香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examination 1st Term, 2008- 2009

Course Code & Title : **CSC3130: Formal languages and automata theory**

Time allowed : **2 hours**

Student I.D. No. : .... Seat No. : ....

Problem 1: ....

Problem 2: ....

Problem 3: ....

Problem 4: ....

Total: ____________

Write your answers in the spaces provided. When given multiple choices circle one answer. Remember that it is always to your advantage to explain your answer. An incorrect answer with no explanation may be given no credit. A partially correct answer with a reasonable explanation is more likely to receive credit.

Problem 1 (42 points)

For each of these statements, say if it is true or false. Give a proof or provide a counterexample for your answer.

(a) (6 points) There is a 2-state NFA for the language $(01)^*$.

true **false**

(b) (6 points) There is a 2-state DFA for the language $(01)^*$.

true **false**

(c) (6 points) If L is regular over $\Sigma = \{0, 1\}$, then L' is also regular, where

$$L' = \{x : x \in L \text{ and } x \text{ starts and ends with the same symbol}\}.$$

true **false**

(d) (6 points) The grammar $S \rightarrow aSb \mid a$ is $LR(0)$.

true **false**

(e) (6 points) The following language is decidable:

$$L = \{\langle R \rangle : R \text{ is a regular expression for the language } (0+1)^*\}$$

true **false**

(f) (6 points) The language $L = \{wxw^R : x, w \in \Sigma^*\}$ is context-free over alphabet $\Sigma = \{a, b\}$.

true **false**

(g) (6 points) The language $L = \{wxw^R x^R : x, w \in \Sigma^*\}$ is context-free over alphabet $\Sigma = \{a, b\}$.

true **false**

Problem 2 (20 points)

Prove the following statements. You can use these facts:

- $A_{TM} = \{(\langle M\rangle, w) : M$ is a TM that accepts $w\}$ is undecidable.
- $ALL_{TM} = \{\langle M\rangle : M$ is a TM that accepts all inputs$\}$ is not recognizable.

(a) (12 points) $L_1 = \{(\langle M_1\rangle, \langle M_2\rangle, \langle M_3\rangle, x)$: at least two of TM M_1, M_2 and M_3 accept $x\}$ is undecidable but is recognizable.

(b) (8 points) $L_2 = \{\langle M\rangle : M$ is a TM accepts all inputs that end in $0\}$ is not recognizable.

Problem 3 (18 points)

For each of the following languages, say whether it is decidable or not. Justify your answer by describing an appropriate Turing Machine (algorithm), or using the following facts:

- $ALL_{CFG} = \{\langle G \rangle : G \text{ is a CFG that accepts all inputs}\}$ is not decidable.
- $AMBIG = \{\langle G \rangle : G \text{ is an ambiguous CFG}\}$ is not decidable.

(a) (9 points) $L_1 = \{(\langle G_1 \rangle, \langle G_2 \rangle) : G_1 \text{ and } G_2 \text{ are CFGs that generate the same strings}\}$.

decidable **undecidable**

(b) (9 points) $L_2 = \{\langle G \rangle : G \text{ is a CFG such that the language of } G \text{ is infinite}\}$.

decidable **undecidable**

Problem 4 (20 points)

The Computer Science department at CUHK has n students and offers m different classes. Each student is enrolled in some subset of the classes (zero, one, or more). At the end of the semester the dean wants to gather a group of at most k students so that each one of the classes is represented. A student may represent several classes he or she is enrolled in.

(a) (4 points) Formulate this problem as a language L.

(b) (6 points) Show that L is in NP.

(c) (10 points) Show that L is NP-hard. (Use the fact that vertex cover is NP-hard. Your classes may be small.)

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