香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examinations 2001 - 2002

Course Code & Title : CSC 3130 Formal Languages and Automata Theory

Time allowed : 2 hours .... minutes

Student I.D. No. : .... Seat No. : ....

1. Decide whether the following statements about languages over the alphabet $\Sigma=\{0,1\}$ are true or false. Give brief justifications to your answers.
   (a) Every finite language is regular. (5%)
   (b) Every non-empty regular language has at least one proper subset that is also regular. (5%)
   (c) Let $L = L_1 \cup L_2$. If both L and L_1 are regular, L_2 must also be regular. (5%)

2. Consider the following language L over the alphabet $\Sigma=\{a,b,c\}$:
$$L = \{\ a^i b^j c^k \mid i \neq j \text{ and } j \neq k \text{ and } i \neq k\ \}$$
   (a) Suppose we want to prove that L is not context free by using the Pumping Lemma directly. Let $n>0$ be the constant in the Pumping Lemma, and we consider the string $\alpha=a^n b^{n+2} c^{n+4}$ that is in L. However, we can indeed find u, v, w, x and y such that $\alpha=uvwxy$, $|vwx|\leq n$, $|vx|\geq 1$ and $uv^iwx^iy \in L$ for all $i\geq 0$. Give an example of such u, v, w, x and y. (5%)
   (b) To prove that L is not context free, we can make use of the Ogden's Lemma that is a stronger version of the Pumping Lemma:

   Ogden's Lemma: If L is a CFL, there is a constant $n>0$ such that for each $\alpha \in L$ where $\alpha=\alpha_1\alpha_2\alpha_3$ and $|\alpha_2|=n$, α_2 can be written as $uvwxy$ such that $|vwx|\leq n$, $|vx|\geq 1$ and $\alpha_1 uv^iwx^iy\alpha_3 \in L$ for all $i\geq 0$.

   Prove that L is not context free by using the Ogden's Lemma. Hint: Consider a string with $n!$ more a's (or c's) than b's where n is the constant in the Ogden's Lemma. (11%)

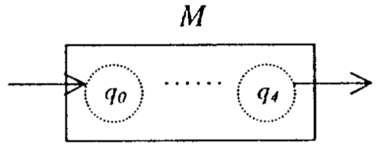
3. Consider the set S of all recursively enumerable languages over the alphabet $\Sigma=\{0,1\}$:
   (a) Is S closed under complementation? Explain. (8%)
   (b) Is S closed under intersection? Explain. (8%)

4. Consider a Turing machine $M=(\{q_0,q_1,q_2,q_3,q_4\},\{0,1\},\{0,1,\#\}, \delta, q_0, \#,\{q_4\})$ where the transition function δ is:

$\delta(q_0, 0) = (q_0, 0, R)$	$\delta(q_1, 0) = (q_2, 1, L)$	$\delta(q_2, 0) = (q_2, 0, L)$	$\delta(q_3, \#) = (q_4, \#, R)$
$\delta(q_0, 1) = (q_0, 1, R)$	$\delta(q_1, 1) = (q_1, 0, L)$	$\delta(q_2, 1) = (q_2, 1, L)$	
$\delta(q_0, \#) = (q_1, \#, L)$	$\delta(q_1, \#) = (q_3, 1, L)$	$\delta(q_2, \#) = (q_4, \#, R)$	

   (a) Draw the transition diagram of M. (5%)
   (b) What will be the output (tape contents) for the following input: (4%)
      (i) (all blanks)
      (ii) 1
      (iii) 10
      (iv) 111
   (c) What is the function of M? (4%)
   (d) In the following, we will use a black box to represent M:

M



By making use of M, we can construct a Turing machine M' that, given a unary number $x \geq 1$ (so there will be at least one "1" in the input), can convert x to a binary number y as in the following examples:

Input:	Output:
...##1111##...	...##100#1111##...
^	^
...##1##...	...##1#1##...
^	^
...##11##...	...##10#11##...
^	^

Notice that the input unary number x should remain in the output and the binary number y is written on its left hand side separated by a blank. The tape head should be pointing to the leftmost digit of y when M' accepts. Give the transition diagram of M'. (12%)

5. Consider the Turing machine $M=(\{q_0,q_1,q_2,q_3\},\{a,b\},\{a,b,\#\}, \delta, q_0, \#,\{q_3\})$ where the transition function δ is:

$$\delta(q_0, a) = (q_1, a, R) \quad \delta(q_0, b) = (q_2, b, R) \quad \delta(q_1, b) = (q_1, b, R)$$
$$\delta(q_2, a) = (q_2, a, R) \quad \delta(q_1, \#) = (q_3, a, L) \quad \delta(q_2, \#) = (q_3, b, L)$$

Now, we want to construct an MPCP instance $I=(A,B)$ such that I has a solution if and only if M accepts the input "abb". Some of the corresponding pairs in list A and B are given as follows:

i	A	B
1	$\#$	$\#q_0abb\#$
2	a	a
3	b	b
4	$\#$	$\#$
5	aq_3	q_3
6	bq_3	q_3
7	q_3a	q_3

i	A	B
8	q_3b	q_3
9	aq_3a	q_3
10	bq_3b	q_3
11	aq_3b	q_3
12	bq_3a	q_3
13	$q_3\#\#$	$\#$

(a) Complete the construction of the MPCP instance I by adding more corresponding pairs to list A and B according to the transition function of M. (7%)

(b) Does I have a solution? If yes, give the sequence of indices used in the construction of the solution. (5%)

6. (a) Given a Turing machine M for the following language over the alphabet $\Sigma=\{0,1\}$:

$L=\{ (k,w) \mid$ Turing machine T_k on input w will halt with the tape head pointing to a blank symbol. $\}$

Show how you can make use of M to check whether a Turing machine T_k will halt on an input w. Hint: You need to modify T_k to $T_{k'}$ before making use of M. (8%)

(b) Given that the following language H over the alphabet $\Sigma=\{0,1\}$ is non-recursive:

$H=\{ (k,w) \mid$ Turing machine T_k on input w will halt. $\}$

Show that L is also non-recursive. (8%)

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