香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examination First Term, 2012 - 2013

Course Code & Title : CSCI 3130 Formal Languages and Automata Theory

Time allowed : 2 hours 0 minutes

Student I.D. No. : _____ Seat No. : _____

1. Give examples of languages L_1 and L_2 over $\Sigma = \{0, 1\}$ that satisfy the following descriptions. Explain your answers briefly. (15%)
   (a) L_1 is regular, L_2 is non-regular, and $L_1 \cup L_2$ is regular.
   (b) L_1 is regular, L_2 is non-regular, and $L_1 \cup L_2$ is non-regular.
   (c) L_1 is non-regular, L_2 is non-regular and $L_1 \cup L_2$ is regular

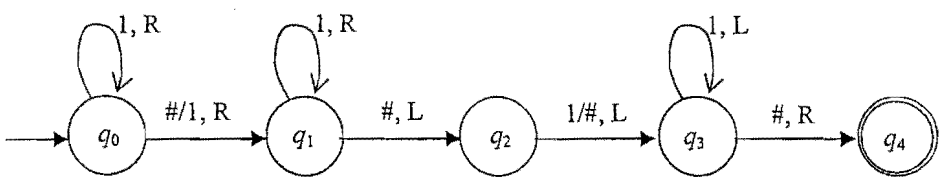
2. Decide whether the following languages are context free. Prove your answers. (20%)
   (a) $\{a^i b^j a^i \mid i, j \geq 0\}$
   (b) $\{a^i b^{2i} a^i \mid i \geq 0\}$

3. Consider the following grammar G for language L over an alphabet Σ = {a, b, +, (,)} with an end-of-line marker #: (15%)
   $S \rightarrow A\#$
   $A \rightarrow A + T \mid T$
   $T \rightarrow a \mid b \mid (A)$
   (a) Give all the LR(0) items of G.
   (b) Show the parsing of the string "$(a + b)\#$".

4. Consider the following language L: (15%)
   $\{(k, q) \mid$ Turing machine T_k with input string "101" will enter state q in $T_k\}$
   (a) Is L recursively enumerable? Why?
   (b) Is L recursive? Why?

5. For the following Post Correspondence Problem, find a solution or show that there is no solution. (15%)
   (a) $A = (a, bb, a)$; $B = (aa, b, bb)$
   (b) $A = (b, aa, bab, ab)$; $B = (ba, b, aa, ba)$

6. Consider the following Turing machine T: (20%)



   (a) Trace the operation of T on the tape content "...#111#11#..." where the tape head points to the leftmost "1" at the beginning. Show the tape content, position of the tape head and the state at each step.

(b) What does T do on two input numbers in unary representation separated by a blank?
(c) Construct a Modified Post Correspondence Problem (MPCP) instance $I = (A, B)$ such that I has a solution if and only if T accepts the input string "111".
(d) Does I have a solution? If yes, give the sequence of indices used in the construction of the MPCP solution. Otherwise, explain why a solution does not exist.

End of Paper