香 港 中 文 大 學
The Chinese University of Hong Kong

Course Examinations 2002 - 2003

Course Code & Title : CSC 3130 Formal Languages and Automata Theory

Time allowed : 2 hours .... minutes

Student I.D. No. : .... Seat No. : ....

Answer all the questions.

1. Consider the regular language L over the alphabet $\Sigma=\{0,1,2\}$:

$$L = \{ w \mid w \text{ does not contain the substring "01"} \}$$

   (a) Draw a DFA for L. (5%)
   (b) Show that every ε-NFA that accepts a non-empty language can be converted into an ε-NFA in which the start state has no incoming edges and there is only one final state that has no outgoing edges. Give an example by converting the DFA you constructed in part (a) to an ε-NFA M that satisfies the above conditions. (5%)
   (c) Consider an ε-NFA M_1 for a language L_1 over an alphabet Σ that satisfies the conditions in part (b). Explain how you can construct an ε-NFA from M_1 that also satisfies the conditions in part (b) for the following languages:
   (i) $\{ w \mid xw \in L_1 \text{ for some } x \in \Sigma^* \}$ (5%)
   (ii) $\{ w \mid wy \in L_1 \text{ for some } y \in \Sigma^* \}$ (5%)
   (iii) $\{ w \mid xwy \in L_1 \text{ for some } x, y \in \Sigma^* \}$ (5%)
   For part (iii), give an example using the ε-NFA M you constructed in part (b).

2. Consider the following context free grammar G:

$$S \rightarrow aSb \mid aaSbbb \mid ab \mid aabbb$$

   (a) Describe the language L generated by G. (5%)
   (b) Show that G is an ambiguous grammar. (5%)
   (c) Write an unambiguous grammar for L. (5%)
   (d) Explain why the grammar you gave in part (c) is unambiguous. (6%)

3. (a) Consider a Turing machine that can check whether a binary number x where $x \geq 0$ is even or odd as follows:

Input	Output
...##1101##...	...##11010##...
^ (under the leftmost 1)	^ (under the final 0)
...##110##...	...##1101##...
^ (under the leftmost 1)	^ (under the final 1)

   Note that the tape head will point to the leftmost digit of the input at the beginning. At the end, M will write a "1" (when x is even) or a "0" (when x is odd) right after the input x and the tape head will point to this "0" or "1" answer. Give such a Turing machine M where M **will never write a blank "#" and never move left from its initial tape head position**. (8%)

(b) Construct an MPCP instance I=(A,B) such that I has a solution if and only if M accepts the input "1". (8%)

(c) Does I have a solution? If yes, give the sequence of indices used in the construction of the solution. (6%)

(d) It is known that the special case of PCP in which the alphabet has only two symbols is also undecidable. Show how to reduce an instance in PCP to an instance in this special PCP such that the latter will have a solution if and only if the former has one. (8%)

4. (a) We have discussed in class that the universal language problem *ULP*:

Given (k,w), determine if the Turing machine T_k will accept w.

is undecidable. Show that a special case of the universal language problem ULP_1:

Given k, determine if the Turing machine T_k will accept a blank input.

is also undecidable by reducing an instance in *ULP* to an instance in ULP_1. (8%)

(b) Consider a problem *WriteSymbol*:

Given (k,a), determine if the Turing machine T_k on a blank input will ever write the symbol "a".

Show that *WriteSymbol* is also undecidable by reducing an instance in ULP_1 to an instance in *WriteSymbol*. (8%)

(c) Surprisingly, the following problem *WriteNonblank*:

Given k, determine if the Turing machine T_k on a blank input will ever write a non-blank symbol.

is found to be solvable. Explain how you can modify a given Turing machine T_k to a Turing machine T_k' such that T_k', on a blank input, will always halt and it will halt at an accepting state if and only if T_k on a blank input will write a non-blank symbol. (8%)
(Hint: You need to consider the following three cases: (i) when T_k writes a non-blank symbol, (ii) when T_k halts without writing any non-blank symbol, and (iii) when T_k keeps going without writing any non-blank symbol. In the last case, the fact that the number of states in T_k must be finite is important.)

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