{13}$ can not be represented as terminating decimal

(c). $\frac{7}{20} = \frac{p}{q}$

$q = 20 = 4 \times 5 = 2^2 \times 5$

Hence, divisor of q can be represented in terms of 2 and 5

Hence $\frac{7}{20}$ can be represented as terminating decimal.

UNIT DIGIT OF ANY POWER

1. The unit digit of any power of 0, 1, 5 and 6 is 0, 1, 5 and 6 respectively

For example:

(i) The unit digit of $(20)^{53}$ is 0.
(ii) The unit digit of $(81)^{43}$ is 1
(iii) The unit digit of $(65)^{123}$ is 5
(iv) The unit digit of $(216)^{273}$ is 6.

2. The unit digit of $(4)^{\text{odd number}}$ is 4
The unit digit of $(4)^{\text{even number}}$ is 6
The unit digit of $(9)^{\text{odd number}}$ is 9

The unit digit of $(9)^{\text{even number}}$ is 1

For example:

The unit digit of $(34)^{57}$ is 4
The unit digit of $(29)^{32}$ is 1

3. The unit digit of the power of 2, 3, 7 and 8 follow a cyclic pattern i.e. they repeat after every 4 steps therefore to find the unit digit of any power of 2, 3, 7 and 8 we first divide the power by 4 and look at the remainder, the remainder can be 0, 1, 2 or 3 and to find the unit digit use the following table.

Remainder	Unit digit of 2	Unit digit of 3	Unit digit of 7	Unit digit of 8
1	$2(2^1 = 2)$	$3(3^1 = 3)$	$7(7^1 = 7)$	$8(8^1 = 8)$
2	$4(2^2 = 4)$	$9(3^2 = 9)$	$9(7^2 = 49)$	$4(8^2 = 64)$
3	$8(2^3 = 8)$	$7(3^3 = 27)$	$3(7^3 = 343)$	$2(8^3 = 512)$
0	$6(2^4 = 16)$	$1(3^4 = 81)$	$1(7^4 = 2401)$	$6(8^4 = 4096)$

For example:
The unit digit of $(72)^{431}$ is ……
Step – 1 $431 \div 4$ remainder is 3
Step – 2 $2^3 = 8$ $\therefore$ unit digit of $(72)^{431}$ is 8
The unit digit of $(128)^{1024}$ is
Step – 1 $1024 \div 4$, remainder is o
Step – 2 $8^4 = 4096$ $\therefore$ unit digit of $(128)^{1024}$ is 6

Illustration 7:

Find the unit digit of $(14)^{124} \times (29)^{123}$

Solution: Unit digit of $(14)^{124} \times (29)^{123}$ = unit digit of $(4^{124} \times 9^{123)}$
The unit digit of $(4)^{124}$ is 6
The unit digit of $(9)^{123}$ is 9
Now $6 \times 9 = 5\underline{4}$
$\therefore$ Unit digit of $(14)^{124} \times (29)^{123}$ is 4.

FINDING THE REMAINDER IN DIVISIONS INVOLVING POWER OF NUMBERS

The method to find remainder in divisions involving power of numbers is explained below with the help of an example.

Illustration 8:

Find the remainder of 3^{43} when divided by 4.

Solution: Firstly we find the pattern that the remainder follow when the successive powers of 3 are divided by 4.
Remainder of 3^1 when divided by 4 = 3
Remainder of 3^2 when divided by 4 = 1
Remainder of 3^3 when divided by 4 = 3
$\therefore$ The remainder repeats after 2 steps and it is 3 when exponent of 3 is odd and it is 1 when exponent of 3 is even. $\therefore$ Requited remainder = 3.

RULES PERTAINING TO $a^n + b^n$ or $a^n - b^n$:

Rule 1– $a^n - b^n$ is always divisible by $a - b$ n is even or odd.
Rule 2– $a^n - b^n$ is divisible by $a + b$ if n is even.
Rule 3– $a^n + b^n$ is divisible by $a + b$ if n is odd.

Illustration 9:

Find the remainder when $(23)^{17} + (15)^{17}$ is divided by 19.

Solution: By Rule – 3 $(23)^{17} + (15)^{17}$ is divisible by $23 + 15 = 38$ and 38 is also divisible by 19.
$\therefore$ when $(23)^{17} + (15)^{17}$ is divisible by 19 the remainder will be zero.

KEY TO EXERCISES

Exercise 1:

(ii) 4

Exercise 2:

(i) (a) 14, 490
(b) 16, 448

(ii) 12 hrs

(iii) 17

FORMULAE AND CONCEPTS AT A GLANCE

- Given two positive integers a and b there exists unique integers q and r such that $a = bq + r$ where $0 \leq r < b$.
- Every composite number can be uniquely expressed as a product of prime number.
- The decimal representation of a rational number is either terminating or non-terminating but recurring.
- The decimal representation of irrational numbers is non-terminating and non-repeating.
- **Even and odd numbers**

  Multiples of 2 are called even numbers. e.g., 10, 20, 26 etc. are even numbers. Numbers which are not multiples of 2 are called odd numbers.

  (i) Any odd numbers can be written in the form of $2n + 1$ or $2n - 1$ (where n is a natural number) and any even number can be written as $2n$.

  (ii) Sum of

  2 even numbers even

  2 odd numbers even

  (iii) Difference of

  2 odd numbers even

  2 even numbers even

SOLVED PROBLEMS

SUBJECTIVE

Section – A

Problem 1: *Find HCF of 3675 and 42 using Euclid's division algorithm.*

Solution: Step 1: $3675 = 42 \times 87 + 21$
Remainder is not zero. Now 42 becomes the new dividend and 21 becomes the new divisor.
Step 2: $42 = 21 \times 2 + 0$
Remainder becomes zero here
$\therefore$ the divisor at this step is the HCF of 3675 and 42
$\therefore$ HCF (3675, 42) = 21.

Problem 2: *Using Euclid's division lemma, prove that difference of squares of 2 odd natural numbers is a multiple of 8.*

Solution: Let the 2 odd natural numbers be m and n.
By division lemma,
$m = 2u + 1$, $n = 2v + 1$
$m^2 - n^2 = (m - n)(m + n)$
$= [2u + 1 - (2v + 1)][2u + 1 + 2v + 1]$
$= 4(u - v)(u + v + 1)$ ...(1)
Now, if u and v are both even or both odd, then $u - v$ will also be even and hence (1) is a multiple of 8.
Otherwise, when one of u and v is even and one is odd, then $u + v + 1$ will be even and hence (1) is a multiple of 8.

Problem 3: *Check whether 7^n can end with the digit '0' for any natural number n.*

Solution: If 7^n ends with a zero, then 2 or 5 must be a factor of 7^n. But by fundamental theorem of arithmetic, 7^n has only factor 7.
$\therefore$ 7^n cannot end with a zero for any natural number n.

Problem 4: *Find LCM and HCF of the following numbers using prime factorization:*
(a) 14, 22, 32 *(b) 18, 81, 90*

Solution:
(a) $14 = 2 \times 7$
$22 = 2 \times 11$
$32 = 2^5$
HCF = 2
LCM = $32 \times 11 \times 7 = 2464$
(b) 18, 81, 90
$18 = 2 \times 3^2$
$81 = 3^4$
$90 = 2 \times 3^2 \times 5$
HCF = $3^2 = 9$
LCM = $3^4 \times 2 \times 5 = 810$.

Problem 5: Given that HCF (189, 144) = 9, find their LCM.

Solution: HCF × LCM = product of 2 numbers
$9 \times \text{LCM} = 189 \times 144$

$\therefore$ LCM = $\frac{189\times144}{9}=3024$

Problem 6: *If a is an irrational number, then prove that –a is also an irrational number.*

Solution: Let a be an irrational number. Say, if –a is not an irrational number then –a is a rational number. Now, –(–a) = a is an irrational number. But we know that if (–a) is a rational number then –(–a) = a will also be a rational number, which is a contradiction. Hence, our supposition is wrong. Therefore –a is an irrational number.

Problem 7: *Prove that $2+\sqrt{2}$ is not a rational number.*

Solution: Let $a=2+\sqrt{2}$ be a rational number. Squaring both sides,

$a^2=4+2+2\cdot2\cdot\sqrt{2}=6+4\sqrt{2}$

$\Rightarrow\ \frac{a^2-6}{4}=\sqrt{2}$

$\because$ a is a rational number $\therefore$ a^2 is also a rational number

$\therefore\ \frac{a^2-6}{4}$ is also a rational number.

But we know that an irrational number can never be equal to a rational number.

Hence $\sqrt{2}=\frac{a^2-6}{4}$ is not possible.

Because $\sqrt{2}$ is an irrational number, our supposition $2+\sqrt{2}$ is a rational number is wrong. Hence $2+\sqrt{2}$ is an irrational number.

Problem 8: *Without performing the long division process state whether the following rational numbers will have terminating decimal expansion or a non-terminating repeating decimal expansion*

(i) $\frac{17}{8}$ *(ii)* $\frac{64}{455}$ *(iii)* $\frac{29}{343}$ *(iv)* $\frac{15}{1600}$ *(v)* $\frac{13}{3125}$ *(vi)* $\frac{23}{2^3 5^2}$

Solution: (i) $\frac{17}{8}=\frac{17}{2^3}$

So denominator 8 is of form $2^m\times5^n$, where m, n are non-negative integers.

Hence $\frac{17}{8}$ has a terminating decimal expansion.

(ii) $\frac{64}{455}=\frac{64}{5\times7\times13}$ Non-terminating repeating decimal expansion

(iii) $\frac{29}{343}=\frac{29}{3^5}$ Non-terminating repeating decimal expansion

(iv) $\frac{15}{1600}=\frac{3}{320}=\frac{3}{64\times5}=\frac{3}{2^6\times5^1}$ Terminating decimal expansion

(v) $\frac{13}{3125}=\frac{13}{5^5}$ Terminating decimal expansion

(vi) $\frac{23}{2^3 5^2}$ Terminating decimal expansion

Problem 9: *Let a, b, c and d be positive rationals such that* $a+\sqrt{b}=c+\sqrt{d}$, *then show that either a = c and b = d.*
or b and d are squares of rationals.

Solution:

Case I : a = c

As $a+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow \sqrt{b}=\sqrt{d}$

$\Rightarrow b = d$

Case II: $a \neq c$

then, there exists a rational number
x such that a = c + x

Now $a+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow c+x+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow x+\sqrt{b}=\sqrt{d}$ ...(i)

$\Rightarrow x^2+b+2\sqrt{b}x=d$ (on squaring both sides)

$\Rightarrow \sqrt{b}=\dfrac{d-b-x^2}{2x}$ = rational number

Hence b is square of a rational number
Again from (i)

$\sqrt{d}=\sqrt{b}+x$

$\because$ b is square of a rational number
Hence d is also square of a rational number.

Problem 10: *Prove that* $\sqrt{5}+\sqrt{3}$ *is an irrational number.*

Solution:

Let us suppose that $\sqrt{5}+\sqrt{3}$ is a rational number

Then $\sqrt{5}+\sqrt{3}=\dfrac{a}{b}$, where $b \neq 0$ and a, b are integers

$$\Rightarrow \left(\sqrt{5}\right)^2=\left(\frac{a}{b}-\sqrt{3}\right)^2$$

$$\Rightarrow 5=\frac{a^2+3b^2-2ab\sqrt{3}}{b^2}$$

$$\Rightarrow \frac{a^2+3b^2-5b^2}{2ab}=\sqrt{3}$$

$$\Rightarrow \frac{a^2-2b^2}{2ab}=\sqrt{3}$$

Since a and b are integers hence $\dfrac{a^2-2b^2}{2ab}$ must be a rational number which is a contradiction.

Hence $\sqrt{5}+\sqrt{3}$ is irrational.

Problem 11: *Which is greater?.*
(i) $\sqrt{2}$ or $\sqrt[3]{3}$, *(ii)* $\sqrt{3}$ or $\sqrt[4]{10}$, *(iii)* $\sqrt[4]{5}$ or $\sqrt[3]{4}$

Solution:

(i) LCM of 2 and 3 is 6

Thus, $\sqrt{2}=\sqrt[6]{2^3}=\sqrt[6]{8}$

and $\sqrt[3]{3}=\sqrt[6]{3^2}=\sqrt[6]{9}$

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$\therefore \sqrt[6]{9} > \sqrt{2}$

Hence, $\sqrt[3]{3} > \sqrt{2}$

(ii) LCM of 1 and 4 is 4

Thus, $\sqrt{3} = \sqrt[4]{3^2} = \sqrt[4]{9}$

and $\sqrt[4]{10} = \sqrt[4]{10}$

$\Rightarrow \sqrt[4]{10} > \sqrt{3}$

(iii) LCM of 4 and 3 is 12

Thus, $\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

and $\sqrt[4]{3} = \sqrt[12]{4^4} = \sqrt[12]{256}$

hence, $\sqrt[3]{4} > \sqrt[4]{5}$

Problem 12: *Simplify and express the result in its convert form* $\sqrt[3]{4} \times \sqrt[3]{22}$.

Solution: $\sqrt[3]{4} \times \sqrt[3]{22} = \sqrt[3]{4 \times 22} = \sqrt[3]{2^3 \times 11} = 2\sqrt[3]{11}$

Problem 13: $\left(3\sqrt{5} - 5\sqrt{2}\right)\left(4\sqrt{5} + 3\sqrt{2}\right)$ *simplify it.*

Solution: $\left(3\sqrt{5} - 5\sqrt{2}\right)\left(4\sqrt{5} + 3\sqrt{2}\right)$

$= [12 \times \left(\sqrt{5}\right)^2 + 9\sqrt{5} \times \sqrt{2} - 20\sqrt{2} \times \sqrt{5} - 15\left(\sqrt{5}\right)^2]$

$= (12 \times 5 + 9\sqrt{10} - 20\sqrt{10} - 15 \times 2)$

$= (60 - 30) + (9 - 20)\sqrt{10} = 30 - 11\sqrt{10}$

Problem 14: *Represent* $0.\overline{57}$ *in the form of* $\frac{p}{q}$.

Solution: $\frac{p}{q} = 0.\overline{57}$ ...*(i)*

$100\frac{p}{q} = 57.\overline{57}$ ...(ii)

Subtract (i) by (ii)

$99\frac{p}{q} = 57$

$\frac{p}{q} = \frac{57}{99}$

Problem 15: *Which is greater* $\sqrt{3}$ *or* $\sqrt[3]{5}$ *?*

Solution: LCM of 2 and 3 is 6

Thus $\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[6]{25}$

$\Rightarrow \sqrt{3} > \sqrt[3]{5}$

Section – B

Problem 1: *If* $y = 3^{\frac{1}{3}} + 3$ *then value of* $y^3 - 9y^2 + 27y$.

Solution: Given $y = 3^{\frac{1}{3}} + 3$

$\Rightarrow y - 3 = 3^{\frac{1}{3}}$

Taking the cubes on both sides

$\Rightarrow (y-3)^3 = \left(3^{\frac{1}{3}}\right)^3$

$\Rightarrow y^3 - 9y^2 + 27y - 27 = 3$

$\Rightarrow y^3 - 9y^2 + 27y = 30$

Problem 2: *Simplify this* $\sqrt{\sqrt[2]{2^x \sqrt[x^2]{3^{x^3} \sqrt[x^3]{6^{x^6} \sqrt[x^4]{9^{x^{10}}}}}}}$

Solution: $\sqrt{2^{\frac{x}{x}} \times 3^{\frac{x^3}{x^3}} \times 6^{\frac{x^6}{x^6}} \times 9^{\frac{x^{10}}{x^{10}}}}$

$\sqrt{2 \times 3 \times 6 \times 9}$

$= 18$

Problem 3: $\sqrt{7+2\sqrt{6}} + \sqrt{7-2\sqrt{6}}$ *simplify to* $p + \sqrt{q}$ *form*

Solution: $\sqrt{7+2\sqrt{6}} = \sqrt{6+1+2\sqrt{6}} = \sqrt{\left(\sqrt{6}+1\right)^2}$

$\sqrt{7-2\sqrt{6}} = \sqrt{6+1-2\sqrt{6}} = \sqrt{\left(\sqrt{6}-1\right)^2}$

By adding both

$= \sqrt{6} + 1 + \sqrt{6} - 1 = 2\sqrt{6}$

Problem 4: *If* $x = 1 + 5^{\frac{1}{3}} + 5^{\frac{2}{3}}$, *then find the value of* $x^3 - 3x^2 - 12x - 16$.

Solution: $x = 1 + 5^{\frac{1}{3}} + 5^{\frac{2}{3}}$

$x - 1 = 5^{\frac{1}{3}} + 5^{\frac{2}{3}}$

cubing both sides we get

$(x-1)^3 = 5 + 5^2 + 3 \times 5(x-1)$

$x^3 - 1 - 3x(x-1) = 30 + 15x - 15$

$x^3 - 3x^2 - 12x - 16 = 0$

Problem 5: *If $p = 7 - 4\sqrt{3}$ then value of $\frac{p^2+1}{7p}$.*

Solution: If $p = 7 - 4\sqrt{3}$

$$\frac{1}{p} = \frac{1}{7-4\sqrt{3}}$$

by rationalizing

$$\frac{1}{p} = \frac{7+4\sqrt{3}}{49-48}$$

$$\therefore \frac{1}{p} = 7+4\sqrt{3}$$

$$\Rightarrow p + \frac{1}{p} = 7-4\sqrt{3}+7+4\sqrt{3}$$

$$\because p+\frac{1}{p} = 14$$

Now, $\frac{p^2+1}{p} = 14$

$$\Rightarrow \frac{p^2+1}{7p} = 2$$

OBJECTIVE

Level – I

Multiple Choice Questions (Single Option Correct)

Problem 1: *Given that HCF (306, 657) = 9, find LCM (306, 657)*
(A) 22333 *(B) 22338*
(C) 33228 *(D) none of these*

Solution: **(B)** We know, HCF (a, b) × LCM (a, b) = a × b
i.e., 9 × LCM (306, 657) = 306 × 657
i.e., LCM (306, 657) = $\frac{306 \times 657}{9} = 22338$.

Problem 2: *Find the number of zeroe's in the end of 6^n for any natural number n.*
(A) 10 *(B) 100*
(C) 0 *(D) none of these*

Solution: **(C)** If 6^n ends with 0, then 2 and 5 must be a factor of 6^n. But by fundamental theorem of arithmetic, 6^n has factors 2 and 3.
$\therefore$ 6^n cannot end with 0, for any $n \in N$.

Problem 3: *By applying division lemma, find out HCF (36168, 210).*
(A) 8 *(B) 9*
(C) 6 *(D) none of above*

Solution: **(C)** 36168 = 210 × 172 + 48
210 = 48 × 4 + 18
48 = 18 × 2 + 12
18 = 12 × 1 + 6
12 = 6 × 2 + 0
$\because$ remainder, 0 came at the last step.
$\therefore$ HCF(36168, 210) = 6.

Problem 4: *As express each of the following numbers as a product of its prime factors find minimum value of 'n' in 7^n*
60025, 756, 6615
(A) 1 *(B) 2*
(C) 4 *(D) none of these*

Solution: **(A)** $60025 = 5^2 \times 7^4$
$756 = 2^2 \times 3^3 \times 7$
$6615 = 3^3 \times 5 \times 7^2$

Problem 5: *As express each of the following numbers as a product of its prime factors find HCF of the following*
60025, 756, 6615
(A) 35 *(B) 120*
(C) 7 *(D) none of these*

Solution: **(C)** $60025 = 5^2 \times 7^4$
$756 = 2^2 \times 3^3 \times 7$
$6615 = 3^3 \times 5 \times 7^2$

Problem 6: *As express each of the following numbers as a product of its prime factors find the maximum value of k in 3^k.*
60025, 756, 6615
(A) 2 *(B) 3*
(C) 1 *(D) none of these*

Solution: **(B)** $60025 = 5^2 \times 7^4$
$756 = 2^2 \times 3^3 \times 7$
$6615 = 3^3 \times 5 \times 7^2$

Problem 7: *Simplify:* $6\sqrt{3}+5\sqrt{12}$.
(A) $3\sqrt{16}$ *(B)* $16\sqrt{3}$
(C) $15\sqrt{12}$ *(D) none of above*

Solution: **(B)** $6\sqrt{3}+5\sqrt{12}$
$= 6\sqrt{3}+5\sqrt{4\times3}$
$= 6\sqrt{3}+10\sqrt{3}$
$= 16\sqrt{3}$

(Fill in the Blanks)

Problem 8: *HCF of 81 and 237 is ___________ .*

Solution: $237 = 81 \times 2 + 75$
$81 = 75 \times 1 + 6$
$75 = 6 \times 12 + 3$
$6 = 3 \times 2 + 0$
Hence HCF = 3.

Problem 9: *If HCF of 210 and 55 is 5, then LCM of 210 and 55 = ___________ .*

Solution: $\text{HCF} \times \text{LCM} = 210 \times 55$
$\Rightarrow 5 \times \text{LCM} = 210 \times 55$
$\Rightarrow \text{LCM} = 210 \times 11 = 2310.$

(True or False)

Problem 10: *Since HCF of 2 and 4 is 2. Hence HCF of 4 and 16 is 4.*

Solution: **(True)** If HCF of a and b is c, then HCF of a^2 and b^2 is c^2.

Problem 11: *If HCF of a and b is h and LCM of a and b is l, then h : a :: b : l.*

Solution: **(True)** HCF × LCM = a × b.

Problem 12: $\frac{21}{175}$ *has a terminating decimal expansion.*

Solution: **(True)** $\frac{21}{175} = \frac{21}{5^2 \times 7} = \frac{3}{5^2}$

Level – II

Problem 1: *HCF of 87 and 99 is*
(A) 9 *(B) 3*
(C) 27 *(D) none of these*

Solution: **(B)** 87 = 3 × 29
99 = 3^2 × 11
∴ HCF = 3.

Problem 2: *A number of the form 6q + 2 (q ∈ N) is*
(A) always odd *(B) always even*
(C) depends on q *(D) none of these*

Solution: **(B)** 6q + 2 = 2(3q + 1)
i.e. 6q + 2 has 2 as a factor
∴ always even.

Problem 3: *LCM of 93 and 102 is*
(A) 3162 *(B) 9486*
(C) 1581 *(D) none of these*

Solution: **(A)** 93 = 3 × 31
102 = 3 × 34
∴ LCM = 3 × 31 × 34
= 3162.

Level – III

Problem 1: *Which of the following rational numbers has a terminating decimal expansion ?*
(A) $\frac{17}{343}$ *(B)* $\frac{41}{63}$
(C) $\frac{81}{95}$ *(D) none of these*

Solution: **(D)** If the denominator has a prime factorization of the form of 2^m5^n, then the rational number has a terminating decimal expansion.
Here, 343 = 7^3
63 = 3^2 × 7
95 = 5 × 19
None of the denominators has factorization of the form 2^m5^n.

Problem 2: *Which of the following real numbers are not rational ?*

(A) 0.30100200040000 … *(B)* $0.\overline{123}$

(C) 0.1231437 *(D) none of these*

Solution: **(A).** Real numbers whose decimal representation is non-terminating and non-repeating are irrational numbers.

Problem 3: *If* $x = \frac{1}{5+2\sqrt{6}}$ *then* $x^2 - 10x + 1 =$

(A) 1 *(B) – 1*

(C) 0 *(D) 10*

Solution: **(C).** $x = \frac{1}{5+2\sqrt{6}}$

$x = \frac{5-2\sqrt{6}}{25-24}$ (by rationalizing)

$\Rightarrow x = 5-2\sqrt{6}$

$x + \frac{1}{x} = 10$

$x^2 - 10x + 1 = 0$

ADD TO YOUR KNOWLEDGE

(i)

- When $(ak \pm 1)^n$ is divided by a then

(a) remainder is 1 if n is even.
(b) remainder is $(a - 1)$ if n is odd

- $a^n + b^n$ is always divisible by $a + b$ if n is odd.
- $a^n - b^n$ is always divisible by $a + b$ if n is even.
- $a^n - b^n$ is always divisible by $a - b$ if n is either even or odd.

(ii)

- LCM of fractions = $\dfrac{\text{LCM of numberators}}{\text{HCF of denominators}}$
- HCF of fractions = $\dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

(iii) Product of two numbers = Product of their LCM and HCF

(iv)

- Greatest number that will divide x, y and z leaving remainder p, q and r respectively.

Required number = HCF of $(x - p)$, $(y - q)$, $(z - r)$

- Greatest number that will divide x, y and z leaving the same remainder in each case.

Required number = HCF of $|x - y|$, $|y - z|$ and $|z - x|$

- Least number which when divided by x, y and z leaves the remainder p, q and r respectively and $x - p = y - q = z - r = k$

Required number = (LCM of x, y and z) - k

- Every even positive integer can be written as $2n$ and every odd integer can be written as $2n + 1$ where n is any whole number.
- Any positive integer can be written as $3n$ or $3n + 1$ or $3n + 2$ for some integer n.
- Square of any integer can be written as $3m$ or $3m + 1$.

CHAPTER PRACTICE PROBLEM

1. If the HCF of 408 and 1032 is expressible in the form $1032 \times m - 408 \times 5$. Find m.

2. Find the largest number which exactly divides 280 and 1245 and leaves remainders of 4 and 3.

3. Prove that there is no natural number for which 4^n ends with the digit zero.

4. Find the last digit of the sum $\lfloor 1 + \lfloor 2 + ... + \lfloor 49$.

5. There are two positive integers X & Y. When X is divided by 237, the remainder is 192. When Y is divided by 117 the quotient is the same but the remainder is 108. Find the remainder when the sum of X & Y is divided by 118.

6. Find the remainder when $43^{33} - 23^{33}$ is divided by 5.

7. How many composite numbers from 71 to 76 have the same number of divisors?

8. Prove that if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

9. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

10. If a, b, c and d are four positive real numbers such that abcd = 1, what is the minimum value of (1 + a)(1 + b)(1 + c)(1 + d).

11. A number when divided by a divisor leaves a remainder of 27. Twice the number divided by the same divisor leaves a remainder of 3. Find the divisor?

12. If N is two digit prime number. When digits are interchanged, we get another prime number M. If N + M = 176 then find N – M.

13. A number when divided by a divisor leaves a remainder of 5 and when divided by twice the divisor leaves a remainder of 45. Find the divisor.

14. Find the rightmost non-zero digit of the number $(30)^{2720}$.

15. Find the product of divisors of 7056.

ASSIGNMENT

SUBJECTIVE

Section A

1. State Euclid's division lemma.

2. Find LCM of 36, 40, 48.

3. State the fundamental theorem of arithmetic.

4. Write 98 as a product of its prime factors.

5. Write the prime factor of 2700.

6. Write the condition to be satisfied by q so that a rational number $\frac{p}{q}$ has a terminating decimal expansion.

7. Which two of the following rational numbers have a terminating decimal representation ?
(i) $\frac{81}{80}$ (ii) $\frac{91}{81}$ (iii) $\frac{64}{125}$ (iv) $\frac{97}{90}$

8. Without performing the long division process write whether the rational number $\frac{13}{3125}$ has a terminating decimal or a non-terminating repeating decimal.

9. Write the condition satisfied by 455 so that a rational number $\frac{64}{455}$ has a non-terminating repeating decimal expansion.

10. Classify the following numbers as rational or irrational
(a) $3+\sqrt{6}$ (b) π (c) $\left(\sqrt{5}+3\right)^2$
(d) 2.3030030003...... (e) $\frac{7}{2\sqrt{5}}$ (f) $\left(6-\sqrt{6}\right)\left(6+\sqrt{6}\right)$

11. Write whether the number $\frac{\sqrt{13}}{625}$ has a terminating decimal or a non-terminating decimal.

12. Write whether the number $\frac{\sqrt{325}}{625\sqrt{52}}$ has a terminating decimal or a non-terminating decimal.

13. Is $3-\sqrt{7}$ irrational ?

14. Can a number of the form 5^n have a zero at its end ?

15. Check whether 6^n can end with the digit 0 for any natural number n.

16. Find HCF of 154 and 660.

17. Given HCF (273, 133) = 7, find their LCM.

18. Given that HCF (306, 657) = 9, find LCM (306, 657).

19. Find the HCF and LCM of 24, 36, 48 and hence check that HCF $\times$ LCM is not equal to product of three given numbers.

20. Prove that HCF(24, 4) = HCF(12576, 4052).

21. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

22. Find the unit digit of $19^{2013} + 11^{2013} - 6^{2013}$.

23. What can you say about the prime factorization of the denominator of the rational number $34.\overline{5678}$.

24. Show that every positive even integer is of the form 2n and every positive odd integer is of the form 2n + 1.

25. Find the least number which when divided by 2, 3, 4, 5, 6, leaves remainder of 1 in each case but when divided by 7 leaves no remainder.

26. Wilson theorem states that if n is a prime number that n divides $(n - 1)! + 1$ using this find the smallest divisor of $12! + 6! + 12! \times 6! + 1!$.

27. Show that any positive integer is of the from 3q or 3q + 1 or 3q + 2 for some integer q.

28. A garden has 48 guava trees, 60 pineapple trees and 96 mango trees. These have to be arranged in rows such that each row has same numbers of trees and all are of same type, Find the minimum number of such rows that can be formed ?

29. Show that $3 + 2\sqrt{5}$ is irrational.

30. If the HCF of 210 and 55 is expressible in form $210 \times 5 + 55y$, find y.

Section B

1. Which among the following is greatest $\sqrt{7}+\sqrt{3},\ \sqrt{5}+\sqrt{5},\ \sqrt{6}+2$?

2. Express each of the following as a product of its prime factors:
(a) 6615 (b) 756 (c) 60025

3. Does there exist any irrational number in between any two rational numbers ?

4. Find the least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18.

5. Solve: $\left(28-10\sqrt{3}\right)^{1/2}-\left(7+4\sqrt{3}\right)^{-1/2}$

6. A man has 1044 candles. After burning, he can make a new candle from 9 stubs left behind. Find the maximum number of candles that can be made.

7. The difference of $10^{25}-7$ and $10^{24}+x$ is divisible by 3 find x.

8. What will be last digit of $(73)^{75^{64^{76}}}$

9. Find the least number that is divisible by all the natural numbers between 1 and 10. (both inclusive)

10. Use Euclid's division algorithm to find HCF of 96 and 294.

11. Which two of the following real numbers are irrational ?
(A) 0.3421234676 ... (B) 0.1010010001... (C) $0.\overline{1372}$ (D) 0.24579

12. Show that any positive odd integer is of the form of 4q + 1 or 4q + 3 where q is any integer.

13. Two runners A and B halt after 3 hrs and 5 hrs respectively while running. After how many hours will both of them halt together for the first time?

14. Find the remainder when the product of any three consecutive integers is divided by 6.

15. Given two numbers 32 and 128. Find their HCF and LCM show that HCF $\times$ LCM = 32 $\times$ 128.

16. Find LCM of 6, 72 and 120 using the prime factorization method.

17. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

14. Find the remainder when the product of any three consecutive integers is divided by 6.

15. Given two numbers 32 and 128. Find their HCF and LCM show that HCF $\times$ LCM = 32 $\times$ 128.

16. Find LCM of 6, 72 and 120 using the prime factorization method.

17. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

18. What is the volume of sphere of radius 3 and hence conclude that the volume of this sphere is terminating decimal or non-terminating decimal?

19. Show that any positive odd integer of the form 8p + 1 or 8p + 3 or 8p + 5 or 8p + 7 where p is some integer.

20. Find the HCF of $\frac{9}{10}, \frac{12}{25}, \frac{18}{35}$ and $\frac{21}{40}$.

21. Prove that $\sqrt{5}$ is an irrational number.

22. Prove that $\sqrt{5}+\sqrt{7}$ is an irrational number.

23. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case.

24. Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

25. Find the greatest numbers that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively.

26. 4 bells toll together at 9.00 a.m. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours ?

27. Two numbers are in the ratio 17 : 13. If their H.C.F. is 15. What are the numbers ?

28. Three persons running around a circular track, can complete one revolution in 2, 4 and 5.5 hours respectively. When will they meet at the starting point?

29. Show that any prime number can be expressed in the form of $6k \pm 1$ $(k \in N)$ for number greater than 3.

30. How many numbers up to 100 are co-prime to 19?

Section C

Numerical Based Questions (Single Digit Answer 0 to 9)

1. Find the remainder when $1^{2013}+2^{2013}+3^{2013}+\ldots 2012^{2013}$ is divisible by 2013.

2. Find the remainder when $2^{100}+3^{100}+4^{100}+5^{100}$ is divided by 7.

3. What is the unit digit of $2^{3^{4^{5}}}$?

4. Show $2^{36}-1$ is divisible by 9 and if $2^{36}-1$ = 68 a19476735 then find the value of a.

5. Find the unit digit of the expression $(1!)^1+(2!)^3+(3!)^3+\ldots\ldots+(10!)^{10}$.

Numerical Based Questions

6. Find the remainder when 7^{12} is divided by 47.

7. If $n > 2$, then show that n^5-5n^3+4n is divisible by 120.

8. Show that square of any positive integer is of the form 3m or 3m + 1.

9. Show that $2^{105}+3^{105}$ is divisible by 5, 7, 11, 25 but not by 13.

10. Find the remainder when $(12^{13}+23^{13})$ is divided by 11.

OBJECTIVE

LEVEL – I

Multiple Choice Questions (Single Option Correct)

1. Which of the following statements is false:
(A) sum of 2 irrational numbers is always irrational
(B) product of 2 irrational numbers may be rational or irrational
(C) sum of 2 irrational numbers may be rational or irrational
(D) none of these

2. Which of the following pairs of numbers are coprime ?
(A) 13, 17 (B) 81, 18
(C) 108, 26 (D) 42, 56

3. HCF of 135 and 1575 is
(A) 45 (B) 5
(C) 3 (D) 1

4. Which of the following is irrational ?
(A) $\sqrt{81}$ (B) $0.\overline{273}$
(C) 0.321010397 ... (D) 0.3251

5. Which of the following statements is false ?
(A) product of HCF and LCM of 3 numbers is equal to product of those 3 numbers
(B) a number of the form $5^n + 5$ always ends with zero (n is a natural number)
(C) every real number is rational or irrational
(D) $\frac{93}{25}$ has terminating decimal representation

6. $4.\overline{12}$ is equivalent to
(A) $\frac{103}{25}$ (B) $\frac{138}{47}$
(C) $\frac{136}{33}$ (D) none of these

7. If p and q are two co-prime number, then their H.C.F. is equal to
(A) p (B) q
(C) 1 (D) pq

8. The greatest number that will exactly divide 162, 257 and 324 leaving the remainder 6, 10 and 12 respectively, is
(A) 13 (B) 12
(C) 17 (D) 11

9. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is:
(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}\sqrt{3}}{2}$
(C) 1.5 (D) 1.4

10. $(x^n - a^n)$ is completely divisible by $(x - a)$ when
(A) n is any natural number (B) n is prime
(C) n is an even number (D) n is an odd number

11. The L.C.M. and H.C.F. of two rational number are equal, then the numbers must be
(A) prime
(B) co-prime
(C) composite
(D) equal

12. The H.C.F. and L.C.M. of two numbers are 9 and 90 respectively. If one of the number is 45, then the other number is
(A) 27
(B) 18
(C) 30
(D) none of these

13. Which of the following rational number have terminating decimal?
(A) $\frac{16}{225}$
(B) $\frac{5}{18}$
(C) $\frac{2}{21}$
(D) $\frac{7}{250}$

14. If n is a natural number, then $6^n - 5^n$ always ends with
(A) 1
(B) 3
(C) 5
(D) 7

15. If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3q$; p, q being prime number, then L.C.M. (a, b) is
(A) pq
(B) p^3q^3
(C) p^3q^2
(D) p^2q^2

(Fill in the Blanks)

16. HCF of 143 and 85________.

17. HCF of 343 and 81 ________.

(True or False)

18. A number of the form 3q + 1 (q is any integer) is always even.

19. $0.34\overline{31}$ is irrational.

20. Division of 2 rational numbers always gives a rational number.

(Match the Following)

21. Match the following:

Column – I		Column – II	
(A)	HCF of 92 and 21 is	(i)	1008
(B)	LCM of 1008 and 63 is	(ii)	7224
(C)	LCM of 1032 and 301	(iii)	1
(D)	HCF of 203 and 145	(iv)	29

22. Match the following:

Column – I		Column – II	
(A)	Decimal representation of an irrational number is always	(i)	Irrational number
(B)	$\left(\sqrt{2}+\frac{1}{\sqrt{3}}\right)^2$	(ii)	Rational number
(C)	The decimal representation of 8/27 is	(iii)	Non-repeating, non-terminating
(D)	Zero is	(iv)	Non-terminating

LEVEL – II

Multiple Choice Questions (Single Option Correct)

1. Four bells commence tolling together. They toll at intervals of 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$ seconds respectively. After what intervals will they toll together again?
(A) 1 min 30 sec (B) 1 min 45 sec
(C) 1 min 12 sec (D) none of these

2. The unit digit of $(6717)^{103}$ is
(A) 3 (B) 7
(C) 9 (D) 2

3. The right most non-zero digit of $(80)^{1365}$ is
(A) 4 (B) 2
(C) 8 (D) 6

4. Suppose a and b are positive integers neither of which is a multiple of 3 then the remainder when $a^2 + b^2$ is divided by 3
(A) must be 0 (B) must be 1
(C) must be 2 (D) may be 1 or 2 but not 0

5. If a and b are real numbers with a > 1 and b > 0 such that $ab = a^b$ and $a/b = a^{3b}$, then the value of a is
(A) $\sqrt{2}$ (B) 2
(C) 1/2 (D) 4

6. The expression $\left[\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+......+\frac{1}{n(n+1)}\right]$ for any natural number is
(A) always greater than 1 (B) always less than 1
(C) always equal to 1 (D) all are equal

7. Find the remainder when 2^{89} is divided by 89?
(A) 1 (B) 2
(C) 87 (D) 88

8. A boy writes all the numbers from 100 to 999. The number of zeros that he uses is 'a', then number of 5's that he uses is 'b' and the number of 8's he uses is 'c'. What is the value of b + c – a?
(A) 280 (B) 380
(C) 180 (D) 80

Multiple Choice Questions (Multiple Options Correct)

9. $2^{105} + 3^{105}$ is divisible by which of the following
(A) 7 (B) 11
(C) 25 (D) 13

10. The remainder when 17^{15} is divided by 9 is k then k is divisible by
(A) 4 (B) 8
(C) 2 (D) 9

11. The sum of two co-prime numbers is 43 and their L.C.M. is 450, which of the following are numbers

(A) 18 (B) 30
(C) 25 (D) 15

12. Let n be the first 2006 positive integers are divisible by all of the numbers 1, 2, 3, 4, 5 and 7 then n will be divisible by

(A) 1 (B) 2
(C) 3 (D) 4

13. $8^6 - 5^6$ is divisible by

(A) 91 (B) 49
(C) 129 (D) 13

14. The sum of all the factors of 45000 which area exactly the multiples of 10 is

(A) 152295 (B) 141960
(C) 600 (D) 1100

LEVEL – III

1. It is being given that $2^{32} + 1$ is completely divisible by a whole number which of the following numbers is completely divisible by this number.
(A) $2^{16} + 1$ (B) $2^{16} - 1$
(C) 7×2^{33} (D) $2^{96} + 1$

2. What will be the remainder when $67^{67} + 67$ is divided by 68
(A) 1 (B) 63
(C) 66 (D) 67

3. What is the unit digit in $7^{95} - 3^{58}$.
(A) 0 (B) 4
(C) 6 (D) 7

4. Find the remainder when 2^{31} is divided by 5.
(A) 1 (B) 2
(C) 3 (D) 4

5. Let N be the greatest number that will divide 1305, 4665 and 6905 leaving the same remainder in each case. The sum of the digits in N is
(A) 4 (B) 5
(C) 6 (D) 8

6. If n is an odd natural number $3^{2n} + 2^{2n}$ is always divisible by
(A) 13 (B) 5
(C) 17 (D) 19

7. Find the remainder when $7^{21} + 7^{22} + 7^{23} + 7^{24}$ is divided by 25.
(A) 0 (B) 2
(C) 4 (D) 6

8. If $N = 901 \times 902 \times 903$. If N is divided by 25 the remainder is:
(A) 0 (B) 2
(C) 6 (D) 8

9. How many three digit numbers have exactly 3 factors?
(A) 2 (B) 1
(C) 10 (D) 7

10. The unit digit of $(1 + 9 + 9^2 + 9^3 + \ldots.. + 9^{2006})$ is
(A) 0 (B) 1
(C) 9 (D) 4

11. The largest number amongst the following that will perfectly divide $101^{100} - 1$ is
(A) 100 (B) 10000
(C) 100^{100} (D) 100000

12. If n is natural number, then find the remainder when $(37)^{n+2} + 16^{n+1} + 30^n$ is divided by 7
(A) 0 (B) 1
(C) 2 (D) 3

KEY AND ANSWERS TO CPP AND ASSIGNMENT

CHAPTER PRACTICE PROBLEM

1.	2	2.	138
4.	3	5.	64
6.	0	7.	2
8.	16	11.	51
12.	18	13.	40
14.	1	15.	$(84)^{45}$

ASSIGNMENT

SUBJECTIVE

Section A

2. 720
4. 2×7^2
5. $2^2 \times 3^3 \times 5^2$
6. q has to be of the form $2^m \times 5^n$ where m and n are non-negative integers.
7. (i), (iii)
8. Terminating
10. (a) Irrational (b) Irrational (c) Irrational
    (d) Irrational (e) Irrational (f) Rational
11. non-terminating
12. terminating
13. Yes
14. No
15. No
16. 22
17. 5187
18. 22338
19. 12 and 144
21. 35
22. 4
23. not of the form $2^m \times 5^n$
25. 301
26. 7
28. 17
30. – 19

Section B

1. $\sqrt{5} + \sqrt{5}$
2. (a) $3^3 \times 5 \times 7^2$ (b) $2^2 \times 3^3 \times 7$ (c) $5^2 \times 7^4$
3. yes
4. 364
5. 3
6. 130
7. 2
8. 3
9. 2520
10. 6
11. (A), (B)
13. 15 hrs.
14. 0
15. HCF = 32, LCM = 128
16. 360
20. 3/1400
23. 16
24. 64
25. 63
26. 5 time
27. 255, 195
28. 44 hr
30. 95

Section C

Numerical Based Questions (Single Digit Answer 0 to 9)

1.	0	2.	5
3.	2	4.	7
5.	7		

Numerical Based Questions

6.	17	10.	2

OBJECTIVE

LEVEL – I

Multiple Choice Questions (Single Option Correct)

1.	A	**2.**	A	**3.**	A
4.	C	**5.**	A	**6.**	C
7.	C	**8.**	A	**9.**	C
10.	A	**11.**	D	**12.**	B
13.	D	**14.**	A	**15.**	C

(Fill in the Blanks)

16.	1	**17.**	1

(True or False)

18.	F	**19.**	F	**20.**	F

(Match the Following)

21.	(A) – (iii)	(B) – (i)	(C) – (ii)	(D) – (iv)
22.	(A) – (iii)	(B) – (i)	(C) – (iv)	(D) – (ii)

LEVEL – II

Multiple Choice Questions (Single Option Correct)

1.	B	**2.**	A	**3.**	C
4.	C	**5.**	D	**6.**	B
7.	B	**8.**	B		

Multiple Choice Questions (Multiple Options Correct)

9.	A, B, C	**10.**	A, B, C	**11.**	A, C
12.	B, D	**13.**	A, B, C, D	**14.**	B

LEVEL – III

1.	D	**2.**	C	**3.**	B
4.	C	**5.**	A	**6.**	A
7.	A	**8.**	C	**9.**	D
10.	B	**11.**	B	**12.**	A