香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examination 1st Term, 2005- 2006

Course Code & Title : CSC 3130 Formal Languages and Automata Theory

Time allowed : 2 hours minutes

Student I.D. No. : Seat No. :

1 (15%) In each of the following cases, find a string of minimum length in {0,1}* that is not in the language of the given regular expression:
(a) 1*(01)*0*
(b) (0* + 1*)(0* + 1*)(0* + 1*)
(c) 0*(100*)*1*

2 (10%) Show that every regular language can be accepted by a deterministic PDA with only two states and has no ε-transitions. Explain your answer.

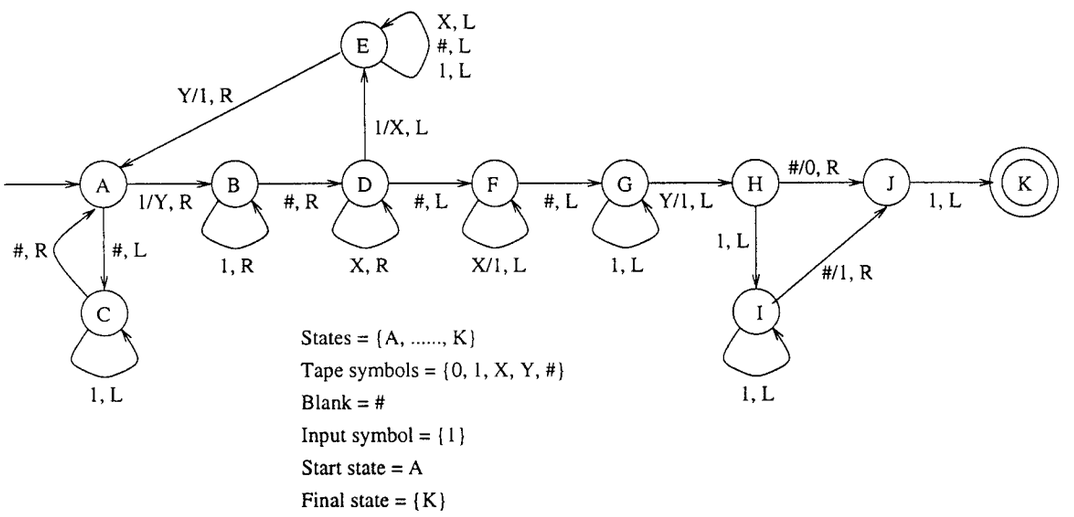
3 (16%) For each of the following Post Correspondence Systems, find a solution or show that it has no solutions:
(a) $A = (aa, bb, abb)$; $B = (aab, ba, b)$
(b) $A = (b, aa, bab, ab)$; $B = (ba, b, aa, ba)$

4 (27%) Consider the following problem P:

> Given an arbitrary Turing machine T_k, determine whether T_k will halt on at least one input string.

(a) Show that P is recursively enumerable.
(b) Show that if P is decidable, the classical halting problem is also decidable.
(c) What can you say about problem P? Is it decidable? Why?

5 (32%) Consider the following Turing machine M_1:



(a) Suppose the input is $1^a\#1^b$ where $a, b > 0$. What will be the output and where will be the tape head when M_1 halts?

(b) Construct a Turing machine M_2 that compares whether two non-zero unary numbers are the same. For example:

Input:	Output:
...##111#11111##...	...##0111#11111##...
^	^
...##111#111##...	...##1111#111##...
^	^

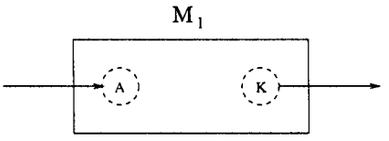
Given an input $1^a\#1^b$ where $a, b > 0$, M_2 will write a '0' in front of the first number 1^a and point to it if $a \neq b$; otherwise, it will write a '1' in front of the first number 1^a and point to it, while the two given numbers 1^a and 1^b remain unchanged on the tape.

(c) Make use of M_1 and M_2 to construct a Turing machine M that determines whether a given non-zero unary number is a prime number. For example:

Input:	Output:
...##11111111##...	...##011111111##...
^	^
...##11111##...	...##111111##...
^	^

Given an input 1^a where $a > 0$, M will write a '1' in front of the input 1^a and point to it if a is a prime; otherwise, it will write a '0' in front of the input 1^a and point to it, while the given number 1^a remains unchanged on the tape.

Notice that you can draw M_1 and M_2 as black boxes. For example, the given Turing machine in part (a) can be drawn as:



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