

**MID-SEMESTER EXAMINATION, SEPTEMBER-2017****CALCULUS - I (MTH-1001)****Programme: B.Tech****Semester: 1st****Full Marks: 30****Time: 2 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Use limit laws to evaluate the limit of a function and demonstrate the existence of limit and continuity of functions.CO-1	1.L3,L3,L3 2.L2,L3,L3 3.L3,L3,L4	1.a,b,c 2.a,b,c 3.a,b,c	18
Compute slope of tangent lines and derivative by different techniques and apply the concept of derivatives for linearization of functions and solve various physical and engineering problems.CO-2	4.L1,L3,L3 5.L3,L3,L3	4.a,b,c 5.a,b,c	12

\*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

**Answer all questions. Each question carries equal mark.**

1. (a) If a rock is thrown upward on the planet Mars with a velocity of 10m/s, its height in meters  $t$  seconds later is given by  $y=10t-1.86t^2$  then compute the average velocity over the time interval  $[1,1.1]$ . 2
- (b) Sketch the graph of an example of a function  $f$  that satisfies the given condition:- 2  
 $\lim_{x \rightarrow 3^+} f(x)=4, \lim_{x \rightarrow 3^-} f(x)=2, \lim_{x \rightarrow 2} f(x)=2$   
 $f(3)=3$  and  $f(-2)=1$ .
- (c) Determine the infinite limit  $\lim_{x \rightarrow 3^+} \ln(x^2 - 9)$ . 2

2. (a) Explain why  $\lim_{x \rightarrow 8} [x]$  does not exist and also illustrate it graphically. 2
- (b) If  $2x \leq g(x) \leq x^4 - x^2 + 2$  for all  $x$  then determine  $\lim_{x \rightarrow 1} g(x)$ . 2
- (c) Apply  $\varepsilon, \delta$  definition of limit to prove that  $\lim_{x \rightarrow -2} \left( \frac{x}{2} + 3 \right) = 2$  2
3. (a) A particle moves along a straight line with equation of motion  $S = f(t) = 100 + 50t - 4.9t^2$ , where  $S$  is measured in meters and  $t$  in seconds. Apply the definition of derivatives to find the velocity and speed when  $t = 5$ . 2
- (b) Compute  $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$ . 2
- (c) Examine the continuity of the function 2
- $$f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases} \text{ at } a = 0.$$
4. (a) Find equation of both lines that are tangent to the curve  $y = 1 + x^3$  and parallel to the line  $12x - y = 1$ . 2
- (b) If  $f(x) = e^x g(x)$ , where  $g(0) = 2$  and  $g'(0) = 5$ , determine  $f'(0)$ . 2
- (c) Use the graph of  $f(x) = 5x - 9x^2$  to sketch the graph of  $f'(x)$ . 2
5. (a) Compute the given derivative by finding the first few derivatives and observing the pattern that occurs  $\frac{d^{99}}{dx^{99}}(\sin x)$  2
- (b) Use implicit differentiation to compute an equation of the tangent line to the curve at the given point  $x^2 + xy + y^2 = 3$ ,  $(1, 1)$ . 2
- (c) Use logarithmic differentiation to determine the derivative of the function  $x^y = y^x$ . 2

\*End of Questions\*