Question 1

Q1:

A) $x[n] = \cos(0{,}01\pi n) = \cos(2\pi\, 0{,}005\, n)$

$F = 0{,}005 = \frac{5}{1000} = \frac{1}{200}$ $\boxed{N = 200}$

it is periodic because F is rational

$\tilde{F} = F = \frac{1}{200}$

B) $x[n] = \cos(\pi \frac{30}{105} n) = \cos(2\pi \frac{30}{210} n) = \cos(2\pi \frac{1}{7} n)$

$F = \frac{1}{7}$ is rational so $x[n]$ is periodic

$\tilde{F} = F = \frac{1}{7}$ $\boxed{N = 7}$

Question 1

C) $x[n] = \cos\left(2\pi \frac{142}{100} n\right)$

$F = \frac{142}{100}$ is rational so it is periodic $N = 100$

$|\tilde{F}| = \frac{142}{100} - 1 = \frac{49}{100}$

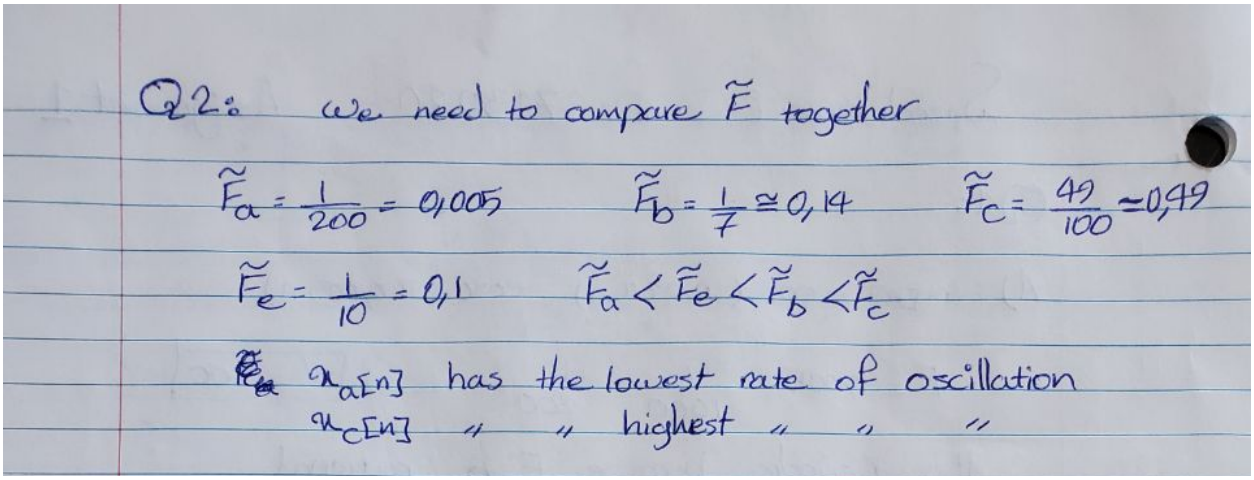
D) $x[n] = \sin(3n) = \sin\left(2\pi \frac{3}{2\pi} n\right)$

$F = \frac{3}{2\pi}$ is irrational so it is not periodic

E) $x[n] = e^{j\pi \frac{62}{10} n} = e^{j2\pi \frac{31}{10} n} \Rightarrow F = \frac{31}{10}$ is rational

$\therefore x[n]$ is periodic $\boxed{N = 10}$ $|\tilde{F}| = \frac{31}{10} - 2 = \boxed{\frac{1}{10}}$

Question 2



Q2: We need to compare $\tilde{F}$ together

$\tilde{F}_a = \frac{1}{200} = 0{,}005$ $\quad \tilde{F}_b = \frac{1}{7} \cong 0{,}14$ $\quad \tilde{F}_c = \frac{49}{100} = 0{,}49$

$\tilde{F}_e = \frac{1}{10} = 0{,}1$ $\quad \tilde{F}_a < \tilde{F}_e < \tilde{F}_b < \tilde{F}_c$

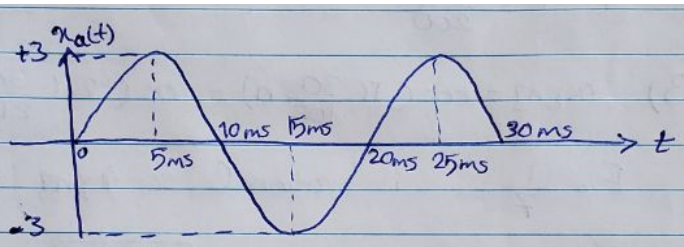
$x_{a[n]}$ has the lowest rate of oscillation

$x_{c[n]}$ " " highest " " "

Question 3

Q3:

A)

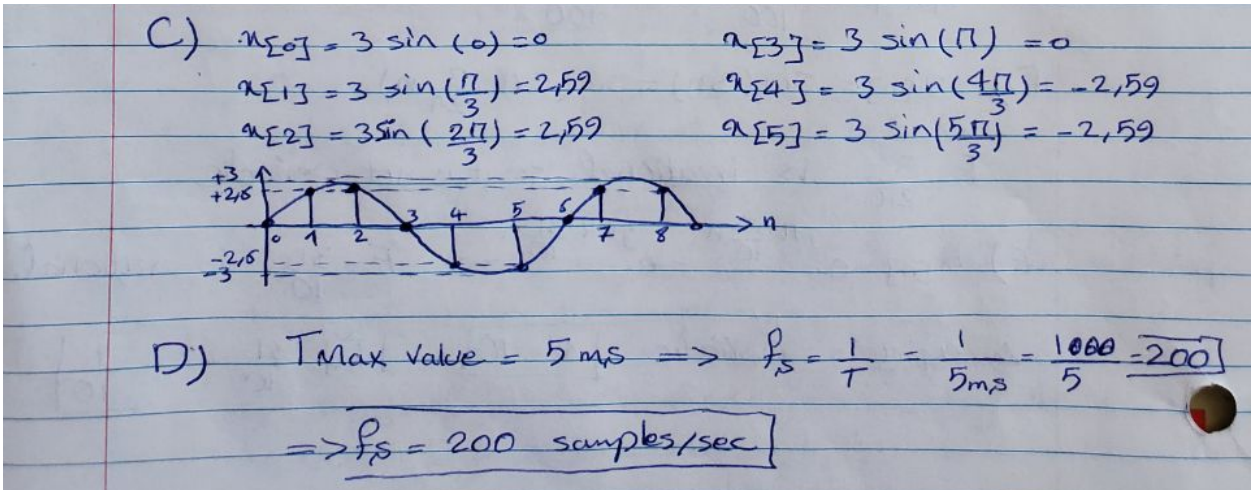


B) $\frac{f}{fs} = 300$ samples/sec $\quad x_a(t) = 3\sin(2\pi 50t)$

$$x[n] = x_a(n/f_s) = 3\sin\left(2\pi \frac{50}{300} n\right) = 3\sin\left(2\pi \frac{1}{6} n\right)$$

$\boxed{F = \frac{1}{6}}$ $\quad \boxed{N = 6}$ the fundamental period

Question 3

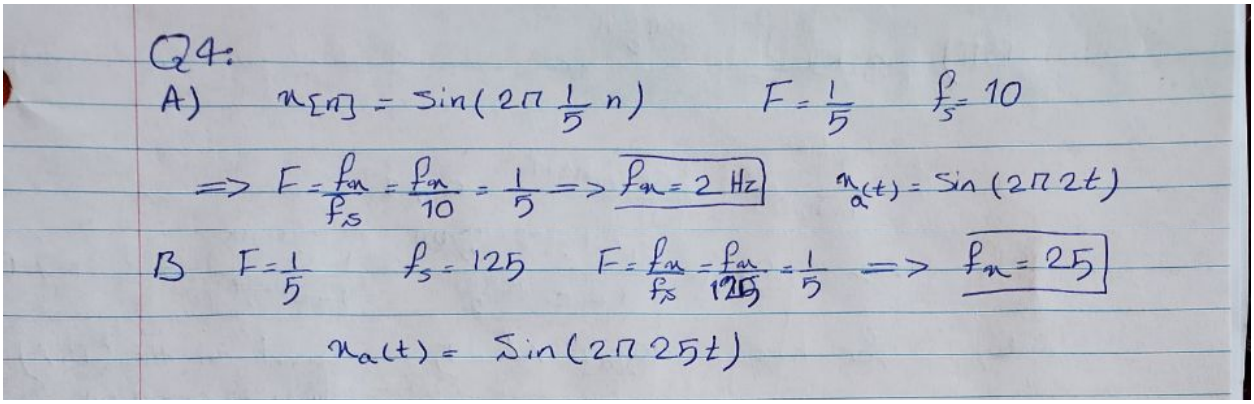


e) for this part, we need to create a DC signal. So, we need to have sample at zero value or every 10 ms.

T = 10ms

F_s= 1/10 ms = 100 sample/sec

Question 4



Q4:

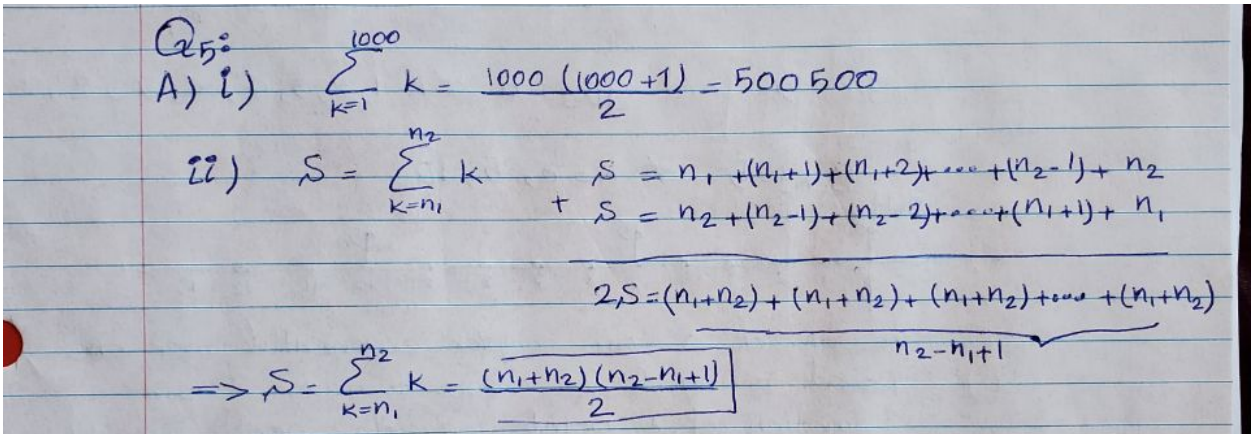
A) $x[n] = \sin(2\pi \frac{1}{5} n)$ $\quad F = \frac{1}{5}$ $\quad f_s = 10$

$\Rightarrow F = \frac{f_m}{f_s} = \frac{f_m}{10} = \frac{1}{5} \Rightarrow \boxed{f_m = 2\ Hz}$ $\quad x_a(t) = \sin(2\pi 2t)$

B $F = \frac{1}{5}$ $\quad f_s = 125$ $\quad F = \frac{f_m}{f_s} = \frac{f_m}{125} = \frac{1}{5} \Rightarrow \boxed{f_m = 25}$

$x_a(t) = \sin(2\pi 25t)$

Question 5



Q5:

A) i) $\sum_{k=1}^{1000} k = \frac{1000\,(1000+1)}{2} = 500500$

ii) $S = \sum_{k=n_1}^{n_2} k$

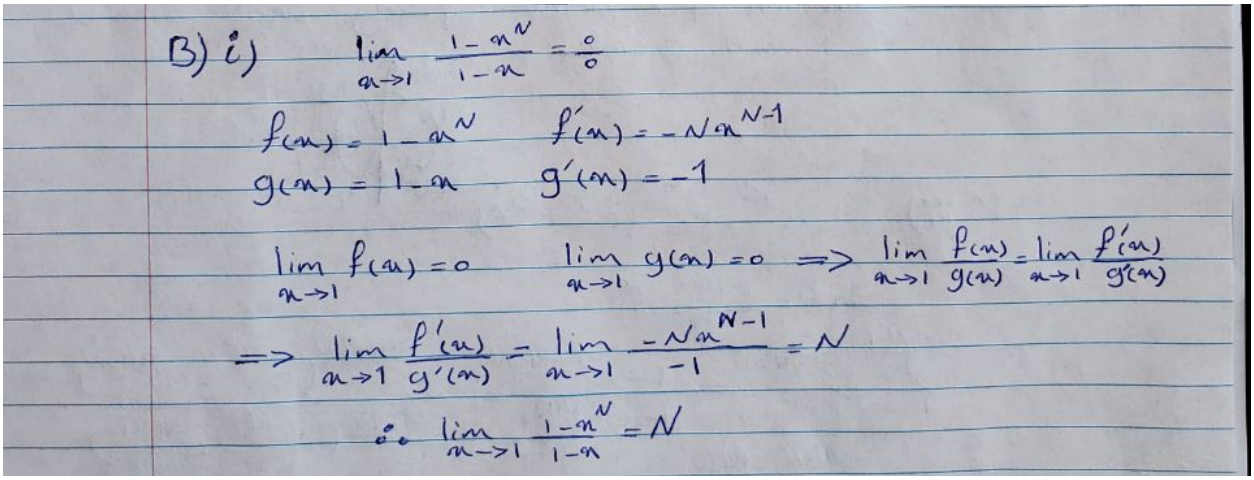
$S = n_1 + (n_1+1) + (n_1+2) + \cdots + (n_2-1) + n_2$

$+\ S = n_2 + (n_2-1) + (n_2-2) + \cdots + (n_1+1) + n_1$

$2S = \underbrace{(n_1+n_2) + (n_1+n_2) + (n_1+n_2) + \cdots + (n_1+n_2)}_{n_2-n_1+1}$

$\Rightarrow S = \sum_{k=n_1}^{n_2} k = \boxed{\frac{(n_1+n_2)(n_2-n_1+1)}{2}}$

Question 5



B) i) $\lim_{x \to 1} \frac{1-x^N}{1-x} = \frac{0}{0}$

$f(x) = 1-x^N \quad f'(x) = -Nx^{N-1}$

$g(x) = 1-x \quad g'(x) = -1$

$\lim_{x \to 1} f(x) = 0 \quad \lim_{x \to 1} g(x) = 0 \Rightarrow \lim_{x \to 1} \frac{f(x)}{g(x)} = \lim_{x \to 1} \frac{f'(x)}{g'(x)}$

$\Rightarrow \lim_{x \to 1} \frac{f'(x)}{g'(x)} = \lim_{x \to 1} \frac{-Nx^{N-1}}{-1} = N$

$\therefore \lim_{x \to 1} \frac{1-x^N}{1-x} = N$

Bii) $g[k]=\sum_{n=0}^{4} e^{-j2\pi\frac{k}{5}n}$

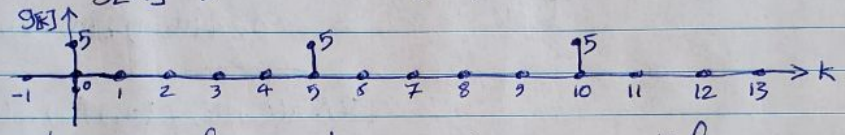
$$\sum_{n=n_1}^{n_2} a^n = \frac{a^{n_1}-a^{n_2+1}}{1-a} \qquad a\neq 1$$

$$\Rightarrow \quad g[k]=\frac{1-e^{-j2\pi\frac{k}{5}(5)}}{1-e^{-j2\pi\frac{k}{5}}}=\frac{1-e^{j2\pi k}}{1-e^{-j\frac{2\pi k}{5}}}=0 \qquad k\neq 0$$

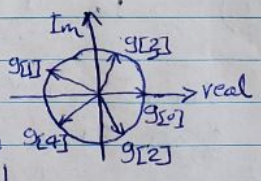
for $k=0$ $\quad g[0]=N=5$ based on the (Bi) the last question

$g[0]=5$ and $g[k]=0 \quad \forall k=1,2,3,4,6,7,8,9$

$g[k]=5 \quad \forall k=0,5,10$

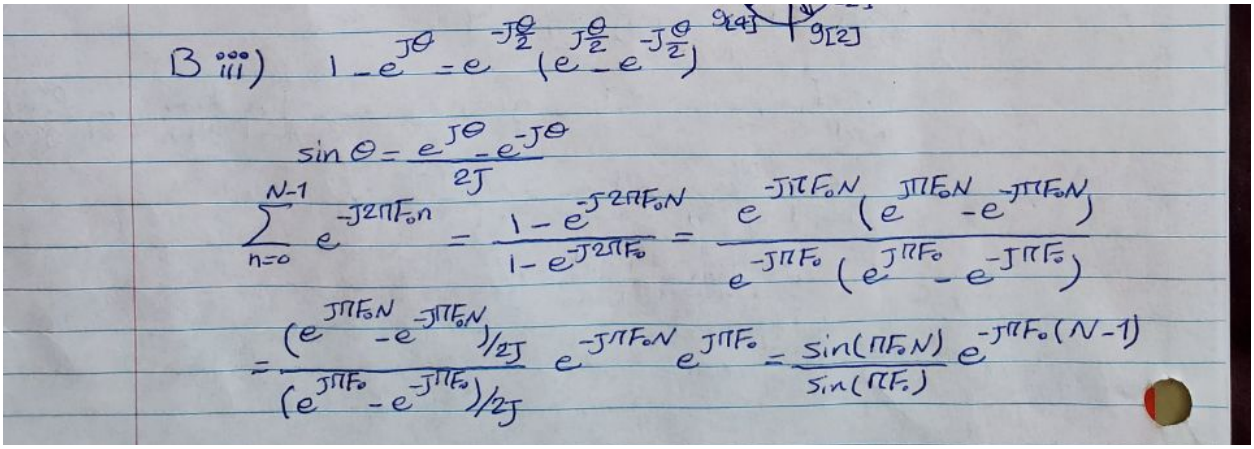


the sum of a dt. complex exponential over one period is equal to zero. because it is a sumation of N points equally spread around the unit circle.

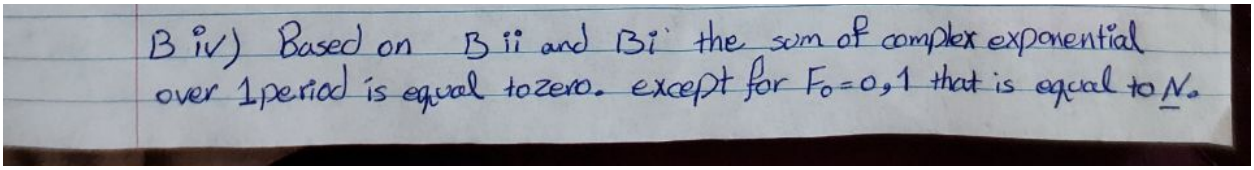


(B ...) $\quad j\theta \quad -j\frac{\theta}{2} \quad j\frac{\theta}{2} \quad -j\frac{\theta}{2}$

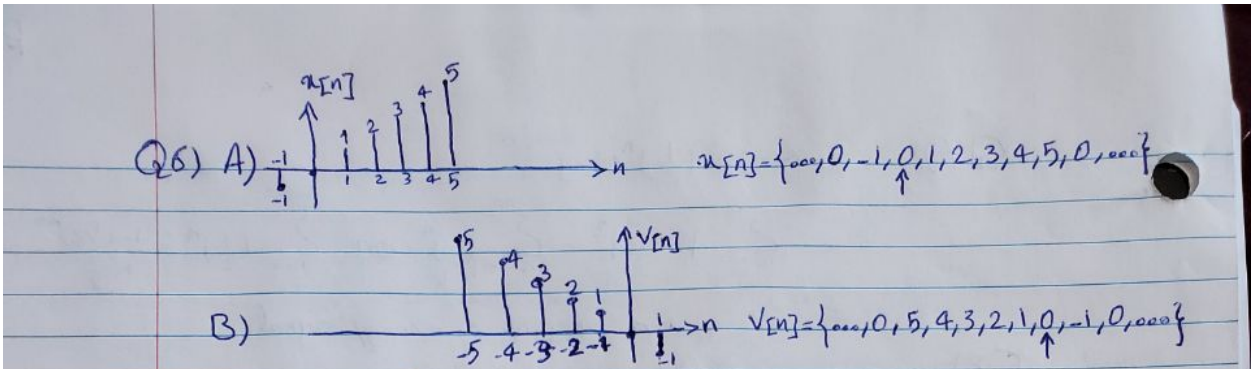
Question 5


B iii) $1 - e^{j\theta} = e^{-j\frac{\theta}{2}}\left(e^{j\frac{\theta}{2}} - e^{-j\frac{\theta}{2}}\right)$
g[4] g[2]
$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$
$\sum_{n=0}^{N-1} e^{-j2\pi F_0 n} = \frac{1 - e^{-j2\pi F_0 N}}{1 - e^{-j2\pi F_0}} = \frac{e^{-j\pi F_0 N}\left(e^{j\pi F_0 N} - e^{-j\pi F_0 N}\right)}{e^{-j\pi F_0}\left(e^{j\pi F_0} - e^{-j\pi F_0}\right)}$
$= \frac{\left(e^{j\pi F_0 N} - e^{-j\pi F_0 N}\right)/2j}{\left(e^{j\pi F_0} - e^{-j\pi F_0}\right)/2j} e^{-j\pi F_0 N} e^{j\pi F_0} = \frac{\sin(\pi F_0 N)}{\sin(\pi F_0)} e^{-j\pi F_0 (N-1)}$

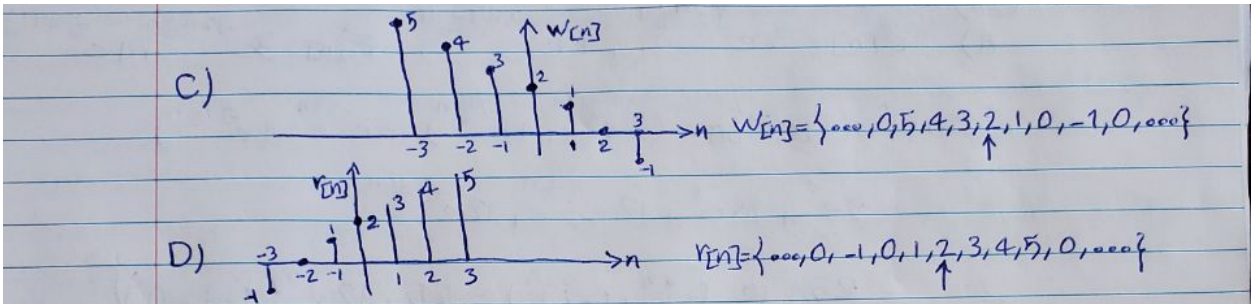
Question 5


B iv) Based on B ii and Bi the sum of complex exponential
over 1period is equal tozero. except for $F_0 = 0, 1$ that is equal to N.

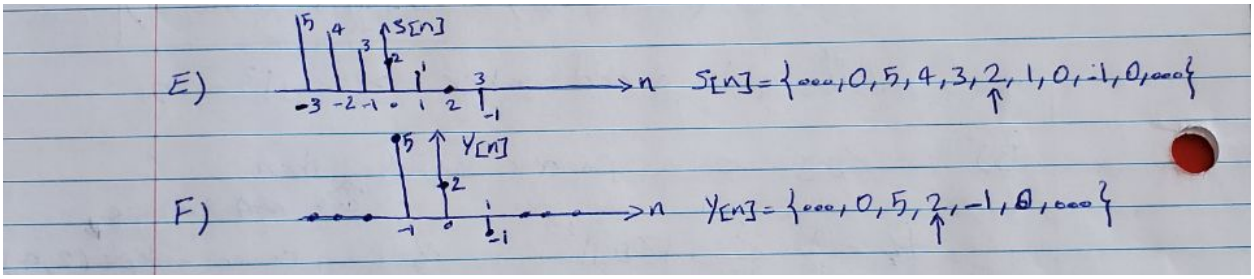
Question 6



Question 6


C)
w[n]
5
4
3
2
1
-3
-2
-1
1
2
3
-1
n
w[n]={...,0,5,4,3,2,1,0,-1,0,...}
D)
r[n]
2
3
4
5
-3
-2
-1
1
2
3
n
r[n]={...,0,-1,0,1,2,3,4,5,0,...}

Question 6



Question 6

g) $w[n] = v[n-2]$

$v[n] = x[-n] \Rightarrow v[n-2] = x[-(n-2)] = x[2-n]$

$\Rightarrow \boxed{w[n] = x[2-n]}$

$s[n] = r[-n]$

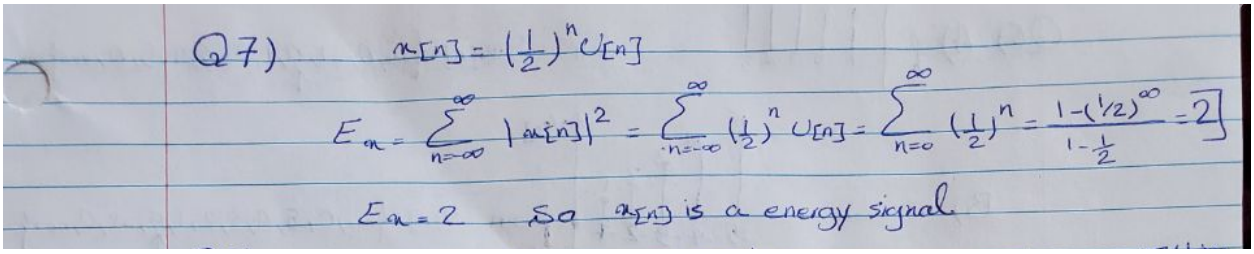
$r[n] = x[n+2] \Rightarrow r[-n] = x[(-n)+2] = x[2-n]$

$\Rightarrow \boxed{s[n] = x[2-n]}$

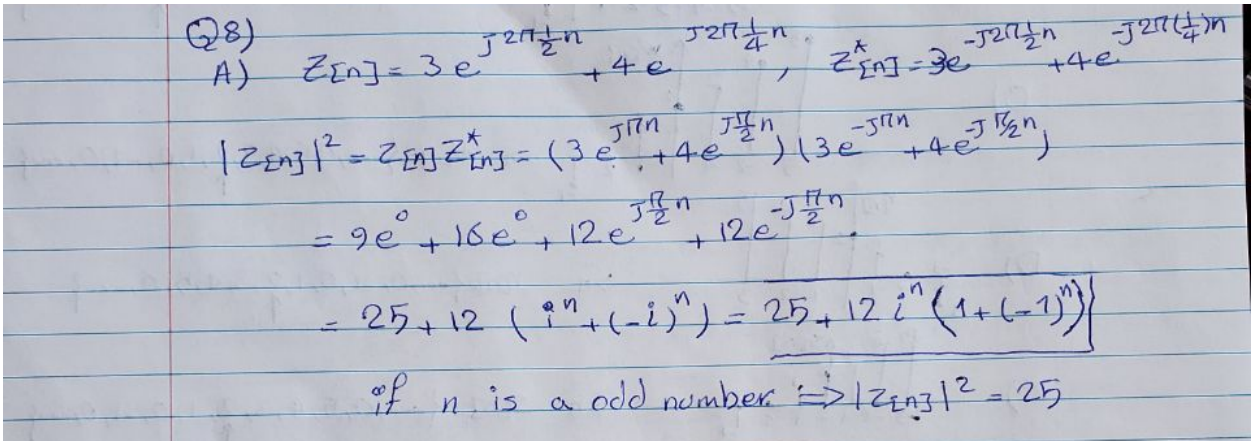
H) Advance $x[n]$ by two samples, then fold the resulting sequence about the 'n=0' axis.

fold $x[n]$ about the 'n=0' axis, then Delay the resulting sequence by two time samples

Question 7


Q7) $x[n] = (\frac{1}{2})^n u[n]$
$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} (\frac{1}{2})^n u[n] = \sum_{n=0}^{\infty} (\frac{1}{2})^n = \frac{1-(1/2)^\infty}{1-\frac{1}{2}} = 2$
$E_x = 2$ so $x[n]$ is a energy signal.

Question 8



Q8)

A) $z[n] = 3e^{j2\pi\frac{1}{2}n} + 4e^{j2\pi\frac{1}{4}n}$, $z^*[n] = 3e^{-j2\pi\frac{1}{2}n} + 4e^{-j2\pi(\frac{1}{4})n}$

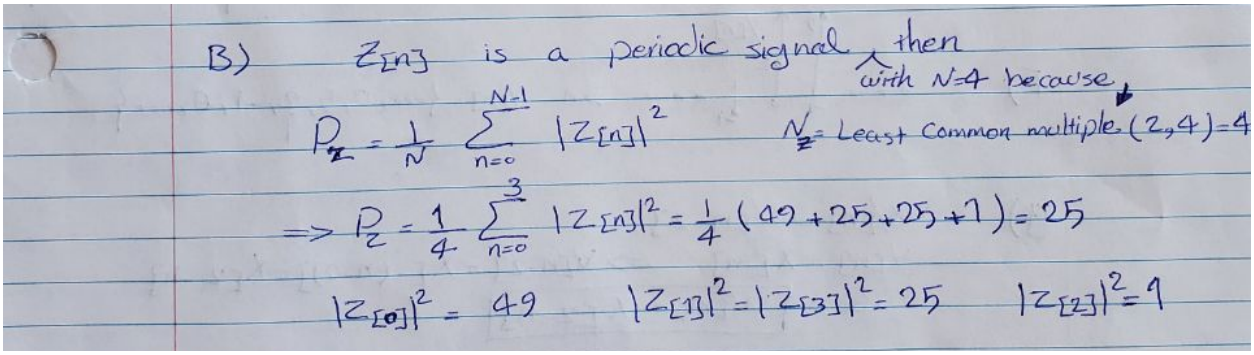
$|z[n]|^2 = z[n]z^*[n] = (3e^{j\pi n} + 4e^{j\frac{\pi}{2}n})(3e^{-j\pi n} + 4e^{-j\frac{\pi}{2}n})$

$= 9e^{0} + 16e^{0} + 12e^{j\frac{\pi}{2}n} + 12e^{-j\frac{\pi}{2}n}$

$= 25 + 12(i^n + (-i)^n) = \underline{25 + 12\, i^n\left(1 + (-1)^n\right)}$

if n is a odd number $\Rightarrow |z[n]|^2 = 25$

Question 8


B) z[n] is a periodic signal, then
with N=4 because
$P_z = \frac{1}{N}\sum_{n=0}^{N-1} |z[n]|^2$
N_z = Least Common multiple (2,4) = 4
$\Rightarrow P_z = \frac{1}{4}\sum_{n=0}^{3} |z[n]|^2 = \frac{1}{4}(49+25+25+1) = 25$
$|z[0]|^2 = 49$ $|z[1]|^2 = |z[3]|^2 = 25$ $|z[2]|^2 = 1$

Question 9