Formula Sheet for ECE4531 Midterm
Stevenson

NAME ______________________

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1. Quadratic Formula

The roots of the polynomial $ax^2 + bx + c$ can be found via the quadratic formula as:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence the polynomial can be rewritten as: $ax^2 + bx + c = a(x - x_1)(x - x_2)$

2. Convolution:

convolution sum: $y[n] = h[n] * x[n] \quad \Rightarrow \quad y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$

3. Summations:

$$\sum_{n=n_1}^{n_2} a^n = \frac{a^{n_1} - a^{(n_2+1)}}{1-a} \quad \Rightarrow \quad \text{if } |a| < 1, \text{ then}: \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

4. Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x \qquad \sin x \sin y = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$$

$$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right) \qquad \cos x \cos y = \frac{1}{2}\left(\cos(x-y) + \cos(x+y)\right)$$

$$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right) \qquad \sin x \cos y = \frac{1}{2}\left(\sin(x-y) + \sin(x+y)\right)$$

5. Cross-correlation and Autocorrelation

The ***cross-correlation***, $r_{xy}[\ell]$, of **two energy sequences**, $x[n]$ and $y[n]$, is defined as:

$$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]\,y^*[n-\ell] = x[\ell] * y^*[-\ell]$$

where $y^*[n]$ denotes the complex conjugate of $y[n]$.

The ***cross-correlation***, $r_{xy}[\ell]$, of **two power sequences**, $x[n]$ and $y[n]$, is defined as:

$$r_{xy}[\ell] = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x[n]\,y^*[n-\ell]$$

If $x[n]$ and $y[n]$ are both periodic with period N, their cross correlation, $r_{xy}[\ell]$, can be computed as:

$$r_{xy}[\ell] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\,y^*[n-\ell]$$

The ***autocorrelation*** function, $r_{xx}[\ell]$, of a sequence $x[n]$ is the cross-correlation of the sequence with itself.