The N-point DFTs of two real-valued sequences can be calculated simultaneously if we combine the two real-valued sequences into a single complex-valued sequence.

$$x_1[n] \xleftrightarrow{\text{DFT}, N} X_1[k] \qquad \text{if } x_1[n] \text{ is real, then } X_1[k] = X_1^*[-k]$$

$$x_2[n] \xleftrightarrow{\text{DFT}, N} X_2[k] \qquad x_2[n] \text{ real} \Rightarrow X_2[k] = X_2^*[-k]$$

$$x[n] = x_1[n] + j\,x_2[n] \xleftrightarrow{\text{DFT}, N} X[k] = X_1[k] + j\,X_2[k]$$

$$X[k] = \text{Re}\{X_1[k]\} + j\,\text{Im}\{X_1[k]\} + j\left(\text{Re}\{X_2[k]\} + j\,\text{Im}\{X_2[k]\}\right)$$
$$= \left(\text{Re}\{X_1[k]\} - \text{Im}\{X_2[k]\}\right) + j\left(\text{Re}\{X_2[k]\} + \text{Im}\{X_1[k]\}\right)$$

$$X^*[-k] = \left(\text{Re}\{X_1[-k]\} - \text{Im}\{X_2[-k]\}\right) - j\left(\text{Re}\{X_2[-k]\} + \text{Im}\{X_1[-k]\}\right)$$
$$= \left(\text{Re}\{X_1[k]\} + \text{Im}\{X_2[k]\}\right) - j\left(\text{Re}\{X_2[k]\} - \text{Im}\{X_1[k]\}\right)$$

the real part of a Hermitian function is even: $\text{Re}\{X_1[k]\} = \text{Re}\{X_1[-k]\}$
the imag part of a Hermitian function is odd: $\text{Im}\{X_1[k]\} = -\text{Im}\{X_1[-k]\}$

$$X[k] + X^*[-k] = 2\,\text{Re}\{X_1[k]\} + 2j\,\text{Im}\{X_1[k]\} = 2\,X_1[k]$$

$$X[k] - X^*[-k] = -2\,\text{Im}\{X_2[k]\} + 2j\,\text{Re}\{X_2[k]\} = 2j\left(\text{Re}\{X_2[k]\} + j\,\text{Im}\{X_2[k]\}\right)$$
$$= 2j\,X_2[k]$$

$$\Rightarrow \begin{cases} X_1[k] = \frac{1}{2}\left(X[k] + X^*[-k]\right) = \frac{1}{2}\left(X[k] + X^*[N-k]\right), & k = 0,\dots,N-1 \\ X_2[k] = \frac{1}{2j}\left(X[k] - X^*[-k]\right) = \frac{1}{2j}\left(X[k] - X^*[N-k]\right), & k = 0,\dots,N-1 \end{cases}$$

Example

$$x_1[n] = \{1 \quad 1 \quad 1 \quad 0\}$$

$$x_2[n] = \{1 \quad 2 \quad 3 \quad 4\}$$

$$x[n] = x_1[n] + j x_2[n] = \{1+j1 \quad 1+j2 \quad 1+j3 \quad j4\}$$

The 4-pt. FFT of $x[n]$ is found in MATLAB AS:

>> x=[1+j1, 1+j2, 1+j3, j4], X= fft(x)

X= [3+j10, -2-j3, 1-j2, 2-j1]

$$X[k] = \{\underbrace{3+j10}_{X[0]} \quad \underbrace{-2-j3}_{X[1]} \quad \underbrace{1-j2}_{X[2]} \quad \underbrace{2-j1}_{X[3]}\}$$

$$X_1[k] = \frac{1}{2}\left(X[k] + X^*[-k]\right) = \frac{1}{2}\left(X[k] + X^*[4-k]\right)$$

$$X_1[0] = \frac{1}{2}\left(X[0] + X^*[4]\right) = \frac{1}{2}\left(X[0] + X^*[0]\right) = \frac{1}{2}\left(2Re\{X[0]\}\right) = 3$$

$$X_1[1] = \frac{1}{2}\left(X[1] + X^*[4-1]\right) = \frac{1}{2}\left(-2-j3 + 2+j1\right) = \frac{1}{2}\left(-j2\right) = -j$$

$$X_1[2] = \frac{1}{2}\left(X[2] + X^*[4-2]\right) = \frac{1}{2}\left(2Re\{X[2]\}\right) = Re\{X[2]\} = 1$$

$$X_1[3] = X_1^*[4-3] = X_1^*[1] = +j$$

using the fact that $X_1[k]$ is Hermitian since $x_1[n]$ is real

$$X_2[k] = \frac{1}{2j}\left(X[k] - X^*[-k]\right) = \frac{1}{2j}\left(X[k] - X^*[4-k]\right)$$

$$X_2[0] = \frac{1}{2j}\left(X[0] - X^*[0]\right) = \frac{1}{2j}\left(2j\, Im\{X[0]\}\right) = Im\{X[0]\} = 10$$

$$X_2[1] = \frac{1}{2j}\left(X[1] - X^*[3]\right) = \frac{1}{2j}\left((-2-j3) - (2+j1)\right) = \frac{1}{2j}\left(-4-j4\right) = -2+j2$$

$$X_2[2] = \frac{1}{2j}\left(X[2] - X^*[2]\right) = \frac{1}{2j}\left(2j\, Im\{X[2]\}\right) = Im\{X[2]\} = -2$$

$$X_2[3] = X_2^*[4-3] = X_2^*[1] = -2-j2$$

using the fact that $X_2[k]$ is Hermitian