1. Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$
$$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$
$$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$
$$\sin(x + \pi/2) = \cos(x)$$
$$\cos(x - \pi/2) = \sin(x)$$
$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$
$$\cos x \cos y = \frac{1}{2}(\cos(x - y) + \cos(x + y))$$
$$\sin x \cos y = \frac{1}{2}(\sin(x - y) + \sin(x + y))$$
$$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

2. Summations:

$$\sum_{n=n_1}^{n_2} a^n = \frac{a^{n_1} - a^{(n_2+1)}}{1 - a} \qquad \sum_{n=n_1}^{n_2} n = \frac{(n_2 - n_1 + 1)(n_1 + n_2)}{2}$$

3. Special Functions (continuous-domain):

rectangle function

$$\mathrm{rect}(x) \equiv \begin{cases} 1, & |x| < ½ \\ 0, & |x| > ½ \end{cases}$$

sinc function

$$\mathrm{sinc}(x) \equiv \frac{\sin(\pi x)}{\pi x}$$

triangle function

$$\Lambda(x) \equiv \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

Dirac Delta function

$$\delta(x) \equiv \lim_{\tau \to 0} \frac{1}{\tau}\mathrm{rect}\left(\frac{x}{\tau}\right)$$

Dirac comb function

$$\mathrm{comb}(x) \equiv \sum_{n=-\infty}^{\infty} \delta(x - n)$$

4. Special Functions (discrete-domain)

unit step function

$$u[n] \equiv \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Kronecker Delta

$$\delta[n] \equiv \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Kronecker comb

$$\mathrm{comb}_N[n] = \sum_{k=-\infty}^{\infty} \delta[n - kN]$$

5. Convolution Sum and Convolution integral:

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k] \qquad x(t) * h(t) = \int_{-\infty}^{\infty} x(a)h(t - a)da$$

6. Cross-correlation and Autocorrelation

The ***cross-correlation***, $r_{xy}[\ell]$, of **two energy sequences**, $x[n]$ and $y[n]$, is defined as:

$$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]\,y^*[n-\ell] = x[\ell] * y^*[-\ell]$$

where $y^*[n]$ denotes the complex conjugate of $y[n]$.

The ***cross-correlation***, $r_{xy}[\ell]$, of **two power sequences**, $x[n]$ and $y[n]$, is defined as:

$$r_{xy}[\ell] = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x[n]\,y^*[n-\ell]$$

If $x[n]$ and $y[n]$ are both periodic with period N, their cross correlation, $r_{xy}[\ell]$, can be computed as:

$$r_{xy}[\ell] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\,y^*[n-\ell]$$

The ***autocorrelation*** function, $r_{xx}[\ell]$, of a sequence $x[n]$ is the cross-correlation of the sequence with itself.

7. Partial Fraction Expansion:

If $Y(z) = \dfrac{\sum_{k=0}^{m} c_k z^k}{(z-p_1)^{\ell}(z-p_{\ell+1})\cdots(z-p_N)}$ where $m < N$ and $p_{\ell+1},\ldots,p_N$ are distinct, then

the partial fraction expansion of $Y(z)$ can be expressed as:

$$Y(z) = \frac{A_1}{z-p_1} + \frac{A_2}{(z-p_1)^2} + \ldots + \frac{A_\ell}{(z-p_1)^\ell} + \frac{A_{\ell+1}}{z-p_{\ell+1}} + \ldots + \frac{A_N}{z-p_N}$$

where the coefficients, $A_1,\ldots,A_N$ can be found as:

$$A_\ell = \left((z-p_1)^\ell Y(z)\right)\Big|_{z=p_1}$$

$$A_{\ell-m} = \frac{1}{m!}\left(\frac{d^m}{dz^m}\left[(z-p_1)^\ell Y(z)\right]\right)\Big|_{z=p_1},\ m = 1,\ldots,\ell-1$$

$$A_k = \left((z-p_k)Y(z)\right)\Big|_{z=p_k},\ k = \ell+1,\ldots,N$$

8. Z-Transform: Let $X(z)$ denote the Z Transform of the signal $x[n]$. Then:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \qquad \text{and} \qquad x[n] = \oint_C X(z)z^{n-1}dz$$

Selected Z Transform Pairs

$x[n]$	$X(z)$	ROC
$\delta[n]$	1	All z
$u[n]$	$\dfrac{1}{1 - z^{-1}}$	$\lvert z\rvert > 1$
$a^n u[n]$	$\dfrac{1}{1 - az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$na^n u[n]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\dfrac{1}{1 - az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\cos(2\pi F_0 n)u[n]$	$\dfrac{1 - z^{-1}\cos(2\pi F_0)}{1 - 2z^{-1}\cos(2\pi F_0) + z^{-2}}$	$\lvert z\rvert > 1$
$\sin(2\pi F_0 n)u[n]$	$\dfrac{z^{-1}\sin(2\pi F_0)}{1 - 2z^{-1}\cos(2\pi F_0) + z^{-2}}$	$\lvert z\rvert > 1$
$\cos(2\pi F_0 n + \theta)u[n]$	$\dfrac{\cos(\theta) - z^{-1}\cos(2\pi F_0 - \theta)}{1 - 2z^{-1}\cos(2\pi F_0) + z^{-2}}$	$\lvert z\rvert > 1$
$a^n\cos(2\pi F_0 n)u[n]$	$\dfrac{1 - az^{-1}\cos(2\pi F_0)}{1 - 2az^{-1}\cos(2\pi F_0) + a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Selected Z Transform Properties

In stating the following Z-Transform properties, we assume that: $x[n]$ has Z Transform $X(z)$ with $\mathrm{ROC}_X = \{z : r_1 < |z| < r_2\}$; $x_1[n]$ has Z Transform $X_1(z)$ with ROC_1; and $x_2[n]$ has Z Transform $X_2(z)$ with ROC_2.

Property	Time Domain	Z Domain	ROC
Linearity	$g[n] = ax_1[n] + bx_2[n]$	$G(z) = aX_1(z) + bX_2(z)$	$\mathrm{ROC}_G \supseteq (\mathrm{ROC}_1 \cap \mathrm{ROC}_2)$
Time-reversal	$g[n] = x[-n]$	$G(z) = X(z^{-1})$	$\mathrm{ROC}_G = \left\{z : \frac{1}{r_2} < \lvert z\rvert < \frac{1}{r_1}\right\}$
Time-shifting	$g[n] = x[n - n_0]$	$G(z) = z^{-n_0}X(z)$	$\mathrm{ROC}_G = \mathrm{ROC}_X$ with possible exceptions at $z = 0$ and $z = \infty$
TD: multiplication by a^n ZD: z-scaling by a	$g[n] = a^n x[n]$	$G(z) = X\left(\frac{z}{a}\right)$	$\mathrm{ROC}_G = \{z : \lvert a\rvert r_1 < \lvert z\rvert < \lvert a\rvert r_2\}$
TD: multiplication by n ZD: differentiation	$g[n] = nx[n]$	$G(z) = -z\frac{dX(z)}{dz}$	$\mathrm{ROC}_G = \mathrm{ROC}_X$
TD: convolution ZD: mutiplication	$g[n] = x_1[n] * x_2[n]$	$G(z) = X_1(z)X_2(z)$	$\mathrm{ROC}_G \supseteq (\mathrm{ROC}_1 \cap \mathrm{ROC}_2)$

Note:
$\mathrm{ROC}_X = \{z : r_1 < |z| < r_2\}$ is read as "the ROC of $X(z)$ is the set of all z such that the magnitude of z is greater than r_1 but less than r_2"
$\mathrm{ROC}_G \supseteq (\mathrm{ROC}_1 \cap \mathrm{ROC}_2)$ is read as "the ROC of G(z) includes at least the intersection of the ROC's of $X_1(z)$ and $X_2(z)$"

9. Discrete-time Fourier Transform: Let $G_{\text{DTFT}}(F)$ denote the Discrete-Time Fourier Transform of the signal $g[n]$. Then:

$$G_{\text{DTFT}}(F) = \sum_{n=-\infty}^{\infty} g[n]e^{-j2\pi Fn} \qquad \text{and} \qquad g[n] = \int_1 G_{\text{DTFT}}(F)e^{j2\pi Fn}dF$$

Selected Discrete-Time Fourier Transform Properties

Property	Mathematical Description
Sum of $g[n]$	$\sum_{n=-\infty}^{\infty} g[n] = G_{\text{DTFT}}(0)$
Area under $G_{\text{DTFT}}(F)$	$g[0] = \int_1 G_{\text{DTFT}}(F)dF$
Time Shifting	$g[n-n_0] \leftrightarrows G_{\text{DTFT}}(F)e^{-j2\pi Fn_0}$
Frequency Shifting	$g[n]e^{j2\pi F_0 n} \leftrightarrows G_{\text{DTFT}}(F-F_0)$
Linearity	$ag_1[n] + bg_2[n] \leftrightarrows aG_{1,\text{DTFT}}(F) + bG_{2,\text{DTFT}}(F)$
Conjugate Functions	$g^*[n] \leftrightarrows G^*_{\text{DTFT}}(-F)$
Multiplication in time	$g[n]h[n] \leftrightarrows (G_{\text{DTFT}}(F)\text{rect}(F)) * H_{\text{DTFT}}(F) = G_{\text{DTFT}}(F) * (H_{\text{DTFT}}(F)\text{rect}(F))$
Convolution in time	$g[n] * h[n] \leftrightarrows G_{\text{DTFT}}(F)H_{\text{DTFT}}(F)$ note: at least one of the two signals ($g[n]$ or $h[n]$) must be an energy signal

Selected Discrete-Time Fourier Transform Pairs

$$\delta[n] \leftrightarrows 1$$

$$\sum_{k=-\infty}^{\infty} N\delta[n-kN] = N\text{comb}_N[n] \leftrightarrows N\text{comb}(FN) = \sum_{k=-\infty}^{\infty}\delta\left(F - \frac{k}{N}\right)$$

$$\exp(j2\pi an) \leftrightarrows \delta(F-a) * \text{comb}(F)$$

10. Discrete Fourier Transform: Let $G_{\text{DFT},N}[k]$ denote the N-point DFT of the signal $g[n]$. Then:

$$G_{\text{DFT},N}[k] = \sum_{n=0}^{N-1} g[n]e^{-j2\pi\frac{k}{N}n} \quad k = 0, \ldots, N-1 \qquad \text{and} \qquad g[n] = \frac{1}{N}\sum_{k=0}^{N-1} G_{\text{DFT},N}[k]e^{j2\pi\frac{k}{N}n} \quad n = 0, \ldots, N-1$$

11. Discrete Fourier Series

If $g[n]$ periodic with period N, then $g[n]$ can be expressed in terms of its DFS representation as follows:

$$g[n] = \sum_{k=0}^{N-1} c_k e^{j2\pi\frac{k}{N}n} \qquad \text{where} \qquad c_k = \frac{1}{N}\sum_{n=0}^{N-1} g[n]e^{-j2\pi\frac{k}{N}n} \tag{1}$$