Be sure to read the course information handout before starting this assignment.

1. Let $x[n]$ be a discrete-time sinusoid, as defined below. For each $x[n]$: find the associated equivalent discrete-time frequency, F_e, such that $-0.5 < F_e \leq 0.5$; state whether $x[n]$ is periodic or not; and if periodic, determine the fundamental period. Recall that the fundamental period of a discrete-time sinusoid must be an integer.

(2 pts.) a) $x[n] = \cos(0.01\pi n)$

(2 pts.) b) $x[n] = \cos\left(\pi \frac{30}{105} n\right)$

(2 pts.) c) $x[n] = \cos\left(2\pi \frac{149}{100} n\right)$

(2 pts.) d) $x[n] = \sin(3n)$

(2 pts.) e) $x[n] = \exp\left(j\pi \frac{62}{10} n\right)$

(3 pts.) 2. Which of the signals in question 1 has the highest rate of oscillation? the lowest rate of oscillation? Explain the procedure you used to determine your answers.

3. Consider the following continuous-time sinusoidal signal:

$$x_a(t) = 3\sin(100\pi t) = 3\sin(2\pi 50 t)$$

(2 pts.) a) Sketch the signal $x_a(t)$ for $0 \leq t \leq 30\text{ms}$. Your sketch should accurately convey the time locations of all zero-crossings, peaks, and valleys.

(2 pts.) b) If $x_a(t)$ is sampled at a rate of $f_s = 300$ samples/sec., determine the discrete-time frequency, F, of the resulting discrete-time sequence, $x[n]$. What is the fundamental period of $x[n]$?

(2 pts.) c) Compute the sample values in one period of $x[n]$. Illustrate these samples on the same sketch you drew for part (a).

(2 pts.) d) Find a value for the sampling rate, f_s, of part (b) which would yield a sequence, $x[n]$, whose maximum value is 3.

(1 pts.) e) Find a value for the sampling rate, f_s, of part (b) which would yield a sequence, $x[n]$, that is not periodic.

4. Consider the discrete-time signal: $x[n] = \sin\left(2\pi \frac{1}{5} n\right)$.

Find a continuous-time signal, $x_a(t)$, which yields the sequence $x[n]$, when sampled at a rate of:

(2 pts.) a) $f_s = 10$ samples/second.

(2 pts.) b) $f_s = 125$ samples/second.

5. The following formulae for summing arithmetic and geometric series are useful to know. Knowing how to derive them is also quite useful as it allows you to derive other similar formulae as well as to rederive these formulae when you forget them.

a) Formula 1 (arithmetic series): If $S = \sum_{k=1}^{N} k$, then $S = \frac{N(N+1)}{2}$. This is easily derived by writing the terms in the sum in two different orders as shown below:

$$\left.\begin{array}{lllllllllll} S & = & 1 & + & 2 & + & \ldots & + & N-1 & + & N \\ S & = & N & + & N-1 & + & \ldots & + & 2 & + & 1 \end{array}\right\}\ \text{two ways of ordering the terms in S}$$

$$2S = \underbrace{(N+1) + (N+1) + \ldots + (N+1) + (N+1)}_{N \text{ terms}} \quad \text{result of adding the two equations above}$$

Thus $2S = N(N+1)$ or $S = N(N+1)/2$. End of Derivation.

(1 pts.) *i.* Find $\sum_{k=1}^{1000} k$.

(3 pts.) *ii.* Assume $n_2 > n_1$. Derive a closed-form expression for $\sum_{k=n_1}^{n_2} k$.

b) Formula 2 (geometric series): $\sum_{k=0}^{N-1} x^k = \frac{1-x^N}{1-x}$

To derive this formula. Let $S = \sum_{k=0}^{N-1} x^k$. Then:

$$\begin{array}{llllllll} & S = 1 & +x & + \ldots & +x^{N-2} & +x^{N-1} & \\ \text{and} & xS = & x & + \ldots & +x^{N-2} & +x^{N-1} & +x^N \end{array}$$

Subtracting the above two equation yields $S - xS = 1 - x^N$ which yields $S = \frac{1-x^N}{1-x}$.

(2 pts.) *i.* Use l'Hospital's Rule to evaluate $\lim_{x \to 1}\left(\frac{1-x^N}{1-x}\right)$. Show your work. As a check, recall that your result should agree with the $\lim_{x \to 1} \sum_{n=0}^{N-1} x^n$.

(4 pts.) *ii.* Let $g[k] = \sum_{n=0}^{4} e^{-j2\pi\frac{k}{5}n}$. Without using a calculator or Matlab, evaluate $g[k]$ for integer k. Show your work and provide justification for your answers. Make a stem plot of $g[k]$ vs. k, $k = 0, 1, 2, \ldots 10$.

(4 pts.) *iii.* Show that: $\sum_{n=0}^{N-1} e^{-j2\pi F_0 n} = \frac{\sin(\pi F_0 N)}{\sin(\pi F_0)} e^{-j\pi F_0(N-1)}$. Provide details.

Hint: $1 - e^{-j\theta}$ can be written as $e^{-j\theta/2}\left(e^{j\theta/2} - e^{-j\theta/2}\right)$ and $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$.

(2 pts.) *iv.* Determine all values of F_0 in the interval $0 \le F_0 \le 1$, for which the sum of part (*iii*) is equal to zero? Show your work. Provide justification.

6. Let $x[n]$ be defined as shown to the right: $x[n] = \begin{cases} n, & -1 \le n \le 5 \\ 0, & \text{otherwise} \end{cases}$
then define: $v[n] \equiv x[-n]$, $w[n] \equiv v[n-2]$,
$r[n] \equiv x[n+2]$, $s[n] = r[-n]$, and $y[n] = s[3n]$.

(2 pts.) a) Make a stem plot of $x[n]$. Also provide the sequence representation of $x[n]$.

(2 pts.) b) Make a stem plot of $v[n]$. Also provide the sequence representation of $v[n]$.

(2 pts.) c) Make a stem plot of $w[n]$. Also provide the sequence representation of $w[n]$.

(2 pts.) d) Make a stem plot of $r[n]$. Also provide the sequence representation of $r[n]$.

(2 pts.) e) Make a stem plot of $s[n]$. Also provide the sequence representation of $s[n]$.

(2 pts.) f) Make a stem plot of $y[n]$. Also provide the sequence representation of $y[n]$.

(3 pts.) g) Express the signals $w[n]$ and $s[n]$ in terms of the signal $x[\cdot]$.

(2 pts.) h) Rewrite the following sentences substituting the appropriate word (***advance*** or ***delay***) in place of the word ***shift***.

A plot of $x[2-n] = x\big[-(n-2)\big]$ can be obtained from a plot of $x[n]$ using either of the following two procedures:
1) ***shift*** $x[n]$ by two time samples, then fold the resulting sequence about the "n=0" axis.
2) fold $x[n]$about the "n=0" axis, then ***shift*** the resulting sequence by two time samples.

(3 pts.) 7. Find the energy, E_x , of the signal $x[n] = \left(\frac{1}{2}\right)^n u[n]$, where $u[n]$ is the unit step sequence.

8. Let $z[n] = 3e^{j2\pi\left(\frac{1}{2}\right)n} + 4e^{j2\pi\left(\frac{1}{4}\right)n}$.

(3 pts.) a) Find $|z[n]|^2$. Show your work. ***Hint:*** $|z[n]|^2 = z[n]\,z^*[n]$.
Squared magnitudes are always real and nonnegative.
Does that describe your solution?

(3 pts.) b) Find P_z , the average power of the sequence $z[n]$.
Hint: $z[n]$ is periodic. What is its period?

9. Discrete-time signals can always be expressed as a sum of Kronecker delta functions.

(2 pts.) a) The sequence representation of the d.t. signal, $v[n]$, is shown to the right. $v[n] = \{\ldots,\ 0,\ 5,\ \underset{\uparrow}{3},\ 1,\ 0,\ \ldots\}$
Express the signal $v[n]$ as a sum of Kronecker delta functions.

(2 pts.) b) Let $y[n] = \sum_{k=-1}^{3} (k+2)\delta[n-k]$. Provide the sequence representation of $y[n]$.