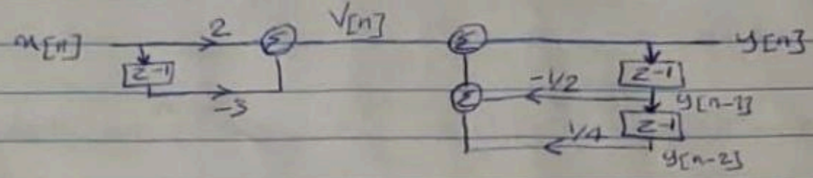
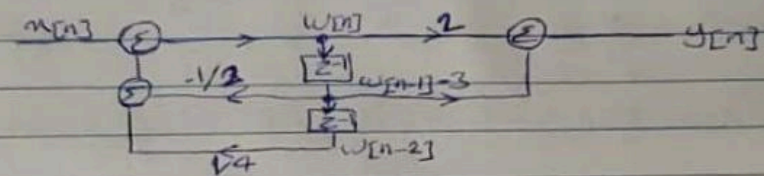
Q1-a

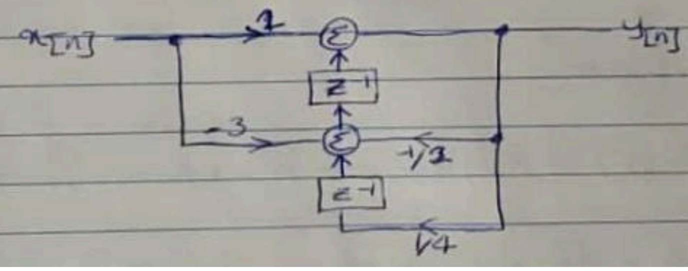
$$y[n] = 2x[n] - 3x[n-1] - \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2] \quad , \quad v[n] = 2x[n] - 3x[n-1]$$



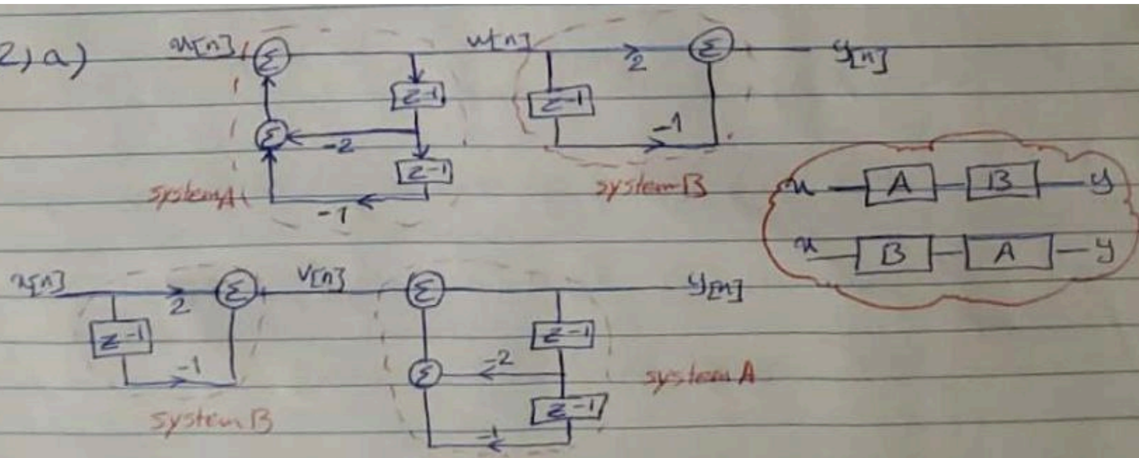
b)



c)



Q2) a)



$\Rightarrow v[n] = 2x[n] - x[n-1]$, $y[n] = v[n] - 2y[n-1] - y[n-2]$

$\Rightarrow y[n] = 2x[n] - x[n-1] - 2y[n-1] - y[n-2]$

b) from different eqn: (in zero-input response)

$y_{zi}[n] = -2y_{zi}[n-1] - y_{zi}[n-2] \Rightarrow$ $y_{-1} = y[-1] = 1$ $y_{-2} = y[-2] = 2$

$n=0 \Rightarrow y_{zi}[0] = -2\overset{1}{y_{zi}[-1]} - \overset{2}{y_{zi}[-2]} = -2-2 = -4$

$n=1 \Rightarrow y_{zi}[1] = -2\overset{-4}{y_{zi}[0]} - \overset{1}{y_{zi}[-1]} = +7$

from structure: $y_{zi}[n] = 2\omega[n] - \omega[n-1]$

$y_{zi}[0] = 2\omega[0] - \omega[-1]$ (1) , $y_{zi}[1] = 2\omega[1] - \omega[0]$ (2)

Also from structure $\omega[n] = \overset{0 \text{ since zi}}{\cancel{x[n]}} - 2\omega[n-1] - \omega[n-2]$

$n=0 \Rightarrow \omega[0] = -2\omega[-1] - \omega[-2]$ (3)

$n=1 \Rightarrow \omega[1] = -2\omega[0] - \omega[-1] = +4\omega[-1] + 2\omega[-2] - \omega[-1]$

$\omega[1] = 3\omega[-1] + 2\omega[-2]$ (4)

(3, 4) and (1, 2) yields:

$y_{zi}[0] = -4\omega[-1] - 2\omega[-2] - \omega[-1] = -5\omega[-1] - 2\omega[-2]$

$y_{zi}[1] = 6\omega[-1] + 4\omega[-2] + 2\omega[-1] + \omega[-2] = 8\omega[-1] + 5\omega[-2$

$\Rightarrow -5\omega[-1] - 2\omega[-2] = -4$

$8\omega[-1] + 5\omega[-2] = 7 \quad \Rightarrow$

$\omega[-1] = \frac{2}{3} \quad \omega[-2] = \frac{1}{3}$

Ass(5) 3

Q3-a)

$$y[n] = 2x[n] + z_1[n-1]$$

$$z_1[n] = y[n] - x[n] + z_2[n-1]$$

$$z_2[n] = -\frac{1}{2} y[n] \Rightarrow z_2[n-1] = -\frac{1}{2} y[n-1]$$

$$z_1[n] = y[n] - x[n] - \frac{1}{2} y[n-1]$$

$$\Rightarrow z_1[n-1] = y[n-1] - x[n-1] - \frac{1}{2} y[n-2]$$

$$\Rightarrow y[n] = 2x[n] - x[n-1] + y[n-1] - \frac{1}{2} y[n-2]$$

b) from eqn: $y_{zi}[0] = y_{zi}[-1] - \frac{1}{2} y_{zi}[-2] = 2 - \frac{1}{2}(-6) = 5$

$$y_{zi}[1] = y_{zi}[0] - \frac{1}{2} y_{zi}[-1] = 5 - \frac{1}{2}(2) = 4$$

from structure: $y_{zi}[n] = z_1[n-1] + \cancel{2x[n]}$ 0

$n=0 \Rightarrow \boxed{y_{zi}[0] = z_1[-1]}$

$n=1 \quad y_{zi}[1] = z_1[0]$

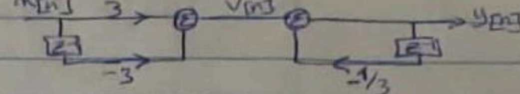
$z_1[0] = y_{zi}[0]$ $z_1[-1]$ $+ z_2[-1]$

$$\Rightarrow \boxed{y_{zi}[1] = z[-1] + z_2[-1]}$$

$$\Rightarrow \boxed{z_1[-1] = 5} \quad , \quad z_1[-1] + z_2[-1] = 4$$

$$\Rightarrow z_2[-1] = \boxed{-1}$$

Q4-a)

like question 2: 

$\Rightarrow \quad y[n] = V[n] - \frac{1}{3} y[n-1] \qquad V[n] = 3x[n] - 3x[n-1]$

$\Rightarrow y[n] = 3x[n] - 3x[n-1] - \frac{1}{3} y[n-1]$

b) $\quad y[n] = 3\omega[n] - 3\omega[n-1]$

$\omega[n] = x[n] - \frac{1}{3}\omega[n-1]$

c) $\quad x[n] = \delta[n] \qquad \omega[-1] = 0$

$\Rightarrow n=0 \Rightarrow \omega[0] = \delta[0] - \frac{1}{3}\omega[-1] = 1$

$\Rightarrow n=1 \Rightarrow \omega[1] = \delta[1] - \frac{1}{3}\omega[0] = -1/3$

$n=2 \Rightarrow \omega[2] = -\frac{1}{3}\omega[1] = \frac{1}{9}$

$\Rightarrow \omega_h[n] = \left(\frac{-1}{3}\right)^n \qquad n \geq 0$

d) $\quad y[n] = 3\omega[n] - 3\omega[n-1]$

$\cancel{n=0 \Rightarrow h[0] = 3\omega[0] - 3\omega[-1] = 3}$

$\cancel{n=1 \Rightarrow h[1] = 3\omega[1] - 3\omega[0] = 3\left(-\frac{1}{3}\right) - 3(1) = -4}$

$\cancel{n=2 \Rightarrow h[2] = 3\omega[2] - 3\omega[1] = 4/3}$

$\omega[n] = \delta[n]$

$\Rightarrow y_h[n] = 3\delta[n] - 3\delta[n-1]$

$= \{\dots, \underset{\uparrow}{3}, -3, \dots\}$

A= 5 5

e) $h[n] = \underbrace{w_h[n]}_{\text{the impulse response of first system}} * \underbrace{y_h[n]}_{\text{the impulse response of the second system}}$

$$\Rightarrow h[n] = (3\delta[n] - 3\delta[n-1]) * \left(\frac{-1}{3}\right)^n u[n]$$

$$\cancel{= 3\left(\frac{-1}{3}\right)^n - 3\left(\frac{-1}{3}\right)^{n-1} = \ldots \left(\frac{1}{3}\right)^{n-1} + \ldots \left(\frac{-1}{3}\right)^{n-2}}$$

$$= 3\left(\frac{-1}{3}\right)^n u[n] - 3\left(\frac{-1}{3}\right)^{n-1} u[n-1]$$

$$h[n] = \begin{cases} 0 & n<0 \\ 3 & n=0 \\ 12\left(\frac{-1}{3}\right)^n & n \geq 1 \end{cases}$$

f) $x[n] = \delta[n]$ $\quad y[n] = h[n]$ $\quad h[-1] = 0$

$$\Rightarrow h[n] = 3\delta[n] - 3\delta[n-1] - \frac{1}{3}h[n-1]$$

$n=0 \Rightarrow h[0] = 3$ ✓ $\qquad n=2 \Rightarrow h[2] = -\frac{1}{3}(-4) = \frac{4}{3}$ ✓

$n=1 \Rightarrow h[1] = -3 - \frac{1}{3}(3) = -4$ ✓

Q5 - a) $x[n] = 2\delta[n+1] + 3\delta[n] + 4\delta[n-1]$

since x is a real signal

$$r_{xx}[\ell] = \sum_{n=-\infty}^{\infty} x[n]\, x^*[n-\ell] = \sum_{n=-\infty}^{\infty} x[n]\, x[n-\ell]$$

$$= \sum_{n} \left(2\delta[n+1] + 3\delta[n] + 4\delta[n-1]\right)\left(2\delta[n-\ell+1] + 3\delta[n-\ell] + 4\delta[n-\ell-1]\right)$$

$$= 4\delta[-\ell] + 6\delta[-1-\ell] + 8\delta[-2-\ell] + 6\delta[1-\ell] + 9\delta[-\ell] + 12\delta[-1-\ell]$$
$$+ 8\delta[2-\ell] + 12\delta[1-\ell] + 16\delta[-\ell]$$

$$= 29\delta[-\ell] + 18\delta[1-\ell] + 8\delta[-2-\ell] + 18\delta[-1-\ell] + 8\delta[2-\ell]$$

$$r_{xx}[\ell] = \{\ldots, 0, 0, 8, 18, \underset{\uparrow}{\overset{\ell=0}{29}}, 18, 8, 0, \ldots\}$$

Q5-b) $y[n] = 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$

y is real

$$r_{yy}[\ell] = \sum_{n=-\infty}^{\infty} y[n]\,y[n-\ell] = \sum_{n} [2\delta[n-1]+3\delta[n-2]+4\delta[n-3]]\,[2\delta[n-\ell-1]$$

$$+3\delta[n-\ell-2]+4\delta[n-\ell-3]] = 4\delta[-\ell] + 6\delta[-\ell-1] + 8\delta[-\ell-2]$$

$$+6\delta[1-\ell] + 9\delta[-\ell] + 12\delta[-\ell-1] + 8\delta[2-\ell] + 12\delta[1-\ell] + 16\delta[-\ell]$$

$$= 29\delta[-\ell] + 18\delta[1-\ell] + 8\delta[2-\ell] + 18\delta[-\ell-1] + 8\delta[-\ell-2]$$

$$= \{\dots, 0, 8, 18, \underset{\uparrow}{29}, 18, 8, 0, \dots\}$$

c) $z[n] = 4\delta[n+1] + 3\delta[n] + 2\delta[n-1]$

z is real

$$r_{zz}[\ell] = \sum_{n=-\infty}^{\infty} z[n]\,z[n-\ell] = \sum_{n=-\infty}^{\infty} (4\delta[n+1]+3\delta[n]+2\delta[n-1])(4\delta[n+1-\ell]$$

$$+3\delta[n-\ell]+2\delta[n-1-\ell]) = 16\delta[-\ell] + 12\delta[-1-\ell] + 8\delta[-2-\ell] + 12\delta[1-\ell]$$

$$+9\delta[-\ell] + 6\delta[-1-\ell] + 8\delta[2-\ell] + 6\delta[1-\ell] + 4\delta[-\ell]$$

$$= 29\delta[-\ell] + 18\delta[1-\ell] + 8\delta[2-\ell] + 18\delta[-\ell-1] + 8\delta[-\ell-2]$$

$$= \{\dots, 0, 8, 18, \underset{\uparrow}{29}, 18, 8, 0, \dots\}$$

d) The autocorrelation of these real signals were equal then shifting and reversing did not affect the autocorrel

y is real

$$y[n] = x[n-2] \Rightarrow r_{yy}[\ell] = \sum_{n} y[n]\,y[n-\ell] = \sum_{n=-\infty}^{\infty} x[n-2]\,x[n-2-\ell]$$

$$n' = n-2 \Rightarrow \sum_{n'=-\infty}^{\infty} x[n']\,x[n'-\ell] = r_{xx}[\ell] = r_{yy}[\ell]$$

$$z[n] = x[-n] \Rightarrow r_{zz}[\ell] = \sum_{n=-\infty}^{\infty} z[n]\,z[n-\ell] = \sum_{n=-\infty}^{\infty} x[-n]\,x[-n+\ell]$$

$$n' = -n \Rightarrow \sum_{n'=-\infty}^{\infty} x[n']\,x[n'-\ell] = r_{xx}[\ell] = r_{zz}[\ell]$$

Q6 $x[n] = \delta[n] + \delta[n-1] + \delta[n-2] \Rightarrow X[z] = z^{0} + z^{-1} + z^{-2}$

$y[n] = \delta[n-2] + \delta[n-3] + \delta[n-4]$

$\bar{y}[n] = y[-n] = \delta[-n-2] + \delta[-n-3] + \delta[-n-4] \Rightarrow \bar{Y}[z] = z^{2} + z^{3} + z^{4}$

y is real

$r_{xy}[\ell] = x[\ell] * y^{*}[-\ell] = x[\ell] * y[-\ell] = x[\ell] * \bar{y}[\ell]$

$R_{xy}[z] = X[z]\,\bar{Y}[z] = z^{4} + 2z^{3} + 3z^{2} + 2z^{1} + z^{0}$

$\Rightarrow r_{xy}[\ell] = \{0, 1, 2, 3, 2, \underset{\uparrow}{1}, 0, \dots\}$

for $\ell = -2$ r_{xy} has the maximum value

$y[n-(-2)] = y[n+2] = \{\dots, 0, \underset{\uparrow}{1}, 1, 1, 0, 0, 0, \dots\} = x[n]$

Q7

$$x[n] = 4\sin\left(2\pi\frac{1}{5}n + \frac{\pi}{4}\right)$$

4 is a real number

$$r_{xx}[\ell] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]x[n-\ell] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]x[n-\ell]$$

$$N = 5 \Rightarrow \frac{1}{5}\sum_{n=0}^{4} 4\sin\left(2\pi\frac{1}{5}n + \frac{\pi}{4}\right)4\sin\left(\frac{2\pi}{5}(n-\ell) + \frac{\pi}{4}\right)$$

$$\Rightarrow \sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\Rightarrow \frac{4\times4}{5\times2}\sum_{n=0}^{4}\cos\left(\frac{2\pi}{5}\ell\right) - \cos\left(\frac{2\pi}{5}2n + \frac{\pi}{2} - \frac{2\pi}{5}\ell\right)$$

it is priodic and sumation ~~of one price~~ is zero

$$r_{xx}[\ell] = \frac{4\times4}{5\times2}\cos\left(\frac{2\pi}{5}\ell\right)\sum_{n=0}^{4}1 = \boxed{8\cos\left(2\pi\frac{1}{5}\ell\right)}$$