Stevenson
Feb. 25, 2021

ECE 4531 Midterm

NAME ______________________________

The value of each question is indicated in the margin. The points total to 50. It's important to pace yourself and to not spend too much time on any one question.
Apart from the formula sheet provided, you should not be consulting any other resource while completing the midterm. The use of a calculator or any other computing device is not allowed.

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1. Let the sequence $h[n]$ be as shown to the right.

$$\boxed{h[n] = \{\underset{\uparrow}{0},\ 3,\ -4,\ 2,\ 0,\ 0\}}$$

(4 pts.) a) Find $r_{hh}[\ell]$, the autocorrelation function of $h[n]$. ***Show your work.*** Express your answer by providing a sequence representation of $r_{hh}[\ell]$.

(2 pts.) b) Let E_h denote the energy of the sequence $h[n]$. Determine the value of E_h. ***Show your work.***

(2 pts.) c) Let $q[n] = h[1 - n]$. Provide a sequence representation of $q[n]$.

$$q[n] = \{\ldots,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \underset{\uparrow}{\_\_},\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \ldots\}$$

(3 pts.) d) Find the input-output equation of an LTI system whose impulse reponse if given by the sequence $h[n]$ shown above. Recall that an input-output equation relates the the system's output, $y[n]$, to the system's input, $x[n]$. Your input-output equation must not refer to any signals other than $y[\cdot]$ and $x[\cdot]$. ***Indicate your procedure.***

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2. Consider an LTI system, whose input, $x[n]$, and output, $y[n]$, satisfy the following LCCDE:

$$y[n] + y[n-1] = (3/4)x[n]$$

(2 pts.) a) Is this system BIBO stable? How do you know?

(3 pts.) b) If your answer to part (a) was "yes", please determine the smallest value of B for which the following statement will always be true:

if $|x[n]| \leq 1$ for all n then $|y[n]| \leq B$ for all n.

If your answer to part (a) was "no", please provide an example of a bounded input $x[n]$, whose response, $y[n]$, will not be bounded.

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(8 pts.) 3. The homogeneous difference equation of a second-order LTI system may be expressed as:

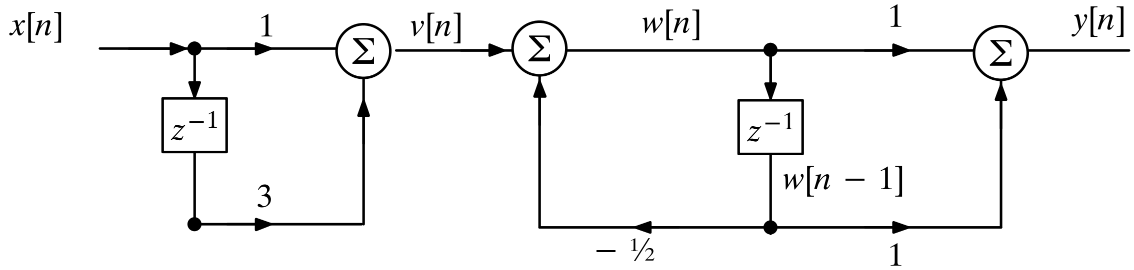
$$y_h[n] + a_1 y_h[n-1] + a_2 y_h[n-2] = 0$$

Given that $y_h[n] = 6(0.5)^n \cos\left(2\pi \frac{1}{8} n + \frac{\pi}{3}\right)$ is a solution to the homogeneous difference equation above, **find the values of the constants a_1 and a_2.**

You must show your work; it must not rely on any memorized formulae.

Hint: what are the characteristic roots, λ_1 and λ_2, of a system with the specified homogenous solution? To answer this question, you may want to use Euler's formula to rewrite $y_h[n]$ as a sum of decaying complex exponentials from which you should be able to identify λ_1 and λ_2, How can you use λ_1 and λ_2 to find a_1 and a_2?

(8 pts.) 4. Consider the system implemented below:



Find the difference equation that relates the system output $y[n]$ to the system input $x[n]$. Although your derivation should refer to intermediate signals, your final difference equation should refer only to the input and output signals, $x[\cdot]$ and $y[\cdot]$.

Please note: ***You must provide details to show how you derived the LCCDE starting from equations that follow directly from*** **individual summers** in the block diagram above or from the individual summers in a manipulated version of the block diagram above. ***If using a manipulated version of the block diagram above, you*** **must justify the equivalence** ***of the manipulated and original block diagrams***; you should also provide labels for intermediate signals of the manipulated block diagram to be used in your derivation.

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5. Consider the LTI system whose input-output relation is described by the following LCCDE:

$$y[n] + 0y[n-1] - \frac{1}{4}y[n-2] = 4x[n] - 10x[n-1]$$

(2 pts.) a) Let λ_1 and λ_2 denote the characteristic roots of the system. Determine values for λ_1 and λ_2. ***Double check your answer before proceeding; you will end up losing more than 2 pts if your answer to this part is incorrect.***

(13 pts.) b) Let $y_s[n]$ denote the system's step response. Find a closed-form expression for $y_s[n]$. ***Work carefully! Procedural errors are costly … and skipping steps often results in costly careless mistakes. Points may also be deducted if your work is poorly organized or difficult to follow.***

Additional space is provided on the next page.

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Use this as additional space for question 5(b).

(3 pts.) c) **As a check of your solution found in part (b)**, evaluate your closed-form expression for $y_s[n]$ to find $y_s[0]$, $y_s[1]$, $y_s[2]$, and $y_s[3]$. Then iterate your difference equation to find the same values. ***Show your work.*** Do the two approaches yield the same values?