Department of Mathematics
Institute of Technical Education & Research, SOA University

Mid-Semester Examination, October-2016
Calculus - I (MTH 1001)

Semester: 1st Semester | Branch: CSE, CSIT, ECE, EE, EEE, CE
Full mark: 30 | Time: 2 Hours

Subject Learning Outcome	*Taxonomy Level	Ques. No.	Marks
Use limit laws to evaluate the limit of a function and demonstrate the existence of limit and continuity of functions.CO-1	1.L3,L4,L4.2.L 2,L3,L3	1.a,b,c 2a,b,c	12
Compute slope of tangent lines and derivative by different techniques and apply the concept of derivatives for linearization of functions and solve various physical and engineering problems.CO-2	3.L3,L3,L3 4.L3,L3,L4	3.a,b,c 4.a,b,c	12
Discuss the mean value theorem and study maximum and minimum values of a function as well as apply L'Hospital's rule to evaluate limit of a function and sketch curves of functions.CO-3	5.L3,L3,L3	5.a,b,c	6

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all five questions.

All questions carry equal marks. All bits of each question carry equal marks.

Q1.

(a) Compute $\lim_{t\to 0}\frac{\sqrt{1+t}-\sqrt{1-t}}{t}$. 2

(b) Examine the continuity of the following function at $x=-2$. 2

$$f(x)=\begin{cases}\frac{1}{x+2} & if \quad x\neq -2\\ 1 & if \quad x=-2\end{cases}$$

(c) Analyze the following statement. 2

$\lim_{x\to -3} f(x)=\infty$ and $\lim_{x\to 4^+} f(x)=-\infty$.

Q2.

(a) Explain that $\lim_{x\to 0}\frac{|x|}{x}$ does not exist. 2

(b) Use limits to find vertical asymptotes of the curve $y=\frac{x^2+1}{3x-2x^2}$. 2

(c) Use the $\in,\delta$ definition of limit to prove that $\lim_{x\to 0}|x|=0$. 2

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Q3.

(a) Compute the derivative of the function, $y = x\tanh^{-1}x + \ln\sqrt{1-x^2}$. 2

(b) Use derivative to find equation of tangent line and normal line to the curve $y = x^2 - x^4$ at the point (1,0). 2

2

(c) A ladder 6m long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, then compute the increase in rate of change of x with respect to θ when $\theta = \frac{\pi}{3}$.

Q4.

(a) Compute y' if $e^y \sin x = x + xy$ by implicit differentiation. 2

(b) The number of yeast cells in a laboratory culture increases rapidly initially but levels off eventually. The population is modeled by the function $n = f(t) = \frac{a}{1+be^{-0.7t}}$ where t is measured in hours. At time $t = 0$ the population is 20 cells and is increasing at a rate of 12 cells/hour. Compute the values of a and b. 2

(c) Analyze the linear approximation of the function $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$. 2

Q5.

(a) Apply the closed interval method to determine the absolute maximum and absolute minimum values of the function $f(x) = x^3 - 3x + 1$ in the interval $[0,3]$. 2

(b) Let a be a fixed point of a function f . If $f'(x) \neq 1$ for all real numbers x, then show that f has at most one fixed point. 2

(c) Compute $\lim_{x \to \frac{\pi}{2}^+} \frac{\cos x}{1-\sin x}$ using L'Hospital's Rule. 2

End of Questions