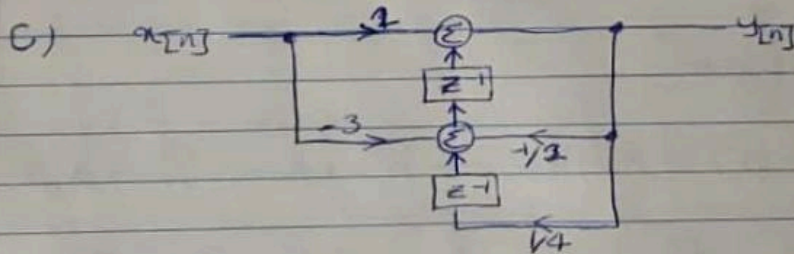
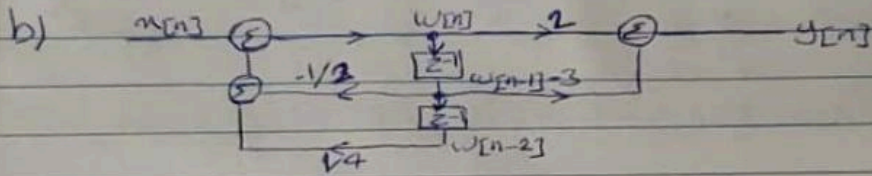
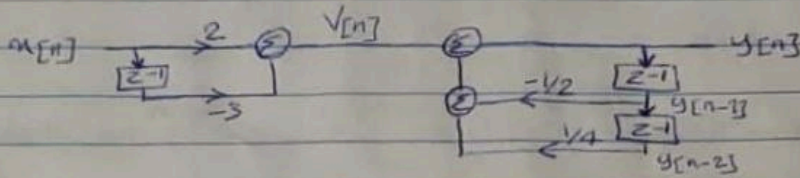
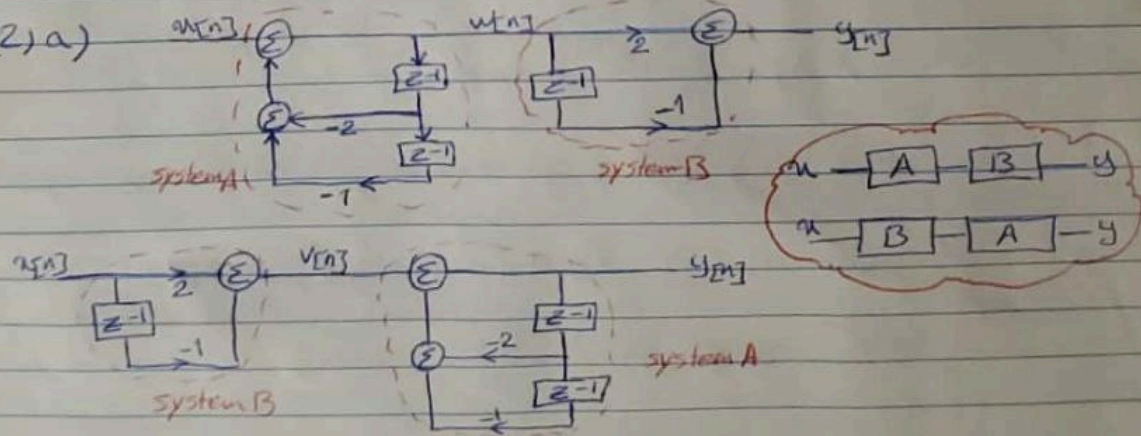


Q1-a

$$y[n] = 2x[n] - 3x[n-1] - \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2], \quad v[n] = 2x[n] - 3x[n-1]$$



Q2) a)



$$\Rightarrow V[n] = 2x[n] - x[n-1], \quad y_2[n] = V[n] - 2y_1[n-1] - y_1[n-2]$$

$$\Rightarrow y_2[n] = 2x[n] - x[n-1] - 2y_1[n-1] - y_1[n-2]$$

b) from different eqn: (in zero-input response)

$$y_{z1}[n] = -2y_{z1}[n-1] - y_{z1}[n-2] \Rightarrow y_{z1}[-1] = y_{z1}[0] = 1 \quad y_{z1}[-2] = y_{z1}[-1] = 2$$

$$n=0 \Rightarrow y_{zi}[0] = -2y_{zi}^{1}[1] - y_{zi}^{2}[2] = -2 - 2 = -4$$

$$n=1 \Rightarrow y_{zi}[1] = -2y_{zi}^{4}[0] - y_{zi}^{1}[1] = +8$$

from structures  $y_{zi}[n] = 2w[n] - w[n-1]$

$$y_{zi}[0] = 2w[0] - w[-1] \quad (1), \quad y_{zi}[1] = 2w[1] - w[0] \quad (2)$$

Also from structure  $w[n] = \overset{\text{since } z_i}{x[n]} - 2w[n-1] - w[n-2]$

$$n=0 \Rightarrow w[0] = -2w[-1] - w[-2] \quad (3)$$

$$n=1 \Rightarrow w[1] = 2w[0] - w[-1] = +4w[-1] + 2w[-2] - w[-1]$$

$$w[1] = 3w[-1] + 2w[-2] \quad (4)$$

(3, 4) and (1, 2) yields:

$$y_{zi}[0] = -4w[-1] - 2w[-2] - w[-1] = -5w[-1] - 2w[-2]$$

$$y_{zi}[1] = 8w[-1] + 4w[-2] + 2w[-1] + w[-2] = 8w[-1] + 5w[-2]$$

$$\Rightarrow -5w[-1] - 2w[-2] = -4$$

$$8w[-1] + 5w[-2] = 8 \Rightarrow$$

$$w[-1] = \frac{28}{3} = \frac{28}{3}, \quad w[-2] = -\frac{1}{3}$$



Q3-a)

$$y[n] = 2x[n] + z_1[n] - 1$$

$$z_1[n] = y[n] - x[n] + z_2[n] - 1$$

$$z_2[n] = -\frac{1}{2}y[n] \Rightarrow z_2[n-1] = -\frac{1}{2}y[n-1]$$

$$z_1[n] = y[n] - x[n] + \frac{1}{2}y[n-1]$$

$$\Rightarrow z_1[n-1] = y[n-1] - x[n-1] + \frac{1}{2}y[n-2]$$

$$\Rightarrow y[n] = 2x[n] - x[n-1] + y[n-1] - \frac{1}{2}y[n-2]$$

b) from eqns  $y_{zi}[0] = y[-1] - \frac{1}{2}y_{zi}[-2] = 2 - \frac{1}{2}(-6) = 5$

$$y_{zi}[1] = y_{zi}[0] - \frac{1}{2}y_{zi}[-1] = 5 - \frac{1}{2}(2) = 4$$

from structure:  $y_{zi}[n] = z_1[n-1] + 2x[n]$

$$n=0 \Rightarrow y_{zi}[0] = z_1[-1]$$

$$n=1 \quad y_{zi}[1] = z_1[0]$$

$$z_1[0] = y_{zi}[0] + z_2[-1]$$

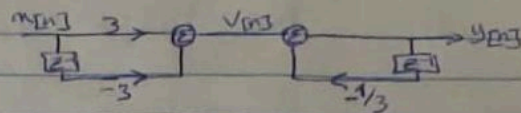
$$\Rightarrow y_{zi}[1] = z_1[0] + z_2[-1]$$

$$\Rightarrow z_1[-1] = 5, \quad z_1[-1] + z_2[-1] = 4$$

$$\Rightarrow z_2[-1] = -1$$

Q4-a)

like question 2:



$$\Rightarrow y[n] = V[n] - \frac{1}{3} y[n-1] \quad V[n] = 3x[n] - 3x[n-1]$$

$$\Rightarrow y[n] = 3x[n] - 3x[n-1] - \frac{1}{3} y[n-1]$$

b)  $y[n] = 3w[n] - 3w[n-1]$

$$w[n] = x[n] - \frac{1}{3} w[n-1]$$

c)  $x[n] = \delta[n] \quad w[-1] = 0$

$$\Rightarrow n=0 \Rightarrow w[0] = \delta[0] - \frac{1}{3} w[-1] = 1$$

$$\Rightarrow n=1 \Rightarrow w[1] = \delta[1] - \frac{1}{3} w[0] = -\frac{1}{3}$$

$$n=2 \Rightarrow w[2] = -\frac{1}{3} w[1] = \frac{1}{9}$$

$$\Rightarrow w[n] = \left(-\frac{1}{3}\right)^n \quad n \geq 0$$

d)  $y[n] = 3w[n] - 3w[n-1]$

$$n=0 \Rightarrow y[0] = 3w[0] - 3w[-1] = 3$$

$$n=1 \Rightarrow y[1] = 3w[1] - 3w[0] = -1$$

$$n=2 \Rightarrow y[2] = 3w[2] - 3w[1] = \frac{4}{3}$$

$$w[n] = \delta[n]$$

$$\Rightarrow y_h[n] = 3\delta[n] - 3\delta[n-1]$$

$$= \left\{ \underset{\uparrow}{-3}, -3, 0 \right\}$$

e)  $h[n] = w[n] * y[n]$

the impulse  
response of first  
system

the impulse  
response of the  
second system

$$\Rightarrow h[n] = (3\delta[n] - 3\delta[n-1]) * (-\frac{1}{3})^n u[n]$$

$$= 3(-\frac{1}{3})^n - 3(-\frac{1}{3})^{n-1} = \cancel{3(-\frac{1}{3})^n} - \cancel{3(-\frac{1}{3})^{n-1}} + (-\frac{1}{3})^{n-2}$$

$$= 3(-\frac{1}{3})^n u[n] - 3(-\frac{1}{3})^{n-1} u[n-1]$$

$$h[n] = \begin{cases} 0 & n < 0 \\ 3 & n = 0 \\ 12(-\frac{1}{3})^n & n \geq 1 \end{cases}$$

f)  $x[n] = \delta[n]$   $y[n] = h[n]$   $h[-1] = 0$

$$\Rightarrow h[n] = 3\delta[n] - 3\delta[n-1] - \frac{1}{3}h[n-1]$$

$$n=0 \Rightarrow h[0] = 3 \checkmark$$

$$n=2 \Rightarrow h[2] = -\frac{1}{3}(-4) = \frac{4}{3} \checkmark$$

$$n=1 \Rightarrow h[1] = -3 - \frac{1}{3}(3) = -4 \checkmark$$



Q5-a)  $x[n] = 2\delta[n+1] + 3\delta[n] + 4\delta[n-1]$  since  $x$  is real signal

$$r_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n] x^*[n-l] = \sum_{n=-\infty}^{\infty} x[n] x[n-l]$$

$$= \sum_n (2\delta[n+1] + 3\delta[n] + 4\delta[n-1])(2\delta[n-l+1] + 3\delta[n-l] + 4\delta[n-l-1])$$

$$= 4\delta[-l] + 6\delta[-1-l] + 8\delta[-2-l] + 6\delta[-l] + 9\delta[-l] + 12\delta[-1-l] \\ + 8\delta[-2-l] + 12\delta[-1-l] + 16\delta[-l]$$

$$= 29\delta[-l] + 18\delta[-1-l] + 8\delta[-2-l] + 18\delta[-1-l] + 8\delta[-2-l]$$

$$r_{xx}[l] = \{0, 0, 0, 0, 8, 18, 29, 18, 8, 0, \dots\}$$

25-b)  $y[n] = 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]$

$y$  is real  $\uparrow$

$$r_{yy}[l] = \sum_{n=-\infty}^{\infty} y[n] y[n-l] = \sum_n [2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3]] [2\delta[n-l-1] + 3\delta[n-l-2] + 4\delta[n-l-3]]$$

$$= 4\delta[l] + 6\delta[l-1] + 8\delta[l-2] + 6\delta[l-1] + 9\delta[l] + 12\delta[l-1] + 8\delta[l-2] + 12\delta[l-1] + 16\delta[l]$$

$$= 29\delta[l] + 18\delta[l-1] + 8\delta[l-2] + 18\delta[l-1] + 8\delta[l-2]$$

$$= \{ \dots, 0, 8, 18, 29, 18, 8, 0, \dots \}$$

c)  $z[n] = 4\delta[n+1] + 3\delta[n] + 2\delta[n-1]$

$z$  is real  $\leftarrow$

$$r_{zz}[l] = \sum_{n=-\infty}^{\infty} z[n] z[n-l] = \sum_{n=-\infty}^{\infty} (4\delta[n+1] + 3\delta[n] + 2\delta[n-1]) (4\delta[n+l+1] + 3\delta[n+l] + 2\delta[n+l-1])$$

$$= 16\delta[l] + 12\delta[l-1] + 8\delta[l-2] + 12\delta[l-1] + 9\delta[l] + 6\delta[l-1] + 8\delta[l-2] + 6\delta[l-1] + 4\delta[l]$$

$$= 29\delta[l] + 18\delta[l-1] + 8\delta[l-2] + 18\delta[l-1] + 8\delta[l-2]$$

$$= \{ \dots, 0, 8, 18, 29, 18, 8, 0, \dots \}$$

d) The autocorrelation of these real signals were equal then shifting and reversing did not affect the autocorrelation

$y$  is real  $\leftarrow$

$$y[n] = x[n-2] \Rightarrow r_{yy}[l] = \sum_n y[n] y[n-l] = \sum_{n=-\infty}^{\infty} x[n-2] x[n-2-l]$$

$$n' = n-2 \Rightarrow \sum_{n'=-\infty}^{\infty} x[n'] x[n'-l] = r_{xx}[l] = r_{yy}[l]$$

$$z[n] = x[-n] \Rightarrow r_{zz}[l] = \sum_{n=-\infty}^{\infty} z[n] z[n-l] = \sum_{n=-\infty}^{\infty} x[-n] x[-n-l]$$

$$n' = -n \Rightarrow \sum_{n'=-\infty}^{\infty} x[n'] x[n'-l] = r_{xx}[l] = r_{zz}[l]$$



$$Q6 \quad x[n] = \delta[n] + \delta[n-1] + \delta[n-2] \Rightarrow X[z] = z^0 + z^{-1} + z^{-2}$$

$$y[n] = \delta[n-2] + \delta[n-3] + \delta[n-4]$$

$$\bar{y}[n] = y[n] = \delta[n-2] + \delta[n-3] + \delta[n-4] \Rightarrow \bar{Y}[z] = z^{-2} + z^{-3} + z^{-4}$$

$$r_{xy}[l] = x[l] * y^*[l] \quad \text{y is real} \quad = x[l] * y[l] = x[l] * \bar{y}[l]$$

$$R_{xy}[z] = X[z] \bar{Y}[z] = z^4 + 2z^3 + 3z^2 + 2z^1 + z^0$$

$$\Rightarrow r_{xy}[l] = \{0, 1, 2, 3, 2, 1, 0, \dots\}$$

for  $l = -2$   $r_{xy}$  has the maximum value

$$y[n-(+2)] = y[n+2] = \{\dots, 0, 1, 1, 1, 0, 0, \dots\} = x[n]$$

Q27

$$x[n] = 4 \sin\left(2\pi \frac{1}{5}n + \frac{\pi}{4}\right)$$

*4 is a real number*

$$r_{xx}[l] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] x[n-l] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] x[n-l]$$

$$N = 5 \Rightarrow \frac{1}{5} \sum_{n=0}^4 4 \sin\left(2\pi \frac{1}{5}n + \frac{\pi}{4}\right) 4 \sin\left(2\pi \frac{1}{5}(n-l) + \frac{\pi}{4}\right)$$

$$\Rightarrow \sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\Rightarrow \frac{4 \times 4}{5 \times 2} \sum_{n=0}^4 \cos\left(\frac{2\pi}{5}l\right) - \cos\left(\frac{2\pi}{5}2n + \frac{\pi}{2} - \frac{2\pi}{5}l\right)$$

*it is periodic and summation of one period is zero*

$$r_{xx}[l] = \frac{4 \times 4}{5 \times 2} \cos\left(\frac{2\pi}{5}l\right) \sum_{n=0}^4 1 = \frac{32}{5} \cos\left(\frac{2\pi}{5}l\right)$$