1. In class, we considered the second-order causal LTI system described by the following LCCDE:

$$y[n] - y[n-1] + \frac{1}{4}y[n-2] = x[n] - x[n-1]$$

A summary of selected findings from class is given below:

characteristic polynomial:	$\lambda^2 - \lambda + 1/4 = 0 \quad \Rightarrow \quad \lambda_1 = \lambda_2 = 1/2$
homogeneous solution:	$y_h[n] = C_1(1/2)^n + C_2 n(1/2)^n$
zero-input soln when: y[−1]=2 and y[−2]=−4	$y_{zi}[n] = 3(1/2)^n + 2n(1/2)^n$

(4 pts.) a) Find the particular solution, $y_p[n]$, when $x[n] = (1/2)^n u[n]$.

(4 pts.) b) Find the zero-state response, $y_{zs}[n]$, $n \geq 0$, of this system when $x[n] = (1/2)^n u[n]$.

(2 pts.) c) Find the zero-input solution, $y_{zi}[n]$, when $y[-1] = 1$, and $y[-2] = 0$.

(2 pts.) d) Find the zero-input solution, $y_{zi}[n]$, when $y[-1] = 0$, and $y[-2] = 1$.

(2 pts.) e) Compare the zero-input (z.i.) response (for the IC's: $y[-1] = 2$, and $y[-2] = -4$) in the summary above to the z.i. responses you found in parts (c) and (d). Explain how the z.i. responses of parts (c) and (d) can be used to find the z.i response provided in the summary of findings above.

(2 pts.) f) Find a closed-form solution for the system response, $y[n]$, $n \geq 0$, when $x[n] = (1/2)^n u[n]$, $y[-1] = 1$, and $y[-2] = 1$. Please simplify as much as possible. Feel free to make use of your earlier work – just be sure to explain what you did.

(4 pts.) g) Check your solution in part (f) by iterating the difference equation to find $y[0]$, $y[1]$, $y[2]$, and $y[3]$. Then compare to the same values based on your closed-form solution. Be sure to show your iteration of the LCCDE and your evaluation of the closed-form solution.

2. Consider the following LTI causal system:

$$y[n] - \frac{3}{2}y[n-1] + \frac{1}{2}y[n-2] = 2x[n] + 3x[n-1]$$

(5 pts.) a) Find a closed-form expression for $h[n]$, the impulse response of the system.

(8 pts.) b) Find a closed-form expression for $y_{\text{step}}[n]$, the step response of the system. Recall that the step response is the zero-state response when the input is a unit step.

(4 pts.) c) Explain why you should find that $h[n] = y_{\text{step}}[n] - y_{\text{step}}[n-1]$. Then verify that your expressions satisfy this relationship. Be sure to show details and appropriate justification of your calculation of the difference $y_{\text{step}}[n] - y_{\text{step}}[n-1]$.

(Assignment 4 is continued on next page.)

3. For each of the LCCDEs listed below, determine whether or not the associated system is BIBO stable or not. Be sure to indicate your procedure. (Note that each system is causal and LTI.)

(3 pts.) a) $y[n] + \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n] - 2x[n-1]$

(3 pts.) b) $y[n] - \frac{1}{\sqrt{2}}y[n-1] + \frac{1}{4}y[n-2] = x[n] - 2x[n-1]$

(3 pts.) c) $y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n] - 2x[n-1]$

4. For certain initial conditions, the zero-input response, $y_{zi}[n]$, of a causal 2nd-order system described by a linear constant coefficient difference equation (LCCDE) is found to be:

$$y_{zi}[n] = 5(1/3)^n \cos\left(2\pi\frac{1}{8}n + \pi/3\right)$$

(3 pts.) a) Determine the system's characteristic roots λ_1 and λ_2.
Hint: Use Euler's formulae to rewrite $y_{zi}[n]$ in the form below; then identify the values of λ_1 and λ_2 by inspection. Please show the details of your work.

$$y_{zi}[n] = C_1(\lambda_1)^n + C_2(\lambda_2)^n$$

(4 pts.) b) Determine the values of the coefficients a_1 and a_2 for which the system's homogenous difference equation is:

$$y[n] + a_1y[n-1] + a_2y[n-2] = 0.$$

Show your work. Simplify the values of a_1 and a_2 as much as possible.