① $H_z(z) = \dfrac{a_2 z^2 + a_1 z + 1}{z^2 + a_1 z + a_2}$

system: real causal stable LTI

a) real information

b) system is stable there fore $|P_1|, |P_2|$ are less than 1
also it is causal then the ROC is the outer part

c) $N(z) = a_2 z^2 + a_1 z + 1 = z^2 (a_2 + a_1 z^{-1} + z^{-2})$

$D(z) = z^2 + a_1 z + a_2 \Rightarrow D(z^{-1}) \overset{\times z^2}{=} z^{-2} + a_1 z^{-1} + a_2 \; {\times z^2}$

$\Rightarrow z^2 D(z^{-1}) = N(z) = a_2 z^2 + a_1 z + 1$

d) $D(p_1) = D(p_2) = 0$

$z^2 D(z^{-1}) = N(z) \Rightarrow z' = z^{-1} \Rightarrow z'^{-2} D(z') = N(z'^{-1})$
$z'^{-1} = z$

$\Rightarrow z^{-2} D(z) = N(z^{-1})$

$z = P_1 \Rightarrow P_1^{-2} \underbrace{D(P_1)}_{0} = N(\frac{1}{P_1}) \Rightarrow N(\frac{1}{P_1}) = 0$

$z = P_2 \Rightarrow P_2^{-2} \underbrace{D(P_2)}_{0} = N(\frac{1}{P_2}) \Rightarrow N(\frac{1}{P_2}) = 0$

e) $|H_{DTFT}(F)|^2 = (H_z(z) H_z(z^{-1}))|_{z=e^{j2\pi F}}$ based on Q6 Ass9

$H_z(z) = \dfrac{z^2 D(z^{-1})}{D(z)}$ $\quad H_z(z^{-1}) = \dfrac{z^{-2} D(z)}{D(z^{-1})}$

$\Rightarrow |H_{DTFT}(F)|^2 = \left[ \dfrac{\cancel{z^2} \cancel{D(z^{-1})}}{\cancel{D(z)}} \cdot \dfrac{\cancel{z^{-2}} \cancel{D(z)}}{\cancel{D(z^{-1})}} \right]_{z=e^{j2\pi F}} = 1$

$|H_{DTFT}(F)| = 1$

f) $\Theta_H(F) = \measuredangle H_{DTFT}(F) = \measuredangle H_z(e^{j2\pi F})$ $\quad$ $\Theta_D(F) = \measuredangle D(e^{j2\pi F})$

$$H_z(z) = \frac{z^2 D(z^{-1})}{D(z)} \qquad H_z(e^{j2\pi F}) = \frac{e^{j4\pi F}\, D(e^{-j2\pi F})}{D(e^{j2\pi F})}$$

$$\Theta_H(F) = \measuredangle D(e^{-j2\pi F}) + 4\pi F - \measuredangle D(e^{j2\pi F}) = 4\pi F - 2\measuredangle D(e^{j2\pi F})$$

$$\measuredangle D(z^{-1}) = -\measuredangle D(z) \Rightarrow \measuredangle D(e^{-j2\pi F}) = -\measuredangle D(e^{j2\pi F})$$

since is real
so it is hermition

Q2) $z_{DTFT}(F) = \sum_{k=-\infty} \delta(F - \frac{k}{N})$ $\qquad N = 10$

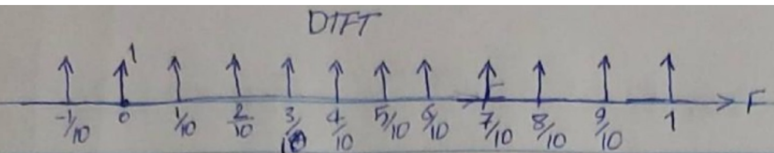
$$z[n] = \int_0^1 Z_{DTFT}(F)\, e^{j2\pi F n}\, dF = \int_0^1 \sum_{k=-\infty}^{\infty} \delta(F - \tfrac{k}{N})\, e^{j2\pi F n}\, dF$$

$$= \sum_{k=-\infty}^{\infty} \int_0^1 \delta(F - \tfrac{k}{N})\, e^{j2\pi F n}\, dF = \sum_{k=-\infty}^{\infty} N e^{j2\pi \frac{k}{N} n} =$$

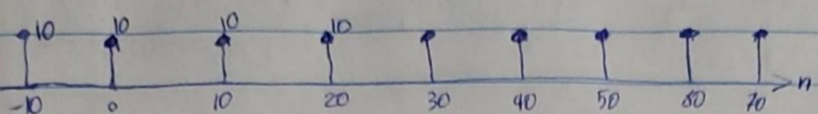
$\int_0^1 \delta(F - \frac{k}{N})\, dF = N$ because there are N delta Function between 0 and 1

$$z[n] = \sum_{k=-\infty}^{\infty} N e^{j2\pi \frac{k}{N} n} = \sum_{k=-\infty}^{\infty} N \delta(n - kN) = \sum_{k=-\infty}^{\infty} 10\, \delta(n - 10k)$$

sumation has only 1 non zero value
in NK and it is equal to 1N otherwise
it is zero because $e^{j2\pi \frac{k}{N} n}$ is periodic with N and sumation on 1 period is 0

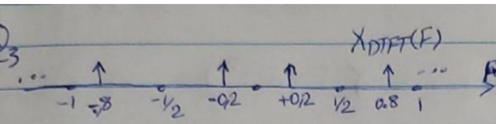


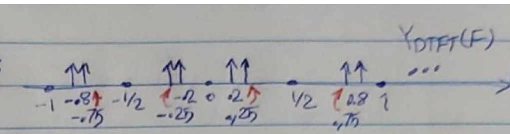

$z_{DTFT}(F) = \sum_{k=-\infty}^{\infty} \delta(F - \frac{k}{N})$




$z[n] = \sum_{k=-\infty}^{\infty} 10\, \delta(n - k10)$

Q3




$x[n] = \cos(2\pi \frac{1}{5} n)$ $\qquad X_{DTFT}(F) = \frac{1}{2}(\delta(F+\frac{1}{5}) + \delta(F-\frac{1}{5})) * comb(F)$

$y[n] = \cos(2\pi\frac{1}{5}n) + \cos(2\pi\frac{1}{4}n)$

$$Y_{DTFT}(F) = \frac{1}{2}\left[\delta(F+\tfrac{1}{5}) + \delta(F-\tfrac{1}{5}) + \delta(F+\tfrac{1}{4}) + \delta(F-\tfrac{1}{4})\right] * comb(F)$$

b) $w[n] = \begin{cases} 1, & 0 \leq n \leq (L-1) \\ 0, & \text{otherwise} \end{cases}, \quad L = 10$

$$W_{DTFT}(F) = \sum_{n=-\infty}^{\infty} w[n] e^{-j2\pi Fn} = \sum_{n=0}^{9} 1 e^{-j2\pi Fn} = \frac{e^{-j2\pi F(0)} - e^{-j2\pi F(10)}}{1 - e^{-j2\pi F}}$$

$$= \frac{1 - e^{-j20\pi F}}{1 - e^{-j2\pi F}} = \frac{e^{-j2\pi F(5)}}{e^{-j\pi F}} \cdot \frac{e^{j2\pi F(5)} - e^{-j2\pi F(5)}}{e^{j\pi F} - e^{-j\pi F}} = \frac{e^{-j\pi F10}}{e^{-j\pi F}} \cdot \frac{\sin(10\pi F)}{\sin(\pi F)}$$

$$= e^{-j\pi F(9)} \frac{\sin(10\pi F)}{\sin(\pi F)}$$

$W_{DTFT}(F+1) = W_{DTFT}(F)$

$$W_{DTFT}(F+1) = e^{-j\pi(F+1)9} \frac{\sin(10\pi(F+1))}{\sin(\pi(F+1))} = e^{-j\pi F9 \underbrace{- j\pi 9}_{-1}} \frac{\sin(10\pi F + 10\pi)}{\sin(\pi F + \pi)}$$

$$= e^{j\pi F9} (\cancel{-1}) \frac{\sin(10\pi F)}{\cancel{-}\sin(\pi F)} = e^{j2\pi F9} \frac{\sin(10\pi F)}{\sin(\pi F)} = W_{DTFT}(F)$$

$\sin(\pi t + \pi) = -\sin(\pi t)$
$\sin(\pi t + 2\pi) = \sin(\pi t)$
$e^{-j\pi a} = -1$ if $\underline{a}$ is odd

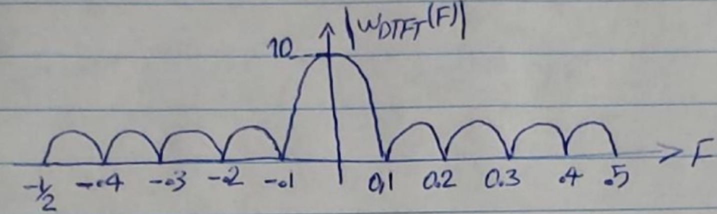
$$W_{DTFT}(0) = \sum_{n=-\infty}^{\infty} w[n] = \sum_{n=0}^{9} 1 = 10$$

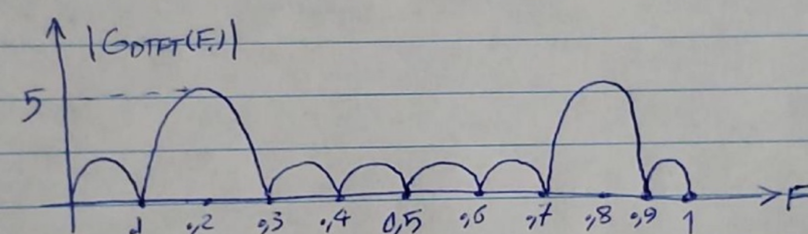
$W_{DTFT}(F) = 0$ when $F = \frac{k}{10}$ $k = \pm 1, \pm 2, \mp 3, \ldots, k \bmod 10 \neq 0$

because $\sin(10\pi F) = 0$ for $F = \frac{k}{10}$ $k = 0, \pm 1, \mp 2, \ldots$
also for $F = 0, 10, 20, \ldots$

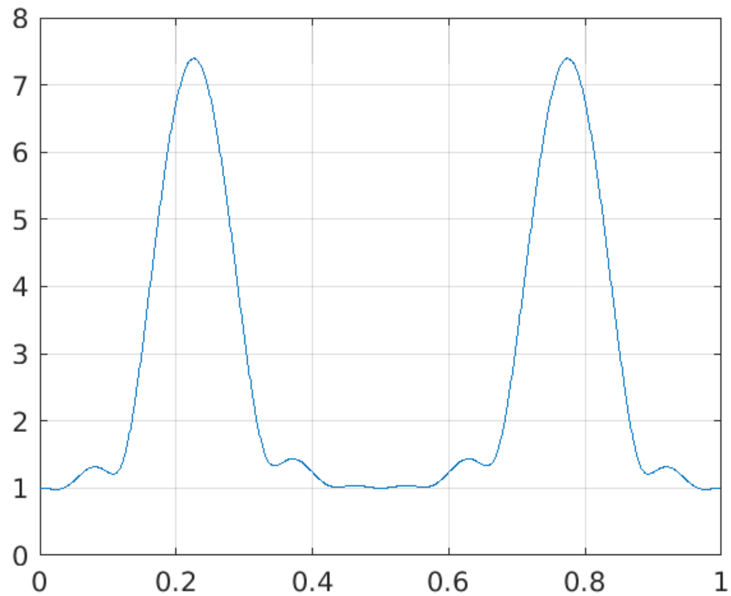
$$W_{DTFT}(F') = 10 \quad \forall\ F' = 0, 10, 20, \ldots$$

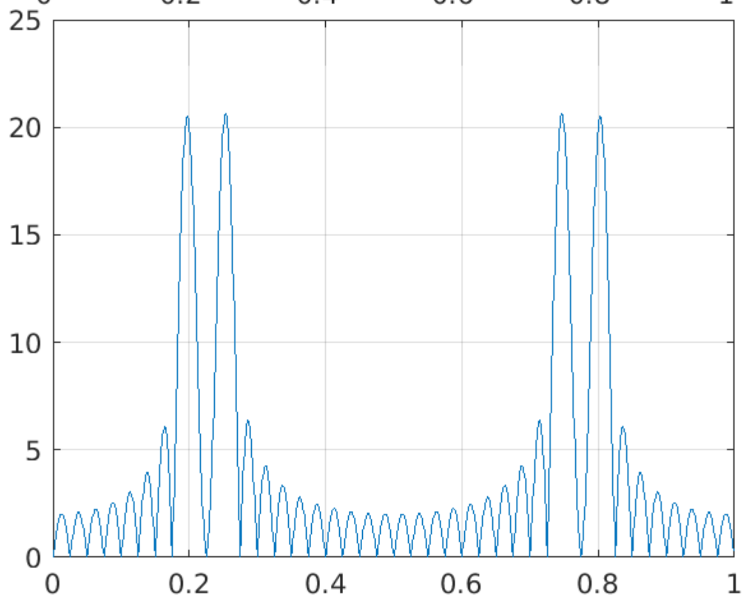
$$|W_{DTFT}(F)| = \left| \frac{\sin(10\pi F)}{\sin(\pi F)} \right|$$




d) yes if I increase the window size, I could resolve two different frequencies otherwise it is not possible


0
1
2
3
4
5
6
7
8
0
0.2
0.4
0.6
0.8
1


0
5
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1