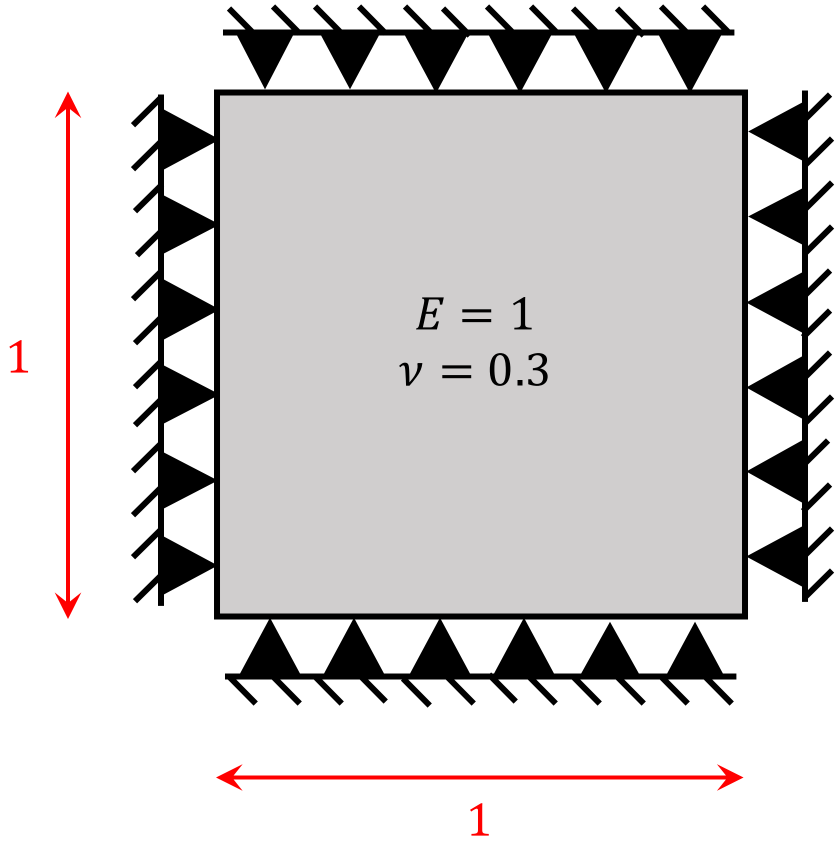
2D PINN for Boundary Value Problem of Linear Elasticity

- Thin plate fixed at the boundary with periodic in-plane loading in the interior.
- Classis case of plane stress boundary value problem of linear elasticity



PDE: Plane stress problem of elasticity

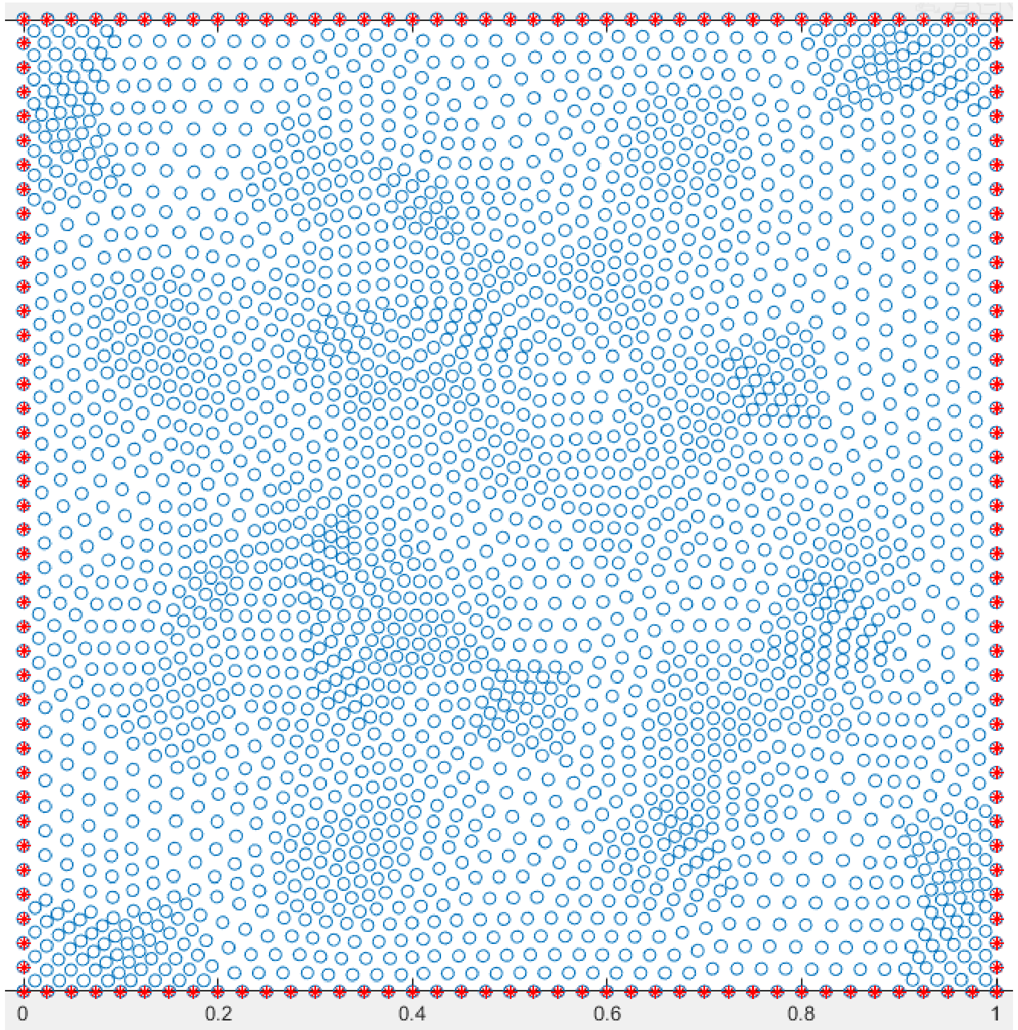
$$G\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + G\left(\frac{1+\nu}{1-\nu}\right)\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial yx}\right] + [\sin(2\pi x)\sin(2\pi y)] = 0$$

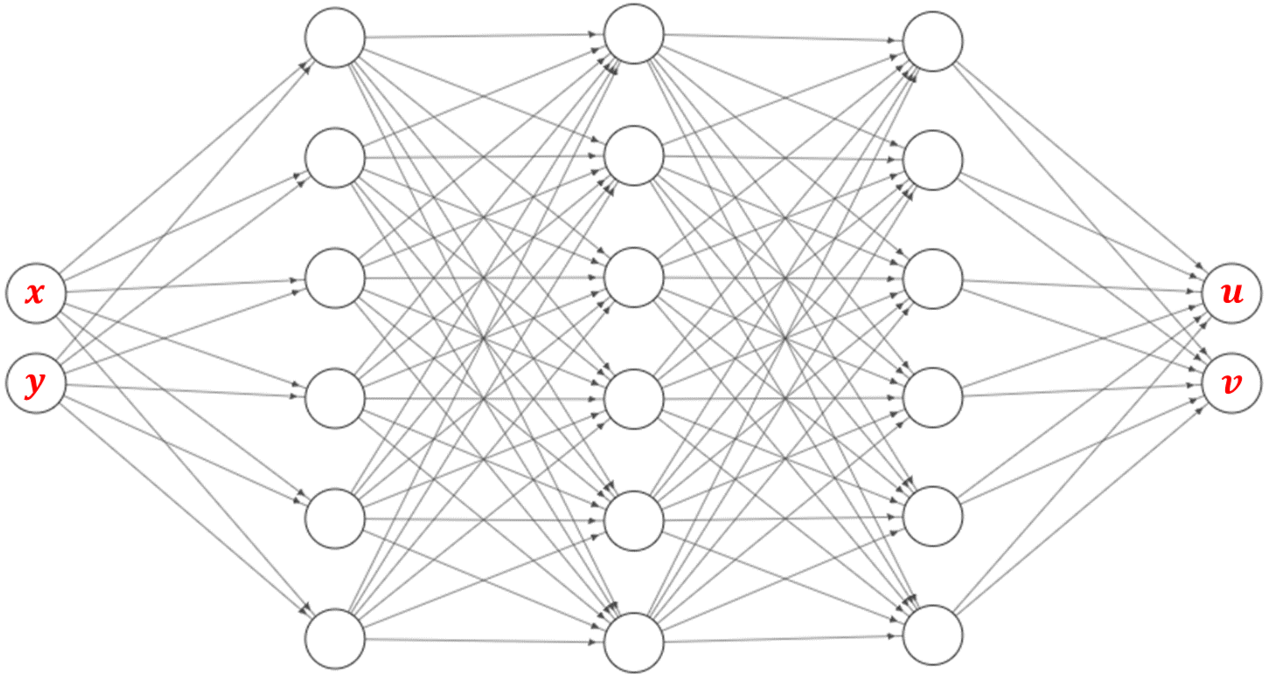
$$G\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + G\left(\frac{1+\nu}{1-\nu}\right)\left[\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial xy}\right] + \underbrace{[\sin(\pi x) + \sin(2\pi y)]}_{\text{Body force } (f_y)} = 0$$

$$G = \frac{E}{2(1+\nu)}$$

Boundary conditions

$u, v = 0$ on all boundary points





No. of hidden layers	5
No. of neurons in each layer	30
Activation function	Tanh
Optimizer	Adam
Learning rate	0.0005
No. of epochs	2000

Given:
Interior collocation points: *'interior_points.mat'*
Boundary points: *'boundary_points.mat'*

Total loss = PDE loss + BC loss

PDE loss = PDE_1 loss + PDE_2 loss

BC loss = BC_u loss + BC_v loss

Assignment questions

1. Solve the 2D plane stress problem of elasticity using PINN using PDE and BC losses. Use stochastic gradient descent for training. Penalize the boundary condition loss with a factor of 10,000. Loss is given by:

$$L(\boldsymbol{\theta}) = \frac{1}{N_f} \|\nabla \cdot \widetilde{\boldsymbol{\sigma}}(x, y, \boldsymbol{\theta}) + \boldsymbol{f}\|_{\Omega}^2 + \frac{1}{N_b} \|\widetilde{\boldsymbol{u}}(x, y, \boldsymbol{\theta}) - \boldsymbol{u}(x, y)\|_{\partial\Omega}^2$$

1. Plot the displacement field contours in $x -$ and $y -$ directions after training the model.
2. Plot the total loss, PDE loss and BC loss versus epochs.
3. Save the trained model (parameters) using *‘torch.save()’*
4. Generate random collocation points sampled from a uniform distribution in the interior (n = 2000 pairs) and boundary of the domain (n = 4*100 pairs). Concatenate all the sampled points and give as input to the trained model.
5. Load the saved model using *‘torch.load()’*
6. Test your trained model on the generated collocation points and plot the displacement field.

PINN based solution