Solution

February 13, 2023

1 Multiclass Classification using Neural Networks

```
[1]: # Scientific and vector computation for python
import numpy as np
import pandas as pd
# Plotting library
import matplotlib.pyplot as plt
```

1.0.1 1. Inspect and plot some portion of the training data. Segregate the data into two separate variables consisting of 'feature matrix' and corresponding 'labels' (first column of the data). Normalize the feature matrix data. Plot some example images along with their descriptive labels

```
[2]: # load the dataset
     fashion_train = pd.read_csv('data/fashion-mnist_train.csv')
     fashion_test = pd.read_csv('data/fashion-mnist_test.csv')
[3]: fashion_train.head()
               pixel1 pixel2 pixel3 pixel4 pixel5 pixel6
[3]:
                                                                    pixel7
                                                                             pixel8
     0
             0
                     0
                              0
                                       0
                                                0
                                                         0
                                                                 0
                                                                          0
                                                                                   9
     1
             1
                     0
                              0
                                       0
                                                0
                                                         0
                                                                 0
                                                                          0
                                                                                   0
     2
             2
                              0
                     0
                                       0
                                                0
                                                         0
                                                                 0
                                                                         14
                                                                                  53
     3
             2
                     0
                              0
                                       0
                                                0
                                                         0
                                                                 0
                                                                          0
                                                                                   0
                     0
                              0
                                       0
                                                0
                                                         0
                                                                 0
             3
                                                                          0
                                                                                   0
                                          pixel777
        pixel9
                    pixel775 pixel776
                                                     pixel778
                                                                pixel779
                                                                           pixel780
     0
              8
                          103
                                      87
                                                 56
                                                             0
                                                                        0
                                                                                   0
     1
              0
                           34
                                       0
                                                  0
                                                             0
                                                                        0
                                                                                   0
                                       0
                                                  0
                                                             0
     2
             99
                            0
                                                                       63
                                                                                  53
     3
                          137
                                     126
                                                140
                                                             0
                                                                      133
                                                                                 224
              0
                                                             0
     4
              0
                            0
                                       0
                                                  0
                                                                        0
                                                                                   0
        pixel781 pixel782 pixel783 pixel784
     0
                0
                           0
                                      0
                                                 0
                0
                           0
                                      0
                                                 0
     1
```

```
    2
    31
    0
    0
    0

    3
    222
    56
    0
    0

    4
    0
    0
    0
    0
```

[5 rows x 785 columns]

```
[4]: # separate the labels and the data

train_labels = fashion_train['label']
train_data = fashion_train.drop('label', axis=1)

test_labels = fashion_test['label']
test_data = fashion_test.drop('label', axis=1)
```

```
[5]: # Normalize the data, as the values are in the range of 0-255 and # we want them in the range of 0-1 for better convergence of the # model while training it, hence we divide the data by 255.

train_data = train_data / 255.0
test_data = test_data / 255.0
```

```
[6]: # function to plot the images
    title ={0: 'T-shirt/top', 1: 'Trouser', 2: 'Pullover', 3: 'Dress', 4: 'Coat', 5:
      6: 'Shirt', 7: 'Sneaker', 8: 'Bag', 9: 'Ankle boot'}
    def plot_grid(n, df):
        labels = df['label'].unique()
        fig, axes = plt.subplots(n, len(labels), figsize=(12, 10))
        for j, label in enumerate(labels):
            selected_rows = df[df['label'] == label]
            for i in range(n):
                 # print(i, j)
                 selected_row = selected_rows.sample(1)
                 axes[0, j].set_title(title[selected_row['label'].values[0]],__
      →fontsize=8)
                 axes[i, j].imshow(selected_row.drop('label', axis=1).values.
      ⇒reshape(28,28), cmap='gray')
                 axes[i, j].axis('off')
        return fig
    fig = plot_grid(8, fashion_train)
    fig.savefig('figures/0101.png')
```



- 1.0.2 2. Classify the dataset using neural networks, with two hidden layers. The number of nodes in the hidden layers is your choice, as discussed in class. Calculate the optimized weights and biases and training set accuracy for the model (take regularization parameter $\lambda=0.1$)
 - We will write the useful functions which will be used in the code.

Cross Entropy:

$$J = -\frac{1}{m} \sum_{i=1}^{m} \sum_{k=1}^{K} y_k^{(i)} \log(\hat{y}_k^{(i)})$$

So let's write a class to calculate the loss and it's prime with respect to y_pred

[7]: # Loss function, we use the cross entropy loss function

class CategoricalCrossentropy:

```
Categorical cross-entropy loss function.
  def __init__(self, 12 = 0) -> None:
      # by default we don't use regularization
      self.12 = 12
  def loss(self, y_true, y_pred, ws):
      m_samples = y_pred.shape[1]
      # we use the log trick to avoid the underflow, (Catelogical cross_{\sqcup}
⇔entropy)
      cost = -np.sum(y_true * np.log(y_pred + 1e-10)) # shape =_
\hookrightarrow (batch size.)
      cost = cost / m_samples
      reg_term = 0
      reg_term += (self.12/(2*m_samples)) * ws
      return cost + reg_term
  def loss_prime(self, y_true, y_pred):
      m_samples = y_pred.shape[1]
       # this is little bit different from the else loss prime,
       # this return the (dJ/dA)*(dA/dz) so we don't need to find the
⇔derivative of sofmax_prime
      cost_prime = (y_pred - y_true) / m_samples
      return cost_prime
```

- Now we will make a class for a fully connected layer in which we will be taking # input nuerons, # output nuerons, 12 regularization as an arguments
- Here we will save the input in the forward_propogation which will be used while backward propogation in updating the weights and biases
- It is a fully connected DenseLayer

$$\mathbf{Z}^{[l]} = \mathbf{W}^{[l]} \mathbf{A}^{[l-1]} + \mathbf{b}^{[l]}$$

where l is the layer number, $A^{[l-1]}$ is the activation of the previous layer, $W^{[l]}$ and $b^{[l]}$ are the weights and biases of the current layer.

$$\mathbf{A}^{[0]} = \mathbf{X}$$
 (input data)

$\mathbf{A}^{[L]} = \hat{\mathbf{Y}}$ (output predictions)

```
[8]: # class for fully connected layer in the neural network
     # it will initialize the weights and biases at the time of initialization
     # and will also implement the forward and backward propagation
     class DenseLayer:
         11 11 11
         Class for fully connected layer in the neural network.
         def __init__(self, n_l_1, n_l, 12=0, bias=True):
             11 11 11
             Initialize the fully connected layer with weights and biases.
             Parameters
             _____
             n_l_1: int
                 Number of neurons in the input layer.
             n l: int
                 Number of neurons in the output layer.
             l2: float, optional
                 L2 regularization parameter.
             bias: bool, optional
                 Whether to use bias in the layer or not.
             11 11 11
             self.bias = bias
             self.weights = np.random.uniform(low=-1, high=1, size=(n_1, n_1_1))
             if self.bias:
                 self.biases = np.random.uniform(
                     low=-1, high=1, size=(n_1, 1)
                 ) # biases (n_[l], 1)
             else:
                 self.biases = 0
             self.12 = 12
```

```
# Total Training parameters
       self.nuerons = n_1
       self.trainable_params = n_1 * (n_1_1 + 1)
  def forward_propagation(self, activate_l_1):
       11 11 11
       Implement forward propagation through the fully connected layer.
       Parameters
       activate_l_1: numpy.ndarray, shape (n_[l-1], batch_size)
           Input data to be propagated through the layer.
      Returns
       numpy.ndarray, , shape (n_[l], batch_size)
           Output of the layer after forward propagation.
       # save the input for backpropagation
      self.input = activate_l_1 # activate (n_[l], batch_size)
      z_1 = (
           np.dot(self.weights, self.input) + self.biases
       ) # z = weights x input + biases = (n_[l], n_[l-1]) x (n_[l-1], l)
\hookrightarrow batch\_size) + (n_[l], 1) = (n_[l], batch\_size)
      self.output = z_l
      \texttt{return self.output} \quad \# \ z \ (n\_[l], \ batch\_size)
  def backward_propagation(self, output_error, learning_rate):
       Implement backward propagation through the fully connected layer.
      Parameters
       output_error: numpy.ndarray
           Error in the output of the layer.
       learning_rate: float
           Learning rate to be used for weight updates.
```

```
Returns
       _____
       numpy.ndarray
           Error to be propagated back to the previous layer.
       # calculating the error with respect to weights before updating the
\hookrightarrow weights
       input_error = np.dot(
           self.weights.T, output_error
       ) # weights x output_error (n_[l], n_[l-1]) x (n_[l], batch_size) = 
\hookrightarrow (n_[l-1], batch_size)
       weights error = np.dot(
           output_error, self.input.T
       ) # output_error x input (n_l, batch_size) x (n_[l-1], batch_size)
\hookrightarrow= (n_[l], n_[l-1])
      m_samples = output_error.shape[1]
       # addition of regularization term, by default both 11 and 12 are 0
      reg_term = (self.12 / m_samples) * self.weights
       # updating the weights and biases
       self.weights -= learning_rate * (
           weights_error + reg_term
       ) # weights -= learning_rate * weights_error
       self.biases -= learning_rate * np.sum(
           output_error, axis=1, keepdims=True
       ) # biases -= learning rate * output error
       return input_error
```

• Let's write a class for the Activation Layer, we will be taking activation class as an argument which will be having two methods activation and activation_prime

$$\mathbf{A}^{[l]} = g^{[l]}(\mathbf{Z}^{[l]})$$

where $g^{[l]}$ is the activation function for layer l.

• in the backward_propogation we will be using the activation_prime to calculate the derivative of the activation function with respect to the Z which will be used in updating the weights and biases

```
[9]: # class for activation functions
# it will implement the forward and backward propagation
# we are going to use three activation functions
# 1. sigmoid
```

```
# 2. relu
# 3. softmax, this is used in the output layer
class Sigmoid:
   def activation(z):
       return 1 / (1 + np.exp(-z))
   def activation prime(z):
       sigmoid_z = Sigmoid.activation(z)
       return sigmoid_z * (1 - sigmoid_z)
class ReLU:
   def activation(z):
       return np.maximum(0, z)
   def activation_prime(z):
       z[z \leftarrow 0] = 0
       z[z > 0] = 1
       return z
class Softmax:
   def activation(z):
       z = np.max(
           z, axis=0, keepdims=True
       ) # axis=0 means coloumn z is the shape of (n_l, batch_size), axis=0
 ⇔batch_size)
       exp_z = np.exp(z)
       return exp_z / np.sum(exp_z, axis=0, keepdims=True)
   def activation_prime(z):
       # we have calculated the dA/dz in the loss_prime itself,
       # that returns (dJ/dA)*(dA/dz) itself so no need to take the derivative
 ⇔of activation here
       return 1
   def softmax_derivative(z):
       softmax_ = Softmax.softmax(z)
       return np.einsum('ij,ik->ij', softmax_, (np.eye(z.shape[0]) - softmax_))
class ActivationLayer:
```

```
Activation layer for neural networks.
   This layer applies an non-linearity to the input data.
  activation (class) : (callable)
       The class of the activation function to be used. The class should have \sqcup
→two methods:
      activation and activation_prime.
  def __init__(self, activation):
       self.activation = activation.activation
       self.activation_prime = activation.activation_prime
       self.activation_name = activation.__name__
  def forward_propagation(self, z_l):
       11 11 11
      Perform the forward propagation of the activation layer.
      Parameters:
       z (numpy.ndarray): The z to the layer.
      Returns:
       numpy.ndarray: The output of the layer after applying the activation \sqcup
\hookrightarrow function.
      self.input = z_1
      activate_l = self.activation(self.input)
      self.output = activate_l
      return self.output # (n_[l], batch_size)
  def backward_propagation(self, output_error, learning_rate):
       Perform the backward propagation of the activation layer.
       Parameters:
       output_error (numpy.ndarray):
           The error that needs to be backpropagated through the layer.
       learning_rate (float):
           The learning rate used to update the weights.
```

- Now we will write a class for the Neural Network which will be taking layers as an argument
 which will be a list of DenseLayer and ActivationLayer objects in the order in which they
 are to be stacked in the neural network.
- It have a method predict which will be used to predict the output for the given input
- In the fit method we will be taking X_train , y_train , epochs , learning_rate , batch_size as an arguments and will be using the Mini-Batch Gradient Descent algorithm to update the weights and biases of the neural network

```
return 12_w
   # predict output for given input
  def predict(self, input_data):
       11 11 11
      Predicts the output for a given input.
      Parameters:
       - input_data (numpy.ndarray):
           A 2D array of shape (m_samples, n_features) representing m_samples,\Box
\neg each with n_features.
       Returns:
       - result (numpy.ndarray):
           A list of 2D arrays of shape (m_samples, n_classes) representing_
\hookrightarrow the predicted output for each of the m_samples. n_classes is the number of \sqcup
output nodes in the neural network.
       # run network over all samples
      output = input_data.T
      for layer in self.layers:
           output = layer.forward_propagation(output)
      return output.T
  def fit(self, X, y, epochs=30, learning_rate=0.01, batch_size=64):
       Train the neural network on the given data.
       Parameters:
       _____
      X (numpy.ndarray):
           The input data.
       y (numpy.ndarray):
           The target data.
       epochs (int):
           The number of epochs to train the model.
       learning_rate (float):
```

```
The learning rate used to update the weights.
       batch_size (int):
           Size of the batch for optimization purpose using Mini-Batch_{\sqcup}
\hookrightarrow Gradient Descent
       Returns:
       list:
          The loss history.
       # if no batch size is provided, use the entire training set in each_{\sqcup}
\rightarrow batch
       if (batch_size is None) or (batch_size > X.shape[0]):
           batch_size = X.shape[0]
       # number of batches
      batches = X.shape[0] // batch_size
       if batches == 0:
           batches = 1
       # train network for given number of epochs
      loss_history = []
      for i in range(epochs):
           loss = 0
           # shuffle training data
           idx = np.random.permutation(X.shape[0])
           X = X[idx]
           y = y[idx]
           for j in range(batches):
               batch_start, batch_end = j * batch_size, (j + 1) * batch_size
               # forward propagation
               output = X[batch_start:batch_end].T # shape (nx , batch_size)
               for layer in self.layers:
                   output = layer.forward_propagation(output)
               # backward propagation
               error = self.loss_.loss_prime(
```

```
[13]: epochs = 500
      layers_sigmoid = [
          # layer-1, nuerons in l=1 are 25
          DenseLayer(784, 25),
          # activation of the first hidden layer 'sigmoid'
          ActivationLayer(Sigmoid),
          # layer-2 (output-layer), nuerons in l=2 are 10
          DenseLayer(25, 10),
          # activation of the output layer 'Softmax
          ActivationLayer(Softmax)
      ]
      # using l2 regularization , lambda_ = 0.1
      loss_ = CategoricalCrossentropy(12=0.1)
      X_train = np.array(train_data)
      y_oh = np.array(pd.get_dummies(train_labels))
     net_sigmoid = Network(layers=layers_sigmoid, use_loss = loss_)
      loss_history_sigmoid = net_sigmoid.fit(X_train, y_oh, epochs=epochs,_
       →learning_rate=0.1, batch_size=128)
```

Epoch 1: loss = 1.768096 Epoch 31: loss = 0.605503

```
Epoch 61: loss = 0.507054
     Epoch 91: loss = 0.451409
     Epoch 121: loss = 0.413130
     Epoch 151: loss = 0.383552
     Epoch 181: loss = 0.359101
     Epoch 211: loss = 0.340386
     Epoch 241: loss = 0.320793
     Epoch 271: loss = 0.305485
     Epoch 301: loss = 0.293002
     Epoch 331: loss = 0.281175
     Epoch 361: loss = 0.268511
     Epoch 391: loss = 0.257053
     Epoch 421: loss = 0.247769
     Epoch 451: loss = 0.236498
     Epoch 481: loss = 0.227099
     Epoch 500: loss = 0.222401
[14]: # optimized weights of both the layers are
      ws = []
      bs = \Pi
      for layer in net_sigmoid.layers:
          # as weights are only assigned in Dense Layer
          if layer.__class__.__name__ == "DenseLayer":
              ws.append(layer.weights)
              bs.append(layer.biases)
[15]: # we got the updated weights and biases for both the layers
      len(ws), len(bs)
[15]: (2, 2)
[16]: # shape of the weights of the first layer is (25, 784)
      # which is equal to the
      # (# nuerons in 1st layer, # in 0th layer)
      print(f"shape of updated weight matrix for first layer: {ws[0].shape}")
      print(f"Shape fo updated bias matrix for layer-1: {bs[0].shape}")
     shape of updated weight matrix for first layer: (25, 784)
     Shape fo updated bias matrix for layer-1: (25, 1)
[17]: # updated weights of first hidden layer
      ws[0]
[17]: array([[ 0.09733237, 0.64211612, 0.43141654, ..., -0.53945321,
               0.81917513, 0.59681227],
             [0.77455723, -0.13199028, 0.20723084, ..., -0.90124562,
              -0.89007832, -0.95372786],
```

```
[0.71167367, 0.17812633, 0.02770294, ..., 0.77028119,
              -0.70029882, 0.61068796],
             [ 0.93071593, -0.32512072, 0.29607617, ..., 0.58656734,
               0.07134936, -0.67555366,
             [-0.68944978, -0.60509642, 0.86117876, ..., 0.28476979,
              -0.06752638, 0.75877546],
             [-0.28574284, -0.12143406, -0.99376385, ..., -0.51540284,
               0.12914181, 0.27426143]])
[18]: # updated biases of the first hidden layer
      bs[0]
[18]: array([[ 1.09723901],
             [-0.29439824],
             [ 0.80394569],
             [ 0.01250182],
             [0.34318807],
             [-1.40486849],
             [0.47254816],
             [-0.71311629],
             [-0.31973412],
             [ 0.35306136],
             [ 0.02699433],
             [-1.37653985],
             [0.25312773],
             [ 1.18998793],
             [-0.53779085],
             [-0.24061911],
             [-0.7950669],
             [-0.48210048],
             [ 0.25832759],
             [-0.81955931],
             [-0.0593232],
             [0.78373251],
             [ 1.69162866],
             [ 0.2379068 ],
             [-0.8356316]])
[19]: # shape of the weights of the first layer is (10, 25)
      # which is equal to the
      # (# nuerons in 2st layer, # in 1st layer)
      print(f"shape of updated weight matrix for first layer: {ws[1].shape}")
      print(f"Shape fo updated bias matrix for layer-1: {bs[1].shape}")
     shape of updated weight matrix for first layer: (10, 25)
     Shape fo updated bias matrix for layer-1: (10, 1)
```

[20]: # updated weights of second output layer ws[1]

```
[20]: array([[ 0.11290728, -0.73596389, 0.4050369, 0.46426625, 0.65540828,
             -1.34739548, -0.61285109, -1.00100304, 3.10525184, -1.18113138,
              0.66015744, -0.70502139, 3.12087622,
                                                   0.37795869, 0.66491928,
             -1.80328058, -1.3282877, -3.10456868, 0.13783313, 1.92478699,
              0.27183049, 2.00879265, 1.52221241, -2.02403651, 0.25395373],
            [-1.54562005, -0.20638878, -0.348165, -0.34103725, 0.87210302,
              1.04816026, 1.88281693, 0.98308368, 1.81843303, -1.30465478,
              2.70555711, 0.28431172, -1.97828873, -1.36997497, 1.14393686,
              2.51066537, -1.08516524, -0.35724532, -1.66683343, -0.55909382,
             -0.17353568, -2.3740337, -2.81547431, 0.69125776, 0.18914549],
            [-0.12225915, -0.95702656, -1.10965318, -0.0831147, -0.8235191,
              0.32138424, 1.70523733, -1.97878828, -2.08489432, 1.9851954,
             -0.98843607, 1.67933124, 1.92885814, -1.8174385, 0.36214101,
              1.55380725, -0.20084358, -2.81469043, -0.65026904, -0.20712445,
              1.32750906, -0.04013955, 0.60806625, -1.78294163, 0.80604073],
            [-0.2264023, -1.42763435, 1.2889511, 0.00964839, 1.44378104,
             -0.46466273, 2.60008765, -2.01880663, 0.68935071, -1.81173528,
             -0.7457708, -0.1894755, -0.89301972, 0.20444824, -1.22071061,
                          2.55062424, -0.0049282, -1.48480399, 0.02559137,
             -2.77228382,
              1.20932987, -0.27461705, -0.38449748, 1.81544701, 1.42018476],
            [-0.31771215, -1.12770044, 0.52870332, 0.45441799, -0.06782859,
              3.32315307, 0.09085441, 0.21685672, -2.55964008, -0.50753101,
             -4.18103891, 1.15825277, -1.02787056, -2.85099339, 1.11968373,
              1.48394873, -0.3777721, 2.33738052, -0.54790802, -1.31954028,
             -0.49610614, -1.47574916, 2.20212292, -2.40325856, 0.13614253],
             [0.83212891, 2.70058426, -0.25334861, 3.64463631, -2.79187033,
             -0.40812296, 0.20749769, -0.0279611, 1.12028818, 1.73415451,
              0.36404344, 0.08872316, 2.1144603, 1.99653281, -0.96665396,
             -0.64563044, 1.48231596, -1.74061986, 3.52292451, 0.40476025,
             -2.2986864 , 1.04561858 , 1.6632179 , 0.94652495 , -1.57550468]
            [0.90896104, 0.36432512, 0.65525468, -1.18058646, 0.5540132,
              0.44296621, -2.98822799, -1.50107615, -0.13536027, -1.19505643,
             -0.3764813 , 0.65895124 , 1.95591732 , -1.45498983 ,
                                                                1.5848723 ,
             -2.678215 , -2.92204044 , -0.10626521 , -1.58376807 , 0.69324541 ,
             -0.15216556, 1.63667766, 3.3535678, 1.05829954, 0.40747782],
            [-0.334517, -2.37437805, 0.03767482, -0.77024691, 0.85790269,
              1.75437202, -0.08612648, 2.92168113, -0.75605866, 1.32223289,
              0.11522473, -1.28373146, -2.18217294, 2.51989342, -2.5640851,
              0.49119728, -0.36016704, 1.52793227, 1.70949389, -1.09296469,
              0.92881864, -2.63937985, -0.22116745, 0.1470785, -3.17362605
            [-2.26303534, -1.19785262, -0.10652274, -1.01538518, -2.71381891,
             -1.82987149, -1.09834257, -0.25145103, -0.02503116, 0.29294229,
              1.24502178, 2.15804841, 0.63344675, 3.54937077, 1.65468867,
              0.21831143, 1.30995909, 1.57876251, -2.0819705, -0.53389697,
```

```
1.79371727, 0.06522304, -2.13830615, -1.68808226, 0.05482755],
             [-1.26979404, -0.15182358, -0.35970662, -1.57226776, 0.1191779,
             -2.37426064, -1.49616705, 1.57861597, -1.61068456, 1.31658228,
              2.19817777, -1.80035683, -2.64920081, -1.66481062, -3.37320626,
              1.88985661, 1.1331821, 1.56401396, 1.91033259, 1.32863181,
             -3.03070372, 1.02860156, -1.96582266, -1.30640229, 1.95234214]])
[21]: # updated biases of second output layer
      bs[1]
[21]: array([[ 0.44534412],
             [-0.79213455],
             [ 0.37066372],
             [-0.52492353],
             [ 0.07146499],
             [0.34065735],
             [ 0.62387333],
             [ 0.10975186],
             [ 0.05616478],
             [-0.50877056]]
```

1.0.3 3. Implement sigmoid and ReLu activation functions and see which performs best. Add a SoftMax activation for the output layer for both cases

```
[11]: # lets write a function to check the accuracy of the model

def accuracy( y_true, y_pred):

"""

Calculates the accuracy of the model.

Parameters:

- y_true (numpy.ndarray):

A 1D array of shape (m_samples, ) representing m_samples each with_

it's label output.

- y_pred (numpy.ndarray):

A 1D array of shape (m_samples, ) representing m_samples each with_

it's label output.

Returns:

- result (float):

The accuracy of the model.
```

```
return np.mean(y_true == y_pred)
[23]: # for the sigmoid as the activation in the hidden layer
      prob_sigmoid = net_sigmoid.predict(X_train)
      prob_sigmoid[:4]
[23]: array([[8.26184805e-01, 5.66628571e-08, 1.74792819e-05, 7.77351227e-05,
              9.73987035e-07, 8.70738127e-07, 1.73715679e-01, 3.65648925e-10,
              2.39910652e-06, 2.00332882e-10],
             [6.94150161e-06, 9.99400130e-01, 1.12551605e-06, 5.73252793e-04,
              1.23867694e-06, 5.96798627e-08, 6.82566342e-07, 1.45606242e-05,
              1.96286832e-06, 4.61121590e-08],
             [2.41388252e-02, 3.91104682e-05, 9.23365325e-01, 7.10811673e-05,
              9.72346062e-04, 1.47407018e-03, 4.99076800e-02, 1.13843251e-07,
              3.11124708e-05, 3.35419241e-07],
             [1.94917520e-01, 4.17640340e-04, 6.42055622e-01, 7.62125840e-05,
              3.68218522e-05, 1.07539113e-04, 1.47720867e-01, 2.09038791e-07,
              1.46625613e-02, 5.00680766e-06]])
[24]: # as the above prediction is the prediction done by softmax which is the
       →probability of each sample belonging to each class
      # so let's take the final label as whose probability is maximum
      def predict_classes( probability):
              11 11 11
              Predicts the class for a given input.
              Parameters:
              - probability (numpy.ndarray):
                  A 2D array of shape (m samples, n features) representing m samples, u
       \neg each with n_features.
              Returns:
              - result (numpy.ndarray):
                  A list of 1D arrays of shape (m_samples, ) representing the
       ⇒predicted output for each of the m_samples.
              11 11 11
              return np.argmax(probability, axis=1)
      pred_sigmoid = predict_classes(prob_sigmoid)
```

Accuracy when sigmoid is used as activation for hidden layer: 0.9283

• Let's train another model for the same parameters except changing the activation function of the Hidden layer from sigmoid to relu

```
[25]: epochs = 500
     layers_relu = [
         # layer-1, nuerons in l=1 are 25
         DenseLayer(784, 25),
         # activation of the first hidden layer 'sigmoid'
         ActivationLayer(ReLU),
         # layer-2 (output-layer), nuerons in l=2 are 10
         DenseLayer(25, 10),
         # activation of the output layer 'Softmax
         ActivationLayer(Softmax)
     ]
     # using l2 regularization , lambda = 0.1
     loss_ = CategoricalCrossentropy(12=0.1)
     X_train = np.array(train_data)
     y_oh = np.array(pd.get_dummies(train_labels))
     net_relu = Network(layers=layers_relu, use_loss = loss_)
     loss_history_relu = net_relu.fit(X_train, y_oh, epochs=epochs, learning_rate=0.
```

```
Epoch 1: loss = 1.223734

Epoch 31: loss = 0.521665

Epoch 61: loss = 0.443733

Epoch 91: loss = 0.402446

Epoch 121: loss = 0.371652

Epoch 151: loss = 0.344593

Epoch 181: loss = 0.322664

Epoch 211: loss = 0.303782

Epoch 241: loss = 0.278880

Epoch 271: loss = 0.265714

Epoch 301: loss = 0.251578
```

```
Epoch 331: loss = 0.232198
Epoch 361: loss = 0.218083
Epoch 391: loss = 0.202282
Epoch 421: loss = 0.198848
Epoch 451: loss = 0.178804
Epoch 481: loss = 0.172237
Epoch 500: loss = 0.165275

[26]: # for the relu as the activation in the hidden layer

prob_relu = net_relu.predict(X_train)
prob_relu[:4]

# get the argmax as the predicted class

pred_relu = predict_classes(prob_relu)

[27]: acc_relu = accuracy(train_labels, pred_relu)
print(f"Accuracy when relu is used as activation for hidden layer: {acc_relu}")
```

Accuracy when relu is used as activation for hidden layer: 0.9419

1.0.4 4. Apply the trained model algorithm on the normalized test dataset and predict the testing accuracy of the model for both sets of activation functions (Use the optimized weights calculated using training data).

```
[28]: # 2. for sigmoid as activation of hidden layer

X_test = np.array(test_data)

prob_sigmoid = net_sigmoid.predict(X_test)
prob_sigmoid[:4]

# get the argmax as the predicted class

pred_sigmoid = predict_classes(prob_sigmoid)

acc_sigmoid = accuracy(test_labels, pred_sigmoid)
print(f"Test Accuracy for sigmoid as activation : {acc_sigmoid}")
```

Test Accuracy for sigmoid as activation: 0.833333333333333333

```
[29]: # let's check the accuracy for both the models

# 1. for relu as activation of hidden layer

X_test = np.array(test_data)
```

```
prob_relu = net_relu.predict(X_test)
prob_relu[:4]

# get the argmax as the predicted class

pred_relu = predict_classes(prob_relu)

acc_relu = accuracy(test_labels, pred_relu)
print(f"Test Accuracy for relu as activation : {acc_relu}")
```

Test Accuracy for relu as activation: 0.9

• Here we can see that test accuracy is better for relu activation function than sigmoid activation function.

1.0.5 5. Use the test data to plot few images along with the model-prediction labels/classes

```
[36]: # function to plot n random images from the dataset X and y, and prints the
       ⇔predicted and true labels.
      def plot_random_images( X, y, n=10):
                  11 11 11
                  Plots n random images from the dataset X and y, and prints the \sqcup
       ⇔predicted and true labels.
                  Parameters:
                  - X (numpy.ndarray):
                      A 2D array of shape (m_samples, n_features) representing_
       \rightarrowm_samples, each with n_features.
                  - y (numpy.ndarray):
                      A 1D array of shape (m_samples, ) representing m_samples each\sqcup
       \neg with it's label output.
                  - n (int):
                      Number of images to plot.
                  Returns:
                  - None
                  titles ={0: 'T-shirt/top', 1: 'Trouser', 2: 'Pullover', 3: 'Dress', |
```

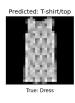
```
6: 'Shirt', 7: 'Sneaker', 8: 'Bag', 9: 'Ankle boot'}
            # get n random indices
            indices = np.random.randint(0, len(X), n)
            # get the images and labels
            images = X[indices]
            labels = y[indices]
            # plot the images
            fig, axes = plt.subplots(1, n, figsize=(20, 15), dpi=150)
            for i in range(n):
                axes[i].imshow(images[i].reshape(28, 28), cmap="gray")
                axes[i].set_title(f"Predicted: {titles[pred_relu[indices[i]]]}")
                axes[i].set_xlabel(f"True: {titles[labels[i]]}")
                axes[i].set_xticks([])
                axes[i].set_yticks([])
            return fig
fig = plot_random_images(X_test, np.array(test_labels), n=7)
fig.savefig("figures/0102.png")
```















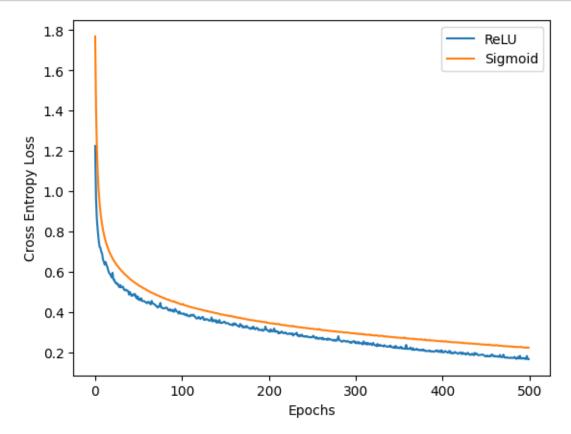
1.0.6 6. Compare how the two NNs fare in terms of prediction accuracy for the same number of optimisation iterations and comment what you can conclude from that

- We can see that for both the NN models the accuracy in training data is good for relu model and it's loss is also far less than the sigmoid model.
- For Test data also the accuracy is better for relu model than sigmoid model.
- So we can conclude that relu activation function is better than sigmoid activation function for this dataset.

1.0.7 7. Plot the evolution of cost function over optimization iterations

```
[37]: plt.plot(loss_history_relu, label="ReLU")
plt.plot(loss_history_sigmoid, label="Sigmoid")
plt.xlabel("Epochs")
plt.ylabel("Cross Entropy Loss")
```

```
plt.legend();
plt.savefig("figures/0103")
```



- So here we can see that for the case of relu activation function the cost function is much less than the sigmoid activation function.
- For test data also the cost function is less for relu activation function. Hence it is not overfitting also.

2 Binary Classification with Nonlinear Decision Boundary

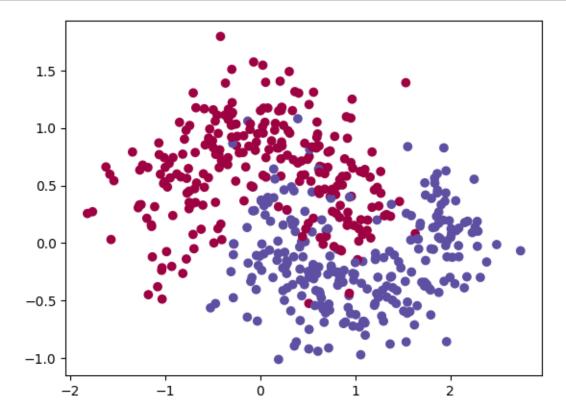
2.0.1 1. Use the following code snippet to generate a 'Binary Classification' dataset

```
[12]: import sklearn
import sklearn.datasets
X, y = sklearn.datasets.make_moons(500, noise=0.30)
import matplotlib.pyplot as plt
```

2.0.2 2. Write a Neural Network code to do Binary Classification using Non-linear Logistic Regression and plot the decision boundary with scattered data-points in the same plot.

```
[13]: # let's plot the data

plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Spectral);
plt.savefig("figures/0201.png")
```



```
[16]: # Lets add the quadratic features to the data
      # i.e x1^2, x2^2, x1*x2
     X_{-} = np.hstack((X, X[:, 0:1] ** 2, X[:, 1:2] ** 2, X[:, 0:1] * X[:, 1:2]))
[17]: X_[:5]
[17]: array([[ 0.51104506, -0.52381803, 0.26116706, 0.27438532, -0.26769462],
             [-0.7745695, 0.56225509, 0.59995791, 0.31613079, -0.43550565],
             [0.54133244, -0.23172011, 0.29304081, 0.05369421, -0.12543761],
             [1.0526018, 0.62230741, 1.10797056, 0.38726652, 0.65504191],
             [-0.83674135, 0.57511969, 0.70013608, 0.33076265, -0.48122642]])
[18]: # new shape of the data
     X_{.}shape
[18]: (500, 5)
[19]: # we will need this time BinaryCrossentropy loss
      class BinaryCrossentropy:
          11 11 11
          Binary cross-entropy loss function.
          def __init__(self, 12 = 0) -> None:
              # by default we don't use regularization
              self.12 = 12
          def loss(self, y_true, y_pred, ws):
              m_samples = y_pred.shape[1]
              cost = (
                  -np.sum(
                      y_true * np.log(y_pred + 1e-10)
                      + (1 - y_true) * np.log(1 - y_pred + 1e-10)
                  / m_samples
              reg_term = 0
              reg_term += (self.12/(2*m_samples)) * ws
              return cost + reg_term
          def loss_prime(self, y_true, y_pred):
```

```
m_samples = y_pred.shape[1]

cost_prime = ((y_pred - y_true) / (y_pred * (1 - y_pred + 1e-10))) /

m_samples

return cost_prime
```

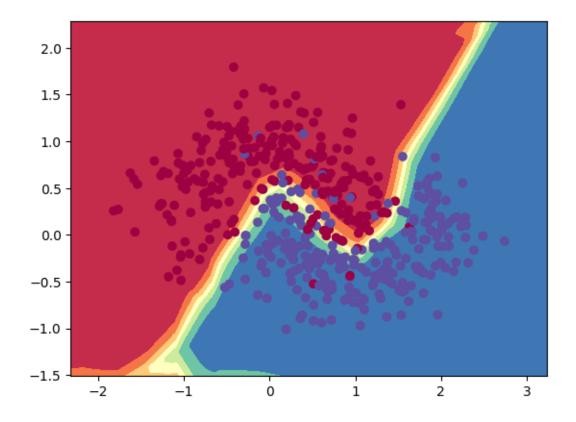
```
[20]: # Loss function, we use the cross entropy loss function
      class CategoricalCrossentropy:
          11 11 11
          Categorical cross-entropy loss function.
          def __init__(self, 12 = 0) -> None:
              # by default we don't use regularization
              self.12 = 12
          def loss(self, y_true, y_pred, ws):
              m_samples = y_pred.shape[1]
              # we use the log trick to avoid the underflow, (Catelogical cross_{\sqcup}
       \rightarrow entropy)
              cost = -np.sum(y_true * np.log(y_pred + 1e-10)) # shape =_
       ⇔(batch_size,)
              cost = cost / m_samples
              reg_term = 0
              reg_term += (self.12/(2*m_samples)) * ws
              return cost + reg_term
          def loss_prime(self, y_true, y_pred):
              m_samples = y_pred.shape[1]
              # this is little bit different from the else loss_prime,
              # this return the (dJ/dA)*(dA/dz) so we don't need to find the
       \rightarrow derivative of sofmax_prime
              cost_prime = (y_pred - y_true) / m_samples
              return cost_prime
```

```
[21]: # let's train the model
      # As this is a binary classification problem, we will use the sigmoid
       →activation function in the output layer.
      # and loss function will be binary cross entropy
      epochs = 100
      layers = [
          # layer-1, nuerons in l=1 are 25
          DenseLayer(5, 25),
          # activation of the first hidden layer 'Relu'
          ActivationLayer(ReLU),
          DenseLayer(25, 25),
          # activation of the second hidden layer 'Relu'
          ActivationLayer(ReLU),
          # layer-2 (output-layer), nuerons in l=2 are 1
          DenseLayer(25, 1),
          # activation of the output layer 'sigmoid'
          ActivationLayer(Sigmoid)
      ]
      # using l2 regularization , lambda = 0.1
      loss_ = BinaryCrossentropy(12=0.1)
      nn = Network(layers, use_loss=loss_)
      # train the model
      loss_history = nn.fit(X_, y, epochs=epochs, learning_rate=0.1, batch_size=64)
     Epoch 1: loss = 0.346795
     Epoch 31: loss = 0.183402
     Epoch 61: loss = 0.175590
     Epoch 91: loss = 0.181081
     Epoch 100: loss = 0.173443
[22]: pred = nn.predict(X_)
      pred_class = np.where(pred > 0.5, 1, 0)
      # accuracy
      accuracy(y, pred_class.reshape(-1))
```

[22]: 0.936

```
[23]: x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
h = 0.01
# Generate a grid of points with distance h between them
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
# Predict the function value for the whole gid
```

```
[25]: # Plot the boundary
      def plot_decision_boundary(model, X, y):
          # Set min and max values and give it some padding
          x_{\min}, x_{\max} = X[:, 0].min() - .5, X[:, 0].max() + .5
          y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
          h = 0.01
          # Generate a grid of points with distance h between them
          xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
          # Add the quadratic features to the data
          final_X = np.hstack((np.c_[xx.ravel(), yy.ravel()], np.c_[xx.ravel(), yy.
       →ravel()] ** 2, (xx.ravel()*yy.ravel()).reshape(-1,1) ))
          # Predict the function value for the whole gid
          Z = model.predict(final_X)
          # Put the result into a color plot
          Z = Z.reshape(xx.shape)
          # Plot the contour and training examples
          plt.contourf(xx, yy, Z, cmap=plt.cm.Spectral)
          plt.scatter(X[:, 0], X[:, 1], c=y, cmap=plt.cm.Spectral)
          return plt
      fig = plot_decision_boundary(nn, X, y)
      fig.savefig("figures/0202.png")
```



• Here we can see that our model is able to classify the data sets very efficiently. Only problem it is facing is at the boundary as few of the points of blue are present in the red.

3 Extra

- I have also made a whole API for the implementation of the Neural Network which can be used for any dataset. It can be found inside the nueral folder.
- There I have 5 py files which are as follows:
- 1. activations.py: It contains the activation functions and their derivatives.
- 2. layers.py: It contains the DenseLayer and ActivationLayer classes.
- 3. losses.py: It contains the Loss class which is used to calculate the loss and it's derivative with respect to the y_pred
- 4. metrics: It contains the Accuracy class which is used to calculate the accuracy of the model.
- 5. nn.py: It contains the NeuralNetwork class which is used to create the neural network model.

- In activations.py I have implemented the following activation functions which can be used in the ActivationLayer class:
- 1. tanh:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

2. relu:

$$relu(x) = max(0, x)$$

3. linear:

$$linear(x) = x$$

4. sigmoid:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

5. hard_sigmoid:

hard_sigmoid(x) =
$$\begin{cases} 0 & x < -2.5 \\ \frac{1}{5}x + 0.5 & -2.5 \le x \le 2.5 \\ 1 & x > 2.5 \end{cases}$$

6. softmax:

$$\operatorname{softmax}(x) = \frac{e^x}{\sum_{i=1}^n e^{x_i}}$$

- In layers.py I have implemented the following classes:
- 1. DenseLayer: It is a fully connected layer which takes # input nuerons, # output nuerons, 11 and 12 regularization as an arguments.

-It also take activation class's object as an argument which can be used for the activation purpose if we don't want to use the ActivationLayer class.

- It also take the type of initialization of the weights and biases as an argument which can be he or xavier or random or zeros or ones.
- 2. ActivationLayer: It takes the activation class's object as an argument which can be any activation. It is callable so we can use a self defined activation function also which is not present in the activations.py file.
- 3. DropoutLayer: It takes the dropout_rate as an argument which is the probability of the neuron to be dropped. It just sets the output of the neuron to zero with the given probability.
- It can be used as a regularizer to prevent overfitting.
- In losses.py I have implemented the following classes:
- 1. MSE: It is the Mean Squared Error loss function which is used for regression problems. It takes the y_true and y_pred as an arguments and returns the loss and derivative of the loss with respect to the y_pred.

- 2. MAE: It is the Mean Absolute Error loss function which is used for regression problems. It takes the y_true and y_pred as an arguments and returns the loss and derivative of the loss with respect to the y_pred.
- 3. BinaryCrossEntropy: It is the Binary Cross Entropy loss function which is used for binary classification problems. It takes the y_true and y_pred as an arguments and returns the loss and derivative of the loss with respect to the y_pred.
- 4. CategoricalCrossEntropy: It is the Categorical Cross Entropy loss function which is used for multi-class classification problems. It takes the y_true and y_pred as an arguments and returns the loss and derivative of the loss with respect to the y_pred.
- In metrics.py I have implemented the following classes:
- 1. Accuracy: It is used to calculate the accuracy of the model. It takes the y_true and y_pred as an arguments and returns the accuracy.
- 2. Precision: It is used to calculate the precision of the model. It takes the y_true and y_pred as an arguments and returns the precision.
- 3. Recall: It is used to calculate the recall of the model. It takes the y_true and y_pred as an arguments and returns the recall.
- 4. F1Score: It is used to calculate the F1 Score of the model. It takes the y_true and y_pred as an arguments and returns the F1 Score.
- 5. r2_score: It is used to calculate the r2_score of the model. It takes the y_true and y_pred as an arguments and returns the r2_score.
- 6. confusion_matrix: It is used to calculate the Confusion Matrix of the model. It takes the y_true and y_pred as an arguments and returns the Confusion Matrix.
- In nn.py I have implemented the following classes:
- 1. NeuralNetwork: It is the main class which is used to create the neural network model.
- let's implement a simple neural network model for the Classification problem.
- import the important libraries from the nueral folder.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from neural.nn import NeuralNetwork
from neural.layers import ActivationLayer, DenseLayer, DropoutLayer
from neural.losses import CategoricalCrossentropy
from neural.activations import Sigmoid, ReLU, Softmax
from neural.metrics import Metrics
```

• let's train a classification model on the Fashion-MNIST dataset.

```
[2]: df = pd.read_csv('data/fashion-mnist_train.csv')
   y_train = df['label'].values
   x_train = df.drop('label', axis=1).values/255.

test_df = pd.read_csv('data/fashion-mnist_test.csv')
   y_test = test_df['label'].values
   x_test = test_df.drop('label', axis=1).values/255.
```

```
[3]: # define a callback function to calculate different metrics
     precisions_ = []
     accuracies_ = []
     recalls_ = []
     f1s_{-} = []
     losses_ = []
     def callback(true, predicted, loss, epoch):
         losses_.append(loss)
         pred = Metrics.predict_classes(predicted)
         # print(pred.shape)
         global precisions_
         global accuracies_
         global recalls_
         global f1s_
         precisions_.append(Metrics.precision(true, pred))
         accuracies_.append(Metrics.accuracy(true, pred))
         recalls_.append(Metrics.recall(true, pred))
         f1s_.append(Metrics.f1_score(true, pred))
```

• Fianlly let's create our model

```
model.use_loss(CategoricalCrossentropy(12=0.1))
   # Print the model summary
   model.summary()
   Summary of the Neural Network
   Layer (type)
                Neurons #
                             Input Shape
                                        Output Shape Weights shape
           Param #
   Bias shape
   ______
                                         (784, None)
   Input
                  784
                     0
                            (784, None)
                                        (25, None)
   DenseLayer
                 25
                                                   (25, 784)
   (25, 1)
                  19625
                 10
                             (25, None)
                                        (10, None)
                                                   (10, 25)
   DenseLayer
   (10, 1)
                   260
   _____
   _____
   Total params
   19885
[5]: # one-hot encode the labels as this is a multiclass classification problem
   y_oh = np.array(pd.get_dummies(y_train))
   model.fit(x_train, y_oh, epochs=500, learning_rate=0.01, batch_size=128,__
    ⇒verbose=1, callback=callback)
   Epoch 1-500 =========> cost: 1.7929
   Epoch 11-500 ========> cost: 0.7316
   Epoch 21-500 =========> cost: 0.6176
   Epoch 31-500 =========> cost: 0.5529
   Epoch 41-500 ========> cost: 0.5135
   Epoch 51-500 ========> cost: 0.4885
   Epoch 61-500 =========> cost: 0.4667
   Epoch 71-500 =========> cost: 0.4515
   Epoch 81-500 =========> cost: 0.4373
   Epoch 91-500 =========> cost: 0.4261
   Epoch 101-500 =========> cost: 0.4205
   Epoch 111-500 =========> cost: 0.4054
```

Epoch 121-500 =========> cost: 0.3981 Epoch 131-500 =========> cost: 0.3906 Epoch 141-500 =========> cost: 0.3837 Epoch 151-500 ========> cost: 0.3772

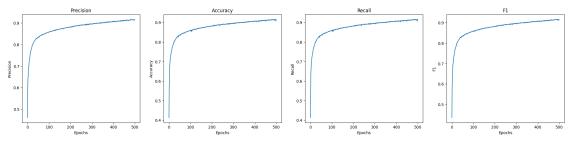
```
Epoch 171-500 =========> cost: 0.3627
   Epoch 181-500 =========> cost: 0.3600
   Epoch 191-500 =========> cost: 0.3588
   Epoch 201-500 =========> cost: 0.3486
   Epoch 211-500 =========> cost: 0.3414
   Epoch 221-500 =========> cost: 0.3387
   Epoch 231-500 =========> cost: 0.3336
   Epoch 241-500 =========> cost: 0.3287
   Epoch 251-500 =========> cost: 0.3230
   Epoch 261-500 =========> cost: 0.3195
   Epoch 271-500 =========> cost: 0.3144
   Epoch 281-500 =========> cost: 0.3149
   Epoch 291-500 =========> cost: 0.3079
   Epoch 301-500 =========> cost: 0.3040
   Epoch 311-500 =========> cost: 0.2994
   Epoch 321-500 =========> cost: 0.2972
   Epoch 331-500 =========> cost: 0.2927
   Epoch 341-500 =========> cost: 0.2931
   Epoch 351-500 =========> cost: 0.2855
   Epoch 361-500 =========> cost: 0.2856
   Epoch 371-500 =========> cost: 0.2791
   Epoch 381-500 =========> cost: 0.2774
   Epoch 391-500 =========> cost: 0.2743
   Epoch 401-500 =========> cost: 0.2716
   Epoch 411-500 =========> cost: 0.2667
   Epoch 421-500 =========> cost: 0.2660
   Epoch 431-500 =========> cost: 0.2685
   Epoch 441-500 =========> cost: 0.2616
   Epoch 451-500 =========> cost: 0.2635
   Epoch 461-500 =========> cost: 0.2544
   Epoch 471-500 =========> cost: 0.2571
   Epoch 481-500 =========> cost: 0.2557
   Epoch 491-500 =========> cost: 0.2477
[6]: def plot_metrics(metrics, titles):
       # plot all the metrics in one plot
       fig, ax = plt.subplots(1, len(metrics), figsize=(25, 5))
       if len(metrics) == 1:
          ax = [ax]
       for i, metric in enumerate(metrics):
          ax[i].plot(metric)
          ax[i].set_title(titles[i])
          ax[i].set_xlabel("Epochs")
          ax[i].set_ylabel(titles[i])
```

Epoch 161-500 =========> cost: 0.3709

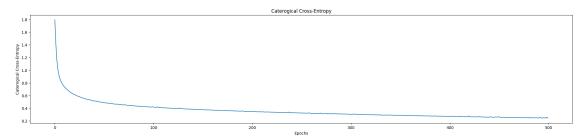
```
plt.show()

plot_metrics([precisions_, accuracies_, recalls_, f1s_], ["Precision", □

→"Accuracy", "Recall", "F1"])
```



[7]: plot_metrics([losses_], ["Caterogical Cross-Entropy"])



• So here we can see that Precision, Accuracy, Recall and F1 Score are all increasing with the epochs.

and Loss is decreasing with the epochs.

so our model is learning. it can be trained for more epochs to get better results.

```
[8]: # let's see the accuracy on the training set

print(f"Training Accuracy: {accuracies_[-1]}")
```

Training Accuracy: 0.9159

```
[9]: prob_test = model.predict(x_test)
  test_pred = Metrics.predict_classes(prob_test)
  print("Test Accuracy: ", Metrics.accuracy(y_test, test_pred))
```

Test Accuracy: 0.9

• As we can see that it is well performing on the test data and can be trained for more epochs also as it is not overfitting.