9. Discrete-time Fourier Transform: Let $G_{\mathrm{DTFT}}(F)$ denote the Discrete-Time Fourier Transform of the signal $g[n]$. Then:

$$G_{\mathrm{DTFT}}(F) = \sum_{n=-\infty}^{\infty} g[n]e^{-j2\pi Fn} \qquad \text{and} \qquad g[n] = \int_0^1 G_{\mathrm{DTFT}}(F)e^{j2\pi Fn}dF$$

Selected Discrete-Time Fourier Transform Properties

Property	Mathematical Description
Sum of $g[n]$	$\sum_{n=-\infty}^{\infty} g[n] = G_{\mathrm{DTFT}}(0)$
Area under $G_{\mathrm{DTFT}}(F)$	$g[0] = \int_1 G_{\mathrm{DTFT}}(F)dF$
Linearity	$ag_1[n] + bg_2[n] \leftrightarrows aG_{1,\mathrm{DTFT}}(F) + bG_{2,\mathrm{DTFT}}(F)$
Time Shifting	$g[n - n_0] \leftrightarrows G_{\mathrm{DTFT}}(F)\, e^{-j2\pi Fn_0}$
Frequency Shifting	$g[n]e^{j2\pi F_0 n} \leftrightarrows G_{\mathrm{DTFT}}(F - F_0)$
Time Reversal	$g[-n] \leftrightarrows G_{\mathrm{DTFT}}(-F)$
Conjugate Functions	$g^*[n] \leftrightarrows G^*_{\mathrm{DTFT}}(-F)$
Multiplication in time	$g[n]h[n] \leftrightarrows \big(G_{\mathrm{DTFT}}(F)\mathrm{rect}(F)\big) * H_{\mathrm{DTFT}}(F) = G_{\mathrm{DTFT}}(F) * \big(H_{\mathrm{DTFT}}(F)\mathrm{rect}(F)\big)$
Convolution in time note: at least one of the two signals ($g[n]$ or $h[n]$) must be an energy signal	$g[n] * h[n] \leftrightarrows G_{\mathrm{DTFT}}(F)H_{\mathrm{DTFT}}(F)$

Selected Discrete-Time Fourier Transform Pairs

$$\delta[n] \leftrightarrows 1$$

$$\sum_{k=-\infty}^{\infty} N\delta[n - kN] = N\mathrm{comb}_N[n] \leftrightarrows N\mathrm{comb}(FN) = \sum_{k=-\infty}^{\infty} \delta\left(F - \frac{k}{N}\right)$$

$$\exp(j2\pi an) \leftrightarrows \delta(F - a) * \mathrm{comb}(F)$$

10. Discrete Fourier Transform: Let $G_{\mathrm{DFT},N}[k]$ denote the N-point DFT of the signal $g[n]$. Then:

$$G_{\mathrm{DFT},N}[k] = \sum_{n=0}^{N-1} g[n]e^{-j2\pi \frac{k}{N}n} \quad k = 0, \ldots, N-1 \qquad \text{and} \qquad g[n] = \frac{1}{N}\sum_{k=0}^{N-1} G_{\mathrm{DFT},N}[k]e^{j2\pi \frac{k}{N}n} \quad n = 0, \ldots, N-1$$

11. Discrete Fourier Series

If $g[n]$ is periodic with period N, then $g[n]$ can be expressed in terms of its DFS representation as follows:

$$g[n] = \sum_{k=0}^{N-1} c_k e^{j2\pi \frac{k}{N}n} \qquad \text{where} \qquad c_k = \frac{1}{N}\sum_{n=0}^{N-1} g[n]e^{-j2\pi \frac{k}{N}n} \tag{1}$$