Mahidol University
Faculty of Science

Parallel Implementations of a Modified Bellman-Ford Shortest-Path Ray Tracing Algorithm on GPU

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Shortest-path problems in Physics

In physics, applying **high frequency approximation**, either wave equation or the Schrödinger equation is transformed to the eikonal equation (1-left).

The eikonal equation is equivalent to the Fermat's principle.

$$|\nabla T| = \frac{1}{v} \quad \longleftrightarrow \quad T = \int_L \frac{1}{v}\,\mathrm{d}l \tag{1}$$

where

- T is traveltime or the first arrival time (the least time),
- L is the ray path or the path that giving the traveltime,
- v is a spatially dependent velocity or a local wave speed,
- $\mathrm{d}l$ is an infinitesimal path length.

Shortest-path ray tracing method (SPR)

Given a domain with a velocity field, and a point source position, we aim to find the traveltimes and ray paths to all receivers in the domain.

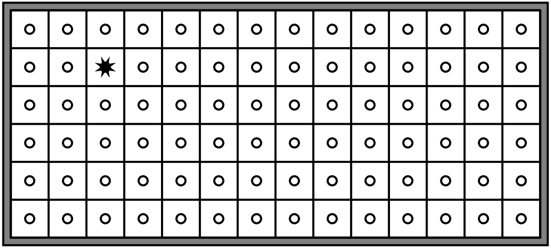


Figure 1: Nodes of grid model for 14×6 cells together with source (black star) at (3,5).

SPR solves the problem by constructing **a network of small ray paths**, then employs **a single-source shortest-path algorithm**.

Radius, Neighbors and Edges

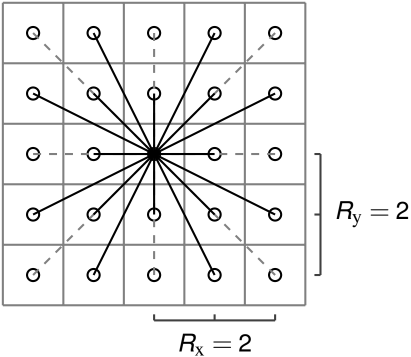


Figure 2: Edges linking a node (0,0) to its neighbors (i, j) for $R_x = 2$ and $R_y = 2$ are shown in which dashed lines indicate that the edges can be discarded (Edge reduction).

Straight line approximation (SPR)

The Fermat's principle is discretized to compute the weight of the graph.

$$T = \int_L \frac{1}{v}\,\mathrm{d}l \approx \sum_{n=1}^{L(i,j)} \frac{\Delta l_n}{v_n} \tag{2}$$

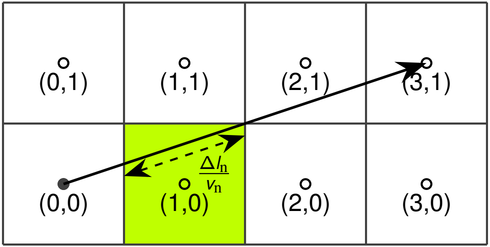


Figure 3: The length of the segment Δl_n and the velocity of the cell v_n are used in computing the wight of the edge that connects the node to its neighbor (3,1).

Single-Source Shortest-Path algorithm

The **Bellman-Ford algorithm** majorly consists of

1. Computing possible traveltimes coming from neighbors,
2. Comparing them with the traveltime that the node is currently holding,
3. Saving the least arrival time, that just found.

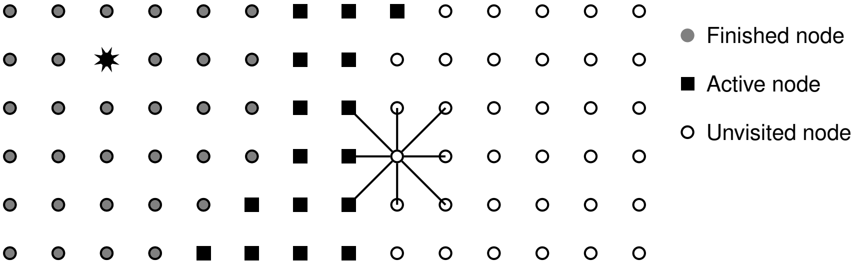


Figure 4: Finished nodes, active nodes, unvisited nodes, and source (star).

Parallel implementation on GPU

Diving the nodes into groups, **update map** is to save the active groups that surround the active region.

- 1 Node : 1 GPU thread
- 1 Group : 1 GPU block

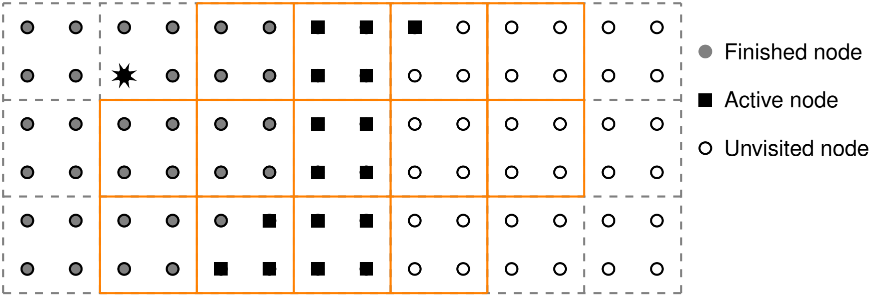


Figure 5: Three types of nodes, source (star), and active groups (orange cells).

Fast iterative method (FIM)

" From programming view point, changing the SPR weight calculation to the Godunov upwind scheme will transform SPR to FIM ".

Applying **upwind finite difference** to the eikonal equation, a quadratic equation is obtained as shown in equation 3—for two dimensional problems.

$$\left[(T_{i,j} - T_{\mathrm{x}}^{\mathrm{min}})^{+}\right]^2 + \left[(T_{i,j} - T_{\mathrm{y}}^{\mathrm{min}})^{+}\right]^2 = h^2 / V_{i,j}^2 \tag{3}$$

where T is traveltime, V is velocity, $h = h_{\mathrm{x}} = h_{\mathrm{y}}$ is grid spacing and

$$T_{\mathrm{x}}^{\mathrm{min}} = \min(T_{i-1,j}, T_{i+1,j}), \quad T_{\mathrm{y}}^{\mathrm{min}} = \min(T_{i,j-1}, T_{i,j+1}), \quad (x)^{+} = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Solving this quadratic equation, we get the Godunov upwind scheme.

Extended scheme (Ex)

Treating the node as local source, **weight is simply traveltime** from this local source to neighbors.

Additional comparisons by the Godunov upwind scheme are taken place on the gray neighbors, using previously computed SPR traveltime.

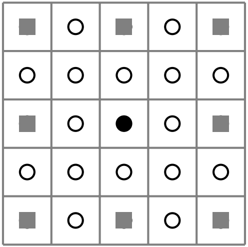
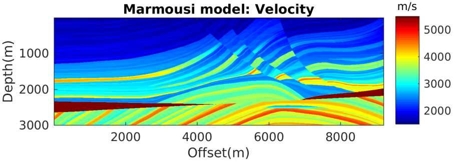
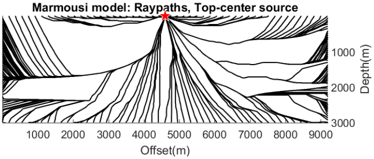


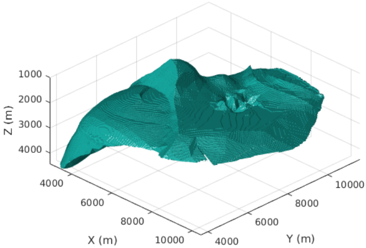
Figure 6: The two-dimensional extended scheme when $R = 2$.

2D Marmousi model (2304x752)

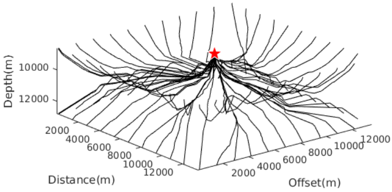




3D SEG salt model (336x336x104)

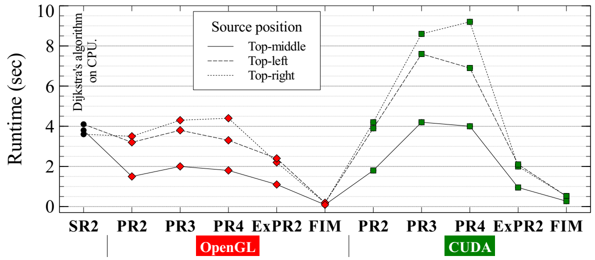


(a) Salt of the SEG-salt model



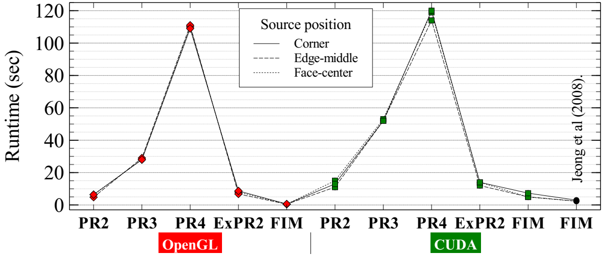
(b) Ray paths

2D Runtime (Marmousi)



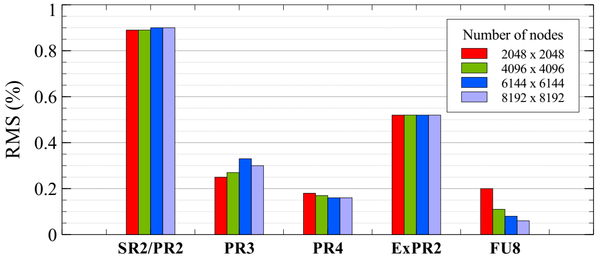
(c) Runtime result of our 2D programs, using NVIDIA GeForce 1050Ti and AMD Ryzen 3 1300X.

3D Runtime (SEG-Salt)



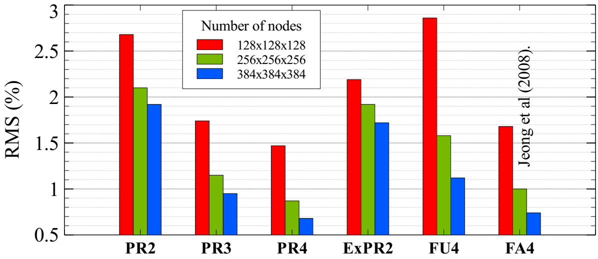
(d) Runtime result of our 3D programs and the original FIM, using NVIDIA GeForce 1050Ti.

2D Deviation
(V(r,θ) = r)



(e) Two-dimensional accuracy test result of a radial gradient model.

3D Deviation
(V(x,y,z) = z)



(f) Three-dimensional accuracy test result of a vertical gradient model.

Conclusion

- **Parallel shortest-path ray tracers are programed on GPU** in order to solve shortest-path problems in physics.
- **Update map and extended scheme are introduced** to improve their performance.
- However, **the original fast iterative method is faster and more accurate.** (although ray paths are not found concurrently)
- Because its accuracy depends on the radius, **the parallel SPR is suitable for a small-scale complicated heterogeneous velocity model**.

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