Assignment 5

1. Consider the LTI system described by the following LCCDE:

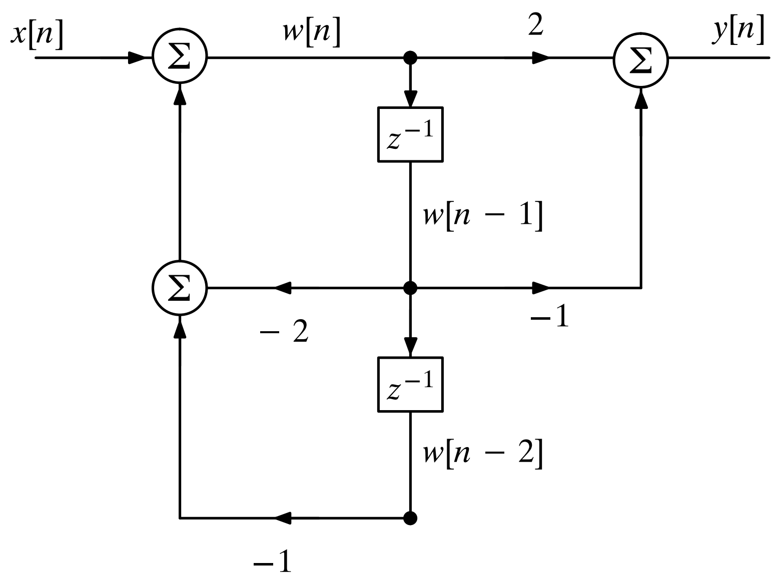
$$y[n] + \frac{1}{2}y[n-1] - \frac{1}{4}y[n-2] = 2x[n] - 3x[n-1]$$

(3 pts.) a) Draw the direct form I structure for implementation of the system described by the LCCDE above.

(3 pts.) b) Draw the direct form II structure for implementation of the system described by the LCCDE above.

(3 pts.) c) Draw the transposed direct form II structure for implementation of the system described by the LCCDE above.

2. Consider the implementation structure shown below.



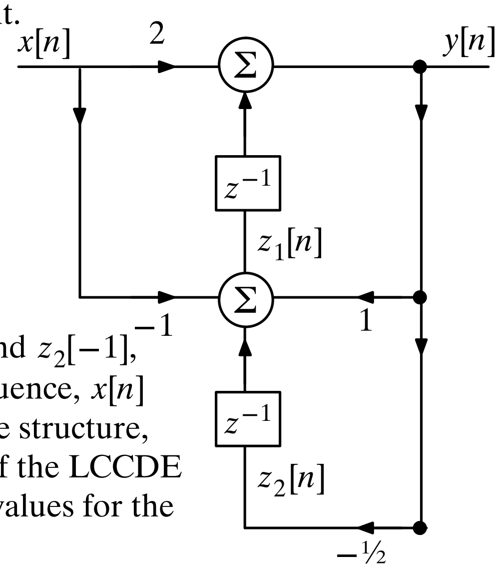
(3 pts.) a) Find an LCCDE that describes the input-output relationship of the system. No terms involving $w[n]$ should appear in your LCCDE. ***Show your work*** (see note below). Note: Rather than simply writing the LCCDE, you must provide details to show how you derived the LCCDE starting from equations that follow directly from the block diagram OR you may explain/illustrate how you manipulated the block diagram (being sure to provide adequate justification for your manipulation) and then provide details to show how you derived the LCCDE starting from equations that follow directly from your manipulated block diagram.

(7 pts.) b) Given the following equivalent initial conditions for the LCCDE of part (a):

$y[-1] = 1$
$y[-2] = 2$

Find values for the initial states, $w[-1]$ and $w[-2]$, in the structure above, such that the structure's output for $n \geq 0$ will agree with the solution of the LCCDE when the IC's are as specified above. ***Show your work.***

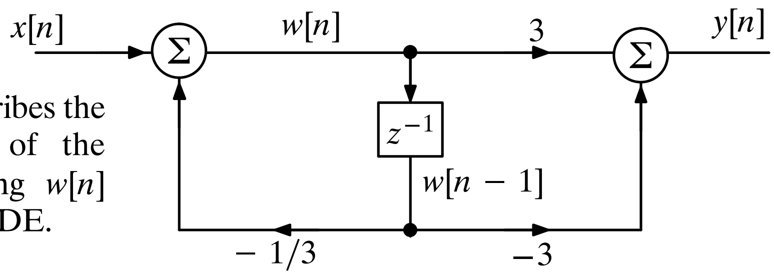
3. Consider the implementation structure shown to the right.



(2 pts.) a) Find an LCCDE that describes the input-output relationship of the system. No terms involving $z_1[n]$ or $z_2[n]$ should appear in your LCCDE. ***Show your work.***

(4 pts.) b) Given the following initial conditions for the LCCDE of part (a):
$y[-1] = 2$ and $y[-2] = -6$
Find equivalent values for the initial states, $z_1[-1]$ and $z_2[-1]$, of the structure above, such that when any input sequence, $x[n]$ is applied to the structure for $n \geq 0$, the output of the structure, $y[n]$, $n \geq 0$, will be in accordance with the solution of the LCCDE for the specified input $x[n]$, $n \geq 0$ and the specified values for the IC's ($y[-1]$ and $y[-2]$). ***Show your work.***

4. Consider the discrete-time system shown to the right:



(1 pt.) a) State the LCCDE that describes the input-output relationship of the system. No terms involving $w[n]$ should appear in your LCCDE.

(2 pts.) b) Find an LCCDE that relates the sequence $w[n]$ to the sequence $x[n]$.

(4 pts.) c) Find a closed-form expression for $w[n]$, $n \geq 0$, when $x[n] = \delta[n]$ and $w[-1] = 0$.

(1 pt.) d) State the LCCDE that relates the sequence $y[n]$ to the sequence $w[n]$.

(4 pts.) e) Use your answers to parts (c) and (d) to find a closed-form expression for the impulse response, $h[n]$, of the system illustrated above. ***Evaluate your closed-form expression to find values for*** $h[0]$ ***and*** $h[1]$.

(3 pts.) f) Check your solution to part (e) by iterating the difference equation found in part (a), with $y[-1] = h[-1] = 0$ to find values for $h[0]$ and $h[1]$. Confirm that iteration of the difference equation yields the same values as your closed-form expression of part (e).

Note: The procedure, described in Handout 9, for finding a closed-form expression for the response of a recursive system is only applicable to systems with $M < N$ and is thus not applicable to the system above. As discussed in class, a closed-form expression for the response of systems with $M \geq N \geq 1$ can always be found by decomposing the overall system into a serial connection of two systems: the first being a recursive system of order N with $M = 0$ (part (b) of this problem) and the second being a nonrecursive FIR system with M memory elements (part (d) of this problem). You can find a closed-form expression for the response of the first system (the recursive one) using the technique on Handout 9 (part (c) of this problem) and then use this expression together with the LCCDE for the FIR system to find a closed-form solution for the overall system (part (e) above).

5. Determine the autocorrelation functions of the energy signals below. **Show your work**.

(2 pts.) a) $x[n] = \{..., 0,\ 2,\ \underset{\uparrow}{3},\ 4,\ 0,\ 0, ...\}$

(2 pts.) b) $y[n] = \{..., \underset{\uparrow}{0},\ 2,\ 3,\ 4,\ 0,\ 0, ...\}$

(2 pts.) c) $z[n] = \{..., 0,\ 0,\ 4,\ \underset{\uparrow}{3},\ 2,\ 0, ...\}$

(6 pts.) d) Note that $y[n] = x[n-2]$ and $z[n] = x[-n]$. In general, how does a time-shift or time-reversal affect the autocorrelation function of a real-valued signal? (to answer this question, you should determine, mathematically, the relationship between $r_{yy}[n]$ and $r_{xx}[n]$ given that $y[n] = x[n-n_0]$; the phrase "in general" implies that your stated relationship should hold regardless of whether $x[n]$ is the signal above or some other energy signal; similarly you will need to determine the relationship between $r_{zz}[n]$ and $r_{xx}[n]$ given that $z[n] = x[-n]$). ***Justify your response for the general case***. (note: your response will be justified for the general case provided you leave $x[n]$ as an arbitrary signal, as opposed to the signal defined above, when determining the requested relationships. Parts (a)−(c) can be used as a check for your findings.)
Hint: How do you express autocorrelation in terms of convolution? Note also that $x[n-2]$ can be expressed as the convolution of $x[n]$ with the impulse response of a system that delays its input by two samples.

(3 pts.) 6. Determine the cross-correlation sequence $r_{xy}[\ell]$ if:

$x[n] = \{..., 0,\ \underset{\uparrow}{1},\ 1,\ 1,\ 0,\ 0,\ 0,\ 0, ...\}$ and $y[n] = \{..., 0,\ 0,\ 0,\ \underset{\uparrow}{0},\ 0,\ 1,\ 1,\ 1,\ 0, ...\}$

At which value of ℓ does the maximum value of $r_{xy}[\ell]$ occur? Confirm that this is the value of ℓ for which the sequence $y[n-\ell]$ is most similar to the sequence $x[n]$.

(5 pts.) 7. Find $r_{xx}[\ell]$ the autocorrelation function of the periodic signal $x[n] = 4\sin\left(2\pi\frac{1}{5}n + \pi/4\right)$.
Show your work.