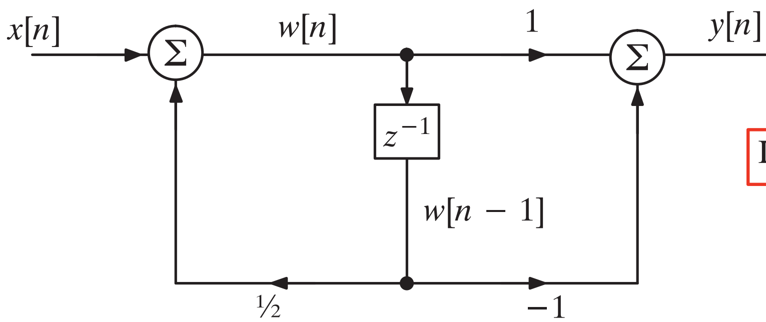
ECE 4531
Stevenson

Example to illustrate how the Direct Form II structure provides a method for handling the analysis of systems when $M \geq N$.

Problem: Consider the system whose Direct Form II structure is shown below. Find the system's difference equation and the system's impulse response

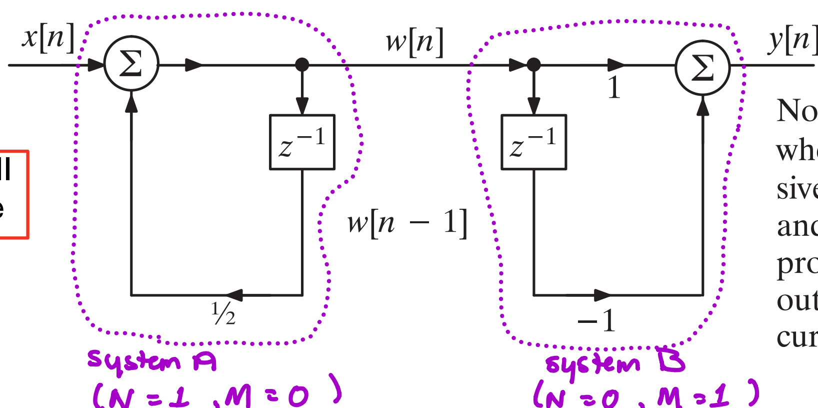


Direct Form II structure

Will first find the LCCDE:

Solution: It is more straightforward to determine the difference equation from the Direct Form I structure than the Direct Form II structure. Unless you can remember how to read off the difference equation directly from the Direct Form II structure, I recommend that you convert back to the Direct Form I structure. We can transition from the Direct Form II structure to the Direct Form I structure by first replicating the storage/delay elements to get the *split Direct Form II structure* shown below. [As suggested by the note below, the split D.F. II structure provides a method for analyzing systems with $M \geq N$, which we will come back to.]

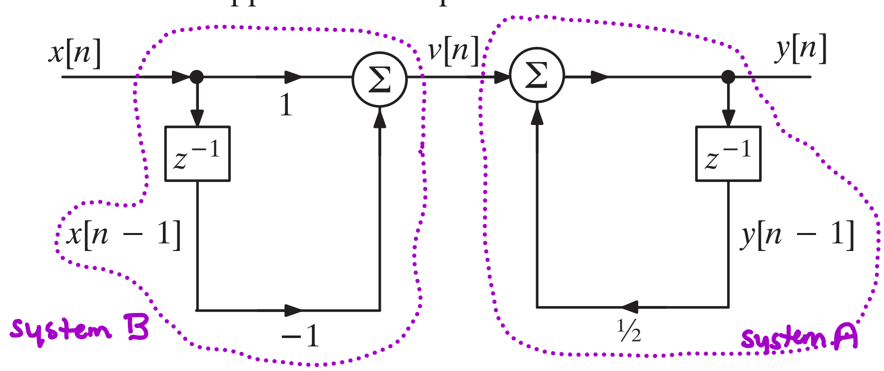
Split DF II Structure



Note: $y[n] = w[n] - w[n-1]$ where $w[n]$ is the output of a recursive system with $N = 1 > M = 0$ and can, thus, be found using the procedure described on the handout entitled "Analysis of causal recursive LCCDE's".

The Direct Form I structure, shown below, is then found by changing the order in which the two systems above are applied to the input.

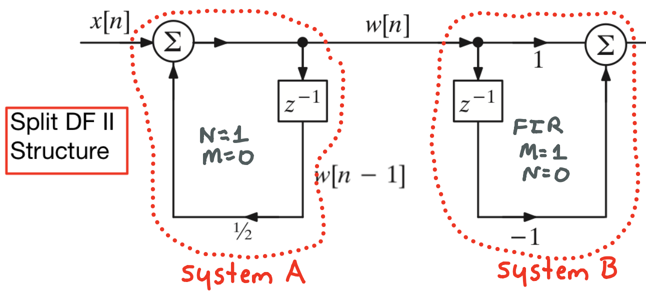
DF I structure



LCCDE: $\boxed{}$ ⇒ M= N=

Outline of the procedure we will use to find the impulse response, $h[n]$:

Our previous analysis technique assumes $N > M$ which is not the case for this system. As noted below, the split D.F. II structure offers a solution. It decomposes the overall system into a recursive system with $N=1$ and $M=0$ (System A), followed by an FIR system (System B) whose impulse response may be written down by inspection.



Note: $y[n] = w[n] - w[n-1]$ where $w[n]$ is the output of a recursive system with $N = 1 > M = 0$ and can, thus, be found using the procedure described on the handout entitled "Analysis of causal recursive LCCDE's".

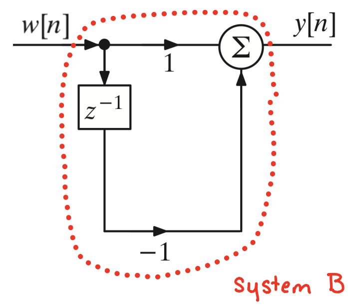
Since System A and System B are both LTI, we may find the impulse response, $h[n]$, of the overall system, as follows:

$$h[n] = h_A[n] * h_B[n] \qquad (1)$$

where: $h_A[n]$ denotes the impulse response of system A

$h_B[n]$ denotes the impulse response of system B

Since system B is an FIR system, we may determine $h_B[n]$ by inspection of its implementation structure:



$$h_B[n] = \{ \qquad \} = \qquad (2)$$

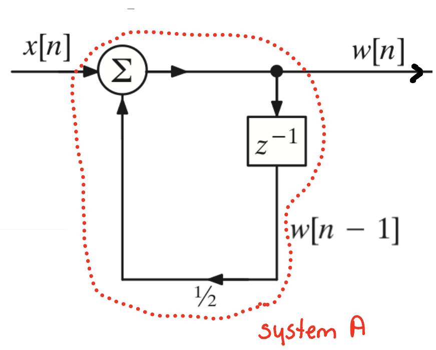
Substituting (2) into (1) allows us to express $h[n]$ in terms of $h_A[n]$ as:

$$h[n] = h_A[n] * (\qquad) =$$

$$\Rightarrow \boxed{h[n] = \qquad} \qquad (3)$$

Furthermore, since $h_A[n]$ is the impulse response of a system with $N = 1 > M = 0$, we may find $h_A[n]$ using our previously established analysis technique.

To find the impulse response of system A:



LCCDE for system A:

char. eqn:

char. root:

general form of homog. soln: $y_h[n] =$

When $N > M$, the impulse response will have the same general form, for $n \geq 0$, as the solution to the system's homogeneous eqn. Thus:

general form for $h_A[n]$: $h_A[n] =$ (4)

Evaluating (3) at $n=0$: $h_A[0] =$ (5)

Difference eqn. satisfied by impulse response: $h_A[n] =$

Iterating diff. eqn. to find $h_A[0]$ yields: $h_A[0] =$

Equating (5) to the value of $h_A[0]$ obtained by iteration, allows us to solve for C_1:

$h_A[0] =$ $\Rightarrow$ $C_1 =$

And substituting the value of C_1 into (4) yields the complete solution for $h_A[n]$:

$h_A[n] =$ (6)

Finally, substituting (6) into (3) yields the impulse response, $h[n]$, of the overall system:

$h[n] = h_A[n] - h_A[n-1] =$