1. Consider the causal system described by the following LCCDE:

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n]$$

(2 pts.) a) Find the system transfer function, $H(z)$.

(5 pts.) b) Find a closed-form expression for the impulse response, $h[n]$, by finding the inverse Z transform of $H(z)$.

(6 pts.) c) Use the Z transform approach to find a closed-form expression for the step response, $y_{\text{step}}[n]$, *i.e.*, the zero-state response of the system when $x[n] = u[n]$.

(6 pts.) d) Use the Z transform approach to find a closed-form expression for the zero-state response, $y_{zs}[n]$, when $x[n] = (1/2)^n u[n]$.

(6 pts.) e) Use the one-sided Z transform to find the zero-input response of the system when $y[-1] = 2$ and $y[-2] = 28$.

(6 pts.) f) Let $y[n]$ denote the response of the system when $x[n] = (1/2)^n u[n]$, $y[-1] = 2$, and $y[-2] = 28$. Find a closed-form expression for $y[n]$. Check your expression by iterating the difference equation (either by hand, or by using the filter function in matlab) to find values for $y[0]$, $y[1]$, $y[2]$, and $y[3]$. Then compare these values to the values of your closed-form expression at $n = 0, 1, 2$, and 3. Please show the details of your calculations/comparisons. If you use matlab to iterate the difference equation or evaluate your closed-form expression, you must provide the matlab commands that you used and the values returned.

2. Consider the following periodic signal:

$$x[n] = 2 + 2\cos(\pi n/4) + \cos(\pi n/2) + 0.5\cos(3\pi n/4)$$

(2 pts.) a) Let N denote the ***fundamental period*** of $x[n]$. Determine the value of N being sure to show your work. (note: N will be the smallest common period of all the signal components in the expression for $x[n]$). If you are at all uncertain, then before proceeding to future parts of this question, you should verify your answer in matlab by plotting $x[n]$ then inspecting the plot to verify that the fundamental period of $x[n]$ is as you claimed.

(3 pts.) b) Find the values of the DFS coefficients, $c_k, k = 0, \ldots N-1$, for this signal. Try doing this without a calculator. ***Hint***: You can identify the values of the DFS coefficients by inspection of $x[n]$ if you use Euler's identity to rewrite $x[n]$ as a sum of complex exponentials and compare the resulting expression for $x[n]$ to the DFS representation for $x[n]$ (using your knowledge of equivalent discrete-time frequencies).

(2 pts.) c) Make a table or plot to illustrate the values of the power spectral density, $\left\{|c_k|^2\right\}$, of $x[n]$.

(4 pts.) d) Use the power spectral density (*i.e.*, the squared magnitude of the DFS coefficients) in conjuction with Parseval's Theorem to find the average power of $x[n]$. ***Leave your answer expressed as a fraction. Does your answer agree with what it should be based on what you know that a time-domain calculation for the average power of $x[n]$ would yield?***
To help you answer this question, note that $x[n]$ is a sum of orthogonal signals on the interval of length N (all of which are periodic with period N). Furthermore, we know that the average power of a sum of orthogonal signals is the sum of the individual avg powers. What is the average power of a cosine? what is the average power of a constant?

3. Given that the DFS coefficients of a discrete-time periodic signal $x[n]$ are given by:

$$c_k = \cos(k\pi/4) + \sin(3k\pi/4), \quad k = 0, \pm 1, \pm 2, \ldots$$

(2 pts.) a) Let N denote the fundamental period of the coefficients, $\{c_k\}$. That is let N denote the smallest integer such that $c_{k+N} = c_k$ for all k. Determine N, being sure to show your work. If you have any doubts, you should use matlab to verify your answer before proceeding to the following parts of the question.

(4 pts.) b) Without the use of a calculator, evaluate the DFS synthesis sum to find $x[n]$.

> ***Hint***: Rewrite the expression above for c_k as a sum of complex exponentials. You can then evaluate the DFS synthesis sum for each complex exponential; then sum the individual results to find $x[n]$. Note that if the DFS coefficients of a 6-periodic signal, $q[n]$, are given by $c_k = \frac{1}{2}e^{j2\pi\frac{2}{6}k}$, $k = 0, 1, \ldots, 5$, then we can find $q[n]$ by evaluating the DFS synthesis sum: $q[n] = \sum_{k=0}^{5} c_k \, e^{+j2\pi\frac{k}{6}n} = \sum_{k=0}^{5} \frac{1}{2}e^{j2\pi\frac{2}{6}k} \, e^{+j2\pi\frac{k}{6}n} = \sum_{k=0}^{5} \frac{1}{2}e^{j2\pi\frac{k}{6}(2+n)}$
> Note that $q[n]$ is periodic with period 6, why? Furthermore, in the interval $0 \le n \le 5$, there is only one value of n for which $q[n]$ is nonzero (the rightmost sum makes it easy to see this); which n is it? What is the value of the sum for this n? Note there will be other values of n outside the interval $0 \le n \le 5$ for which the rightmost sum above is obviously nonzero; what are these values of n? what is the value of the synthesis sum at these values of n?

(4 pts.) c) Since the coefficients $\{c_k\}$ are real, we know that $x[n]$ is Hermitian; this implies that the magnitude of $x[n]$ is an even function of n and the phase of $x[n]$ is an odd function of n. Determine values for the magnitude and phase of $x[n]$ on an interval symmetric about the origin that contains at least one period of $x[n]$. Then use these values to argue/demonstrate that the magnitude of $x[n]$ is even and the phase of $x[n]$ is odd.

(4 pts.) d) Note that the DFS coefficients are also periodic with period $M = 2N$ where N is the fundamental period of the coefficients, as defined in part (a). Define $g[n]$ to be the M-periodic sequence that results from the DFS synthesis sum below. Without the use of a calculator, evaluate the sum below to find $g[n]$, where:

$$g[n] = \sum_{k=0}^{M-1} c_k \, e^{+j2\pi\frac{k}{M}n}$$

Hint: Make use of the same techniques you used in part (b). In any interval of length M, how many nonzero values of $g[n]$ will there be?