Q1 abc

Q1 a) $D(z) = \sum_{n=-\infty}^{\infty} d[n] z^{-n} = \sum_{n=-\infty}^{\infty} (2\delta[n+1] + 3\delta[n] + 4\delta[n-1]) z^{-n}$

$= \sum_{n=-\infty}^{\infty} (2\delta[n+1] z^{-n} + 3\delta[n] z^{-n} + 4\delta[n-1] z^{-n}) = 2z^{1} + 3z^{0} + 4z^{-1}$

$ROC_d = \{ z \in \mathbb{C} : 0 < |z| < \infty \}$

b) $Q(z) = \sum_{n=-\infty}^{\infty} q[n] z^{-n} = \sum_{n=-\infty}^{\infty} (\frac{1}{4})^{n} u[n] z^{-n} = \sum_{n=0}^{\infty} (\frac{1}{4})^{n} z^{-n} = \sum_{n=0}^{\infty} (\frac{z^{-1}}{4})^{n}$

$\underset{|(\frac{1}{4} z^{-1})| < 1}{=} \frac{1}{1 - \frac{1}{4} z^{-1}}$ $ROC_q = \{ z \in \mathbb{C} : |z| > \frac{1}{4} \}$

c) $S(z) = \sum_{n=-\infty}^{\infty} s[n] z^{-n} = \sum_{n=-\infty}^{\infty} (\frac{1}{4})^{n} u[-n] z^{-n} = \sum_{-\infty}^{0} (\frac{z^{-1}}{4})^{n}$

$\underset{|\frac{1}{4} z^{-1}| > 1}{=} \frac{(\frac{1}{4} z^{-1})^{-\infty} - (\frac{1}{4} z^{-1})^{0}}{1 - \frac{1}{4} z^{-1}} = \frac{-1}{1 - \frac{1}{4} z^{-1}}$

$ROC_s = \{ z \in \mathbb{C} : 0 < |z| < \frac{1}{4} \}$

Q1 de

d) $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} (-1)^n u[n] z^{-n} = \sum_{n=0}^{\infty} (-z^{-1})^n$

$|-z^{-1}| < 1 \nearrow = \frac{(-z^{-1})^0 - (-z^{-1})^\infty}{1-(-z^{-1})} = \frac{1}{1+z^{-1}}$

$ROC_x = \{ z \in \mathbb{C} : z > 1 \}$

e) $y[n] = \{\ldots, 0, 0, \underset{\uparrow}{1}, 0, 1, 0, \overline{1}, \overline{0}, \ldots\} = u[\frac{n}{2}]$

$Y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n} = \sum_{n=-\infty}^{\infty} u[n/2] z^{-n} = \sum_{m=-\infty}^{\infty} u[m] z^{-2m} = \sum_{m=0}^{\infty} z^{-2m}$

$|z^{-2}| < 1 \nearrow = \frac{(z^{-2})^0 - (z^{-2})^\infty}{1-z^{-2}} = \frac{1}{1-z^{-2}}$ $\quad |z^2| > 1 \quad |z| > \sqrt{1}$

$ROC_y = \{ z \in \mathbb{C} : |z| > 1 \}$

Q2 abc

Q2 a) $u[n] \longleftrightarrow U(z) = \frac{1}{1-z^{-1}} \quad |z|>1$

$x_1[n] = (\frac{1}{2})^n u[n] \longleftrightarrow X_1(z) = ?$

$g[n] = a^n x[n] \longleftrightarrow G(z) = X(\frac{z}{a}) \quad ROC_G \; |a|r_1<|z|<|a|r_2$

$\Rightarrow X_1(z) = U(\frac{z}{a}) = \frac{1}{1-(\frac{z}{a})^{-1}} = \frac{1}{1-(\frac{z}{1/2})^{-1}}$

$X_1(z) = \frac{1}{1-(2z)^{-1}} = \frac{1}{1-\frac{1}{2}z^{-1}} \quad |z|>|a|$

$ROC_{X_1} = \{z \in \mathbb{C} : |z| > \frac{1}{2}\}$

b) $x_2[n] = k\, x_1[n-5] = k(\frac{1}{2})^{n-5} u[n-5] = k(\frac{1}{2})^{-5}(\frac{1}{2})^n u[n-5]$

$\Rightarrow k = (\frac{1}{2})^5$

$g[n] = k\, x[n-k] \quad x[n] \longleftrightarrow X(z)$

$G(z) \longleftrightarrow k z^{-k} X(z) \quad ROC_G = ROC_X$

$X_2(z) = z^{-5}\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right)(\frac{1}{2})^5 \quad |z|>\frac{1}{2}$

$X_2(z) = (\frac{1}{2})^5 \frac{z^{-5}}{1-\frac{1}{2}z^{-1}} \quad |z|>\frac{1}{2}$

c) $x_3[n] = (\frac{1}{2})^n u[n] - (\frac{1}{2})^n u[n-5] = x_1[n] - x_2[n]$

$X_3(z) \longleftrightarrow X_1(z) - X_2(z) \quad ROC_3 \supseteq ROC_1 \cap ROC_2$

$\Rightarrow X_3(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \left[(\frac{1}{2})^5 \frac{z^{-5}}{1-\frac{1}{2}z^{-1}}\right] \quad |z|>\frac{1}{2}$

$= \frac{1-(2z)^{-5}}{1-(2z)^{-1}} \quad |z|>\frac{1}{2}$

because it has a finite lenght and causal then $|z|>0$

Q2 de

d)

$$\begin{array}{l|l} 1 - (2z)^{-5} & 1 - (2z)^{-1} \\ \hline & 1 + (2z)^{-1} + (2z)^{-2} + (2z)^{-3} + (2z)^{-4} \end{array}$$

$$\mp \cancel{1} \pm (2z)^{-1}$$

$$(2z)^{-1} - (2z)^{-5}$$

$$-\cancel{(2z)^{-1}} \pm (2z)^{-2}$$

$$(2z)^{-2} - (2z)^{-5}$$

$$-\cancel{(2z)^{-2}} \pm (2z)^{-3}$$

$$(2z)^{-3} - (2z)^{-5}$$

$$-\cancel{(2z)^{-3}} \pm (2z)^{-4}$$

$$(2z)^{-4} - (2z)^{-5}$$

$$-\cancel{(2z)^{-4}} \pm \cancel{(2z)^{-5}}$$

$$0$$

$$X_3(z) = \frac{1-(2z)^{-5}}{1-(2z)^{-1}} = 1 + (2z)^{-1} + (2z)^{-2} + (2z)^{-3} + (2z)^{-4}$$

$$= 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{8}z^{-3} + \frac{1}{16}z^{-4} \qquad |z| > 0$$

e)

$$x[n] \longrightarrow X(z) \qquad r_1 < |z| < r_2$$

$$g[n] = x[-n] \qquad G(z) = X(z^{-1}) \qquad ROC_G = \frac{1}{r_2} < |z| < \frac{1}{r_1}$$

$$x_4[n] = \left(\frac{1}{2}\right)^{-n} u[-n] = x_1[-n]$$

$$\Rightarrow X_4(z) = X_1(z^{-1}) = \frac{1}{1 - \frac{1}{2}(z^{-1})^{-1}} = \frac{1}{1 - \frac{1}{2}z}$$

$$ROC_{X_4} = \{ z \in \mathbb{C} : |z| < 2 \}$$

Q3 a

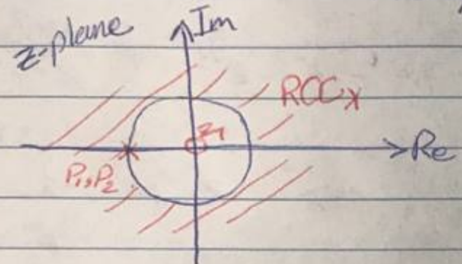
Q3 a) $x[n] = n(-1)^n u[n] = n\,g[n]$

$g[n] = (-1)^n u[n] \longleftrightarrow \frac{1}{1+z^{-1}} = G(z)$ based on Q1 part(d) $|z|>1$

$X(z) = -z\frac{d}{dz}\overset{G}{\cancel{X}}(z)$ $ROC_G = ROC_X$

$X(z) = -z\frac{d}{dz}\left(\frac{1}{1+z^{-1}}\right) = -z\,\frac{-(-z^{-2})}{(1+z^{-1})^2} = \frac{-z^{-1}}{(1+z^{-1})^2}$

$ROC_X = ROC_G$ $|z|>1$



$X(z) = \frac{-z^{-1}}{(1+z^{-1})^2} = \frac{-z^{-1}}{1+2z^{-1}+z^{-2}} = \frac{-z}{z^2+2z+1} = \frac{-z}{(1+z)^2}$

$z_1 = 0$ $P_1 = P_2 = -1$

* because $M<N$, $X(z)$ has ONE zero at $z=0$

the poles are on the unit circle, so the envelop of $x[n]$ remain constant

Q3 b

b) $x_1[n] = u[n] \qquad X_1(z) = \dfrac{1}{1-z^{-1}} \qquad |z|>1$

$x_2[n] = n\,u[n] \qquad X_2(z) = -z\dfrac{d}{dz}X_1(z) \qquad |z|>1$

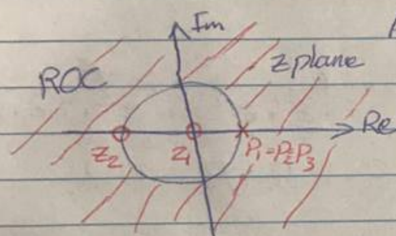
$$X_2(z) = -z \cdot \frac{-(z^{-2})}{(1-z^{-1})^2} = \frac{z^{-1}}{(1-z^{-1})^2} \qquad |z|>1$$

$x_3[n] = n\,x_2[n]$

$$X_3(z) = -z\frac{d}{dz}X_2(z) = -z\left(\frac{-z^{-2}(1-z^{-1})^2 - 2z^{-3}(1-z^{-1})}{(1-z^{-1})^4}\right)$$

$$= \frac{z^{-1}+z^{-2}}{(1-z^{-1})^3} = \frac{z^{-1}+z^{-2}}{1-3z^{-1}+3z^{-2}-z^{-3}} = \frac{z^2+z}{z^3-3z^2+3z-1}$$

$$= \frac{z(z+1)}{(z-1)^3} \Rightarrow z_{1,2} = 0, -1 \qquad P_1 = P_2 = P_3 = 1$$



since $M<N$

Also, there is a zero at $z=0$ ✓ and poles are on the unit circle so the envelop of $x[n]$ is constant

Q3 c

C) $x[n] = n(.9)^n \sin(\frac{\pi n}{3})u[n] = n(0.9)^n \left(\frac{e^{j\pi n/3} - e^{-j\pi n/3}}{2j}\right)u[n]$

$= \frac{n}{2j}(0.9e^{j\pi/3})^n u[n] - \frac{n}{2j}(0.9e^{-j\pi/3})^n u[n]$

$x_1[n] = (0.9e^{j\pi/3})^n u[n] \longleftrightarrow X_1(z) = \frac{1}{1-.9e^{j\pi/3}z^{-1}} \quad |z|>0.9$

$x_2[n] = (0.9e^{-j\pi/3})^n u[n] \longrightarrow X_2(z) = \frac{1}{1-.9e^{-j\pi/3}z^{-1}} \quad |z|>0.9$

$x[n] = \frac{n}{2j}x_1[n] - \frac{n}{2j}x_2[n] = x_3[n] - x_4[n]$

$x_3[n] = \frac{n}{2j}x_1[n] \qquad X_3(z) = \frac{-z}{2j}\frac{d}{dz}X_1(z)$

$X_3(z) = \frac{-z}{2j}\frac{d}{dz}\left(\frac{1}{1-.9e^{j\pi/3}z^{-1}}\right) = \frac{-z}{2j}\left[\frac{-.9e^{j\pi/3}z^{-2}}{(1-.9e^{j\pi/3}z^{-1})^2}\right]$

$X_3(z) = \frac{-z}{2j}\left(\frac{-0.9e^{j\pi/3}}{(z-0.9e^{j\pi/3})^2}\right) = \frac{0.45e^{j\pi/3}z}{j(z-.9e^{j\pi/3})^2} \quad |z|>0.9$

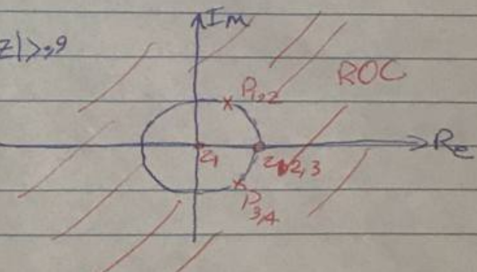
$x_4[n] = \frac{n}{2j}x_2[n] \Rightarrow X_4(z) = \frac{0.45e^{-j\pi/3}z}{j(z-0.9e^{-j\pi/3})^2} \quad |z|>.9$

$\{|z|>.9\} \cap \{|z|>.9\}$

$x[n] = x_3[n] - x_4[n] \longrightarrow X(z) = X_3(z) - X_4(z)$

$X(z) = \frac{.45e^{j\pi/3}z}{j(z-.9e^{j\pi/3})^2} - \frac{0.45e^{-j\pi/3}z}{j(z-.9e^{-j\pi/3})^2} = \frac{.45z\left(e^{j\pi/3}(z-.9e^{-j\pi/3})^2 - e^{-j\pi/3}(z-.9e^{j\pi/3})^2\right)}{(z-.9e^{j\pi/3})^2(z-.9e^{-j\pi/3})^2}$

$= \frac{-.45jz\left(z^2\, 2\sin\frac{\pi}{3} - \overset{2x}{.81}\sin\frac{\pi}{3}\right)}{(z-.9e^{j\pi/3})^2(z-.9e^{-j\pi/3})^2} \quad |z|>.9$



$z_1 = 0 \quad z_{3,2} = .9$

$P_{1,2} = .9e^{j\pi/3} \quad P_{3,4} = 0.9e^{-j\pi/3}$

poles are inside of the unit circle then as $n \to \infty$ the envelop of signal decays to zero

because $M < N$ there is a zero at $z=0$

Q3 d

d) $x[n] = (\frac{1}{2})^n (u[n] - u[n-3]) = x_1[n] - k\, x_1[n-3]$ $\quad k = (\frac{1}{2})^3$

$x_1[n] = (\frac{1}{2})^n u[n] \longrightarrow X_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$ $\quad |z| > \frac{1}{2}$

$x_2[n] = k\, x_1[n-3] \longrightarrow X_2(z) = (\frac{1}{2})^3 \frac{z^{-3}}{1-\frac{1}{2}z^{-1}}$ $\quad |z| > \frac{1}{2}$

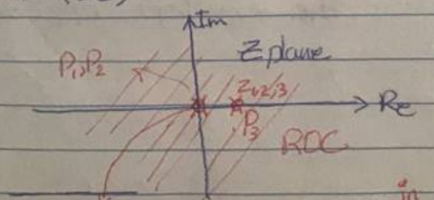
$x[n] = x_1[n] + x_2[n] \longrightarrow X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - [(\frac{1}{2})^3 \frac{z^{-3}}{1-\frac{1}{2}z^{-1}}]$

$= \frac{1-(\frac{1}{2}z^{-1})^3}{1-\frac{1}{2}z^{-1}} = \frac{1-(2z)^{-3}}{1-(2z)^{-1}}$ $\quad |z| > \frac{1}{2}$ but because it has finit lenght and causal $\underline{|z| > 0}$

$X(z) = \frac{z^3 - 1/8}{z^3 - \frac{z^2}{2}}$

$z_{1,2,3} = 1/2$

$P_1 = P_2 = 0 \quad P_3 = \frac{1}{2}$



This circle shows zero is not in ROC

d) the poles are inside of unit circles then the envelop of signal decays to zero as $n \to \infty$