香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examinations 2000 - 2001

Course Code & Title : CSC 3130 Formal Languages and Automata Theory

Time allowed : 2 hours 0 minutes

Student I.D. No. : _____ Seat No. : _____

Answer all the questions:

1. (a) Is $L_1 = \{ a^i b^j a^i b^k \mid i, j, k \geq 0 \}$ a context free language? Prove. (5%)
   (b) Is $L_2 = \{ a^j b^j a^k b^i \mid i, j, k \geq 0 \}$ a context free language? Prove. (5%)
   (c) Is $L_3 = L_1 \cup L_2$ a context free language? Prove. (5%)
   (d) Is $L_4 = L_1 \cap L_2$ a context free language? Prove. (10%)

2. A context-free grammar $G = (V, \Sigma, P, S)$ is called right-linear if each rule is of the form:

   $A \rightarrow u$

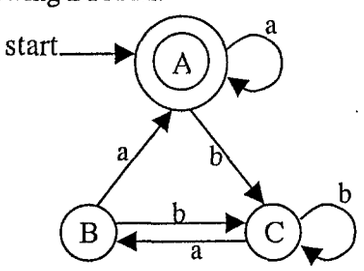
   or $A \rightarrow uB$

   where $A, B \in V$ and $u \in \Sigma^*$.

   (a) Consider the following right-linear grammar G over $\Sigma = \{a, b\}$:

   $S \rightarrow aaS \mid bbS \mid aa$

   Give a regular expression for the language L_1 generated by G. (3%)
   (b) Draw an NFA for L_1 according to the grammar rules in G. (7%)
   (c) Consider the following DFA M:



   Write a regular expression for the language L_2 accepted by M. (3%)
   (d) Write a right-linear grammar for L_2 according to the transitions in M. (7%)
   (e) What is the relationship between right-linear grammars and regular languages? Prove. (5%)

3. (a) Consider a grammar G_1 over $\Sigma = \{a, b\}$:

   $S \rightarrow aX \mid Ya \mid bSb \mid c$
   $X \rightarrow Sa$
   $Y \rightarrow aS$

   Write a regular expression for the language L generated by G_1. (3%)
   (b) Give a rightmost derivation for the string "bacab" using G_1. (4%)
   (c) Show that G_1 is an ambiguous grammar. (6%)
   (d) Consider a LR(0) grammar G_2 over $\Sigma = \{a, b\}$ which is equivalent to G_1:

   $S \rightarrow X$
   $X \rightarrow aXa \mid bXb \mid c$

   Give all the LR(0) items for G_2. (7%)
   (e) Show how the string "bacab" is parsed using G_2. (note: no need to construct the DFA) (10%)

4. (a) Draw a Turing machine that can recognize the language $L = \{wcw \mid w \in \{a, b\}^*\}$. (10%)
   (b) Consider a Turing machine T_m that, given an input k, can determine the behavior of T_k on input k as follow:

   If T_k halts and output x, T_m will halt and output x+1.
   If T_k does not halt, T_m will not halt.

   Is it possible to construct T_m? Explain. (5%)
   (c) Consider a Turing machine T_n that, given an input k, can determine the behavior of T_k on input k as follow:

   If T_k halts and output x, T_n will halt and output 1.
   If T_k does not halt, T_n will halt and output 0.

   Is it possible to construct T_n? Explain. (5%)