1. ***Evaluate the Z-transform sum*** to find the ***Z transforms*** and **ROC**s of the following sequences:

( 3 pts.) a) $d[n] = 2\delta[n+1] + 3\delta[n] + 4\delta[n-1]$

( 3 pts.) b) $q[n] = \left(\frac{1}{4}\right)^n u[n]$

( 3 pts.) c) $s[n] = \left(\frac{1}{4}\right)^n u[-n]$

( 3 pts.) d) $x[n] = (-1)^n u[n]$.

( 3 pts.) e) $y[n] = \left\{\ldots, 0, 0, 0, \underset{\uparrow}{1}, 0, 1, 0, \overline{1, 0}, \ldots\right\}$.

hint: note that for $n \geq 0$: $y[n] = \begin{cases} 1, & n = 2m, \quad m = 0, 1, 2, \ldots \\ 0, & n = (2m+1), \quad m = 0, 1, 2, \ldots \end{cases}$

2. In class, we showed that the $Z$ transform of a unit step function $u[n]$ is given by:

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1$$

( 3 pts.) a) Use the *muliplication-by-$a^n$* property of the Z transform together with $U(z)$ to find the Z-transform of the sequence: $x_1[n] = (1/2)^n u(n)$. ***Don't forget*** to specify the resulting ROC.

( 3 pts.) b) Use your result from part (a) together with the *time-shifting* and *linearity* properties of the $Z$-transform to find the $Z$-transform of the sequence: $x_2[n] = (1/2)^n u[n-5]$. ***Don't forget*** the ROC! (***Hint***: note that $x_2[n] = Kx_1[n-5]$, what is the value of $K$?)

( 3 pts.) c) Use your results from parts (a) and (b) together with the *linearity* property of the $Z$ transform to find the $Z$ transform of the sequence: $x_3[n] = (1/2)^n (u[n] - u[n-5])$. ***Don't forget*** to specify the ROC!

( 3 pts.) d) Note that the sequence $x_3[n] = (1/2)^n (u[n] - u[n-5])$ is as shown below:

$$x_3[n] = \left\{\ldots, 0, \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, 0, 0, \ldots\right\}$$

From this representation of the signal, it is clear that:

$$X_3(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-3} + \frac{1}{16}z^{-4}, \quad |z| > 0$$

In part (c), you should have found an expression for $X_3(z)$ as a ratio of two polynomials (*i.e.*, $X_3(z) = N(z)/D(z)$). Show that the expression you found for $X_3(z)$ in part (c) is equivalent to the one above. Do this by manipulating one expression to find the other. ***Hint***: If starting from the expression in (c), you may want to use long division to show that the denominator polynomial is a factor of the numerator polynomial; if starting from the expression presented in this part, you may want to multiply the expression by 1 (with 1 expressed as $D(z)/D(z)$ where $D(z)$ is the denominator of the expression found in part (c)) to show that you end up with the expression of part (c).

( 3 pts.) e) Use the time-reversal property of the $Z$ transform to find the $Z$ transform of the sequence: $x_4[n] = (1/2)^{-n} u[-n]$. ***Be sure to make it clear how you used the time-reversal property (which Z transform did you start with) to find both $X_4(z)$ and its ROC.***

3. Find the Z transforms and ROC's of the following sequences, identify all finite-valued poles and zeros of the Z transform, and sketch the corresponding pole-zero plots being sure to indicate the ROC on your pole-zero plot. After making the pole-zero plot, think about the pole locations, their multiplicites, and how they relate back to the signal. In order to identify the finite-valued poles and zeros, it is best to express your Z transform in factored form using positive powers of z. Feel free to leave your Z transform in this form.
Note: in deriving the requested Z transforms, you may use the following Z transform pair:

$$a^n u[n] \quad \longleftrightarrow \quad \frac{1}{1 - az^{-1}} \quad \text{with ROC: } |z| > |a|,$$

where $a$ is any real or complex-valued constant, in conjunction with any of the Z transform properties discussed in class (*i.e.,* linearity, time-shifting, time-reversal, multiplication-by-$a^n$ in the time domain or scaling in the $z$ domain, mulitplication-by-$n$ in the time domain or differentiation in the $z$ domain, convolution-in-time or multiplication in the $z$ domain). You **may not** assume knowledge of any other Z transform pair unless you have derived it somewhere else in this assignment.

(6 pts.) a) $x[n] = n(-1)^n u[n]$

( 8 pts.) b) $x[n] = n^2 u[n]$

( 10 pts.) c) $x[n] = n(.9)^n \sin(\pi n/3) u[n]$.
***Hint***: How you approach this problem can make a big difference into how messy it gets. In general it is best to apply the multiplication-by-$n$ property early on in the process. I would also recommend using Euler's formulae to re-write the $\sin(\pi n/3)$ as a sum of two complex exponentials. Be sure to simplify your result.

( 8 pts.) d) $x[n] = (1/2)^n (u[n] - u[n-3])$