香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examinations 2003 - 2004

Course Code & Title : CSC 3130 Formal Languages and Automata Theory

Time allowed : 2 hours 0 minutes

Student I.D. No. : .... Seat No. : ....

1. Consider the following ambiguous CFG G:

$$S \to 0S \mid 1S \mid S0 \mid \varepsilon$$

   (a) Describe in English what the language L generated by G is.
   (b) Give a string in L that has two leftmost derivations. Show the two leftmost derivations.
   (c) Convert G to a PDA accepting by empty stack.
   (d) Convert your answer in (c) to a PDA accepting by final states.
   (e) Give an unambiguous grammar for L. Explain briefly why the grammar given is unambiguous.

2. For each of the following pairs of languages, choose the correct relationship from the following four choices:
   Type 1: L_1 is a proper subset of L_2.
   Type 2: L_2 is a proper subset of L_1.
   Type 3: L_1 and L_2 are the same.
   Type 4: L_1 and L_2 have no containment relationship, i.e., there are strings in L_1 but not in L_2 and there are strings in L_2 but not in L_1.

   (a) L_1 is the language of the CFG with productions $S \to 0S1 \mid 1S0 \mid \varepsilon$.
   L_2 is the language of the regular expression (0+1)*.
   (b) L_1 is the language of the CFG with productions $S \to AS \mid SB \mid \varepsilon$, $A \to 0$ and $B \to 1$.
   L_2 is the language of the regular expression 0*1*.
   (c) L_1 is the language of the regular expression (0+1)*11(0+1)*.
   L_2 is the language of the regular expression (0*1*11)*0*110*1*.
   (d) L_1 Is the language accepted by the NFA $(\{p, q\}, \{0, 1\}, \delta, q, \{q\})$ with transitions:
   $$\delta(q, 0) = \{p\},\ \delta(q, 1) = \{\},\ \delta(p, 0) = \{p, q\},\ \delta(p, 1) = \{p\}.$$
   L_2 is the language of the CFG with productions $S \to AS \mid \varepsilon$, $A \to 0B$ and $B \to 0B \mid 1B \mid 0$.

3. Prove by the Pumping Lemma that the following language is not context-free:

$$L = \{0^i 1^j \mid j = i^3\}$$

4. Give the transition diagram of a Turing machine M that, given two non-zero binary numbers x and y of the same length, will determine whether $x \geq y$:

Input:	Output:
...##101#011##...	...##1#101#011##...
^	^ (since $101_2 \geq 011_2$)
...##0010#0101##...	...##0#0010#0101##...
^	^ (since $0010_2 < 0101_2$)
...##11#11##...	...##1#11#11##...
^	^ (since $11_2 \geq 11_2$)

Notice that the input binary numbers x and y should remain in the tape, the answer is written on the left hand side separated by a blank and the tape head should be pointing to the answer when M accepts. Explain briefly your construction.

5. Consider the following instance of the Modified Post's Correspondence Problem:

Index	A	B
1	10	101
2	101	011
3	110	100

   (a) Explain why the above MPCP instance has no solution.
   (b) Give a fourth pair so that there are solutions. Give one solution.

6. (a) Consider the following language:

$$L_{\text{forever}} = \{(k, w) \mid \text{Turing machine } T_k \text{ runs forever on input } w\}$$

   Explain why if L_{forever} is recursive, the universal language L_u is also recursive.

   (b) Assume that it is undecidable whether a given context-free grammar is ambiguous. A CFG G is called *super-ambiguous* if there is at least one string w in $L(G)$ where w has at least 10 different leftmost derivations. Explain why it is also undecidable whether a given context-free grammar is super-ambiguous.

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