Review

The DFS coefficients of an N-periodic signal $x_p[n]$, may be calculated as:

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p[n] \, e^{-j2\pi \frac{k}{N} n}$$

Given that: $x[n] = \begin{cases} x_p[n], & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$

- The DTFT of $x[n]$ is defined as:

$$X_{DTFT}(F) = \sum_{n=-\infty}^{\infty} x[n] \, e^{-j2\pi F n} = \sum_{n=0}^{N-1} x[n] \, e^{-j2\pi F n}$$

- The N-point DFT of $x[n]$ is defined as:

$$X_{DFT,N}[k] = \sum_{n=0}^{N-1} x[n] \, e^{-j2\pi \frac{k}{N} n} = N c_k$$

$$= X_{DTFT}\left(\tfrac{k}{N}\right)$$

$$= X_z\left(e^{j2\pi \frac{k}{N}}\right)$$

If the nonzero values of $x[n]$ are confined to the interval $0 \le n \le N-1$:

a) $X_{DFT,N}[k] = N c_k$, and

b) $X_{DFT,N}[k] = X_{DTFT}\left(\frac{k}{N}\right)$

A few other things we know:

- It doesn't matter which period of $x_p[n]$ we use to calculate the DFS coefficients, $\{c_k\}$, of $x_p[n]$. We will always find the same values.

- If $x[n]$ and $y[n]$ have the same N-periodic extensions, their DTFTs will have the same values at $F = k/N$.

$$x_p[n] = x[n] * \text{comb}_N[n] \xleftrightarrow{\text{DTFT}} \left(X_{DTFT}(F)\right)\left(\frac{1}{N}\sum_{k=-\infty}^{\infty} \delta\left(F - \tfrac{k}{N}\right)\right) = \frac{1}{N}\sum_{k=-\infty}^{\infty} X_{DTFT}\left(\tfrac{k}{N}\right) \delta\left(F - \tfrac{k}{N}\right)$$

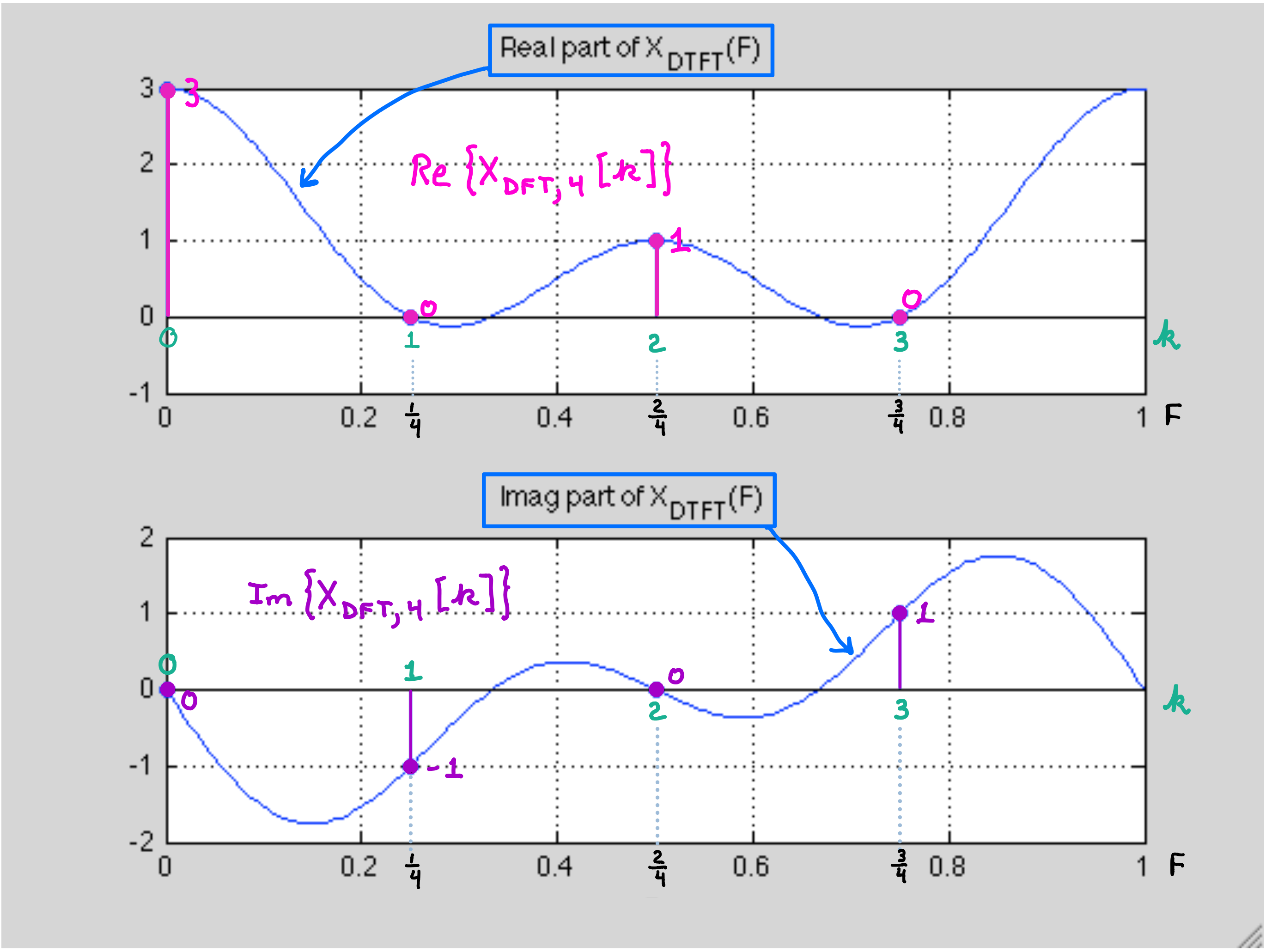
$$x_p[n] = y[n] * \text{comb}_N[n] \xleftrightarrow{\text{DTFT}} \left(Y_{DTFT}(F)\right)\left(\frac{1}{N}\sum_{k=-\infty}^{\infty} \delta\left(F - \tfrac{k}{N}\right)\right) = \frac{1}{N}\sum_{k=-\infty}^{\infty} Y_{DTFT}\left(\tfrac{k}{N}\right) \delta\left(F - \tfrac{k}{N}\right)$$

Last time, we calculated the DTFT and the 4-pt. DFT of the signal $x[n] = \{\underset{\uparrow}{1} \;\; 1 \;\; 1 \;\; 0\}$

We found: $X_{DTFT}(F) = 1 + e^{-j2\pi F} + e^{-j2\pi 2F}$

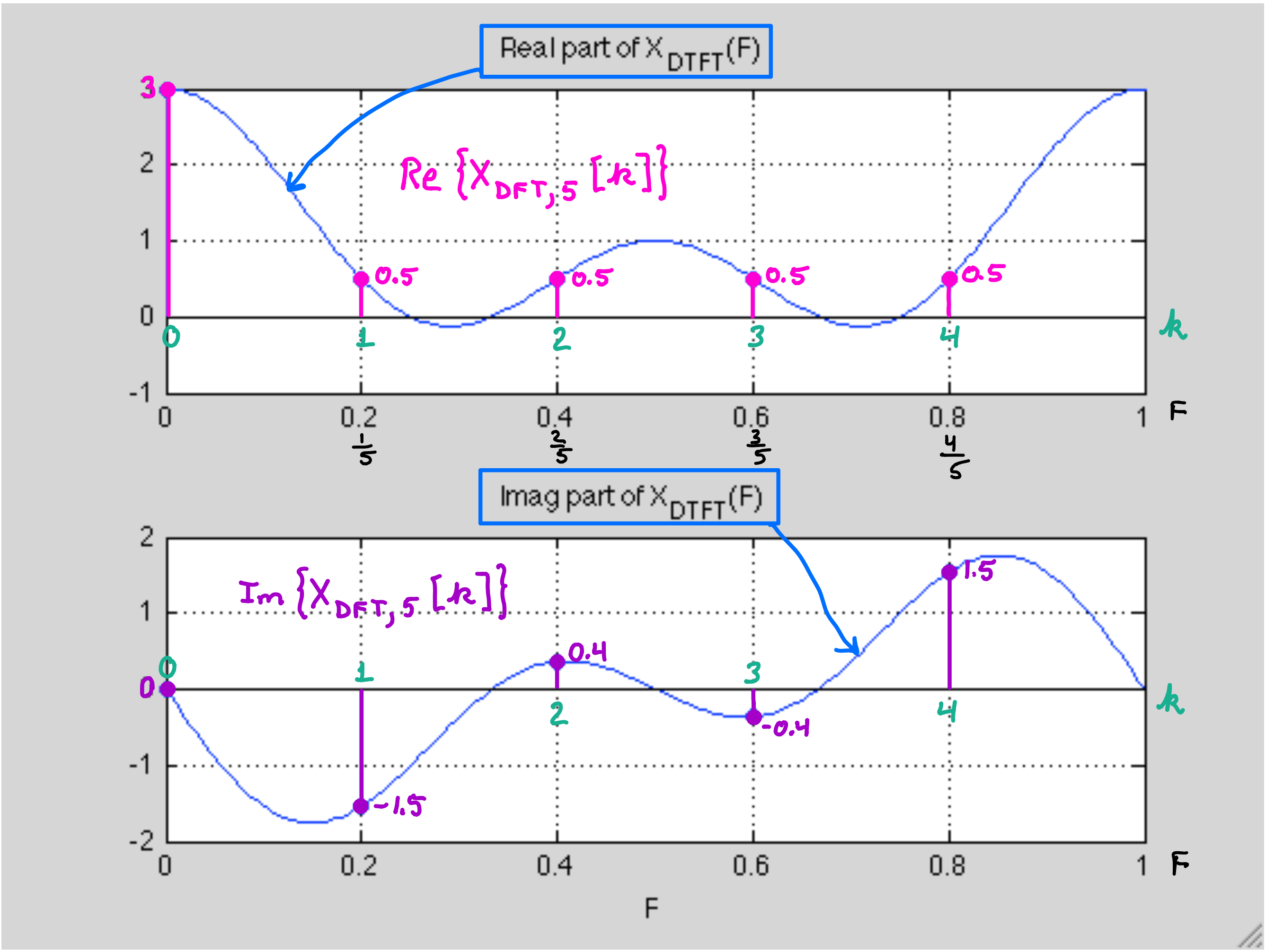
$$X_{DFT,4}[0] = 3, \quad X_{DFT,4}[1] = -j, \quad X_{DFT,4}[2] = 1, \quad X_{DFT,4}[3] = j$$

$$x = \{\underset{\uparrow}{1}, 1, 1\} \quad \Rightarrow \quad X_{\text{DTFT}}(F) = 1 + e^{-j2\pi F} + e^{-j2\pi 2F}$$



x = [1;1;1];	X = fft(x,4);	g = ifft(X);
x =	X =	g =
1	3.0000	1
1	0.0000 - 1.0000i	1
1	1.0000	1
	0.0000 + 1.0000i	0

$$x = \{\underset{\uparrow}{1}, 1, 1\} \quad \Rightarrow \quad X_{\text{DTFT}}(F) = 1 + e^{-j2\pi F} + e^{-j2\pi 2F}$$




```
x = [1;1;1]

x =
     1
     1
     1
```

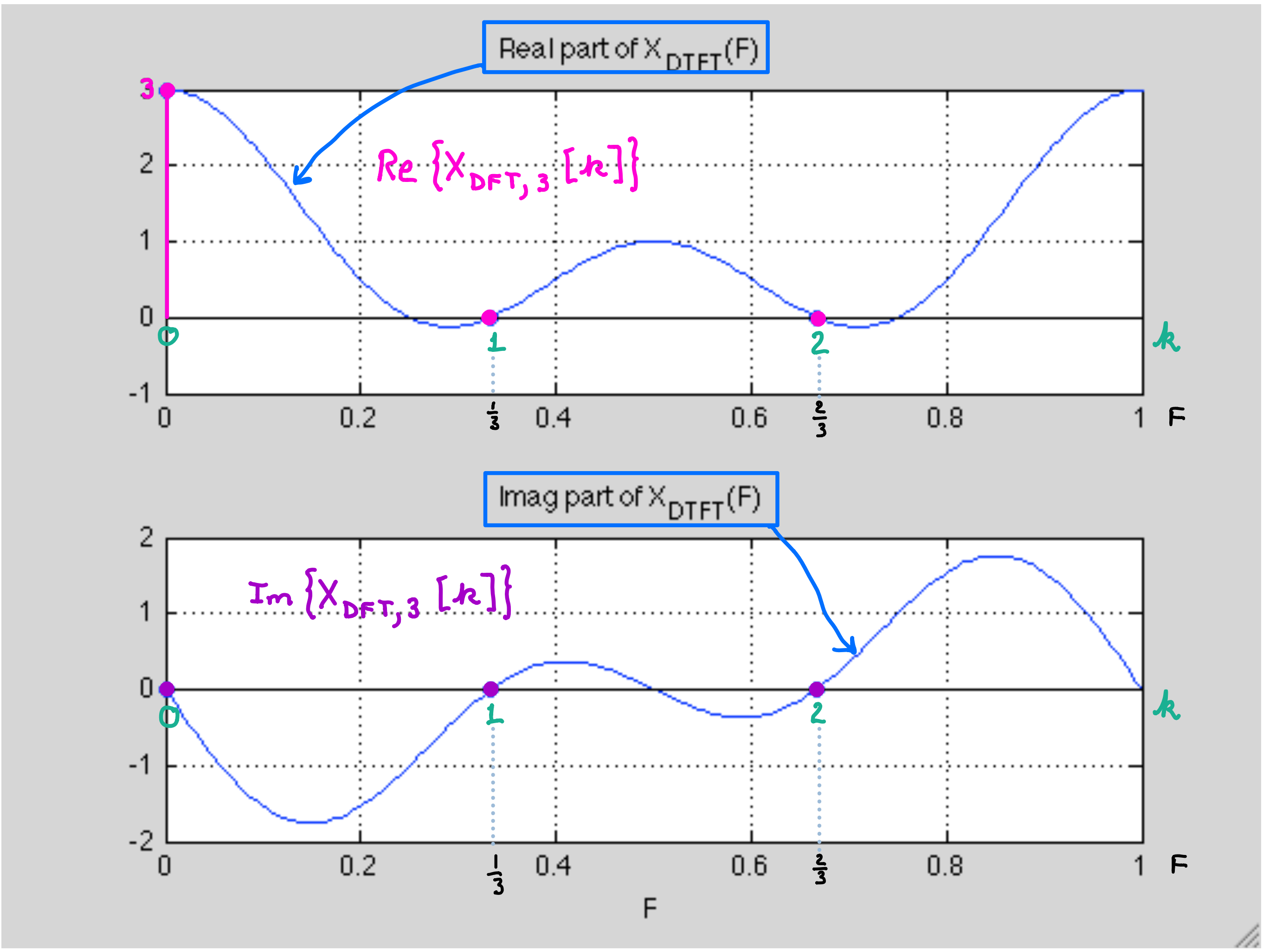
```
X = fft(x,5)

X =
   3.0000
   0.5000 - 1.5388i
   0.5000 + 0.3633i
   0.5000 - 0.3633i
   0.5000 + 1.5388i
```

```
g = ifft(X)

g =
    1.0000
    1.0000
    1.0000
    0.0000
    0.0000
```

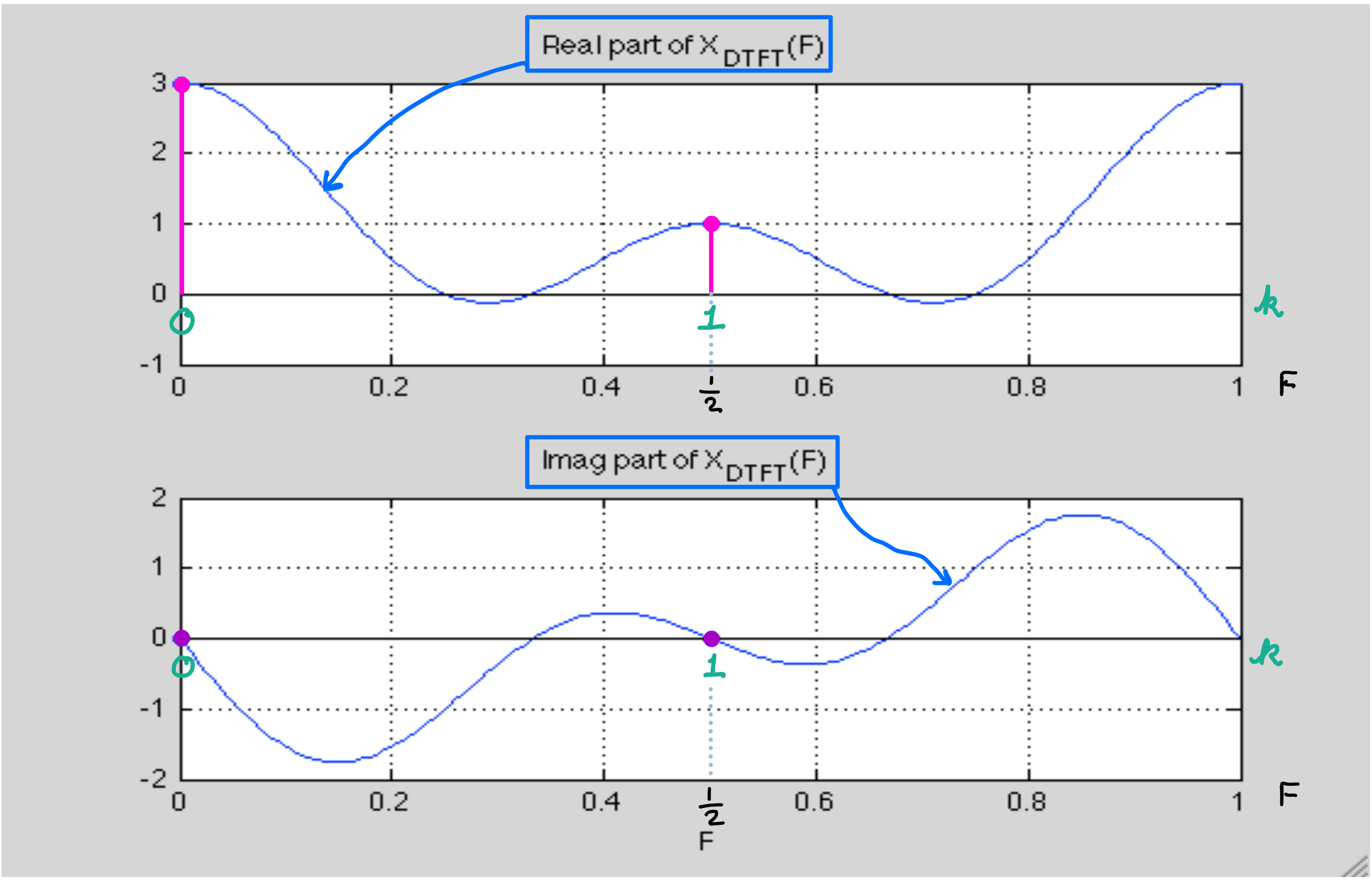
$$x = \{\underset{\uparrow}{1}, 1, 1\} \quad \Rightarrow \quad X_{\text{DTFT}}(F) = 1 + e^{-j2\pi F} + e^{-j2\pi 2F}$$



x = [1;1;1]; X = fft(x,3); g = ifft(X);

x =	X =	g =
1	3	1
1	0	1
1	0	1

$$x = \{\underset{\uparrow}{1}, 1, 1\} \quad \Rightarrow \quad X_{\text{DTFT}}(F) = 1 + e^{-j2\pi F} + e^{-j2\pi 2F}$$



Consider the MATLAB commands below. What will MATLAB return for g?

>> X = [3, 1]; % note: $X = [X_{DTFT}(0), X_{DTFT}(\frac{1}{2})]$

>> g = ifft(X) % ⇒ g = [? ?]

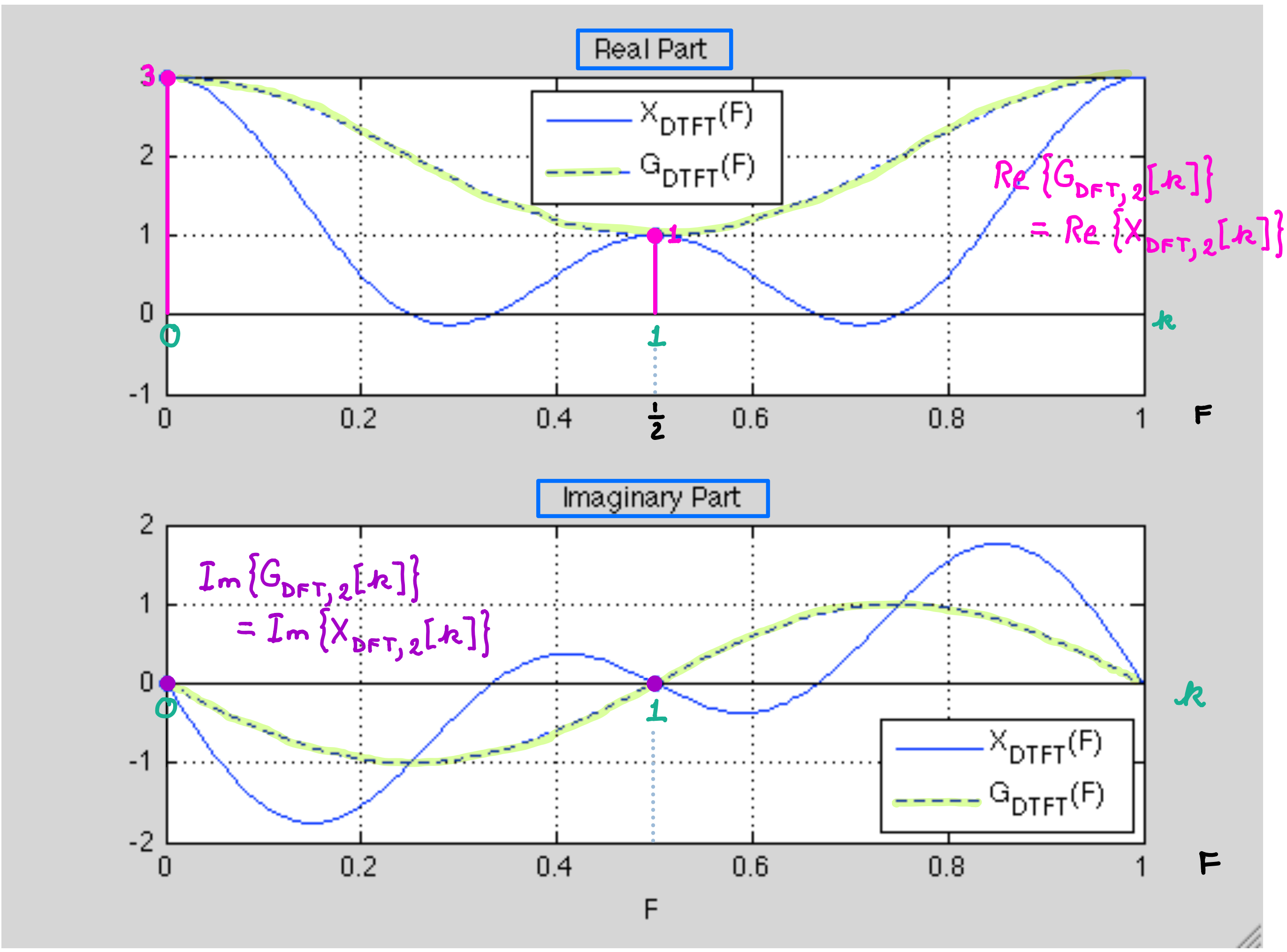
<u>Discussion:</u> Since X has length 2 and no size is specified for the ifft, MATLAB will find the 2-pt. idft of X. The two elements of g will be the values of g[0] and g[1] where g[n] is a length-2 signal whose DTFT satisfies:

$$\left.\begin{aligned} G_{DTFT}(0) &= X_{DTFT}(0) \\ G_{DTFT}(\tfrac{1}{2}) &= X_{DTFT}(\tfrac{1}{2}) \end{aligned}\right\} \Rightarrow$$

The 2 periodic extension of g[n] will be be equal to the 2-periodic extension of x[n].

$$x = \{\underset{\uparrow}{1}, 1, 1\} \quad \Rightarrow \quad X_{\text{DTFT}}(F) = 1 + e^{-j2\pi F} + e^{-j2\pi 2F}$$

$$g = \{\underset{\uparrow}{2}, 1\} \quad \Rightarrow \quad G_{\text{DTFT}}(F) = 2 + e^{-j2\pi F}$$



Since g[n] is the 2-periodic extension of x[n], we know that the DTFTs of g[n] and x[n] will have identical values at F=0 and F=1/2.

This is illustrated by the plots of $X_{DTFT}(F)$ and $G_{DTFT}(F)$ above.