1. In class, we introduced a multiplication-based approach to find the convolution of two finite-length sequences based on polynomial multiplication. For each of parts (a), (b), and (c), below:

   (i) use the multiplication-based approach to find $y[n] = x[n] * h[n]$;

   (ii) inspect the signals $x[n]$, $h[n]$, and $y[n]$ so as to determine the values of:

   $n_{\text{min}}^{(x)}, n_{\text{max}}^{(x)}, \ell_x, n_{\text{min}}^{(h)}, n_{\text{max}}^{(h)}, \ell_h, n_{\text{min}}^{(y)}, n_{\text{max}}^{(y)}$, and ℓ_y

   where: $n_{\text{min}}^{(x)}$ denotes the smallest value of n for which $x[n]$ is nonzero; $n_{\text{max}}^{(x)}$ denotes the biggest value of n for which $x[n]$ is nonzero; $\ell_x = n_{\text{max}}^{(x)} - n_{\text{min}}^{(x)} + 1$ denotes the length of the nonzero portion of the sequence $x[n]$; *etc*.

   (iii) verify that the following three relations hold:

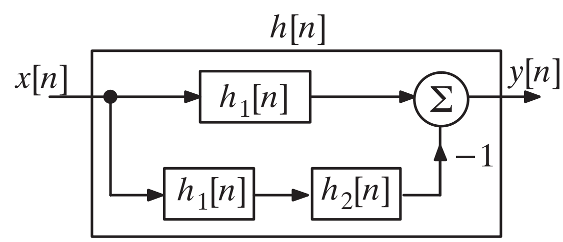
   $n_{\text{min}}^{(y)} = n_{\text{min}}^{(x)} + n_{\text{min}}^{(h)}$, $n_{\text{max}}^{(y)} = n_{\text{max}}^{(x)} + n_{\text{max}}^{(h)}$, and $\ell_y = \ell_x + \ell_h - 1$.

(5 pts.) a) $x[n] = \{\ \dots,\ 0\ \ \underset{\uparrow}{4},\ 4,\ 2,\ 0,\ \dots\ \}$ and $h[n] = \{\ \dots,\ 0,\ 1,\ \underset{\uparrow}{2},\ 3,\ 0,\ 0,\ \dots\ \}$

(5 pts.) b) $x[n] = \begin{cases} 3 - |n|, & |n| \le 2 \\ 0, & \text{otherwise} \end{cases}$ and $h[n] = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}$

(5 pts.) c) $x[n] = \begin{cases} 1, & -3 \le n \le -1 \\ 0, & \text{otherwise} \end{cases}$ and $h[n] = \begin{cases} 1, & 1 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$

2. Let $h[n]$ denote the impulse response of the system shown to the right. Note that the system consists of several LTI subsystems connected as shown. Each subsystem is represented by a block bearing the name of its impulse response. Given that: $h_1[n] = u[n] - u[n-3]$ and $h_2[n] = \delta[n-1]$.



(7 pts.) a) Find and sketch the system's impulse response, $h[n]$. Partial credit will be given for correctly expressing $h[n]$ in terms of $h_1[n]$ and $h_2[n]$ as well as for showing plots or sequence representations of intermediate signals used in determining $h[n]$.

(3 pts.) b) Find the system's input-output equation. Recall that a system's input-output equation relates the output sequence $y[\cdot]$ to the input sequence $x[\cdot]$. It should not make reference to the signal $h[n]$ and should be simplified to the extent possible.

(8 pts.) 3. Let $x[n] = \left(\frac{1}{2}\right)^n u[n]$ and $h[n] = \left(-\frac{1}{2}\right)^n u[n]$.

Find a closed-form expression for the convolution $y[n] = x[n] * h[n]$. For practice, I would like you to ***do this using a mathematical approach***, *i.e.*, substitute the expressions for $x[\cdot]$ and $h[\cdot]$ into the convolution sum and simplify the resulting expression. You can always verify your resulting expression using the sequence approach to determine several values of the convolution sequence (say: $y[0]$, $y[1]$, and $y[2]$). For this problem, I am not requiring you to show the verification but it is recommended that you do it on your own.

4. Consider the causal LTI systems described by the following LCCDE's. For each system, iterate the difference equation to find the system's impulse response, then characterize each system as FIR or IIR. Recall that the impulse response is the response of the relaxed system (*i.e.,* all internal storage registers are assumed to be cleared at $n = 0$, so that $y[-1] = y[-2] = \ldots = 0$) when $x[n] = \delta[n]$. (The impulse response of a causal system is always zero for $n < 0$)

(3 pts.) a) $y[n] = \frac{1}{2}y[n-1] + \frac{1}{2}x[n]$

(3 pts.) b) $y[n] = x[n] - 2x[n-1] + 3x[n-2] + 4x[n-3]$

(3 pts.) c) $y[n] = y[n-1] + x[n] - \frac{1}{3}x[n-1] - \frac{2}{3}x[n-2]$

(9 pts.) 5. Consider again the systems of question 4. For each system, iterate the difference equation to find the system's step response. Recall that the step response is the response of the relaxed system when $x[n] = u[n]$. You should iterate the equation until you see a pattern develop and can propose a closed-form solution for all n greater than where you left off. In each case, you should also determine the value of $y_{\text{step}}[n]$ in the limit as $n \to \infty$.

6. For each LCCDE below: state the associated homogeneous difference equation; find the characteristic roots: λ_1 and λ_2 ; and write the general form of the homogenous solution.

(3 pts.) a) $y[n] + \frac{3}{10}y[n-1] - \frac{1}{10}y[n-2] = x[n] + 5x[n-1]$

(3 pts.) b) $y[n] - \frac{2}{3}y[n-1] + \frac{1}{9}y[n-2] = 2x[n] - 3x[n-1]$

(3 pts.) c) $y[n] - \frac{2}{3}y[n-1] - \frac{1}{3}y[n-2] = 2x[n] - 3x[n-1]$

7. For a second-order system, the two characteristic roots are found to be complex conjugates. In particular: $\lambda_1 = \lambda_2^* = \frac{1}{10}(3 + j4)$.

(3 pts.) a) Find the characteristic equation and homogenous difference equation associated with this system.

(2 pts.) b) Express λ_1 and λ_2 in polar form (*i.e.,* identify the values of the real-valued constants L and β so that $\lambda_1 = Le^{j\beta}$ and $\lambda_2 = Le^{-j\beta}$).

(5 pts.) c) The general solution for the homogeneous difference equation is:

$$y_h[n] = A_1(\lambda_1)^n + A_2(\lambda_2)^n$$

Given that $A_1 = A_2^* = 7e^{j\frac{\pi}{4}}$ and assuming λ_1 and λ_2 to have the values specified above, show the mathematical details/steps involved in converting the expression provided above for $y_h[n]$ to the form: $y_h[n] = C\rho^n \cos(\Omega n + \gamma)$, where C, ρ, Ω, and γ are all real-valued constants.

Once you have completed your conversion, explain how the constants: C, ρ, Ω, and γ are related to the constants: $|\lambda_1| \equiv L$, $\measuredangle\lambda_1 \equiv \beta$, $|A_1|$, and $\measuredangle A_1$.