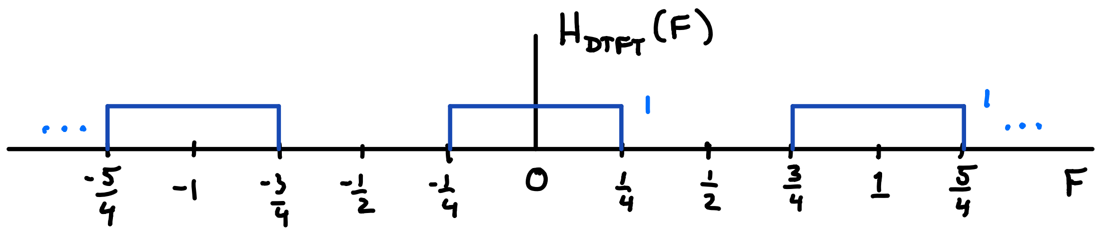
Example Given $H_{DTFT}(F)$ as illustrated below.

Find $h[n]$, the iDTFT of $H_{DTFT}(F)$.

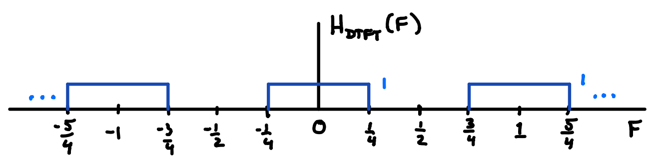


Solution

Approach 1: Use iDTFT integral

$$h[n] = \int_{-1/2}^{1/2} H_{DTFT}(F)\, e^{j2\pi Fn} =$$

Approach 2: use DTFT / CTFT relationships



note that: $H_{DTFT}(F) = X(F) * \text{comb}(F)$ where $X(F) =$

Therefore: $h[n] = x(t)\big|_{t=n}$

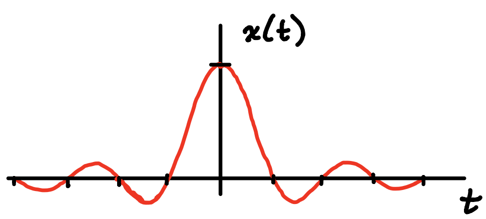
where $x(t) = \mathcal{F}^{-1}\{X(f)\} =$

$$\text{sinc}(t) \xleftrightarrow{\text{CTFT}} \text{rect}(f)$$

$\Downarrow$ freq.-scaling

$$\tfrac{1}{2}\,\text{sinc}\left(\tfrac{t}{2}\right) \xleftrightarrow{\text{CTFT}} \text{rect}(2f)$$

$\Rightarrow h[n] =$



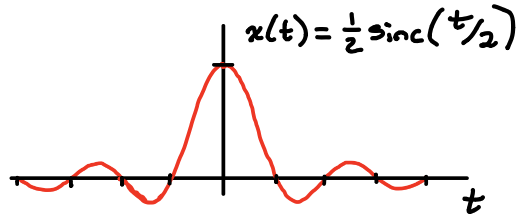
The frequency response of an ideal discrete-time low pass filter (LPF) with cut-off frequency $F_c = \frac{1}{4}$ cycles/sample is given by:

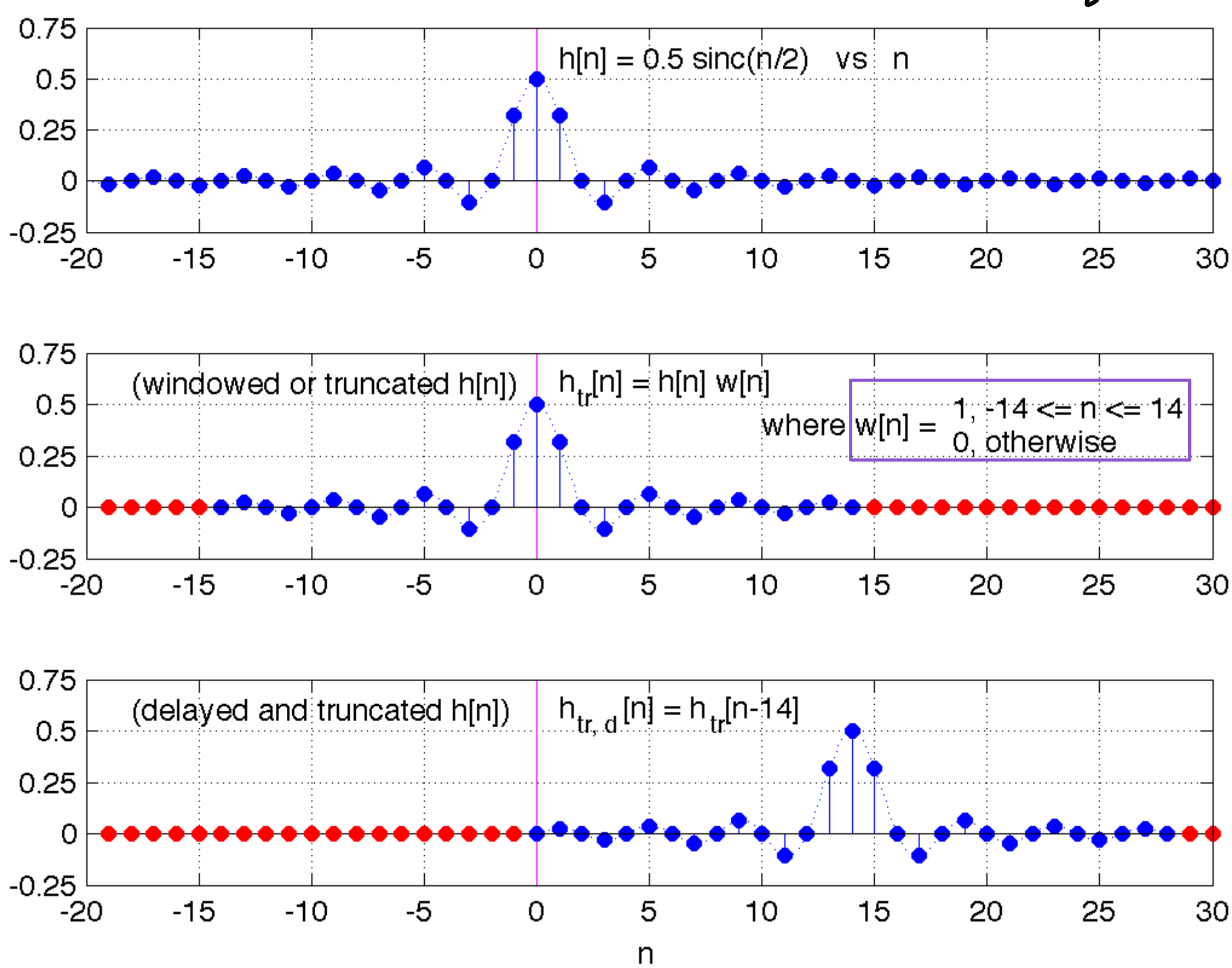
$$H_{\mathrm{DTFT}}(F) = \mathrm{rect}(2F) * \mathrm{comb(F)}$$

The impulse response is given by:

$$h[n] = \frac{1}{2}\mathrm{sinc}(n/2)$$

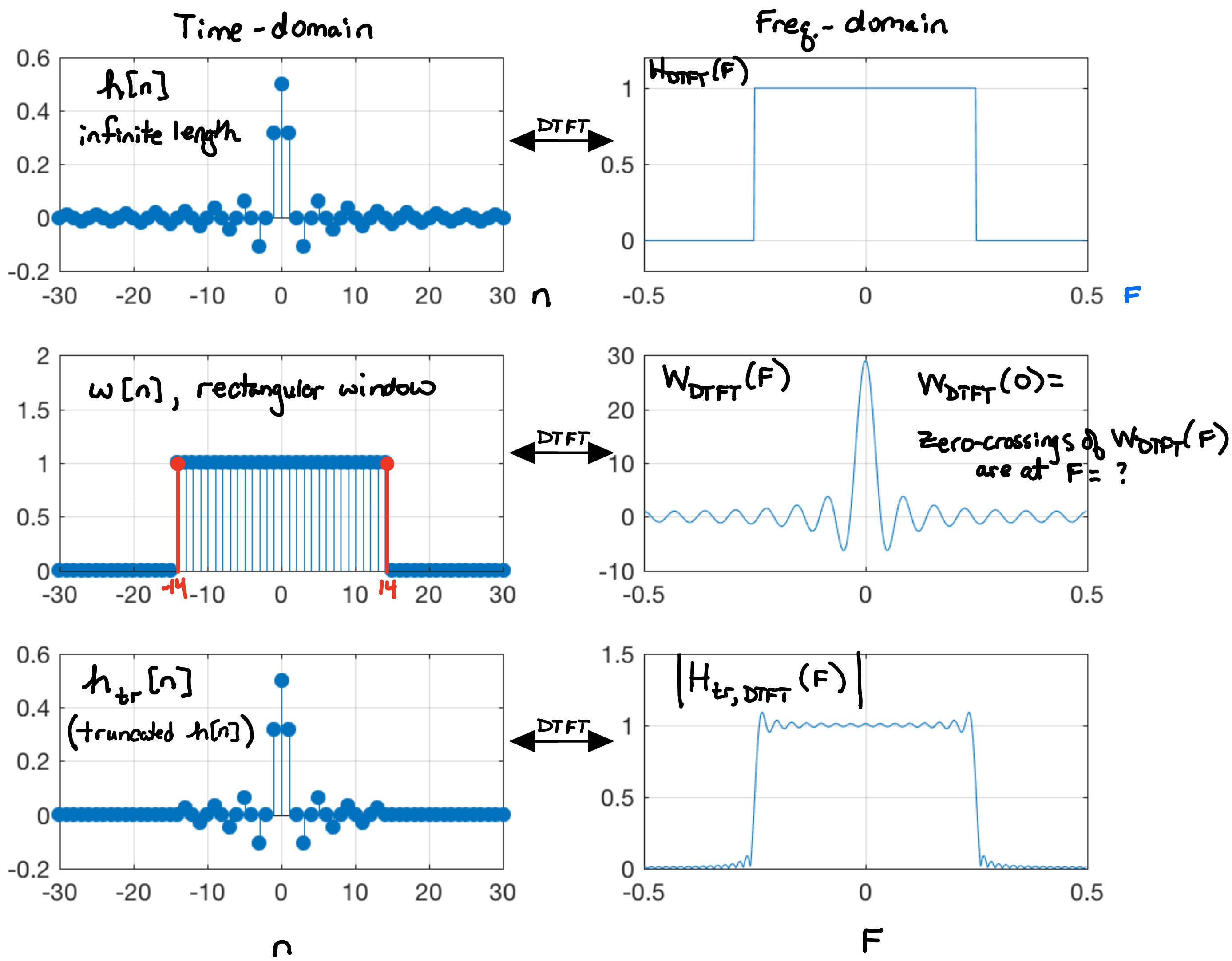
Note that the filter's impulse response is noncausal and is hence not implementable in real time. The ideal filter can be approximated by a filter whose impulse response is the delayed and truncated version of $h[n]$ shown below. What is the effect of the delay? of the window/truncation operation?





How does the magnitude of the DTFT of the truncated and delayed sinc function compare to the frequency response magnitude of an ideal LPF? *i.e.,* how does it compare to the DTFT magnitude of an infinite-length sinc function?

To determine the effect of truncation, note that the truncated sinc function may be viewed as the product of an infinite-length sinc function (top left) with a rectangular window function (middle left), then use the fact that multiplication in time equates to circular convolution of the DTFTs (see DTFT properties- next page).



Note: Since a shift in time will not affect the magnitude of the signal's DTFT, we know that the DTFT magnitude of the truncated and delayed sinc function will be the same as the DTFT magnitude of the truncated sinc function (as shown in the bottom right plot). Thus, as expected, the DTFT magnitude of the truncated and delayed sinc function appears to be a reasonably good approximation to the frequency response magnitude of the ideal low pass filter (top right).
If desired, the magnitude of the ripple can be reduced by using a non rectangular window function whose DTFT has lower sidelobes.

9. Discrete-time Fourier Transform: Let $G_{\text{DTFT}}(F)$ denote the Discrete-Time Fourier Transform of the signal $g[n]$. Then:

$$G_{\text{DTFT}}(F) = \sum_{n=-\infty}^{\infty} g[n]e^{-j2\pi Fn} \qquad \text{and} \qquad g[n] = \int_0^1 G_{\text{DTFT}}(F)e^{j2\pi Fn}dF$$

Selected Discrete-Time Fourier Transform Properties

Property	Mathematical Description
Sum of $g[n]$	$\sum_{n=-\infty}^{\infty} g[n] = G_{\text{DTFT}}(0)$
Area under $G_{\text{DTFT}}(F)$	$g[0] = \int_1 G_{\text{DTFT}}(F)dF$
Linearity	$ag_1[n] + bg_2[n] \leftrightarrows aG_{1,\text{DTFT}}(F) + bG_{2,\text{DTFT}}(F)$
Time Shifting	$g[n-n_0] \leftrightarrows G_{\text{DTFT}}(F)\, e^{-j2\pi Fn_0}$
Frequency Shifting	$g[n]e^{j2\pi F_0 n} \leftrightarrows G_{\text{DTFT}}(F-F_0)$
Time Reversal	$g[-n] \leftrightarrows G_{\text{DTFT}}(-F)$
Conjugate Functions	$g^*[n] \leftrightarrows G^*_{\text{DTFT}}(-F)$
Multiplication in time	$g[n]h[n] \leftrightarrows \left(G_{\text{DTFT}}(F)\text{rect}(F)\right) * H_{\text{DTFT}}(F) = G_{\text{DTFT}}(F) * \left(H_{\text{DTFT}}(F)\text{rect}(F)\right)$
Convolution in time note: at least one of the two signals ($g[n]$ or $h[n]$) must be an energy signal	$g[n] * h[n] \leftrightarrows G_{\text{DTFT}}(F)H_{\text{DTFT}}(F)$

Selected Discrete-Time Fourier Transform Pairs

$$\delta[n] \leftrightarrows 1$$

$$\sum_{k=-\infty}^{\infty} N\delta[n-kN] = N\text{comb}_N[n] \leftrightarrows N\text{comb}(FN) = \sum_{k=-\infty}^{\infty} \delta\left(F - \frac{k}{N}\right)$$

$$\exp(j2\pi an) \leftrightarrows \delta(F-a) * \text{comb}(F)$$

10. Discrete Fourier Transform: Let $G_{\text{DFT},N}[k]$ denote the N-point DFT of the signal $g[n]$. Then:

$$G_{\text{DFT},N}[k] = \sum_{n=0}^{N-1} g[n]e^{-j2\pi\frac{k}{N}n} \quad k = 0,\ldots,N-1 \qquad \text{and} \qquad g[n] = \frac{1}{N}\sum_{k=0}^{N-1} G_{\text{DFT},N}[k]e^{j2\pi\frac{k}{N}n} \quad n = 0,\ldots,N-1$$

11. Discrete Fourier Series

If $g[n]$ is periodic with period N, then $g[n]$ can be expressed in terms of its DFS representation as follows:

$$g[n] = \sum_{k=0}^{N-1} c_k e^{j2\pi\frac{k}{N}n} \qquad \text{where} \qquad c_k = \frac{1}{N}\sum_{n=0}^{N-1} g[n]e^{-j2\pi\frac{k}{N}n} \tag{1}$$