Time Series Analysis

ECE 6563, Electrical and Computer Engineering
University of New Brunswick

Today's agenda

- Administrative/Course Details
- Statistics Background
- What is a Time Series
- Time Series Analysis

Course Details

See course outline

Statistics Background

Given the pre-requisites for this course, it is expected that you are already comfortable with several concepts:

- Statistical Estimation
  - Random processes, observation, probability mass/density functions
  - Estimation of unknown parameters from observations and distributions of the random variables
  - Moments: sample mean, variance, skewness, kurtosis
  - Method of Moments Estimation
  - Maximum Likelihood Estimation

Statistics Background

- Multivariate Distributions
  - Uni- and multi-variate Gaussian/Normal distributions, N(μ,σ2)
  - Joint Distribution = Conditional x Marginal

    $f(x,y) = f(x|y)f(y) = f(y|x)f(x)$
  - For 3 variables X,Y,Z

    $f(x,y,z) = f(x|y,z)f(y,z) = f(x|y,z)f(y|z)f(z)$
  - Independence,
    - e.g. joint distribution is the product of marginal distributions

- Statistical Inference
  - Hypothesis Testing
  - p-values, significance level (prop of type 1 error), Cohen's D
  - Confidence Intervals

What is a Time Series?

A set of observations obtained over a period of time

- The time intervals can be annually, quarterly, monthly, weekly, daily, hourly, etc.

Year	2005	2006	2007	2008	2009	2010
Sales	75.3	74.2	78.5	79.7	80.2	80.9

- For many engineering applications (e.g. sensor data), it may be in milliseconds or microseconds
- Usually at regular (equidistant) time intervals, but not always

A stochastic process, sequence of random variables defined for a probability space

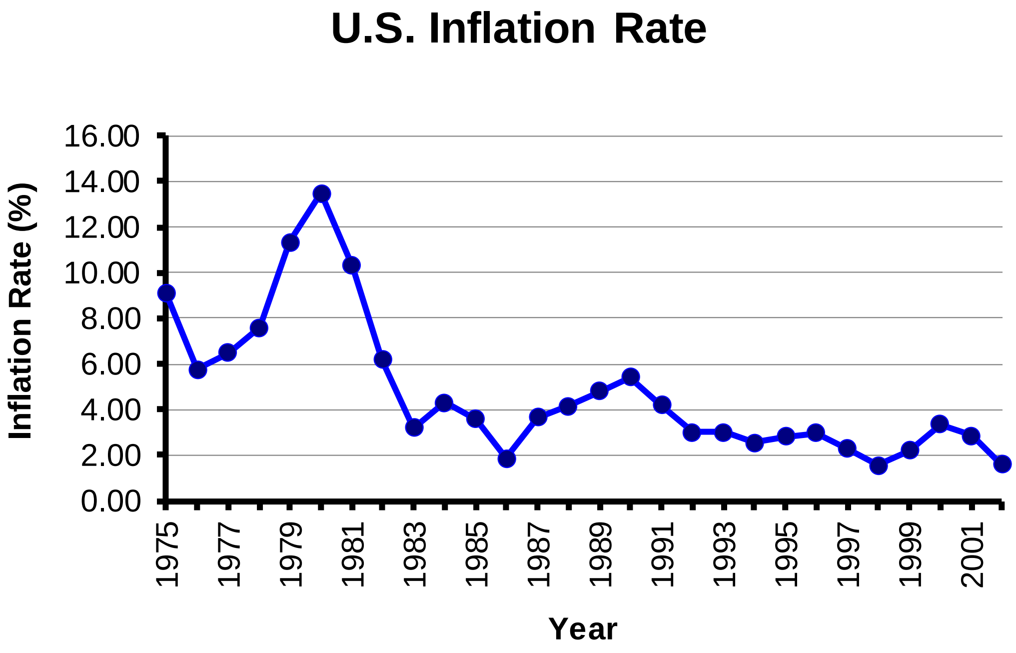
- Examples: sales of wine, accidental deaths, daily temperatures, stock prices, yearly GDP, etc…

What is a Time Series?

A time-series plot (time plot) is a two-dimensional plot of time series data

- The vertical axis measures the variable of interest

- The horizontal axis corresponds to the time periods



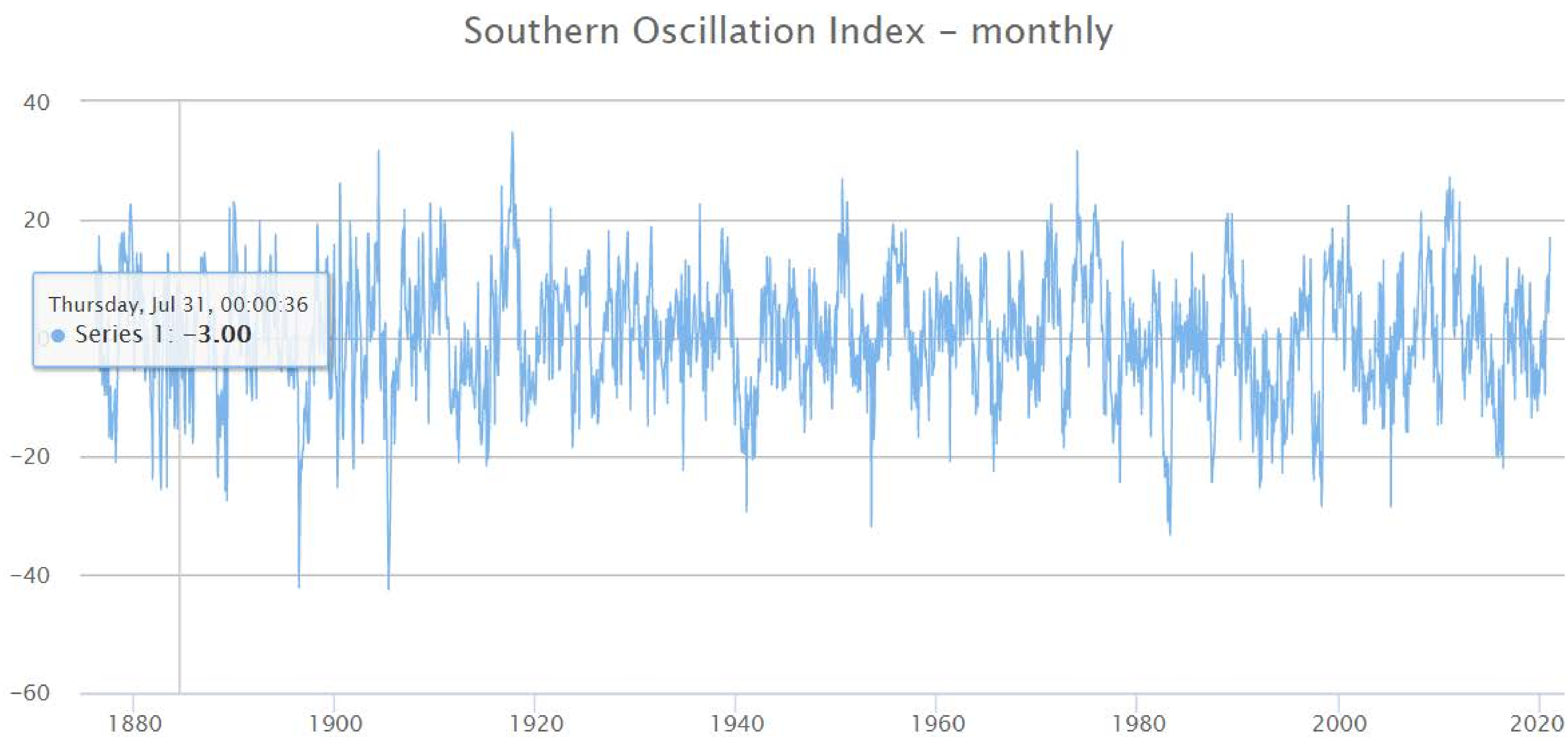
What is a Time Series?

What make time series analysis different from classical statistical analysis?

- There is dependence in the observations. Future points are in some way correlated with previous values
- Ignoring this dependence would lead to inefficient estimates of parameters, poor predictions

Stock Markets

Examples of Time Series



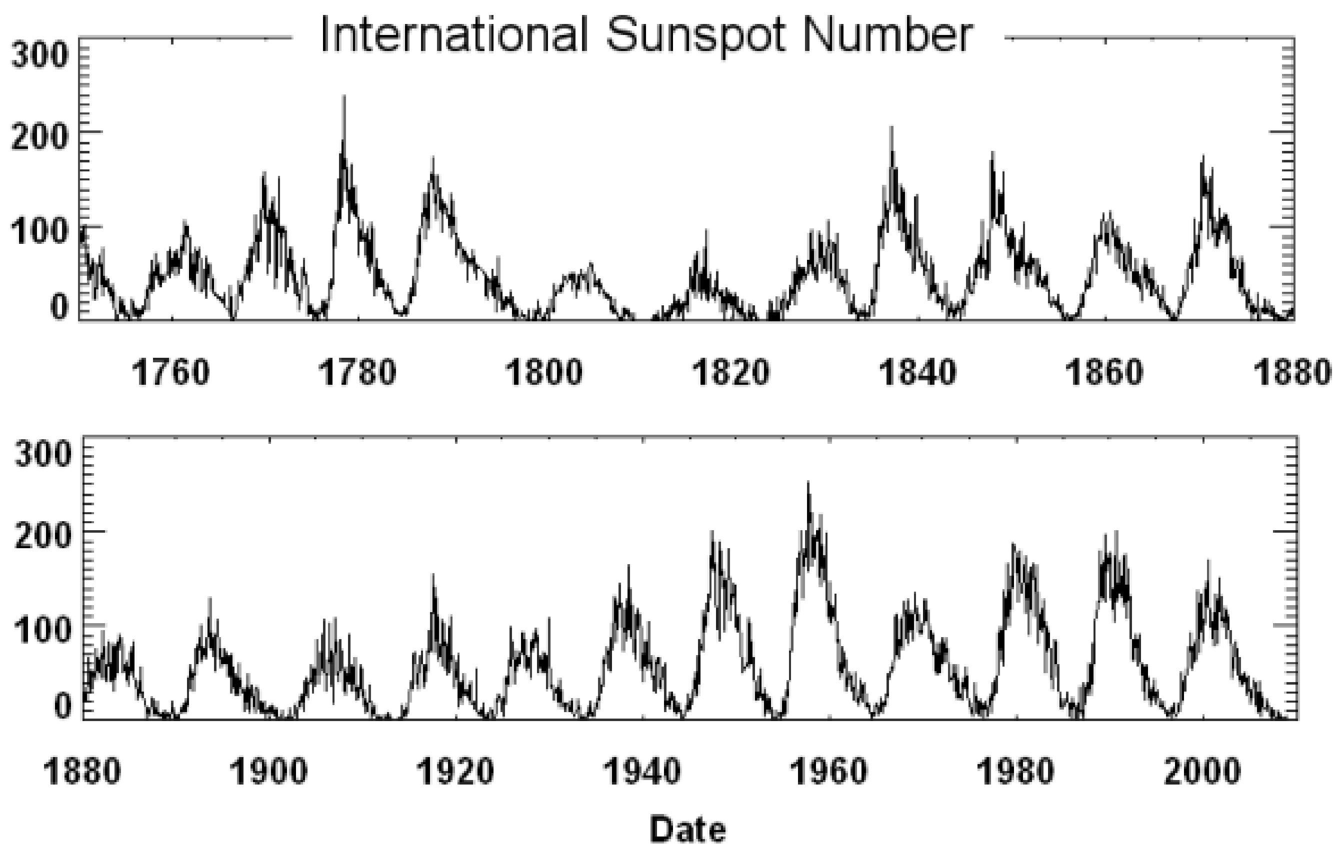
Measure of the Intensity of the El Nino effect in Australia

http://www.bom.gov.au/climate/current/soihtm1.shtml

Examples of Time Series

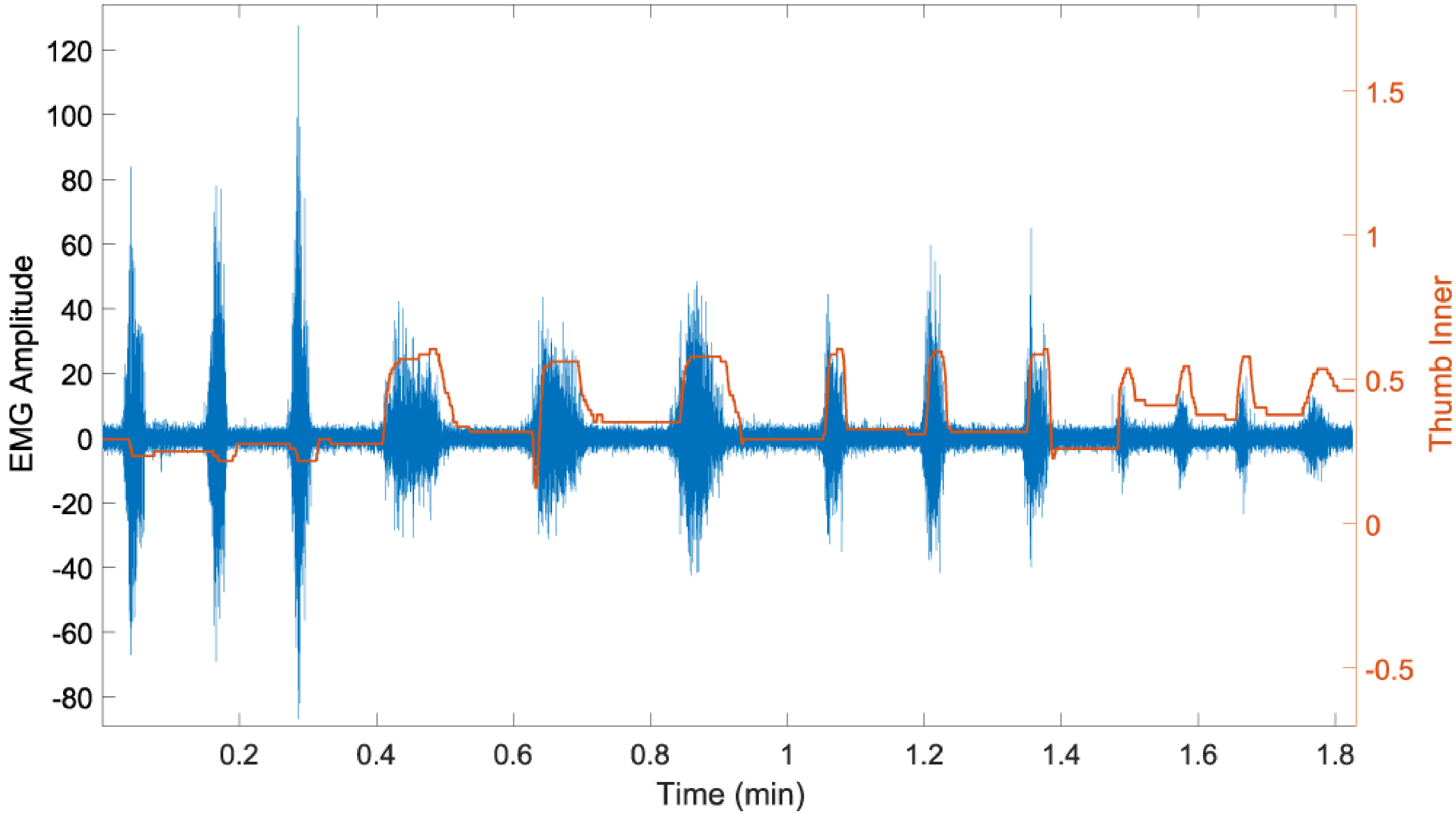
https://www.federalreserve.gov/releases/h10/Hist/dat00_ca.htm

Examples of Time Series



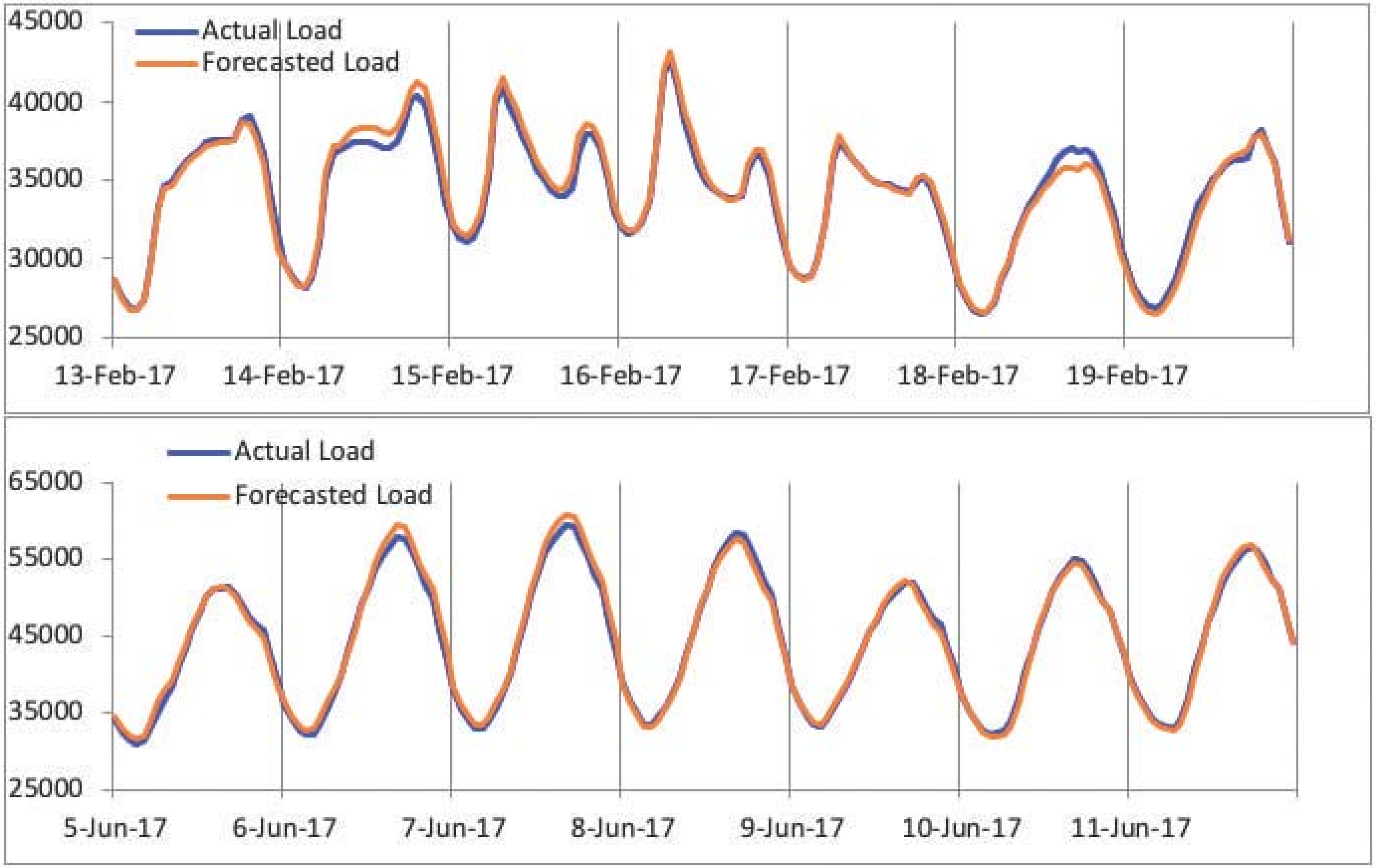
http://www.sidc.be/silso/datafiles

Examples of Time Series



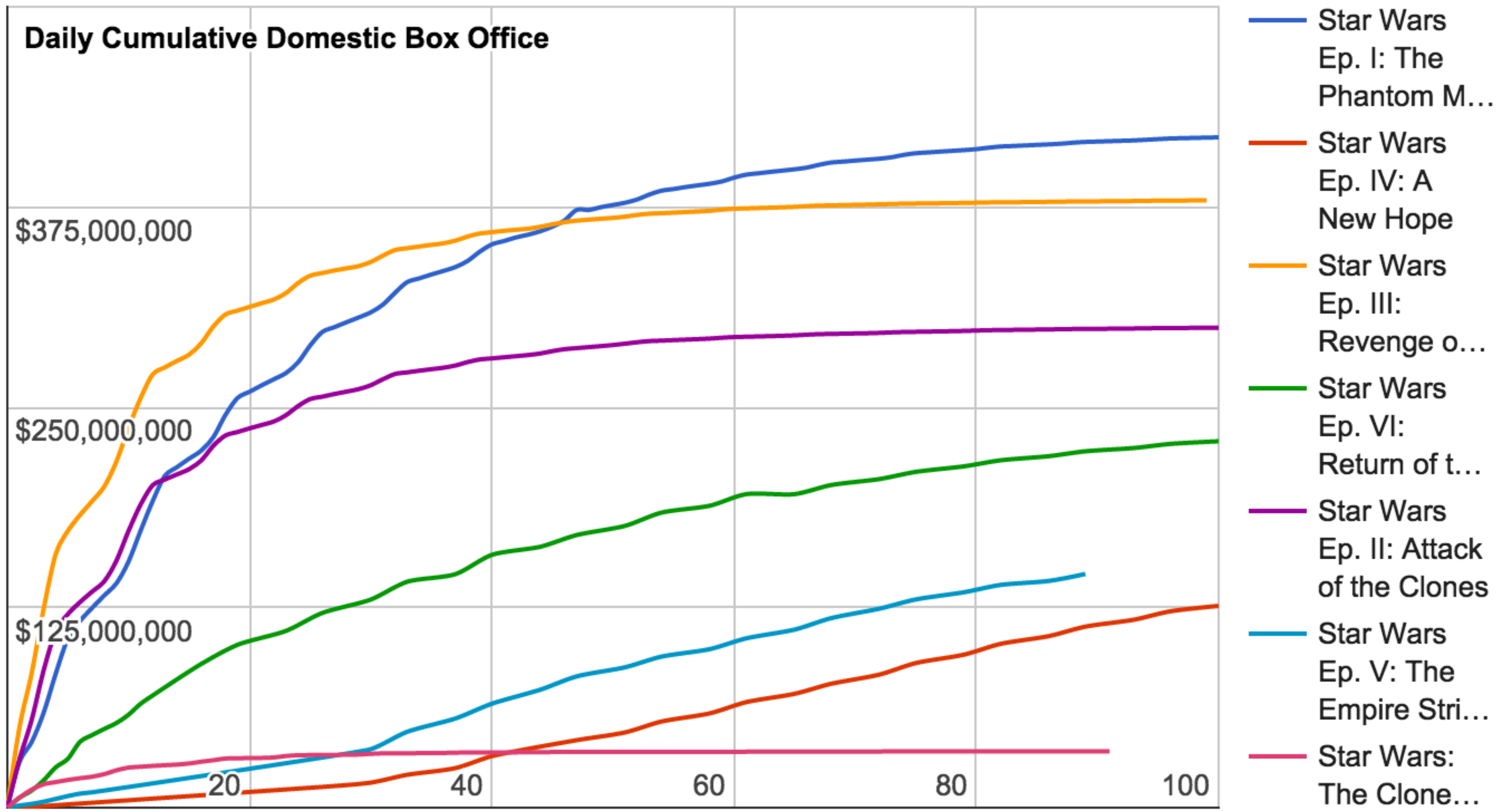
https://www.nature.com/articles/s41597-019-0200-9/figures/4

Examples of Time Series



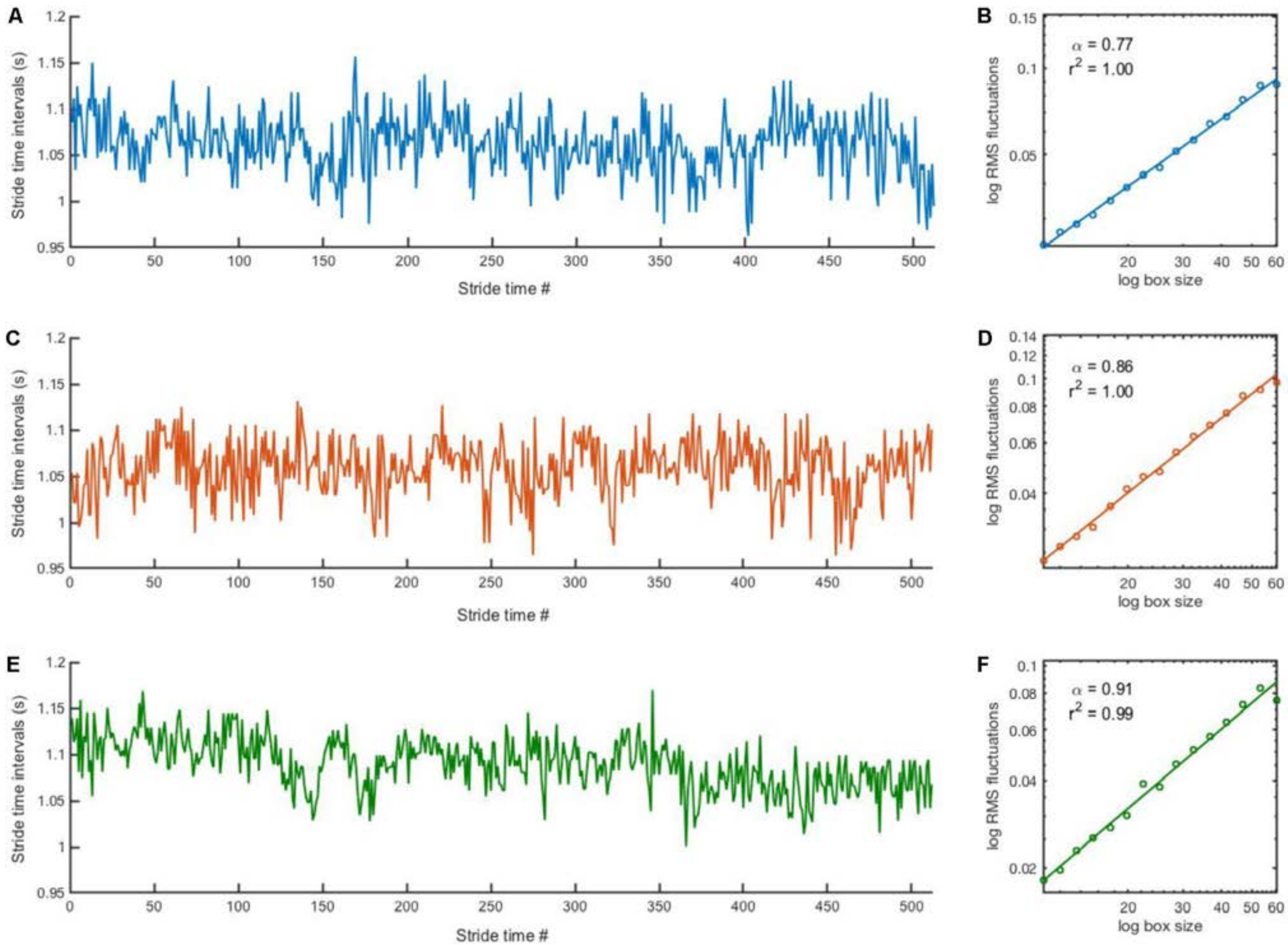
https://www.powermag.com/using-artificial-intelligence-to-develop-electricity-load-forecasts/

Examples of Time Series



http://found-data.blogspot.com/2015/12/movie-data.html

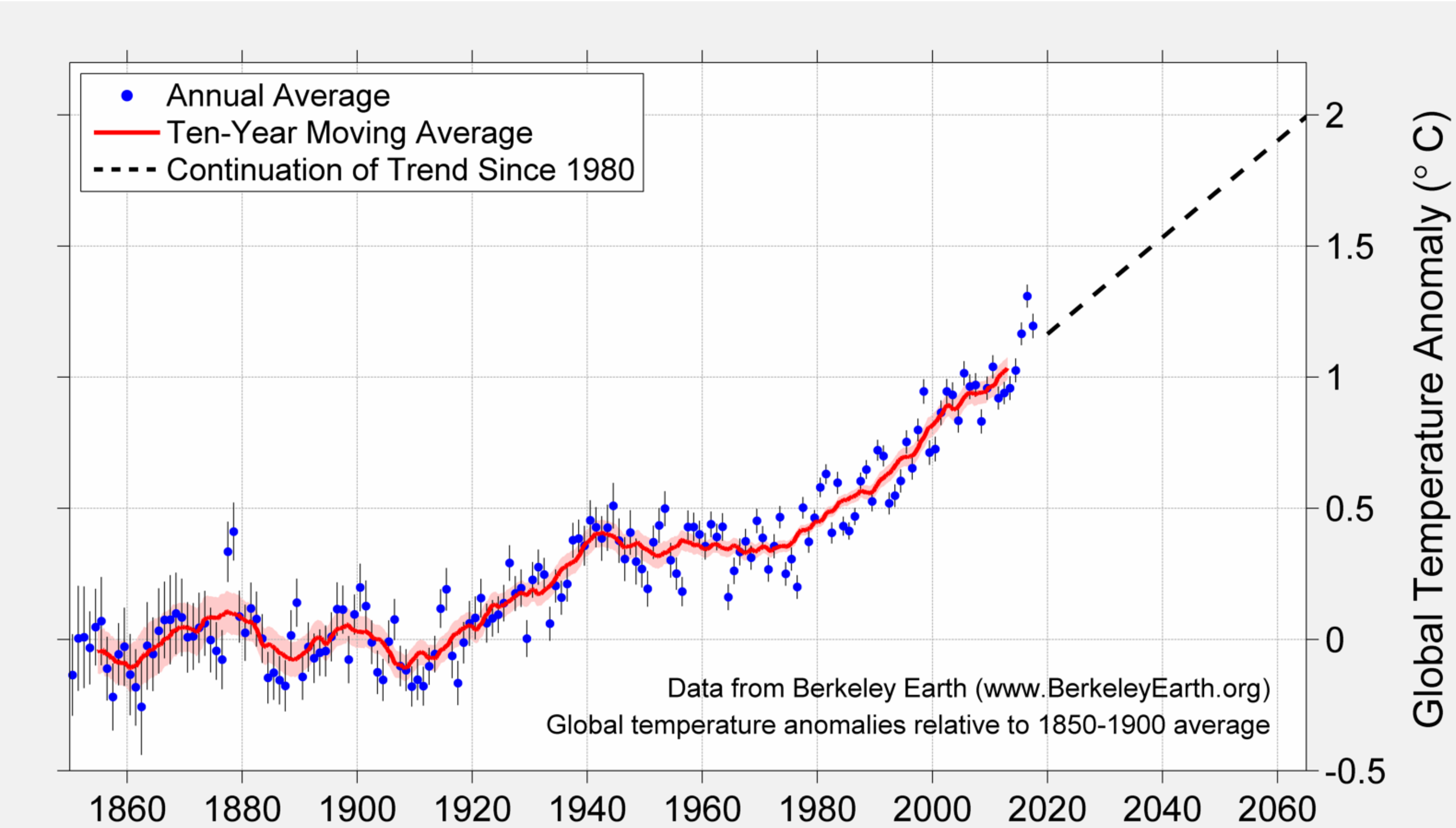
Examples of Time Series



Gait Analysis in Parkinson's Patients using DFA

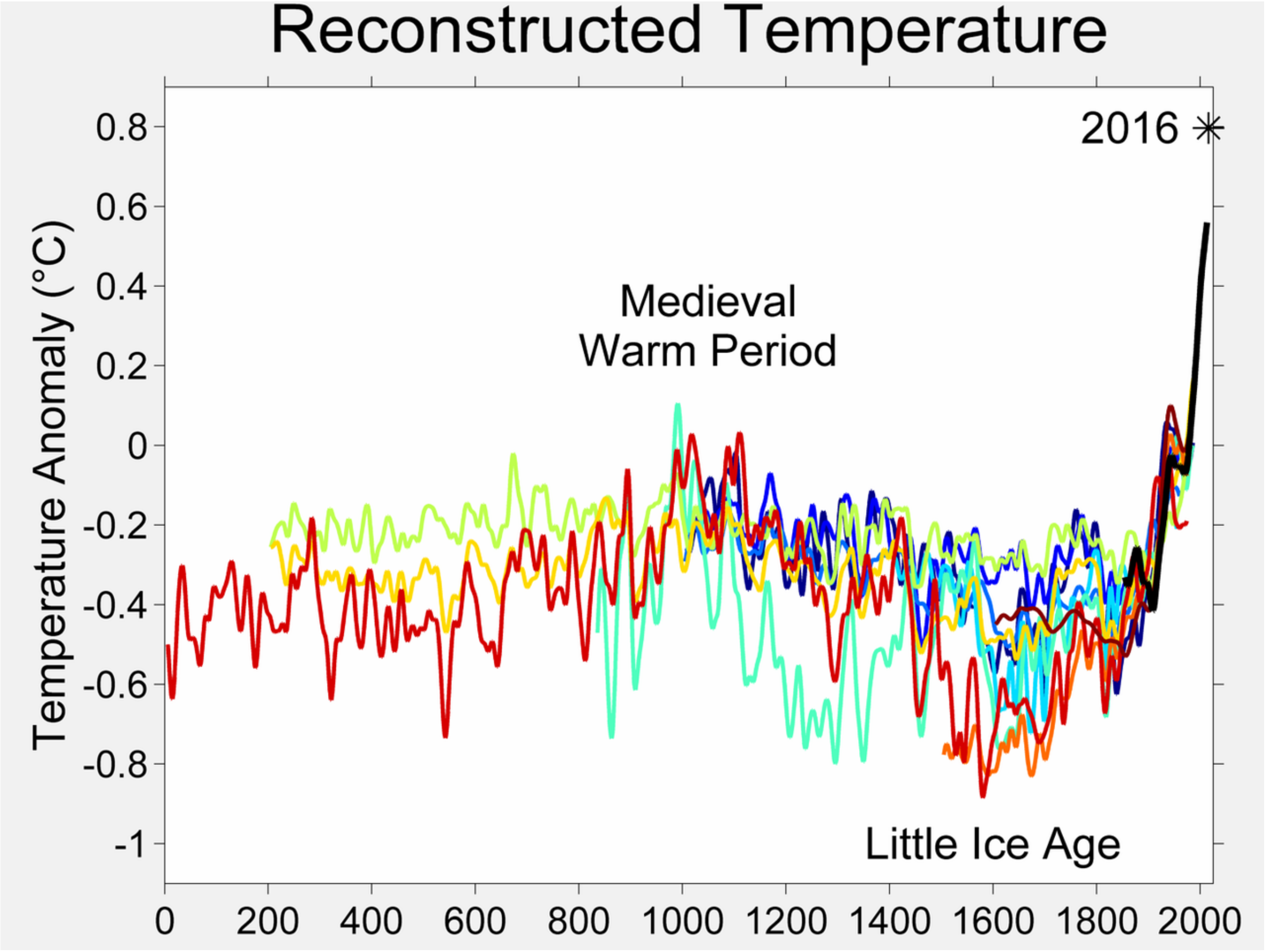
https://www.frontiersin.org/articles/10.3389/fphys.2018.00861/full

Examples of Time Series



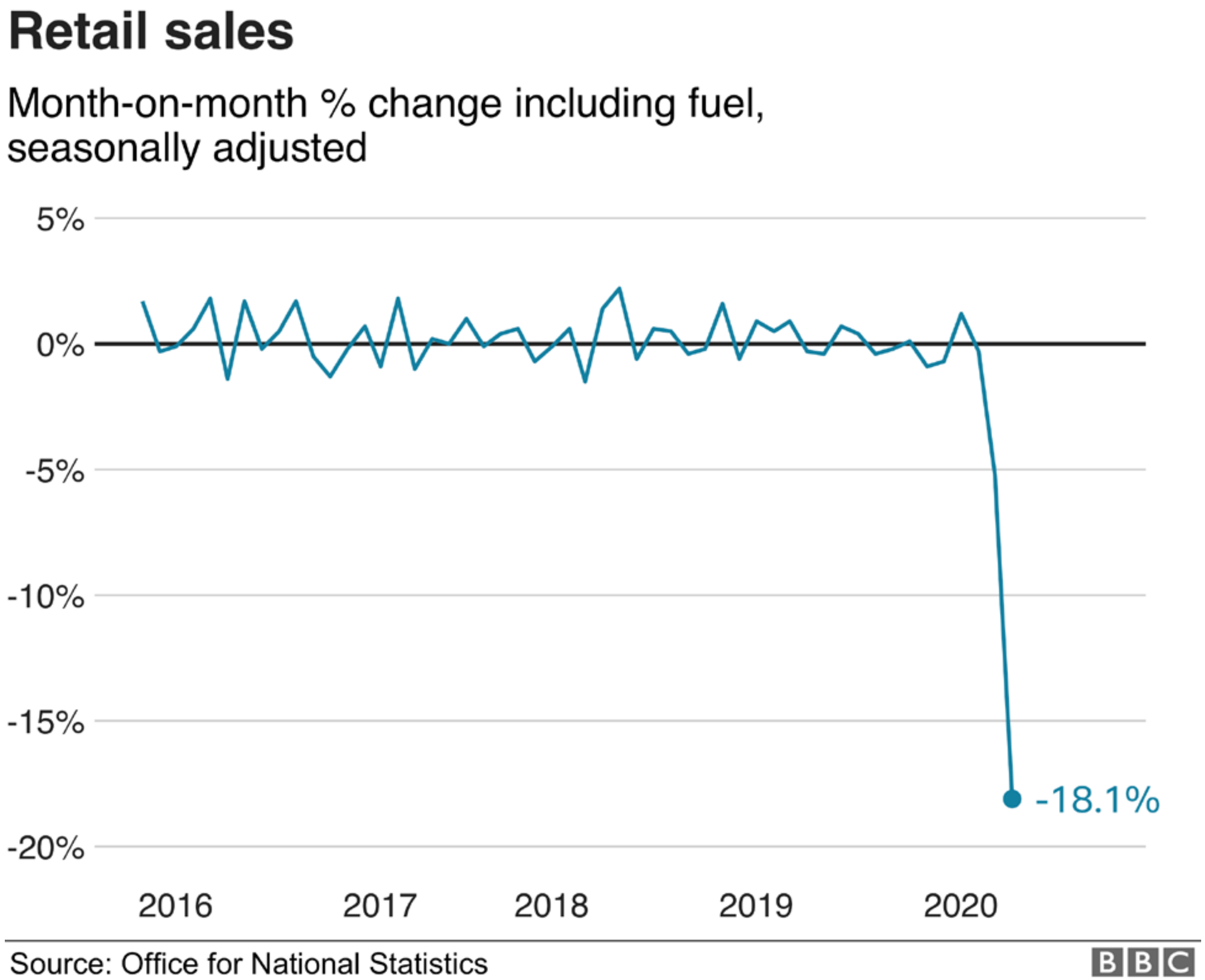
http://berkeleyearth.org/archive/global-temperatures-2017/

Examples of Time Series



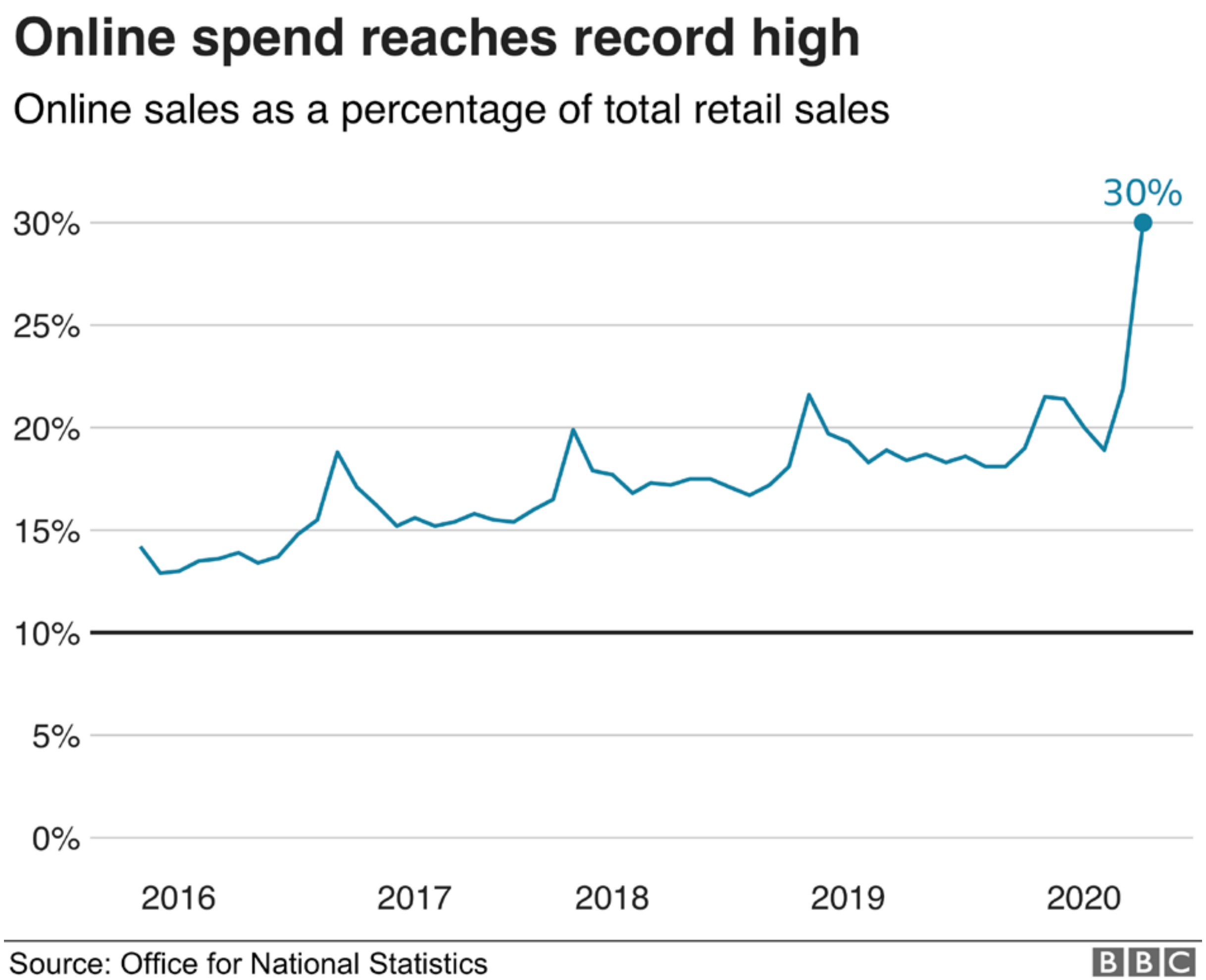
https://simple.wikipedia.org/wiki/Little_Ice_Age#/media/File:2000_Year_Temperature_Comparison.png

Examples of Time Series



https://www.bbc.com/news/business-52766856

Examples of Time Series



https://www.bbc.com/news/business-52766856

Objectives of Time Series Analysis

Why would we want to use time series analysis?

Improve our Understanding

- Plot the data, calculate simple measures (we've all done this)
- Example: Are sales going up? Do I gain weight over the holidays?

Model and Explain Observations

- Can we model the behaviour to describe the time-dependence in the data?
- Example: What is the effect of season on tourism in NB?

Classification

- Can we leverage the dependence to extract features for improved classification
- Example: Auto-Regressive Features in Myoelectric Control

Objectives of Time Series Analysis

Why would we want to use time series analysis?

Forecasting

- Predict the next (or future) values before they happen
- Example: Will it rain tomorrow?

Anomaly Detection

- Detect when something has changed in a modeled system
- Example: Has the stock market changed?

Control Systems

- Adjust some control parameter based on a forecast
- Example: Should we change power production to meet the forecasted demand?

Time Series Modeling

The goal of time series modeling is to explain the behavior of time series

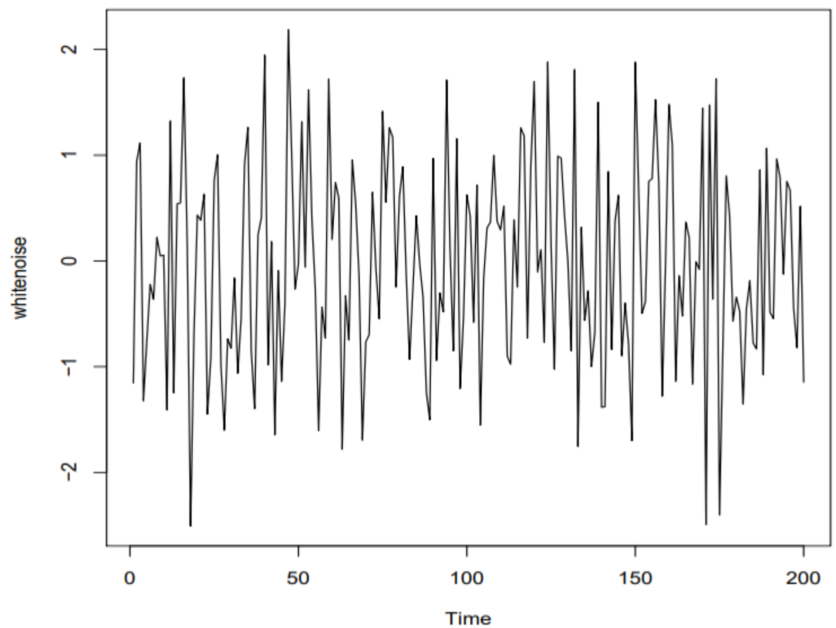
This is conventionally done by breaking the time series (the signal) up into its constituent parts

- Requires estimation/removal/handling of different components

Time-Series Modeling

The most basic time series model is as follows:

$$X_t = \mu_t + Y_t$$



Time-Series Modeling

The most basic time series model is as follows:

$$\boxed{X_t = \mu_t + Y_t}$$

Stochastic process

Time-Series Modeling

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Stochastic process

Note: We will want this to be stationary for future analysis…

Time-Series Modeling

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Stochastic process

random noise

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Stochastic process   mean   random noise

Y_t is the random, residual part of the signal that can't be explained by the μ_t term

Time-Series Modeling

The most basic time series model is as follows:

$$X_t = \mu_t + Y_t$$

Stochastic process

mean

random noise

Y_t is the random, residual part of the signal that can't be explained by the μ_t term

- If we can estimate μ_t, we can isolate these residuals for further analysis
- $Y_t = X_t - \mu_t$

Time-Series Modeling

The residuals, Y_t (or often ε_t) are the unpredictable random, "residual" fluctuations

- "Noise" in the time series

The truly irregular component of cannot be estimated, but there is often some amount of structure (the random component) that can be.

- The focus of time series analysis using stationary ARIMA time series models
- Done after removing as many of the different components as possible

Time-Series Components

So, what are these components?

Time-Series Components

So, what are these components?

Time Series

Residual Component

Irregular,
Erratic, Noise,
or residual
fluctuations

Time-Series Components

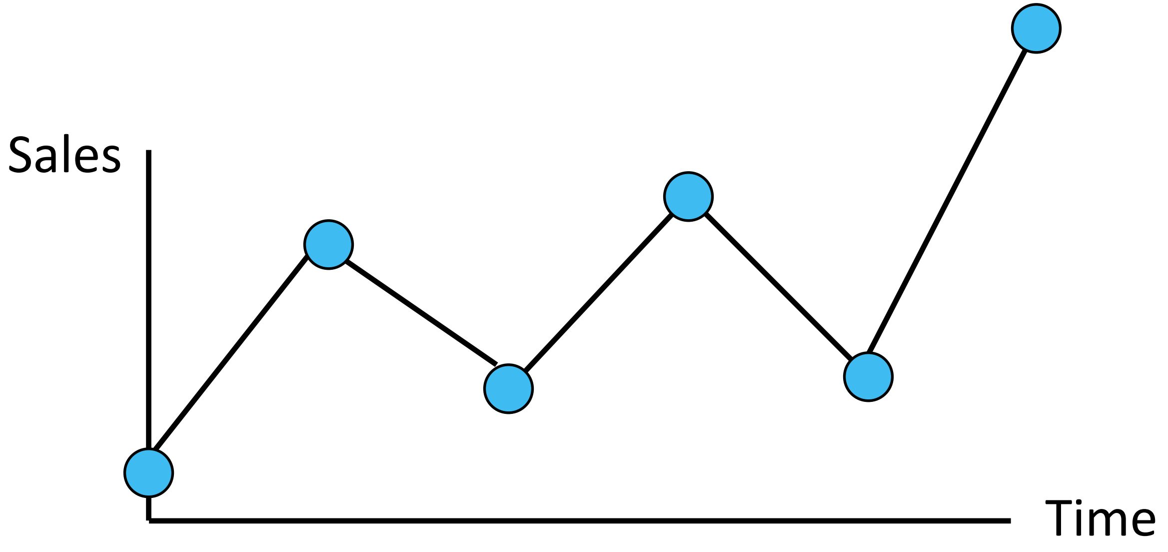
Time Series

- **Trend Component**: Overall, persistent, long-term movement
- **Residual Component**: Irregular, Erratic, Noise, or residual fluctuations

Time-Series Components

Trend: An overall increase or decrease over some period of time

- Overall upward or downward movement
- Typically taken over a length of time substantially longer than the sampling period



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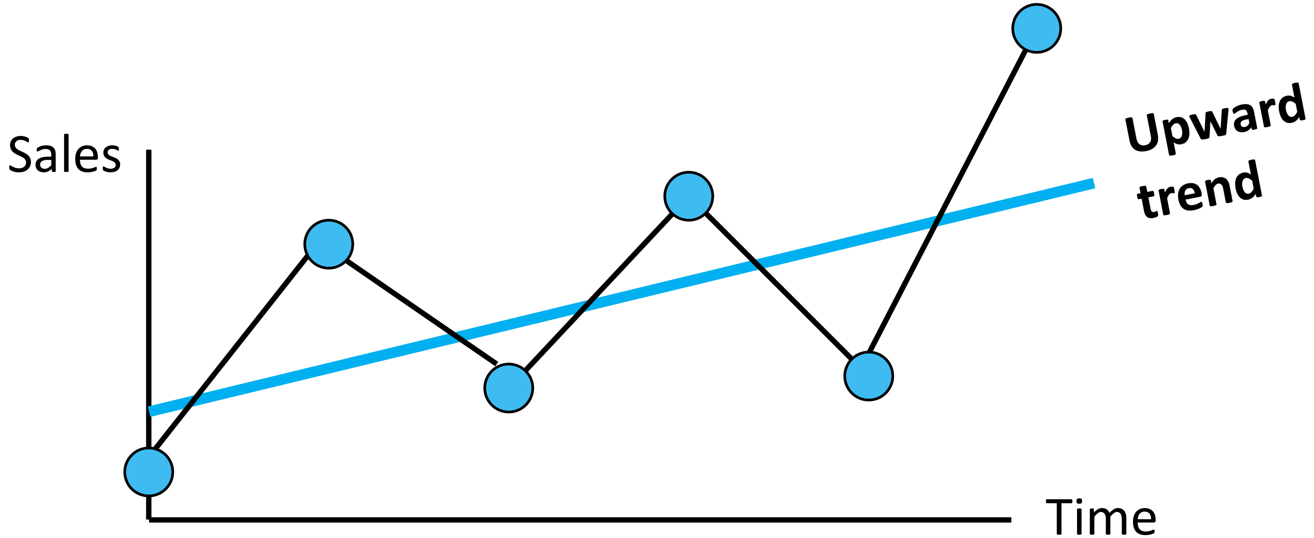
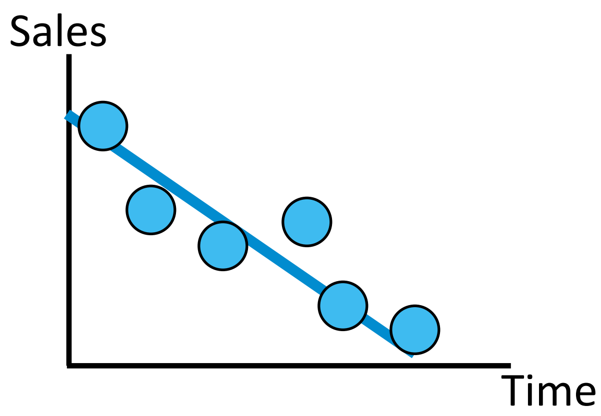


Time-Series Components

Trends can be:

- upward or downward

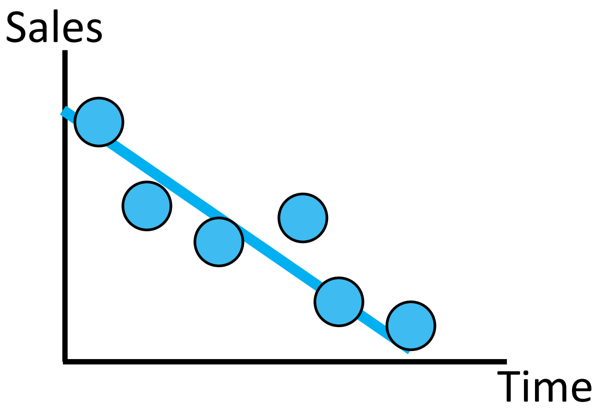


Downward trend

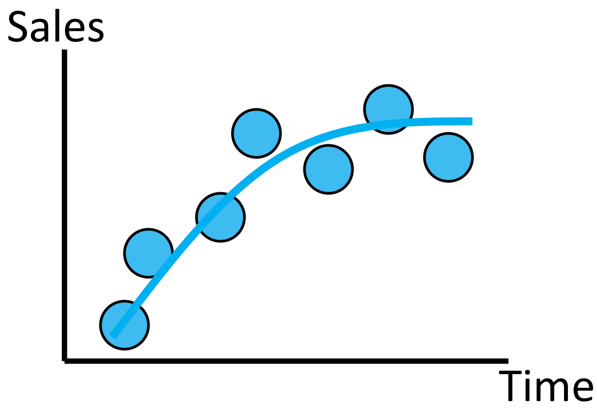
Time-Series Components

Trends can be:

- upward or downward
- linear or nonlinear



Downward
trend

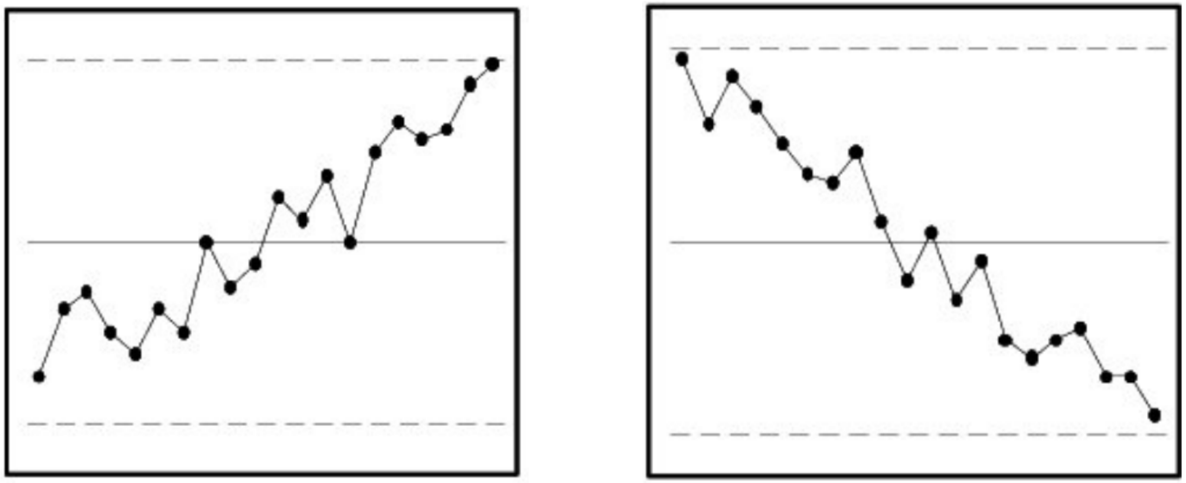


Upward nonlinear
trend

Time-Series Components

A simple time series model that incorporates a trend is as follows:

$$X_t = m_t + Y_t$$



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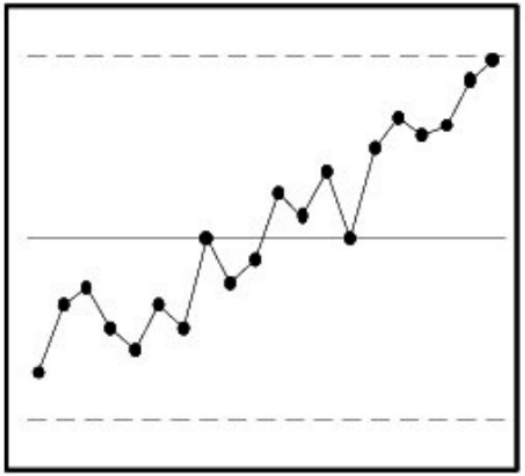

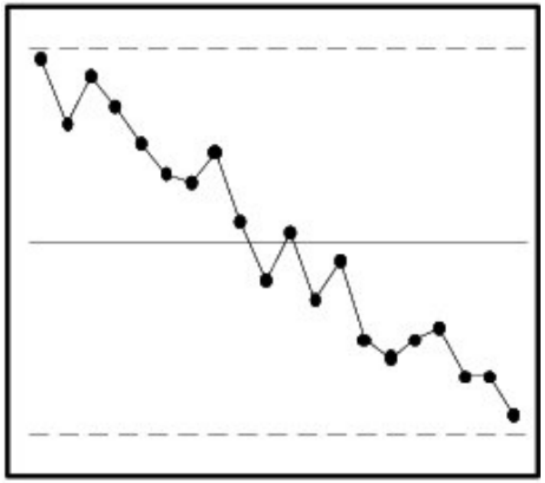

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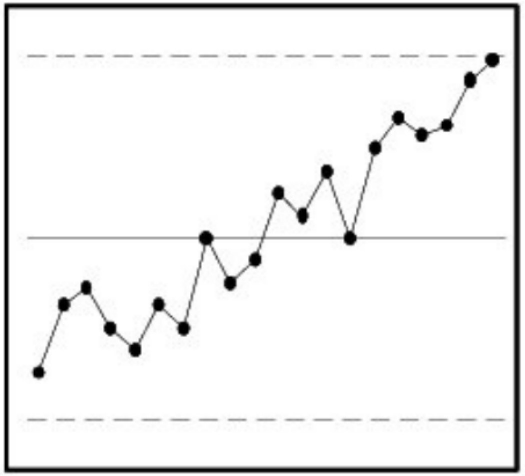
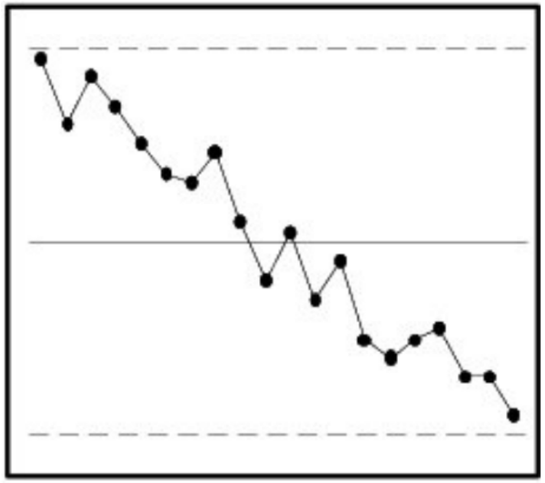
Stochastic process

random noise





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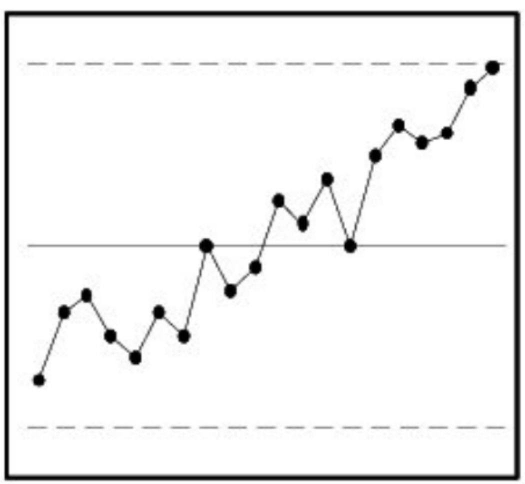

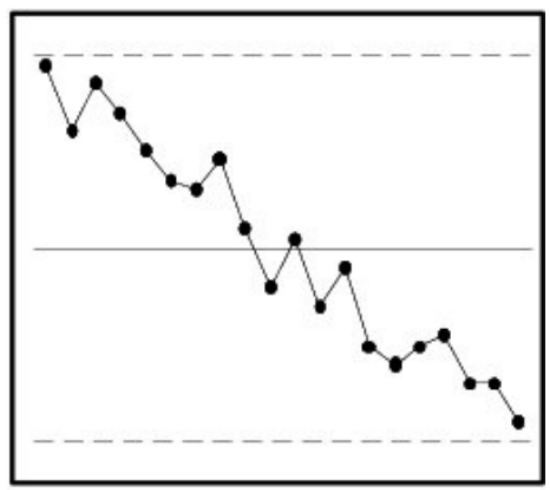

Time-Series Components

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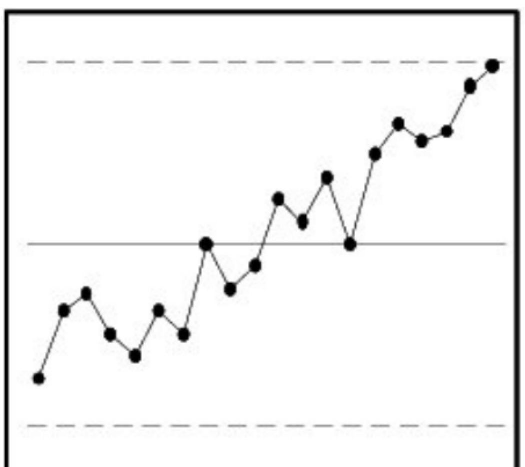
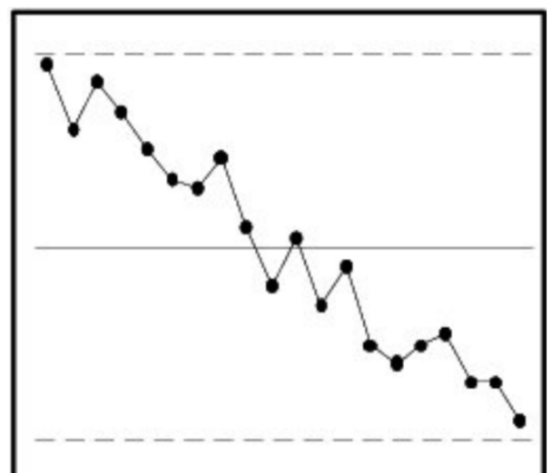
Stochastic process

trend

random noise



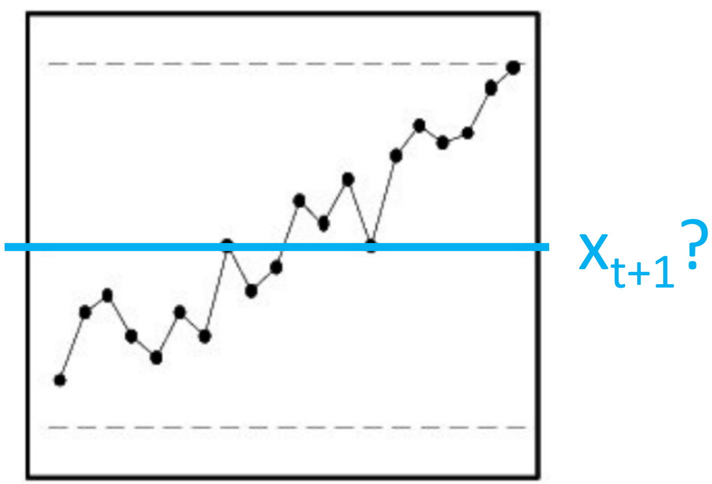


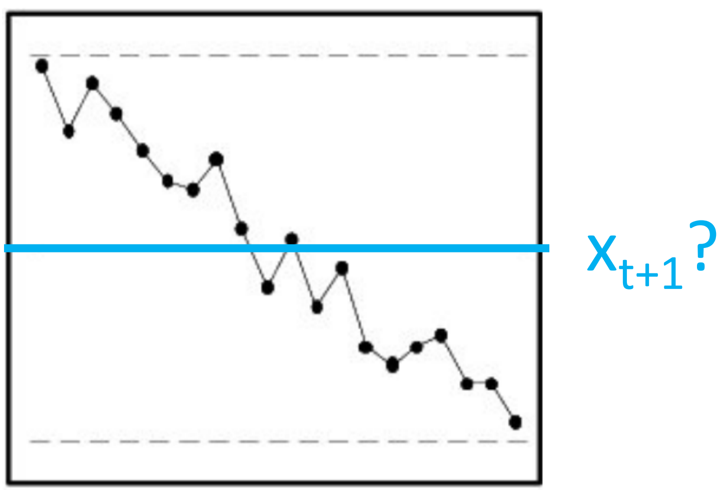
Time-Series Components

If we just used our original $X_t = \mu_t + Y_t$ model, we would not do well predicting future points.

For example, we would have to predict that a future point:

$$x_{t+1} = E[X_t] = \mu_t$$



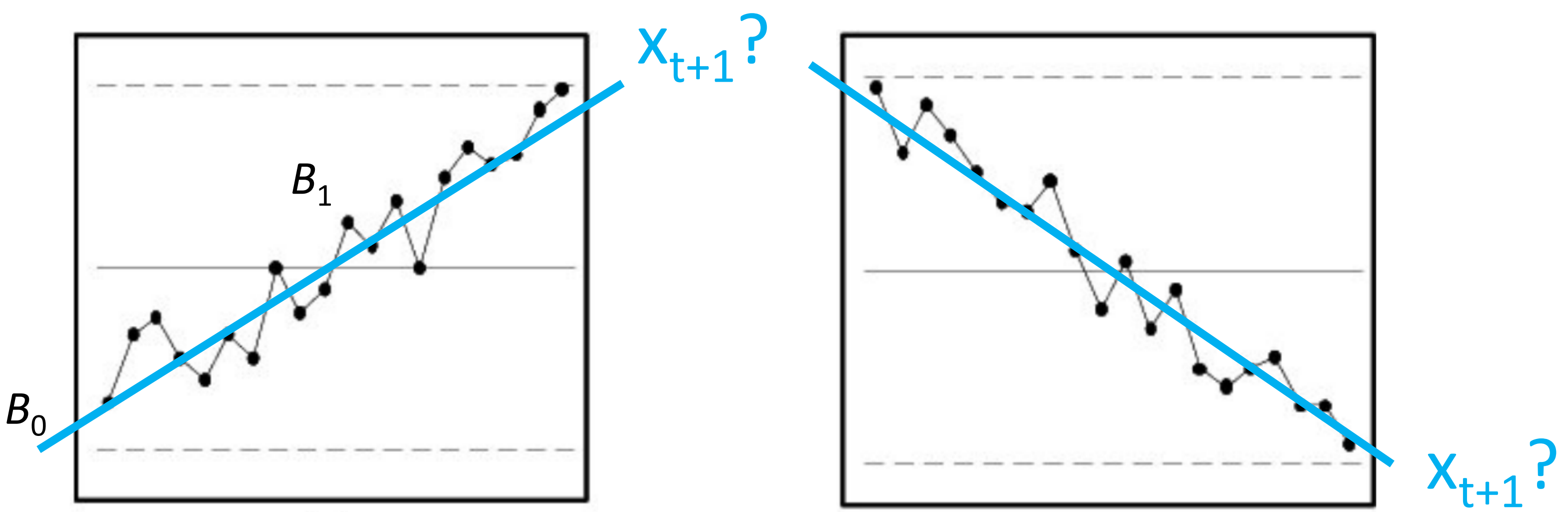


Time-Series Components

Instead, we can use the trend model to improve our prediction

$$X_t = m_t + Y_t$$

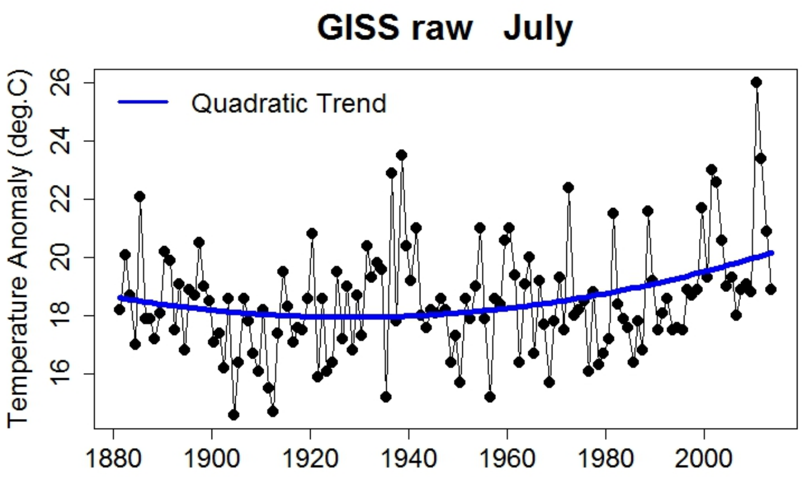
$$X_t = (B_0 + B_1 t) + Y_t$$



Time-Series Components

Note that we aren't limited to a simple linear trend model.

$$X_t = (B_0 + B_1 t + B_1 t^2) + Y_t$$



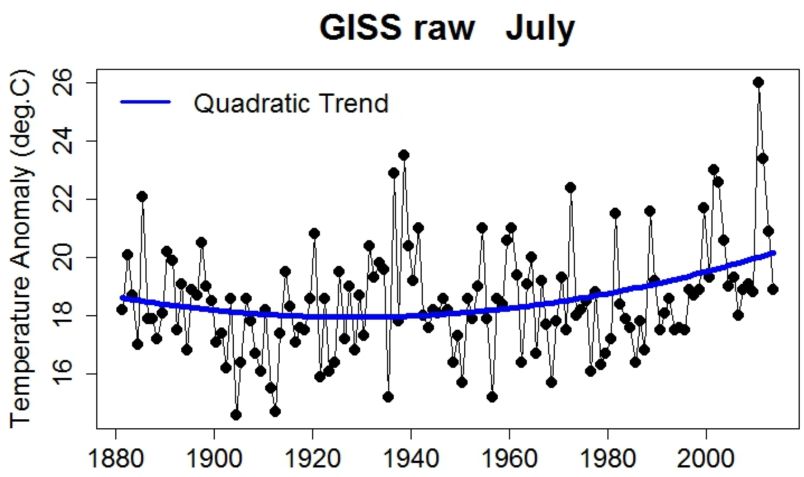
https://tamino.wordpress.com/2014/04/18/non-linear-trends/

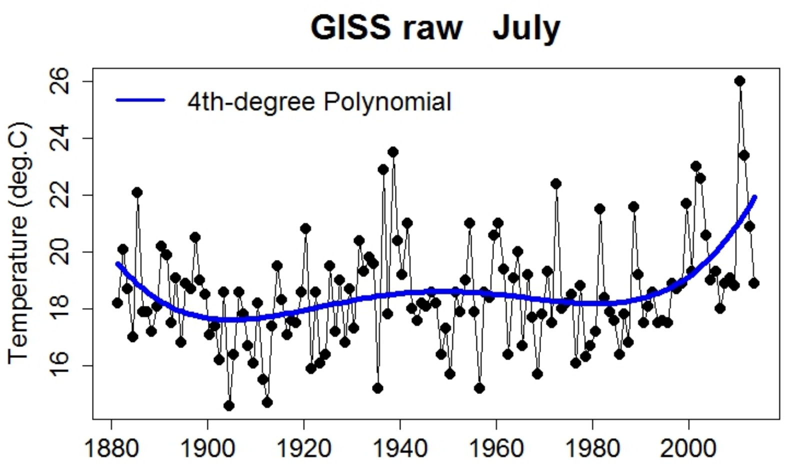
Time-Series Components

Note that we aren't limited to a simple linear trend model.

$$X_t = (B_0 + B_1 t + B_2 t^2) + Y_t$$

But most trends are modeled as either linear, quadratic to avoid overfitting the data





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Time-Series Components

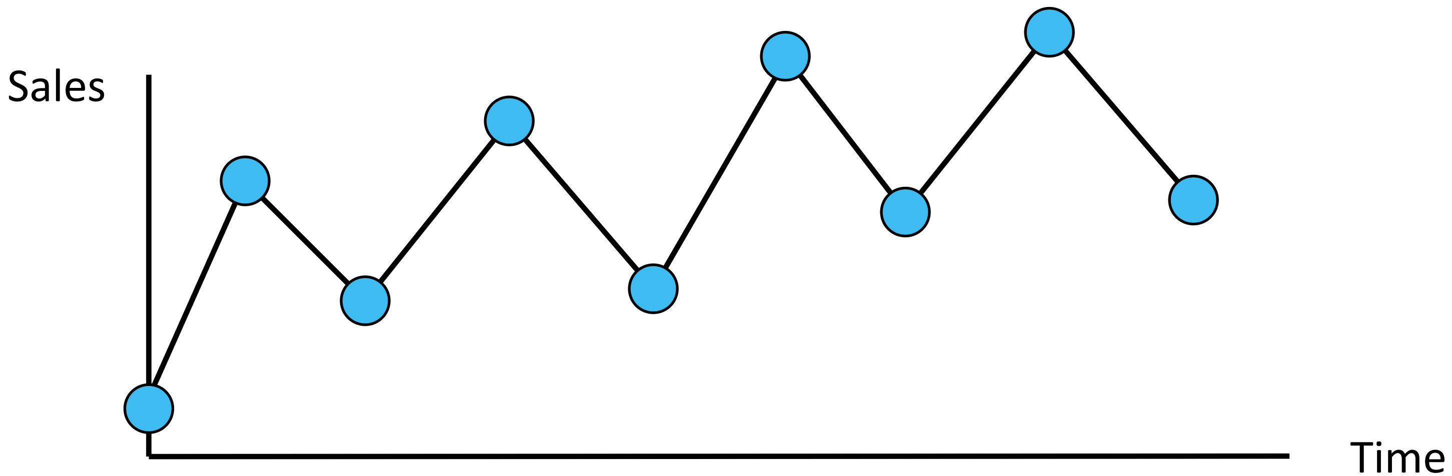
Time Series

- **Trend Component** — Overall, persistent, long-term movement
- **Seasonal Component** — Regular periodic fluctuations
- **Residual Component** — Irregular, Erratic, Noise, or residual fluctuations

Time-Series Components

Seasonal Components are shorter-term (relative to the signal), regular wave-like patterns

- A periodic, repetitive, predictable pattern in level



Time-Series Components

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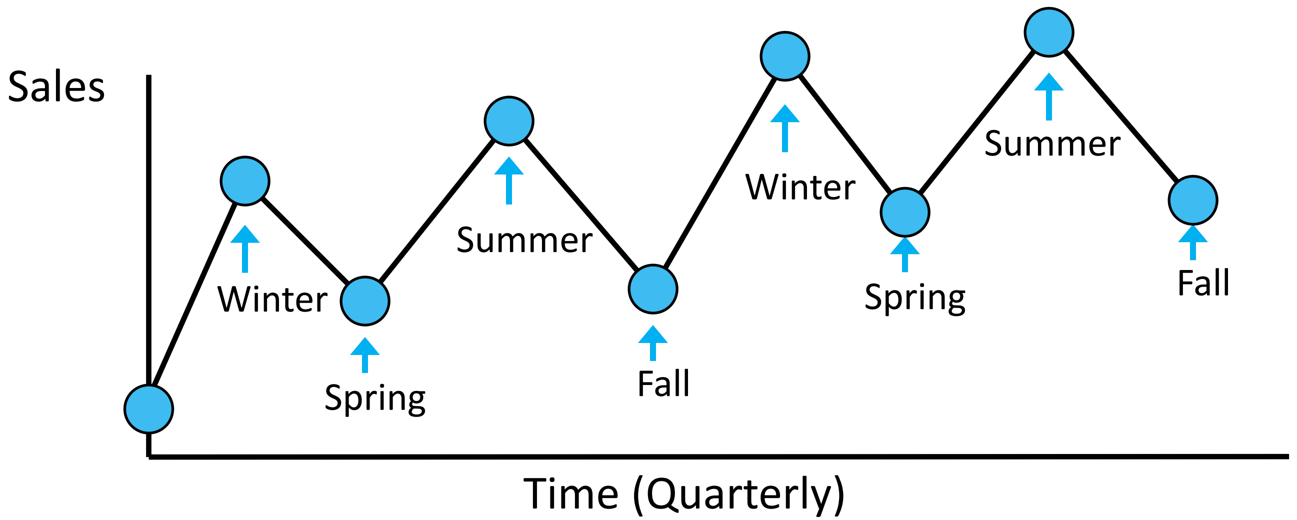
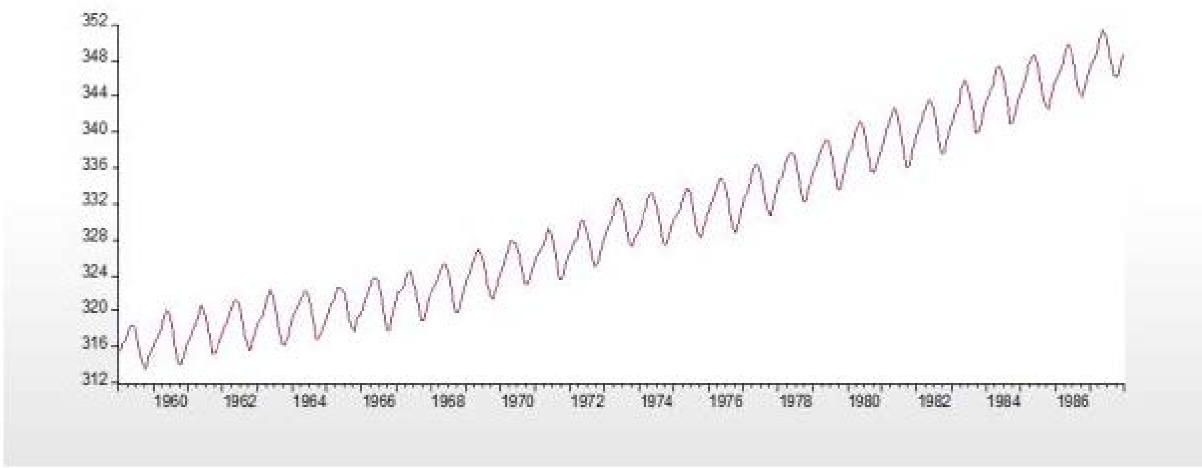
- A periodic, repetitive, predictable pattern in level
- e.g. monthly or quarterly, observed over years



Time-Series Components

A time series model that incorporates trend and seasonality is as follows:

$$X_t = m_t + s_t + Y_t$$



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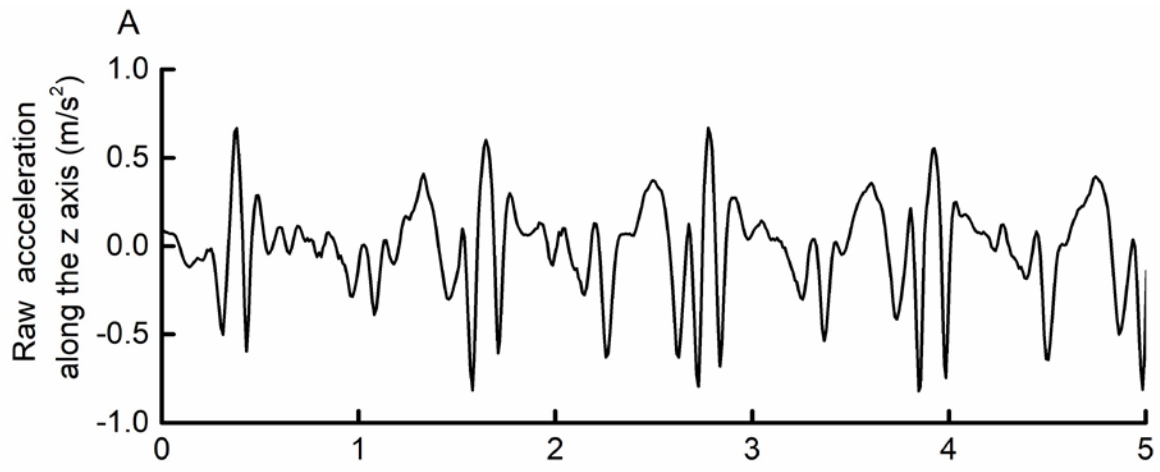
Stochastic process

trend

random noise



https://mhealth.jmir.org/2018/1/e36/

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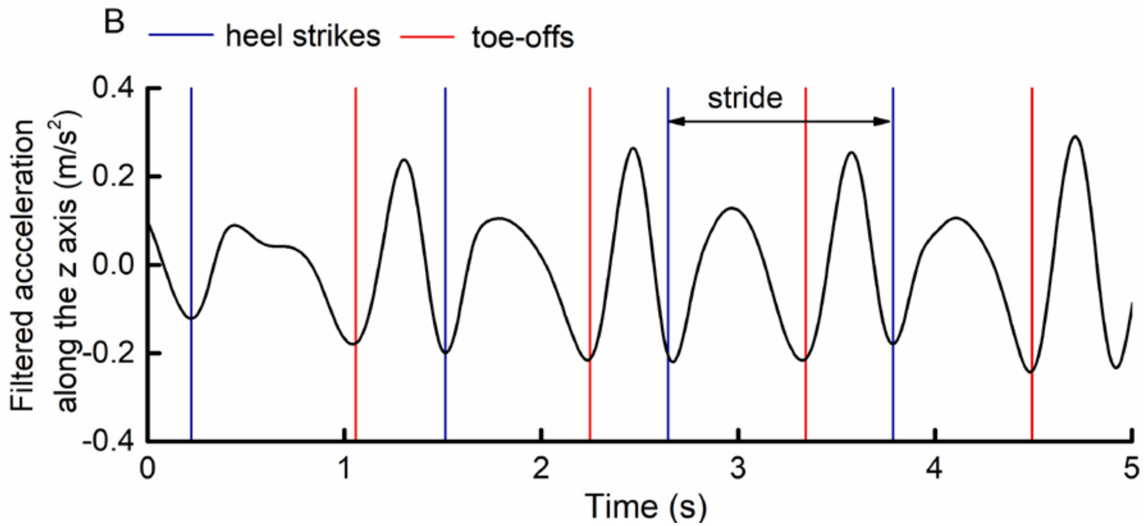

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Stochastic process

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random noise

Because seasonality is periodic, we can represent it as a sinusoidal signal:

$$X_t = m_t + s_t + Y_t = (\beta_0 + \beta_1 t) + \underline{\left[\sum_{j=1}^{k} \left(\alpha_j \cos(\lambda_j t) + \gamma_j \sin(\lambda_j t)\right)\right] + Y_t}$$

Time-Series Components

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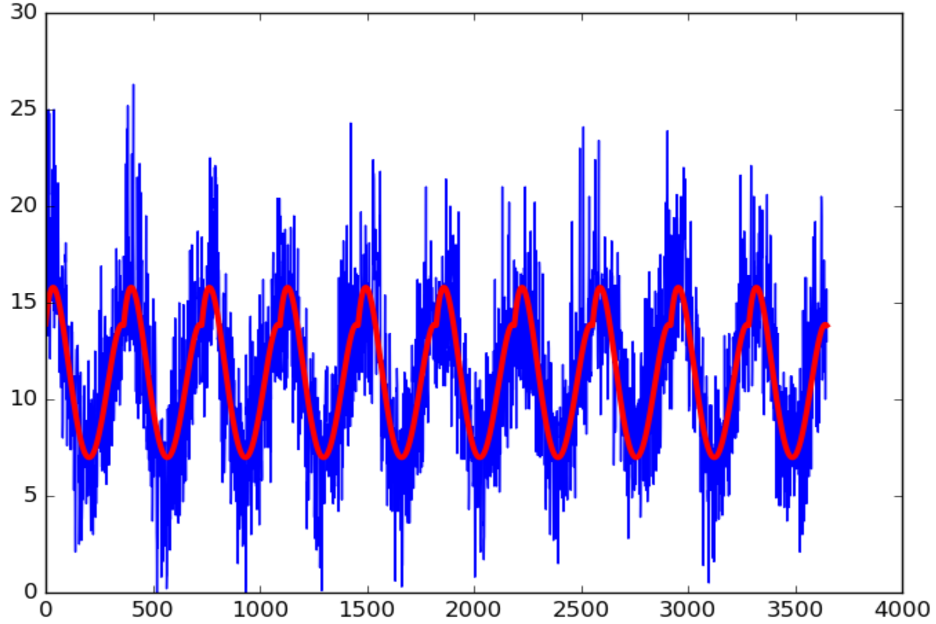

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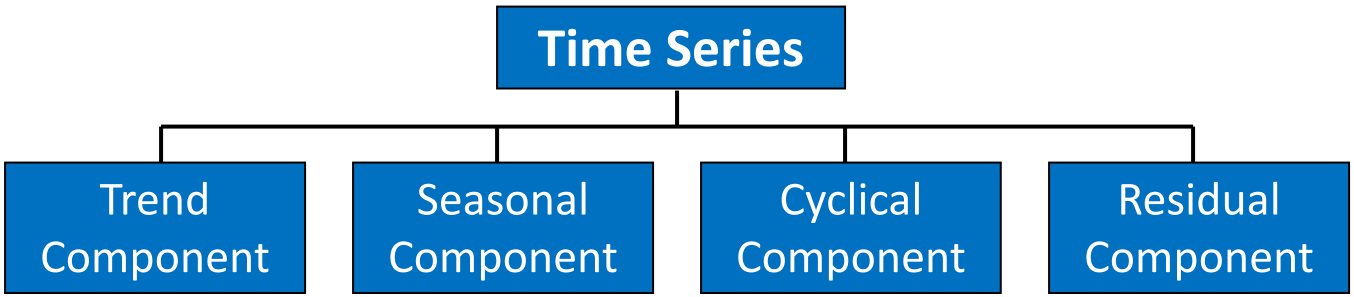


Overall, persistent, long-term movement

Regular periodic fluctuations

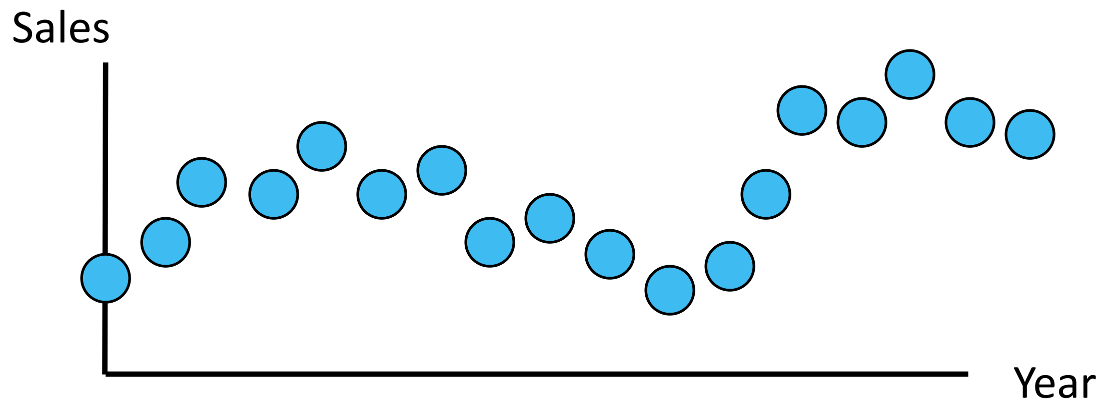
Irregular oscillations or movements

Irregular, Erratic, Noise, or residual fluctuations

Time-Series Components

Cyclical Components are longer-term, irregular wave-like patterns

- Similar to seasonal, but their period may vary in length



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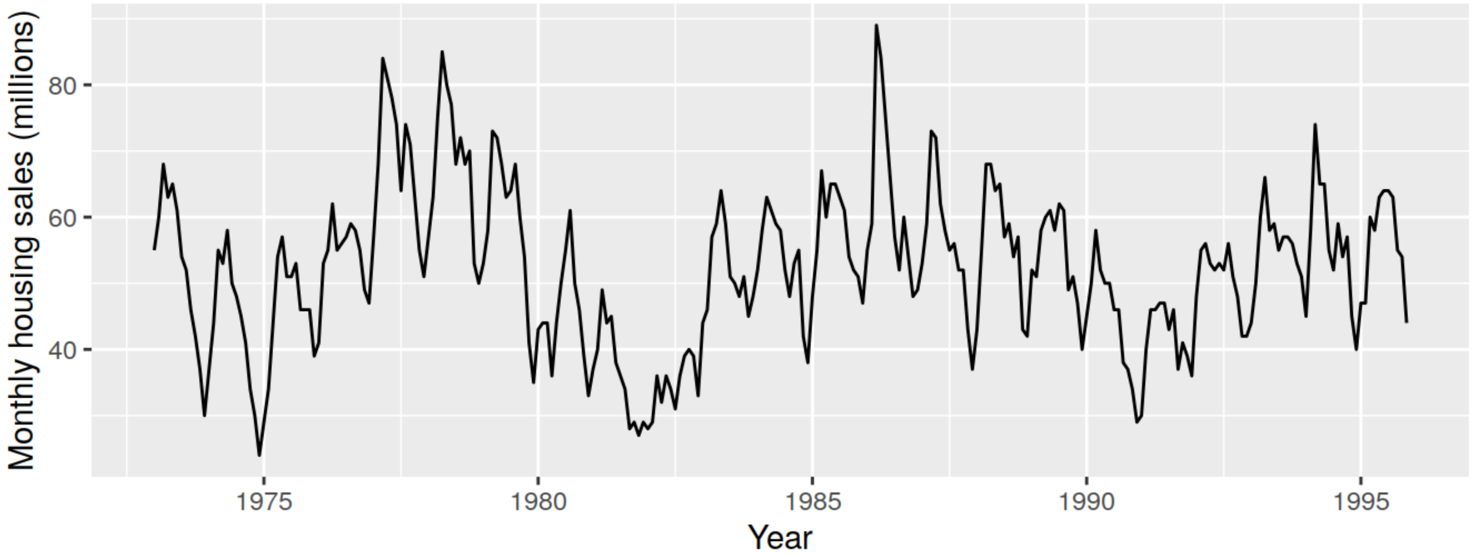
- Usually due to other factors, like economic conditions



https://robjhyndman.com/hyndsight/cyclicts/

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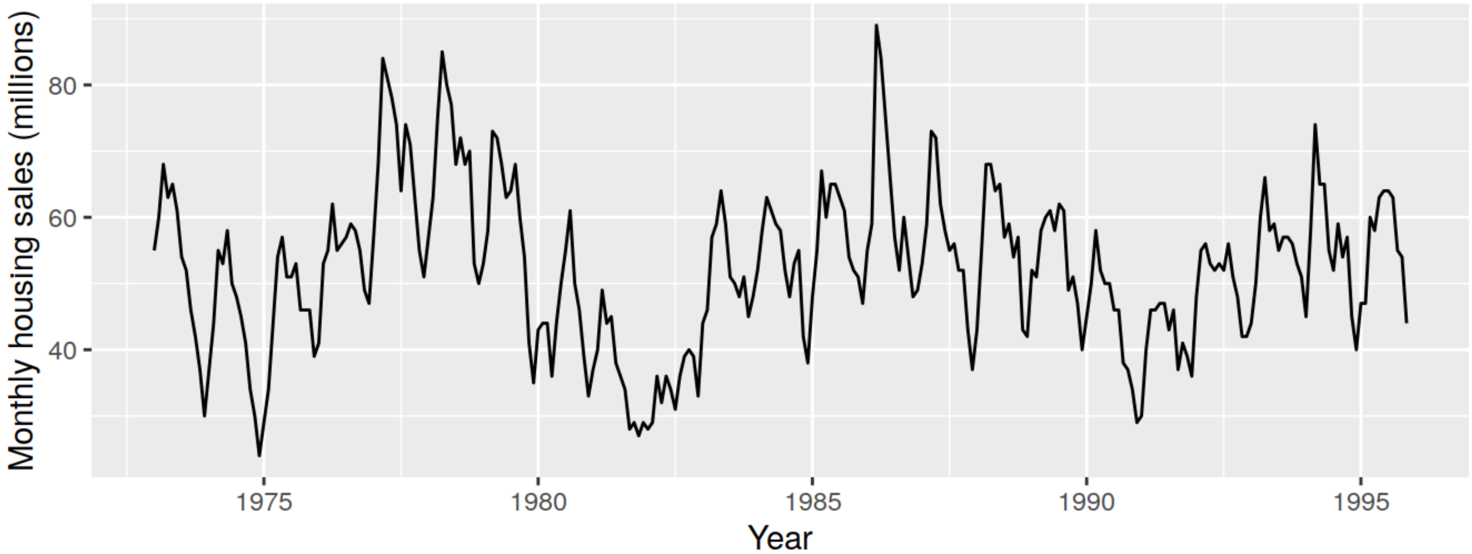


Note the presence of both seasonal AND cyclic behaviours: $X_t = m_t + s_t + c_t + Y_t$

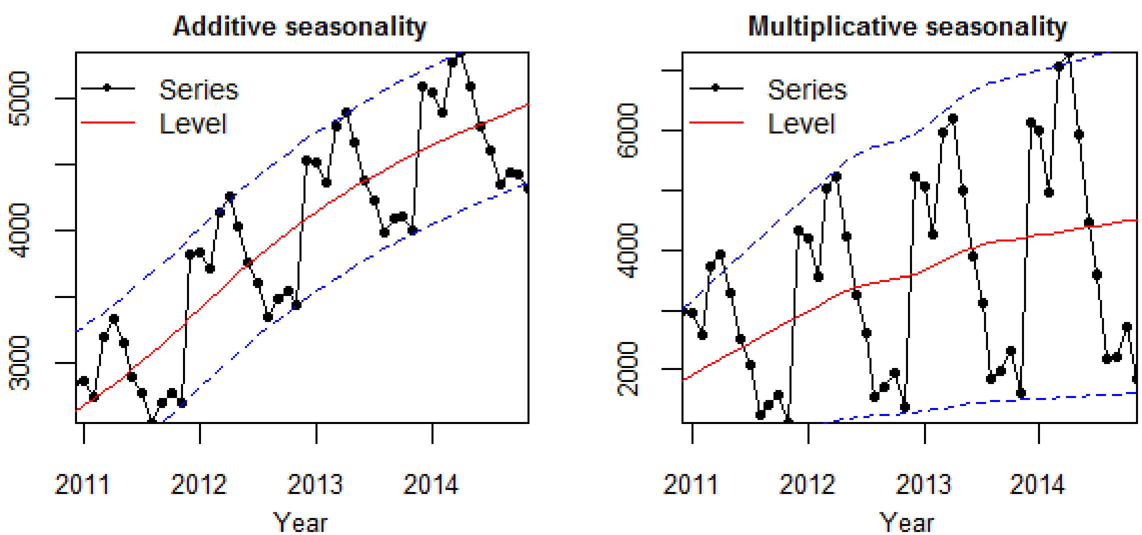
https://robjhyndman.com/hyndsight/cyclicts/

Time-Series Components

Additive Model

So far, our model suggests that all of the components are added together as follows: y(t) = Trend + Seasonality + Noise

- An additive seasonality has the same frequency (width of cycles) and amplitude (height of cycles).
- Example: there are typically 10,000 more flights than the trend in June.



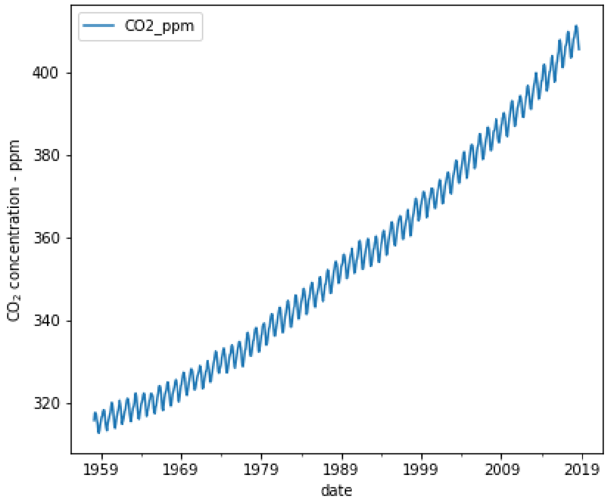
https://kourentzes.com/forecasting/2014/11/09/additive-and-multiplicative-seasonality/

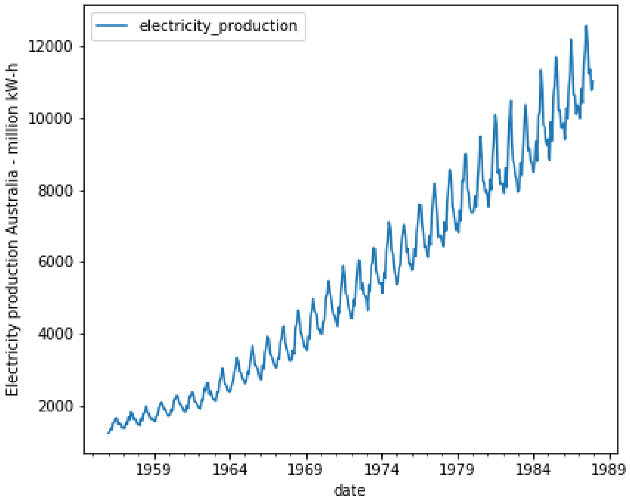
Time-Series Components

Multiplicative Model

A multiplicative model suggests that at least some of the components are multiplied: y(t) = Trend x Seasonality x Noise

- A multiplicative seasonality has an increasing or decreasing frequency and/or amplitude over time.
- Example: there are typically 10% more flights (than the trend) in June.





https://kourentzes.com/forecasting/2014/11/09/additive-and-multiplicative-seasonality/

Summary

In this section, we briefly introduced times series

- What is a time series
- Some examples of time series
- The objectives of time series analysis
- The basic components of a time series

Next time

- How do we estimate/model these components?

Q&A