The LCCDE for one of the systems of Lab 2 is:

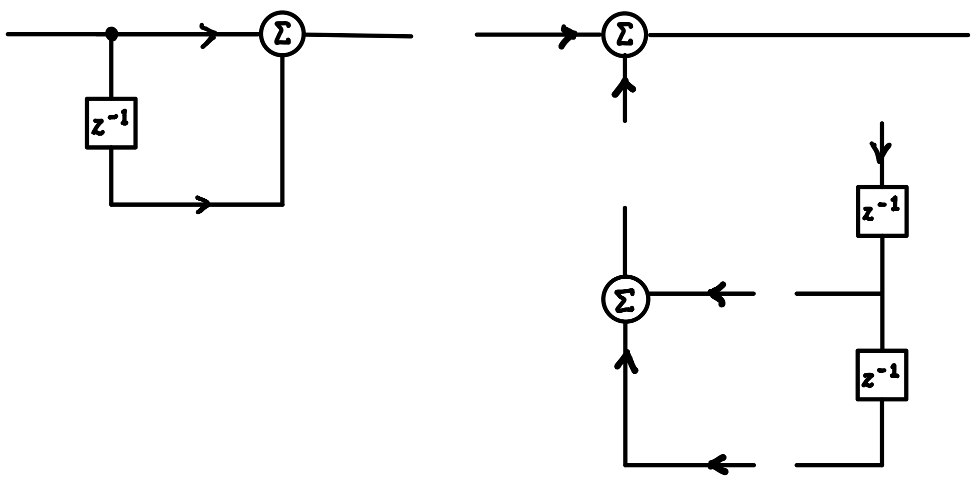
$$\underbrace{1}_{a_0} y[n] - \underbrace{3}_{a_1} y[n-1] - \underbrace{4}_{a_2} y[n-2] = \underbrace{1}_{b_0} x[n] + \underbrace{2}_{b_1} x[n-1] \qquad (1)$$

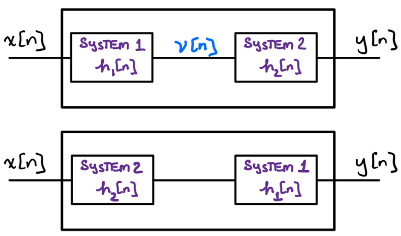
note: Any system described by an LCCDE can be implemented using constant multipliers, summers, and delay elements.

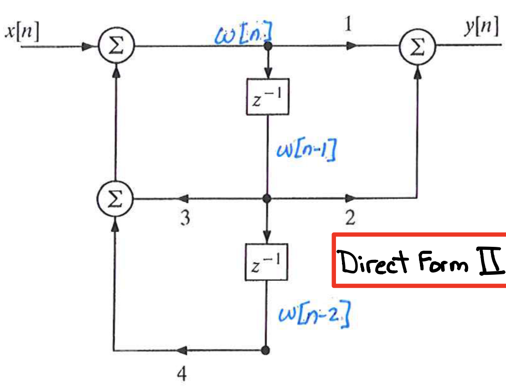
Exercise: Sketch a block diagram illustrating a realization/implementation of system described by eqn (1):

$$y[n] = x[n] + 2x[n-1] + 3y[n-1] + 4y[n-2]$$

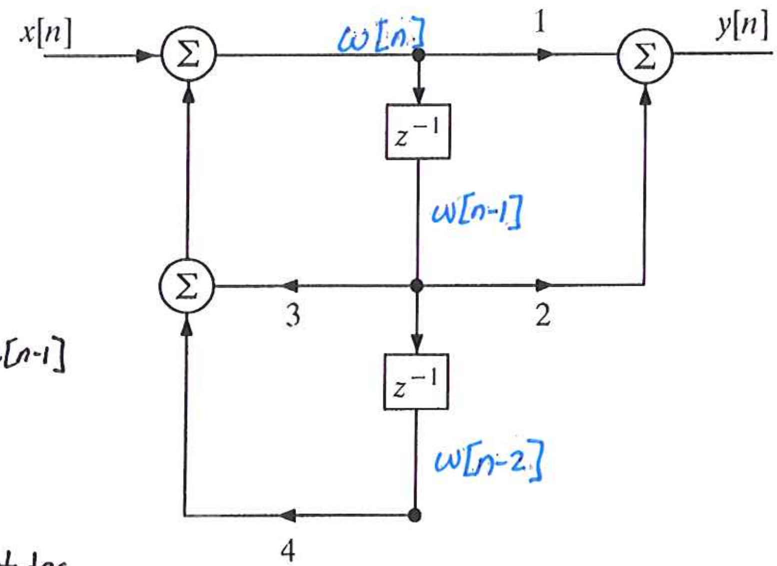
Direct Form I implementation







Direct Form II



The Direct Form 2 structure, shown to the right, implements the LCCDE shown below:

$$y[n] = 3y[n-1] + 4y[n-2] + x[n] + 2x[n-1]$$

Problem

Find values for the initial states, $w[-1]$ and $w[-2]$, so that the structure's output, $y[n]$, will agree with the solution to the LCCDE for the initial conditions (IC's): $y[-1] = 1$ and $y[-2] = -1$

Guiding principle: Choose $w[-1]$ and $w[-2]$ so that the zero-input response of the structure is the same as the zero-input solution to the difference equation for the stated IC values

From difference equation with $y[-1]=1$ and $y[-2]=-1$

$$y_{zi}[n] = 3y_{zi}[n-1] + 4y_{zi}[n-2]$$

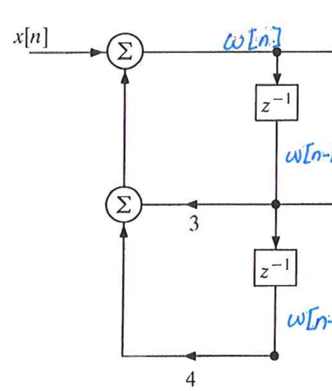
$n=0$: $y_{zi}[0] = 3y_{zi}[-1] + 4y_{zi}[-2] = 3(1) + 4(-1) = -1$

$n=1$: $y_{zi}[1] = 3y_{zi}[0] + 4y_{zi}[-1] = 3(-1) + 4(1) = -3+4 = 1$

From structure:

$y_{zi}[0] = w[0] + 2w[-1]$ (1)

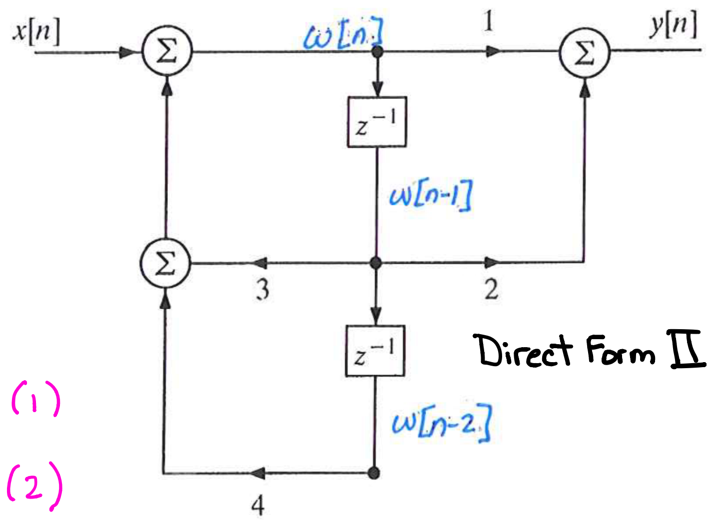
$y_{zi}[1] = w[1] + 2w[0]$ (2)



Direct Form II

(Continued from previous page)

Use structure to express $y_{zi}[0]$ and $y_{zi}[1]$ in terms of $w[-1]$ and $w[-2]$



Direct Form II

$$y_{zi}[n] = w[n] + 2w[n-1]$$

repeated from previous page

$$\Rightarrow \begin{cases} y_{zi}[0] = w[0] + 2w[-1] & (1) \\ y_{zi}[1] = w[1] + 2w[0] & (2) \end{cases}$$

Need to express $w[0]$ and $w[1]$ in terms of $w[-1]$ and $w[-2]$

from structure: $w[n] = 3w[n-1] + 4w[n-2] + x[n]$

$n=0$: $\boxed{w[0] = 3w[-1] + 4w[-2]}$ (3)

$n=1$: $w[1] = 3w[0] + 4w[-1]$

$= 3(\underbrace{3w[-1] + 4w[-2]}_{w[0]}) + 4w[-1]$

$= 9w[-1] + 12w[-2] + 4w[-1]$

$\Rightarrow \boxed{w[1] = 13w[-1] + 12w[-2]}$ (4)

Substituting (3) + (4) into (1) + (2) yields:

$$y_{zi}[0] = \overbrace{3w[-1] + 4w[-2]}^{w[0]} + 2w[-1] = 5w[-1] + 4w[-2] \quad (5)$$

$$y_{zi}[1] = \overbrace{13w[-1] + 12w[-2]}^{w[1]} + 2(\overbrace{3w[-1] + 4w[-2]}^{w[0]}) = 19w[-1] + 20w[-2] \quad (6)$$

Using the values for $y_{zi}[0]$ and $y_{zi}[1]$ obtained from the difference equation yields two equations in two unknowns.

Equating the values for $y_{zi}[0]$ and $y_{zi}[1]$ obtained from the difference equation to those obtained from the Direct Form II structure yields the following two equations in two unknowns, $w[-1]$ and $w[-2]$.

$$y_{zi}[0] = 5\,w[-1] + 4\,w[-2] = -1$$

$$y_{zi}[1] = 19\,w[-1] + 20\,w[-2] = 1$$

Multiplying the top equation by 5 and subtracting from it the bottom equation yields:

$$\begin{array}{r} 25\,w[-1] + 20\,w[-2] = -5 \\ 19\,w[-1] + 20\,w[-2] = 1 \\ \hline 6\,w[-1] = -6 \end{array} \Rightarrow \boxed{w[-1] = -1}$$

Substituting $w[-1] = -1$ into second equation yields

$$19(-1) + 20\,w[-2] = 1 \Rightarrow 20\,w[-2] = 20$$

$$\Rightarrow \boxed{w[-2] = 1}$$

Thus, to implement the difference equation with ICs: $y[-1] = 1, y[-2] = -1$, the initial states of the Direct Form II structure should be set equal to

$$w[-1] = -1 \quad \text{and} \quad w[-2] = 1$$

Transition from Direct Form II to Transposed Direct Form II

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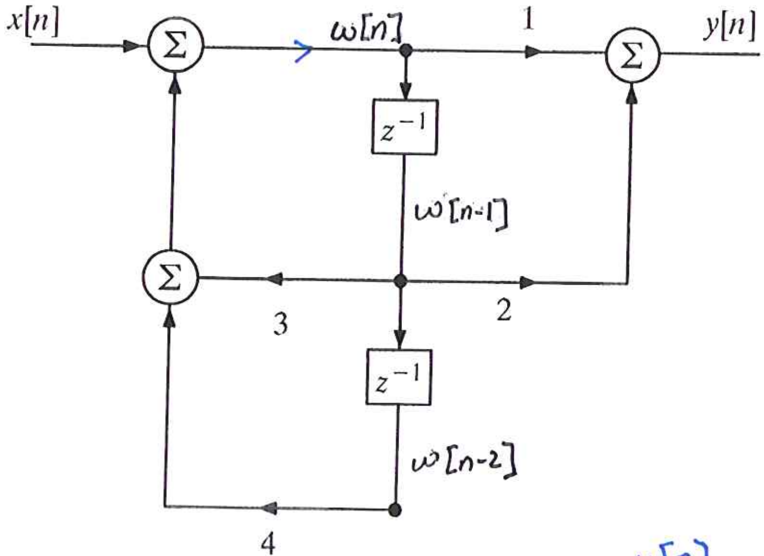
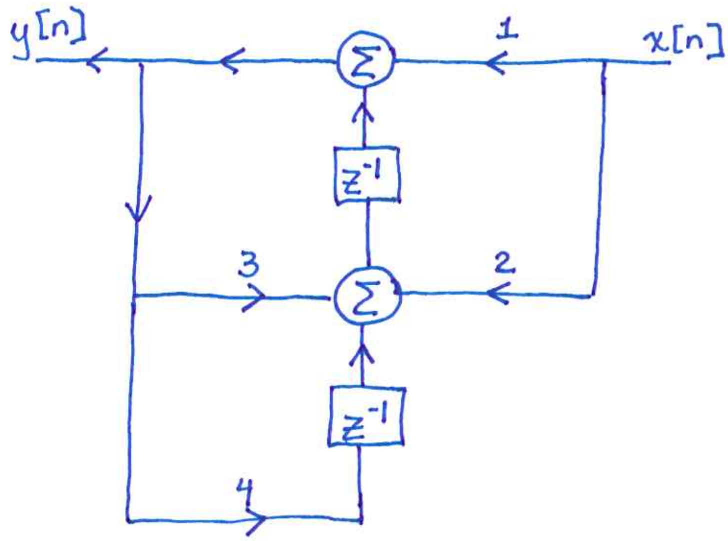


Direct Form 2 structure for LCCDE

$y[n] = 3\,y[n-1] + 4\,y[n-2] + x[n] + 2x[n-1]$

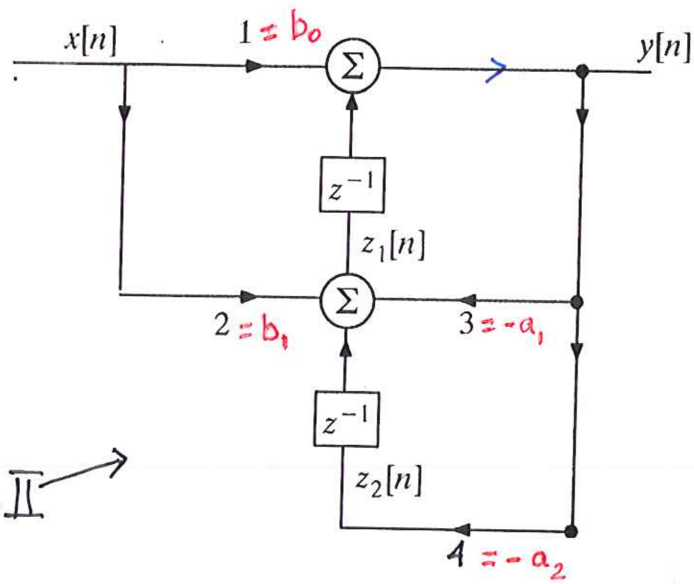
is shown to the left.



To find the transposed D.F. II structure:

- reverse direction of all arrows
- replace summing junctions by branching nodes
- replace branching nodes by summing junctions
- relabel x[n] as y[n] and y[n] as x[n]

- Finally, flip the structure horizontally so that the input is at the left and output on the right.



Transposed Direct Form II

Note: The Matlab function, filter( ), is based on the Transposed Direct Form II implementation.

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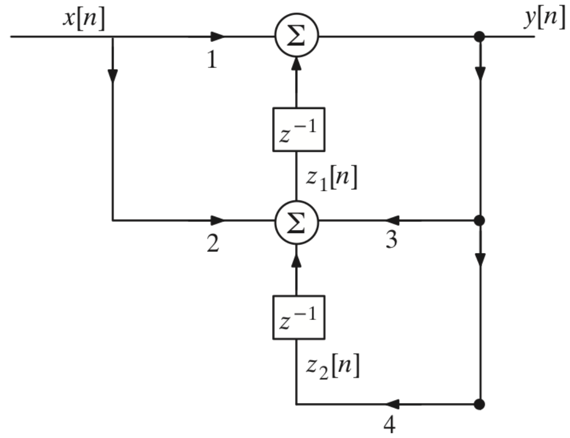
Exercise: Given the Transposed Direct Form II structure below:

1. Find the difference equation realized by the *transposed Direct Form II* structure

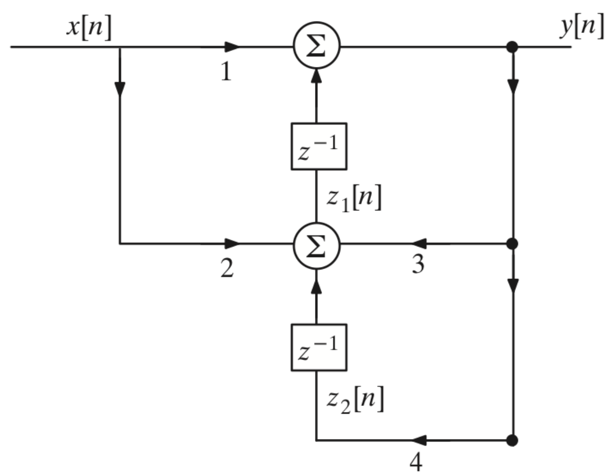


2. Iterate the difference equation to find $y_{zi}[0]$ and $y_{zi}[1]$ when $y[-1] = 1$ and $y[-2] = -1$.

$y_{zi}[0] = 3\, y_{zi}[-1] + 4\, y_{zi}[-2] = 3(\quad) + 4(\quad) =$

$y_{zi}[1] = 3\, y_{zi}[0] + 4\, y_{zi}[-1] = 3(\quad) + 4(\quad) =$

3. Find expressions for $y_{zi}[0]$ and $y_{zi}[1]$ in terms of the initial state values: $z_1[-1]$ and $z_2[-1]$. [note that initial state values always refer to the outputs of the storage registers at $n = 0$.]



4. Use the results of the previous two steps to find initial state values for the implementation structure that will be equivalent to the initial conditions ($y[-1] = 1$ and $y[-2] = -1$) for the difference equation.

In step (2) we found: $y_{zi}[0] = -1$ and $y_{zi}[1] = 1$

In step (3) we found: $y_{zi}[0] = z_1[-1]$

$y_{zi}[1] = 3z_1[-1] + z_2[-1]$

5. What will be returned by the matlab command shown below. How does it relate to your response to question 4?

> filtic([1,2],[1,−3,−4],[1,−1]) % filtic([b_0, b_1], [a_0, a_1, a_2], [$y[-1]$, $y[-2]$])

LCCDE: from step 1

$y[n] = x[n] + 2x[n-1] + 3y[n-1] + 4y[n-2]$

$(1)y[n] + (-3)y[n-1] + (-4)y[n-2] = 1\,x[n] + 2\,x[n-1]$

$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1]$

The matlab command above returns a vector [$z_1[-1]$, $z_2[-1]$] containing the values for the initial states of the Transposed Direct Form II implementation for the LTI system with LCCDE coefficients [b_0, b_1] = [1, 2] and [a_0, a_1, a_2] = [1, -3, -4] such that the implementation will produce the same zero-input response as would be obtained by iterating the zero-input LCCDE with $y[-1] = 1$ and $y[-2] = -1$.

In question 4, we found $z_1[-1] = -1$ and $z_2[-1] = 4$.

Hence, matlab should return

ans = [ ]