香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examination 1st Term, 2007 - 2008

Course Code & Title : CSC 3130 Formal Languages and Automata Theory

Time allowed : 2 hours 0 minutes

Student I.D. No. : Seat No. :

1. Consider a function $f(L) = M$, where L and M are languages over the alphabet $\{0, 1\}$. We say that the function f is "nice" if whenever M is regular, L is regular. For example, the function $f(L) = L^R$ is nice because if L^R is regular, $L = (L^R)^R$ must be regular. In the following, determine whether the function f is nice. In each case, give either a proof (if it is nice) or a counter-example (if it is not nice). For counter-example, it is sufficient to correctly claim a particular language to be non-regular and you do not need to prove it.

   (a) $f(L)$ is the language formed by concatenating L with 00+11.
   (b) $f(L)$ is the language formed from L by removing ε and deleting the last symbol of all the strings, e.g., if $L = \{\varepsilon, 010, 1, 1111\}$, $f(L) = \{01, \varepsilon, 111\}$.

2. Let L be the language consisting of all the strings over $\{a, b\}$ with the same number of a's and b's. Show that the Pumping Lemma is satisfied for L. That is, show that every string $z \in L$ of length n or more has a decomposition that satisfies the conditions of the Pumping Lemma.

3. Consider the following language L:

   $\{(k, w) \mid \text{Turing machine } T_k \text{ will halt at a non-final state on input } w\}$

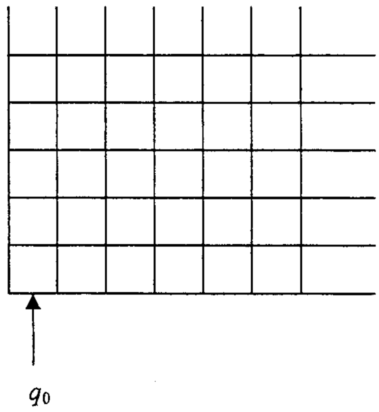
   (a) Is L recursively enumerable? Prove your answer.
   (b) Is L recursive? Prove your answer.

4. Consider the following grammar G:

   $S \to A$
   $A \to aAa \mid aAb \mid ab$

   (a) Give all the LR(0) items of G.
   (b) Is G an LR(0) grammar? Prove your answer.
   (c) Show the parsing of the string $aaabab$ in the language of G.

5. A two-dimensional Turing machine is one in which the tape consists of an infinite two-dimensional array of tape positions:



The transitions are very similar to those in the basic Turing machine model except that instead of just having "L" and "R", the movements can be "U", "D", "L" and "R", denoting up, down, left and right respectively. A computation begins with the tape head reading the lower left corner position (as shown in the figure). Design a two-dimensional Turing machine with input alphabet $\{a\}$ to determine if the tape contains a non-blank symbol. Explain briefly the operations of your Turing machine. Note that the tape head cannot fall off the left and bottom boundaries, and there is no requirement on the final tape content or the final tape head position.

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