1. Let $x[n] = \{\underset{\uparrow}{1}, \; 2, \; 3, \; 0\}$ and let $y[n] = \{\underset{\uparrow}{3}, \; 2, \; 1, \; 0\}$.

(2 pts.) a) Let $z[n] = x[n] * y[n]$, where $*$ denotes the convolution operator. Find $z[n]$.

(6 pts.) b) Let $X_{\text{DFT},4}[k]$ denote the 4-point DFT of $x[n]$. Use the fft algorithm to find $X_{\text{DFT},4}[k]$.

(6 pts.) c) Let $W_{\text{DFT},4}[k] = X_{\text{DFT},4}[k] \times Y_{\text{DFT},4}[k]$ and let $w[n]$ denote the 4-point IDFT of $W_{\text{DFT},4}[k]$. Work in the time-domain to find $w[n]$. You may check your answer using matlab but to get credit you must provide a time-domain solution (*i.e.*, show how you found $w[n]$ without the need to find $W_{\text{DFT},4}[k]$).

(2 pts.) d) Repeat part (c) for the case when all DFT sizes are changed to 8 (*i.e.*, find $w[n]$, the 8-point IDFT of $W_{\text{DFT},8}[k]$ where $W_{\text{DFT},8}[k] = X_{\text{DFT},8}[k] \times Y_{\text{DFT},8}[k]$.). Explain.

2. Every point in the z-plane can be expressed in terms of its magnitude r and its phase $\theta = 2\pi F$ as follows: $z = re^{j\theta} = re^{j2\pi F}$. Determine the values of r and F which describe the point in the z-plane to which the Bilinear Transform maps each of the following s-domain points. Assume a sampling rate of 40 samples per second.

(1 pts.) a) $s = j2\pi 5$

(1 pts.) b) $s = j2\pi 10$

(1 pts.) c) $s = j2\pi 20$

(1 pts.) d) $s = j2\pi 200$

(1 pts.) e) $s = -10 + j2\pi 5$

(1 pts.) f) $s = -20 + j2\pi 5$

(1 pts.) g) $s = -200 + j2\pi 5$

(1 pts.) h) $s = -10 + j2\pi 10$

(1 pts.) i) $s = 10 + j2\pi 5$

(8 pts.) 3. The transfer function of a continuous-time first-order low pass filter (LPF) with 3 dB cutoff frequency f_c Hz is known to be:

$$H(s) = \frac{2\pi f_c}{s + 2\pi f_c}$$

Use the bilinear transform, together with the analog prototype above to design a discrete-time LPF with 3 dB cutoff $F_c = 0.2$ cycles/sample.