Ch.2 POSITION PREDICTION

aims to predict the aircraft pos^n in near future
depending upon the speed, the predic^n is done for about ~~30sec~~ to 1min in future

- accounts for st. & turning flight

- KINEMATICS OF AIRCRAFT
  - described by a set of parameters that are the result of the underlying dynamics
  What we need for TAWS are the foll. parameters:
  (i) Pos^n
  (ii) Horizontal Velocity
  (iii) Flight Path
  (iv) Yaw Rate

  } used to find a suitable simplified dynamics model which forms a basis of prediction

* POSITIONS
  given in the form of:
  Latitude (ϕ) ; longitude (λ) ; Altitude (h)
  $\underbrace{\text{Latitude } (\phi) \text{ ; longitude } (\lambda)}$
  together define the horizontal pos^n
  Dist^n from N/S poles?

- primarily relies on GPS pos^n & altitude

* POSITION
  Latitude (ϕ) ; Longitude (λ) ; altitude (h)

  primary altitude reference : barometer

  The horizontal & vertical datum of the used terrain must match with those of the pos^n obtained.

- HORIZONTAL VELOCITY
  for defining velocity, a local frame needs to be introduced origin of whom is arbitrary
  - defined by 3 dimensional, right handed cartesian system with x_1 axis pointing North
    x_2 " — " — East
    x_3 " — " — Down
  A vector in this system is composed of components in north (n), east (e) & down (d) dir^n.

  Frame: N(orth) - E(ast) - D(own) frame (NED).
  & the velo. vector:

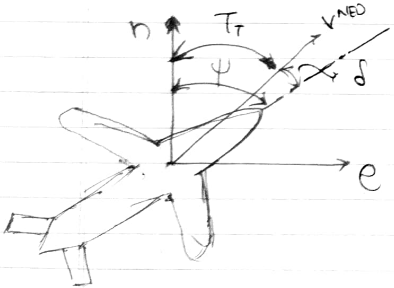
$$V^{NED} = \begin{bmatrix} V_n \\ V_e \\ V_d \end{bmatrix} \qquad (2.1)$$

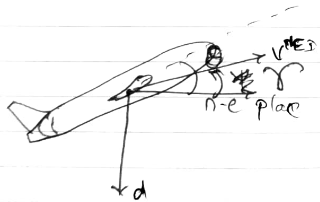
Horizontal velocity $(V_{hor}^{NED}) = \sqrt{V_n^2 + V_e^2 + V_d^2}$ — (2.2)

- FLIGHT PATH - derived from the velocity vector
  └ dirⁿ in which the aircraft is flying wrt NED-frame.

dirⁿ of flight path is split into a horizontal & a vertical component.
vertical component is described by the angle γ
& the horizontal component by the angle T_T

γ ⟶ flight path angle
T_T ⟶ true track angle.





$$\boxed{\gamma = \arcsin \frac{V_d}{\|V^{NED}\|}} \quad (2.3)$$

$$T_T = \arctan \frac{V_e}{V_n}$$

Flight path angle is the angle betⁿ ne plane of the NED frame & the velocity vector

True track angle expresses the dirⁿ of the velocity wrt true North dirⁿ (of our frame).

- Yaw Rate ($\dot{\psi}$) - derived from the yaw (heading) angle of the attitude

attitude — spatial orientation relative to a reference frame, typically defined by the horizon & fixed points in space

— describes the orientation of the body frame (b frame) relative to the NED frame by 3 angles:
- ψ : (Yaw or Heading
- θ : Pitch
- ϕ : Roll or Bank

$\dot{\psi}$ = time derivative of yaw angle.

NOTE : Actual Yaw angle is not of interest, the True track angle is used instead.

Crabbing : The path and the dirⁿ of velo are coincident when the yaw angle = 0.
However, when not, the aircraft is actually crabbing
↳ technique used by pilots to counteract a cross wind.
Airplane yaws in wind, such that relative wind is coming from front.

δ = drift angle

- <u>Sensor Data</u> :

GPS → primary
But yet to increase & improve accuracy & integrity, others, like IRS are recommended

When Multiple sensors are used for determining a parameter, a suitable integrⁿ method (like Kalman filter, complementary filter) are to be used.

GPS

provides : Hor. posⁿ, Altitude, Horizontal & vertical figure of merit (HFOM & VFOM), True Track Angle (T_T) & Flight path angle (γ) as well as Ground speed.

<u>Horizontal posⁿ</u> : latitude & longitude in WGS84 coordinates
- It gives the altitude based on the ellipsoid model & have to be verified, if the model we are using for our terrain databas is different.

HFOM expresses the accuracy of the horizontal posⁿ in nautical miles & VFOM ~~does for~~ expresses the accuracy of the altitude in feet
accuracy of 95% confidence level ; imp factors to determine the reliablity of the supplied info

- True track is provided along with the ground speed
True track & ground speed contain some lag bcoz ~~of~~ these info are derived from positions over time

→ Inertial Referance system (IRS)
gives us:
Horizontal pos^n ; HFOM ; True Track angle.
Flight Path angle, Ground speed ;
Yaw Rate.

- provides info at a much bigger rate
- also denoted as Inertial Measurement System (IMS).

→ ADC (Air data Comp.) & RA (Radio Altimeter
↳ provides barometric altitude
- must be handled with care, as they directly depend & influenced by atm. cond^{n}s & pilot settings.

RA → radio altitude is given ;
which is the relative height of the aircraft above the ground.

- **Aircraft Predic^n of Pos^n :**
- must account for straight flight ($\dot{\psi} = 0$) & turning flight ($\dot{\psi} \neq 0$) as well.

→ 2 imp requirements :
(1.) The FLTA function should be available during all the airborne phases of flight including turning flight
(2.) The lateral search volume should expand as necessary to accomodate turning flight.

Predic^n Algorithm is used to predict the pos^{n}s.
But before that, the type of flight (straight / turning) should be detected

for $\phi \neq 0$ → airborne is in turning flight i.e. it is rolling
As its consequence, its yaw angle (ψ) will change with a certain rate ($\dot{\psi}$)

(ϕ) Roll → rot^n about N axis
(θ) Pitch → rot^n about E axis
(ψ) Yaw → rot^n about D axis

Now, we can use either ϕ or $\dot{\psi}$ to detect whether it is turning or not.

The detec^n uses $\dot{\psi}$;
main reason: ϕ may be erratic during flight in turbulences, while $\dot{\psi}$ is more steady.
Disadvtg: $\dot{\psi}$ lags behind the time.

The detection filters the incoming $\dot{\psi}$ & finally applies a hysteresis fc^n to the filtered $\dot{\psi}$.
The result of this hysteresis fc^n is the flight type.

using the filter & a subsequent hysteresis fc^n is necessary to avoid erratic & wrong detec^n results.
main reason (filter): turbulences influence.

Influences that need to be filtered are the high freq. components compared to the main signal $\dot{\psi}$.

- α-β filter is used
  - often used filter in Navigation applications to smooth the data.
  - closely related to Kalman filter
  - based on the same "predict-update" concept as the Kalman filter.

- **α-β Filter Algorithm:**
  - assumes the system to be described by 2 states, the first being the integr^n of the second.

1^{st} state = x = pos^n
2^{nd} state = v = velocity

The filter assumes the system to be the outcome of a motion with const. velocity.
- Not true for all applic^n.

However by keeping the intg^n interval small, the cond^n of motion with const. velo. can be achieved.

The filter of epoch K, with the measured pos^n x, the estimated pos^n $\hat{x}$ & predicted pos^n $\tilde{x}$ works as follows:

Initializ^n:

$$\hat{x}_0 = \text{initial pos}^n \quad \text{—— (5)}$$
$$\hat{v}_0 = 0 \quad \text{—— (6)}$$

Step1: State predic^n:

$$\tilde{x}_K = \hat{x}_{K-1} + \Delta T\, \hat{v}_{K-1} \quad \text{—— (7)}$$
$$\tilde{v}_K = \hat{v}_{K-1} \quad \text{—— (8)}$$

Step-2: Calculⁿ of residual posⁿ:

$$\hat{r}_k = x_k - \tilde{x}_k \quad (9)$$

Step-3: Measuremᵗ update:

$$\hat{x}_k = \tilde{x}_k + \alpha \hat{r}_k \quad (10)$$

$$\hat{v}_k = \tilde{v}_k + \frac{\beta}{\Delta T} \hat{r}_k \quad (11)$$

- Choosing the suitable Gain Factors:-

α & β — steer the behaviour of the filter
- should lie in the range 0 to 1 in order to have a converging filter.

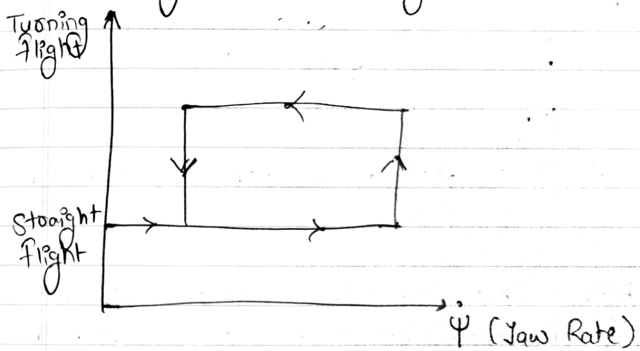
α — controls how new posⁿ measurements are weighted compared to the predicted ones; more it approaches to 1, more the o/p. of the filter ~~appear~~ resembles original data

Nautical Mile: 1 min. of latitude along any line of longitude ~~of~~ on the earth's surface.

1 NM / or / 1 nmi = 1.15078 statute miles
= 1.852 Km

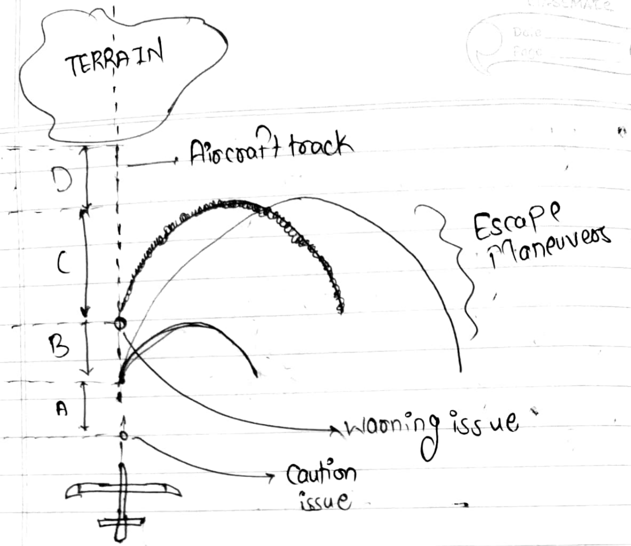
- Hysteresis fcⁿ: used to determine the flight type
  - applied to the filtered yaw rate

flight type is considered to be turning if yaw rate > 0.5 deg/s.
& considered to be straight if yaw rate < 0.5 deg/s.



* Look Ahead Distⁿ (LAD)

The radius of the escape turn depends mainly on the flow bank angle. (ϕ) {the axis passing from the nose to tail has an angle with the NE plane which is ϕ} & the groundspeed (V_{hor}^{NED}).



dist? A → dist? the aircraft travels due to pilot response time.

Imp. → high bank angle results in a small escape turn
& low bank angle results in a large escape turn.

∴ LAD = A+B+C+D

Calculat?: certain pilot response time = t_{pilot}

$$\therefore A = t_{pilot} \cdot V^{NED}_{hor} \quad \text{———} \quad (2.12)$$

the dist?ces B & C correspond to the turning radius R with a certain escape maneuver bank angle ϕ_{escape}:

$$R = \frac{(V^{NED}_{hor})^2}{G \tan \phi_{escape}} \quad \text{———} \quad (2.13)$$

$$\therefore \boxed{LAD = A + 2R + D} \quad \text{———} \quad (2.14)$$

* Straight Flight Prediction Algorithm:

- Predicts the path along an orthodrome for a certain time & is sampled at a certain timestep.
- results in N predicted positions

Followings are the i/ps taken by the algorithm:

(1) Current Aircraft pos?s & Altitude ($\phi_{A/C}$; $\lambda_{A/C}$; $h_{A/C}$)
(2) Current True Track (T_T)
(3) Current Horizontal velocity
(4) Time to predict (t_{pred})
(5) Sample Time (Δt_{pred})

t_{pred} = Look Ahead Time —— (2.15)

$$N = \frac{t_{pred}}{\Delta t_{pred}} \quad \text{—— (2.16)}$$

$$d_{step} = \frac{\Delta t_{pred} \cdot V_{A/C}^{NED}}{R_{EARTH} + h_{A/C}}$$

FOREACH PREDICTED Posn. from $i = 1$ to N;

$$\phi_{pred_i} = \phi_{A/C} + \arcsin\left\{ \sin(\phi_{A/C}) \cdot \cos(i \cdot d_{step}) + \cos(\phi_{A/C}) \cdot \sin(i \cdot d_{step}) \cdot \cos(T_1) \right\} \quad \text{—— (2.18)}$$

$$\lambda_{pred_i} = \lambda_{A/C} + \arctan\left(\frac{\sin(T_1) \cdot \sin(i\, d_{step}) \cdot \cos(\phi_{A/C})}{\cos(i \cdot d_{step}) - \sin(\phi_{A/C}) \cdot \sin(\phi_{pred_i})} \right) \quad \text{—— (2.19)}$$

- **Turning Flight Prediction:** —

- assumes the aircraft to be unaccelerated & bank angle to be const.
  - yields to a const turn radius

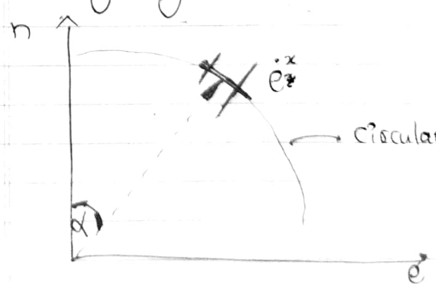
the Radius depends on the horizontal velocity & the Bank angle.

$$R = \frac{\left(V_{Horizontal}^{NED}\right)^2}{g \tan\phi} \quad \text{—— (2.20)}$$

- may also be expressed as:

$$R = \frac{V_{Horizontal}^{NED}}{\dot{\psi}} \quad \text{—— (2.21)}$$

⇒ **Turning Flight Model:** -



$$pos^n : \mathbf{x} = \begin{bmatrix} x_n \\ x_e \end{bmatrix} = R \cdot \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} = R \cdot e^x \quad \text{——— (2.22)}$$

α = angle betn the N
e^x denotes the unit vector
R & α are time dependent

To propagate to a future posn in time,
we use the Taylor series :

$$x_{t_0+dt} = x_{t_0} + \dot{x}_{t_0}\, dt + \frac{1}{2} \ddot{x}_{t_0}\, dt^2 \quad \text{——— (2.23)}$$

as radius is const $\therefore = 0$

$$\dot{x} = \cancel{\dot{R} e^x} + R \dot{e}^x . \alpha^x \quad \text{——— (2.24)}$$

$$\dot{x} = R \cdot \begin{bmatrix} -\sin\alpha \\ \cos\alpha \end{bmatrix} \cdot \dot{\alpha}$$

$$= V^{NED}_{horiz} \begin{bmatrix} -\sin\alpha \\ \cos\alpha \end{bmatrix} \quad \text{——— (2.25)}$$

$R \cdot \dot{\alpha}$ = Radial velocity $= V^{NED}_{hor.}$

&

as flight is unaccelerated $\therefore = 0$

$$\ddot{x} = \cancel{\dot{V}^{NED}_{hor}} . \dot{e}^x + V^{NED}_{hor} . \ddot{e}^x . \dot{\alpha} \quad \text{——— (2.26)}$$

$$\therefore \ddot{x} = V^{NED}_{hor} \begin{bmatrix} -\cos\alpha \\ -\sin\alpha \end{bmatrix} . \dot{\alpha} \quad \text{——— (2.27)}$$

$\therefore$

$$\boxed{x_{t_0+dt} = x_{t_0} + V^{NED}_{hor} \cdot \dot{e}^x dt + \frac{1}{2} V^{NED}_{hor} \dot{\alpha} \ddot{e}^x dt^2 \quad \text{——— (2.28)}}$$

⟹ The Prediction Algorithm :—
result = vector containing the predicted posns.

- uses a cumulated track angle ψ_c to accumulate the track angle change of each orthodrome segment.

Initializn :

$$\left.\begin{aligned} \phi_{pred,0} &= \phi_{A/C} \\ \lambda_{pred,0} &= \lambda_{A/C} \\ \psi_c &= T_T \end{aligned}\right\} \quad \text{——— (2.29)}$$

For each predicted pos^n i from 1 to N :

→ Calculate new centre angle α & update cumulated track angle. ψ_c :

$$\left.\begin{aligned} \alpha &= \psi_c - \pi/2 \\ \psi_c &= \dot{\psi} \cdot \Delta t_{pred} \end{aligned}\right\} \quad \text{—— (2.30)}$$

→ Calculate the orthodrome segment :

$$dx = \begin{bmatrix} dn \\ de \end{bmatrix} = V_{hor}^{NED} \cdot \begin{bmatrix} -\sin(\alpha) \\ \cos(\alpha) \end{bmatrix} \cdot \Delta t_{pred} - \frac{1}{2} \cdot V_{hor}^{NED} \cdot \dot{\psi} \cdot \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} \cdot \Delta t_{pred}^2 \quad \text{—— (2.31)}$$

→ Calculate the new predicted pos^n

$$\rho = \frac{|dx|}{R_{earth} + h_{A/C}} \quad \text{—— (2.32)}$$

$$\vartheta = \arctan\left(\frac{de}{dn}\right) \quad \text{—— (2.33)}$$

$\therefore$

$$\phi_{pred_i} = \phi_{pred_{i-1}} + \arcsin\left(\phi_{pred_{i-1}} \cdot \cos(\rho) + \cos(\phi_{pred_{i-1}}) \cdot \sin(\rho) \cdot \cos(\vartheta)\right) \quad \text{—— (2.34)}$$

&

$$\lambda_{pred_i} = \lambda_{pred_{i-1}} + \arctan\left(\frac{\sin(\vartheta)\sin(\rho)\cos(\phi_{pred_{i-1}})}{\cos(\rho) - \sin(\phi_{pred_{i-1}}) \cdot \sin(\phi_{pred_i})}\right) \quad \text{—— (2.35)}$$

$\dot{\psi}$ — limited to a max — $\dot{\psi}_{max}$
- imp since the horiz

* Predic^n Performance :-
- influenced by
- sensor data quality
- wind (mainly turning.
- Sampling interval

→ uses GPS / IRS for current aircraft pos^n determ^n.
- 95% confidence level

pos^n accuries of 0.04 to 0.15 NM by GPS/IRS using system.

predicted pos^ns are base for the building of search volume

Search vol. considers the accuracy of the aircraft posn.
Appropriate modifics of the search vol. are done/made depending upon the accuracy.