Problem-3

January 24, 2023

1 3. Logistic Regression Problem

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

from scipy.optimize import minimize

plt.rcParams['figure.figsize'] = (10, 6)
plt.rcParams['figure.dpi'] = 150
plt.rcParams['font.size'] = 14
```

1.0.1 1. Inspect and plot some portion of the training data using pandas. Segregate the training and testing data into two separate variables consisting of 'feature values' and corresponding 'predictions' (the prediction column is titled 'RainTomorrow' in the dataset). To simplify the problem a bit, clean the whole data by carrying out the following sub-tasks:

[2]:
```
train_weather = pd.read_csv("data/weather_train.csv")
test_weather = pd.read_csv("data/weather_test.csv")
train_weather.head()
```

[2]:
```
         Date Location  MinTemp  MaxTemp  Rainfall  Evaporation  Sunshine  \
0  01-12-2008   Albury     13.4     22.9       0.6          NaN       NaN
1  02-12-2008   Albury      7.4     25.1       0.0          NaN       NaN
2  03-12-2008   Albury     12.9     25.7       0.0          NaN       NaN
3  04-12-2008   Albury      9.2     28.0       0.0          NaN       NaN
4  05-12-2008   Albury     17.5     32.3       1.0          NaN       NaN

  WindGustDir  WindGustSpeed WindDir9am  … Humidity9am  Humidity3pm  \
0           W           44.0          W  …        71.0         22.0
1         WNW           44.0        NNW  …        44.0         25.0
2         WSW           46.0          W  …        38.0         30.0
3          NE           24.0         SE  …        45.0         16.0
4           W           41.0        ENE  …        82.0         33.0

   Pressure9am  Pressure3pm  Cloud9am  Cloud3pm  Temp9am  Temp3pm  RainToday  \
```

```
0       1007.7       1007.1       8.0       NaN     16.9     21.8         No
1       1010.6       1007.8       NaN       NaN     17.2     24.3         No
2       1007.6       1008.7       NaN       2.0     21.0     23.2         No
3       1017.6       1012.8       NaN       NaN     18.1     26.5         No
4       1010.8       1006.0       7.0       8.0     17.8     29.7         No

   RainTomorrow
0            No
1            No
2            No
3            No
4            No

[5 rows x 23 columns]
```

(a) Convert the predictions in the binary format by using '1' for 'YES' and '0' for 'NO'.

[3]:
```
train_weather = train_weather.replace({"RainTomorrow":{"Yes":1, "No":0}})
train_weather.head()
```

[3]:
```
         Date Location  MinTemp  MaxTemp  Rainfall  Evaporation  Sunshine  \
0  01-12-2008   Albury     13.4     22.9       0.6          NaN       NaN
1  02-12-2008   Albury      7.4     25.1       0.0          NaN       NaN
2  03-12-2008   Albury     12.9     25.7       0.0          NaN       NaN
3  04-12-2008   Albury      9.2     28.0       0.0          NaN       NaN
4  05-12-2008   Albury     17.5     32.3       1.0          NaN       NaN

  WindGustDir  WindGustSpeed WindDir9am  … Humidity9am  Humidity3pm  \
0           W           44.0          W  …        71.0         22.0
1         WNW           44.0        NNW  …        44.0         25.0
2         WSW           46.0          W  …        38.0         30.0
3          NE           24.0         SE  …        45.0         16.0
4           W           41.0        ENE  …        82.0         33.0

   Pressure9am  Pressure3pm  Cloud9am  Cloud3pm  Temp9am  Temp3pm  RainToday  \
0       1007.7       1007.1       8.0       NaN     16.9     21.8         No
1       1010.6       1007.8       NaN       NaN     17.2     24.3         No
2       1007.6       1008.7       NaN       2.0     21.0     23.2         No
3       1017.6       1012.8       NaN       NaN     18.1     26.5         No
4       1010.8       1006.0       7.0       8.0     17.8     29.7         No

   RainTomorrow
0             0
1             0
2             0
3             0
```

```
4             0

[5 rows x 23 columns]
```

- converting the same for the test data as well

[4]:
```
test_weather = test_weather.replace({"RainTomorrow":{"Yes":1 , "No":0}})
test_weather.head()
```

[4]:

	Date	Location	MinTemp	MaxTemp	Rainfall	Evaporation	Sunshine	\
0	11-01-2014	PearceRAAF	19.0	44.5	0.0	NaN	13.1	
1	12-01-2014	PearceRAAF	31.2	44.3	0.0	NaN	12.2	
2	13-01-2014	PearceRAAF	19.3	30.3	0.0	NaN	9.4	
3	14-01-2014	PearceRAAF	14.0	29.7	0.0	NaN	13.1	
4	15-01-2014	PearceRAAF	12.7	27.9	0.0	NaN	12.4	

	WindGustDir	WindGustSpeed	WindDir9am	…	Humidity9am	Humidity3pm	\
0	E	54.0	E	…	27.0	10.0	
1	E	54.0	N	…	10.0	22.0	
2	WSW	46.0	SSW	…	63.0	43.0	
3	WSW	44.0	SSE	…	43.0	29.0	
4	W	50.0	NW	…	48.0	34.0	

	Pressure9am	Pressure3pm	Cloud9am	Cloud3pm	Temp9am	Temp3pm	RainToday	\
0	1015.3	1009.7	NaN	NaN	31.2	42.9	No	
1	1007.7	1007.8	NaN	NaN	40.2	35.8	No	
2	1010.9	1009.5	7.0	2.0	23.2	27.6	No	
3	1012.2	1009.5	0.0	0.0	23.0	29.0	No	
4	1008.4	1008.4	1.0	3.0	23.4	25.7	No	

	RainTomorrow
0	0.0
1	0.0
2	0.0
3	0.0
4	0.0

[5 rows x 23 columns]

(b) Identify and drop the feature columns having datatype 'object'.

- Columns having object datatype are

[5]:
```
obj_dtype_columns = train_weather.select_dtypes(include=['object']).columns
obj_dtype_columns
```

[5]:

```
Index(['Date', 'Location', 'WindGustDir', 'WindDir9am', 'WindDir3pm',
       'RainToday'],
      dtype='object')
```

- Dropping them from both train and test data

[6]:

```python
train_weather = train_weather.drop(columns=obj_dtype_columns, axis=1)
train_weather.head()
```

[6]:

```
   MinTemp  MaxTemp  Rainfall  Evaporation  Sunshine  WindGustSpeed  \
0     13.4     22.9       0.6          NaN       NaN           44.0
1      7.4     25.1       0.0          NaN       NaN           44.0
2     12.9     25.7       0.0          NaN       NaN           46.0
3      9.2     28.0       0.0          NaN       NaN           24.0
4     17.5     32.3       1.0          NaN       NaN           41.0

   WindSpeed9am  WindSpeed3pm  Humidity9am  Humidity3pm  Pressure9am  \
0          20.0          24.0         71.0         22.0       1007.7
1           4.0          22.0         44.0         25.0       1010.6
2          19.0          26.0         38.0         30.0       1007.6
3          11.0           9.0         45.0         16.0       1017.6
4           7.0          20.0         82.0         33.0       1010.8

   Pressure3pm  Cloud9am  Cloud3pm  Temp9am  Temp3pm  RainTomorrow
0       1007.1       8.0       NaN     16.9     21.8             0
1       1007.8       NaN       NaN     17.2     24.3             0
2       1008.7       NaN       2.0     21.0     23.2             0
3       1012.8       NaN       NaN     18.1     26.5             0
4       1006.0       7.0       8.0     17.8     29.7             0
```

[7]:

```python
test_weather = test_weather.drop(columns=obj_dtype_columns, axis=1)
test_weather.head()
```

[7]:

```
   MinTemp  MaxTemp  Rainfall  Evaporation  Sunshine  WindGustSpeed  \
0     19.0     44.5       0.0          NaN      13.1           54.0
1     31.2     44.3       0.0          NaN      12.2           54.0
2     19.3     30.3       0.0          NaN       9.4           46.0
3     14.0     29.7       0.0          NaN      13.1           44.0
4     12.7     27.9       0.0          NaN      12.4           50.0

   WindSpeed9am  WindSpeed3pm  Humidity9am  Humidity3pm  Pressure9am  \
0          26.0          15.0         27.0         10.0       1015.3
1          31.0          30.0         10.0         22.0       1007.7
2          22.0          28.0         63.0         43.0       1010.9
3          17.0          30.0         43.0         29.0       1012.2
4          11.0          31.0         48.0         34.0       1008.4
```

	Pressure3pm	Cloud9am	Cloud3pm	Temp9am	Temp3pm	RainTomorrow
0	1009.7	NaN	NaN	31.2	42.9	0.0
1	1007.8	NaN	NaN	40.2	35.8	0.0
2	1009.5	7.0	2.0	23.2	27.6	0.0
3	1009.5	0.0	0.0	23.0	29.0	0.0
4	1008.4	1.0	3.0	23.4	25.7	0.0

- Checking the columns of train data and test data after dropping the object datatype columns

[8]:
```
assert train_weather.columns.all() == test_weather.columns.all()
```

(c) Identify cells having 'NaN' or 'NA' values and replace them with mean values of their respective columns.

- Training data NaN or NA values

[9]:
```
train_weather.isna().sum()
```

[9]:
```
MinTemp           524
MaxTemp           390
Rainfall         1296
Evaporation     28015
Sunshine        33621
WindGustSpeed    5522
WindSpeed9am     1132
WindSpeed3pm     1756
Humidity9am       790
Humidity3pm      1395
Pressure9am      6995
Pressure3pm      6945
Cloud9am        22448
Cloud3pm        22886
Temp9am           509
Temp3pm          1156
RainTomorrow        0
dtype: int64
```

- Test data NaN or NA values

[10]:
```
test_weather.isna().sum()
```

[10]:
```
MinTemp           171
MaxTemp           139
Rainfall          404
Evaporation     12117
Sunshine        12752
WindGustSpeed     476
WindSpeed9am      155
```

```
WindSpeed3pm       161
Humidity9am        230
Humidity3pm        983
Pressure9am       4203
Pressure3pm       4204
Cloud9am         11637
Cloud3pm         12308
Temp9am            132
Temp3pm            815
RainTomorrow       408
dtype: int64
```

- Filling the NaN or NA values with their respective mean values in both train and test data using fillna() function

[11]:
```
train_weather = train_weather.fillna(train_weather.mean())
train_weather.head()
```

[11]:

	MinTemp	MaxTemp	Rainfall	Evaporation	Sunshine	WindGustSpeed	\
0	13.4	22.9	0.6	5.52876	7.568706	44.0	
1	7.4	25.1	0.0	5.52876	7.568706	44.0	
2	12.9	25.7	0.0	5.52876	7.568706	46.0	
3	9.2	28.0	0.0	5.52876	7.568706	24.0	
4	17.5	32.3	1.0	5.52876	7.568706	41.0	

	WindSpeed9am	WindSpeed3pm	Humidity9am	Humidity3pm	Pressure9am	\
0	20.0	24.0	71.0	22.0	1007.7	
1	4.0	22.0	44.0	25.0	1010.6	
2	19.0	26.0	38.0	30.0	1007.6	
3	11.0	9.0	45.0	16.0	1017.6	
4	7.0	20.0	82.0	33.0	1010.8	

	Pressure3pm	Cloud9am	Cloud3pm	Temp9am	Temp3pm	RainTomorrow
0	1007.1	8.00000	4.487216	16.9	21.8	0
1	1007.8	4.43216	4.487216	17.2	24.3	0
2	1008.7	4.43216	2.000000	21.0	23.2	0
3	1012.8	4.43216	4.487216	18.1	26.5	0
4	1006.0	7.00000	8.000000	17.8	29.7	0

[12]:
```
test_weather = test_weather.fillna(test_weather.mean())
test_weather.head()
```

[12]:

	MinTemp	MaxTemp	Rainfall	Evaporation	Sunshine	WindGustSpeed	\
0	19.0	44.5	0.0	6.096236	13.1	54.0	
1	31.2	44.3	0.0	6.096236	12.2	54.0	
2	19.3	30.3	0.0	6.096236	9.4	46.0	
3	14.0	29.7	0.0	6.096236	13.1	44.0	

```
4     12.7     27.9       0.0     6.096236      12.4           50.0

   WindSpeed9am  WindSpeed3pm  Humidity9am  Humidity3pm  Pressure9am  \
0          26.0          15.0         27.0         10.0       1015.3
1          31.0          30.0         10.0         22.0       1007.7
2          22.0          28.0         63.0         43.0       1010.9
3          17.0          30.0         43.0         29.0       1012.2
4          11.0          31.0         48.0         34.0       1008.4

   Pressure3pm  Cloud9am  Cloud3pm  Temp9am  Temp3pm  RainTomorrow
0       1009.7  3.780706  3.935895     31.2     42.9           0.0
1       1007.8  3.780706  3.935895     40.2     35.8           0.0
2       1009.5  7.000000  2.000000     23.2     27.6           0.0
3       1009.5  0.000000  0.000000     23.0     29.0           0.0
4       1008.4  1.000000  3.000000     23.4     25.7           0.0
```

- After filling the NaN or NA values in train data

[13]:
```
train_weather.isna().sum()
```

[13]:
```
MinTemp          0
MaxTemp          0
Rainfall         0
Evaporation      0
Sunshine         0
WindGustSpeed    0
WindSpeed9am     0
WindSpeed3pm     0
Humidity9am      0
Humidity3pm      0
Pressure9am      0
Pressure3pm      0
Cloud9am         0
Cloud3pm         0
Temp9am          0
Temp3pm          0
RainTomorrow     0
dtype: int64
```

- After filling the NaN or NA values in test data

[14]:
```
test_weather.isna().sum()
```

[14]:
```
MinTemp          0
MaxTemp          0
Rainfall         0
Evaporation      0
Sunshine         0
```

```
WindGustSpeed    0
WindSpeed9am     0
WindSpeed3pm     0
Humidity9am      0
Humidity3pm      0
Pressure9am      0
Pressure3pm      0
Cloud9am         0
Cloud3pm         0
Temp9am          0
Temp3pm          0
RainTomorrow     0
dtype: int64
```

(d) Normalize all the feature values by scaling them between 0 and 1. The values in feature matrix 'X' can be normalized as:

- Normalizing the train and test data using the formula

$$X_{norm} = \frac{X - X_{min}}{X_{max} - X_{min}}$$

- writing a function to normalize the data between 0 and 1

[15]:
```
def normalize(data):
    return (data - data.min()) / (data.max() - data.min())
```

[16]:
```
for col in train_weather.columns:
    if col != "RainTomorrow":
        train_weather[col] = normalize(train_weather[col])
        test_weather[col] = normalize(test_weather[col])
```

- Print the max and min of each column to make sure they are normalized

[17]:
```
for col in train_weather.columns:
    print(col, train_weather[col].max(), train_weather[col].min())
```

```
MinTemp 1.0 0.0
MaxTemp 1.0 0.0
Rainfall 1.0 0.0
Evaporation 1.0 0.0
Sunshine 1.0 0.0
WindGustSpeed 1.0 0.0
WindSpeed9am 1.0 0.0
WindSpeed3pm 1.0 0.0
Humidity9am 1.0 0.0
Humidity3pm 1.0 0.0
Pressure9am 1.0 0.0
```

```
Pressure3pm 1.0 0.0
Cloud9am 1.0 0.0
Cloud3pm 1.0 0.0
Temp9am 1.0 0.0
Temp3pm 1.0 0.0
RainTomorrow 1 0
```

- Checking the same for test data

[18]:
```
for col in test_weather.columns:
    print(col, test_weather[col].max(), test_weather[col].min())
```

```
MinTemp 1.0 0.0
MaxTemp 1.0 0.0
Rainfall 1.0 0.0
Evaporation 1.0 0.0
Sunshine 1.0 0.0
WindGustSpeed 1.0 0.0
WindSpeed9am 1.0 0.0
WindSpeed3pm 1.0 0.0
Humidity9am 1.0 0.0
Humidity3pm 1.0 0.0
Pressure9am 1.0 0.0
Pressure3pm 1.0 0.0
Cloud9am 1.0 0.0
Cloud3pm 1.0 0.0
Temp9am 1.0 0.0
Temp3pm 1.0 0.0
RainTomorrow 1.0 0.0
```

1.0.2 2. Classify the cleaned dataset using binary classification algorithm discussed in the class and calculate the optimized weights and training set accuracy for the model (use Truncated Newton's Method in SciPy for optimization).

- Separating the train data into train_x and train_y and adding a column of ones to train_x for the bias term

[19]:
```
X = train_weather.drop(columns="RainTomorrow")
# insert a column of ones to the beginning of the dataframe
X.insert(0, "Ones", 1)
y = train_weather["RainTomorrow"]
```

- Doing the same for test data

[20]:
```
test_X = test_weather.drop(columns="RainTomorrow")
# insert a column of ones to the beginning of the dataframe
test_X.insert(0, "Ones", 1)
test_y = test_weather["RainTomorrow"]
```

- Defining few important functions for the logistic regression model

[21]:
```
# define the sigmoid function
def sigmoid(z):
    return 1 / (1 + np.exp(-z))


# define the cost function which needs to be minimized
def cost_function(weights, X, y):
    m = len(y)
    y_hat =sigmoid(np.dot(X, weights.T))
    cost = np.sum(-y * np.log(y_hat) - (1 - y) * (np.log(1 - y_hat))) / m
    return cost

# define the gradient function
def gradient(weights, X, y):
    m = len(y)
    y_hat = sigmoid(np.dot(X, weights.T))
    grad = np.dot(X.T, (y_hat - y)) / m
    return grad


# initialize the weights randomly
initial_weights = np.random.rand(X.shape[1])
```

[22]:
```
def accuracy(weights, X, y):
    y_hat = sigmoid(np.dot(X, weights.T))
    y_hat = np.where(y_hat >= 0.5, 1, 0)
    return np.sum(y_hat == y) / len(y)
```

[23]:
```
# Define the call back function for storing the cost values and accuracy values
# during the optimixation of the weights by scipy.optimize.minimize
test_costs = []
train_costs = []
train_acc = []
test_acc = []
def callback(weights):
    train_costs.append(cost_function( weights, X, y))
    test_costs.append(cost_function(weights, test_X, test_y))
    # calculate the training accuracy of the model
    train_accuracy = accuracy(weights, X, y)
    train_acc.append(train_accuracy)

    # calculate the test accuracy of the model
    test_accuracy = accuracy(weights, test_X, test_y)
    test_acc.append(test_accuracy)
```

- Our objective is to minimize the cost function using Truncated Newton's Method in SciPy

for optimization to get the optimized weights

[24]:
```
optimized_result = minimize(
    fun=cost_function,
    args=(X, y),
    x0=initial_weights,
    method='TNC',
    jac=gradient,
    callback=callback,
    tol=1e-10
)
```

- In Above miniminze function,

1. func: The function to minimize
2. args: Extra arguments to func (here, X, y)
3. x0: The initial guess
4. method: The optimization method to use (here, TNC)
5. jac: The gradient of func to be minimized
6. callback: A function to call after each iteration of the optimizer
7. tol: Tolerance for termination

- Now checking how well our model has classified the training data

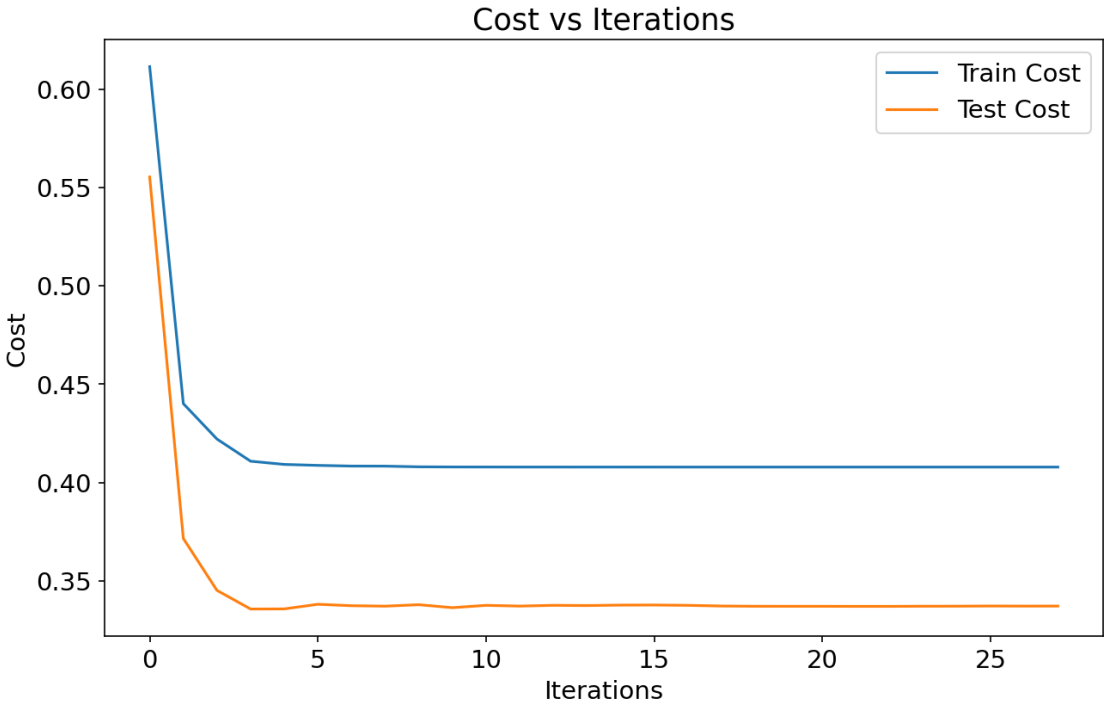
[25]:
```
print(f"Training Accuracy:  {train_acc[-1]}")
```

```
Training Accuracy:  0.822865045522646
```

1.0.3 3. Plot the cost history (J) vs. the number of iterations. [Hint: You can make use of 'callback function' in Optimize.minimize to store the cost history]

- Plottiing the cost history vs number of iterations which we stored using callback function

[26]:
```
plt.plot(train_costs, label="Train Cost")
plt.plot(test_costs, label="Test Cost")
plt.xlabel("Iterations")
plt.ylabel("Cost")
plt.title("Cost vs Iterations")
plt.legend()
fig = plt.gcf()
fig.savefig("figures/0301.png")
```



- So we can see that after almost 5 iterations the cost function is saturated for both training and test data
- We have less cost function's value for training data than test data, this means that our model is not overfitting the training data

1.0.4 4. Apply the trained model on the cleaned test dataset to predict the testing accuracy of the model.

- Our final trained model's weights are

[27]:
```
print(f"Optimized Weights: \n{optimized_result.x}")
```

```
Optimized Weights:
[-5.56372062 -0.68463501  1.00407068  7.15586371  1.76634653 -1.59442006
  5.83119481  1.05492752 -2.37570451  1.14297907  5.25303478  5.31103742
 -6.66614209  0.23726313  1.0158792   0.0673261   0.48519068]
```

- Using the above weights we can predict the test data

[28]:
```
print(f"Test Accuracy:  {test_acc[-1]}")
```

```
Test Accuracy:  0.8471797339566218
```

- The Above accuracy on test data is more than the accuracy on training data, this means that our model is not overfitting the training data
- We can also see the improvement in the accuracy at each iteration

[29]:
```
plt.plot(train_acc, label="Train Cost")
plt.plot(test_acc, label="Test Cost")
plt.xlabel("Iterations")
plt.ylabel("Accuracy")
plt.title("Accuracy vs Iterations")
plt.legend()
fig = plt.gcf()
fig.savefig("figures/0302.png")
```