

香港中文大學
The Chinese University of Hong Kong

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Course Examination Term 1, 2004 - 2005

Course Code & Title : CSC 3130 Formal Languages and Automata Theory

Time allowed : 2 hours 0 minutes

Student I.D. No. : Seat No. :

1. Consider four languages A , B , C and D . We know the following about them:

- i. A can be reduced to B .
- ii. B can be reduced to C .
- iii. D can be reduced to C .

For each of the following statements, indicate whether they are true, false or maybe true (depending on what the languages are). Explain your answers:

- (a) If A is recursive, then B is recursive.
- (b) If C is recursive, then the complement of D is recursive.
- (c) If C is recursively enumerable, then the union of B and D is recursively enumerable.
- (d) If C is recursively enumerable, then the intersection of B and D is recursively enumerable.

2. For each of the following languages, determine whether it is regular, context-free but not regular, or non-context free. Explain your answers:

- (a) The set of all strings in $\{0,1\}^*$ such that in no prefix does the number of '0's exceed the number of '1's by more than two, nor does the number of '1's exceed the number of '0's by more than two.
- (b) The set of all strings in $\{0,1,2\}^*$ such that the number of each symbol is at least 100.
- (c) The set of all strings in $\{0,1,2\}^*$ such that the number of '0's is greater than the sum of the numbers of '1's and '2's.

3. Draw a Turing machine M_1 that computes $\lceil x/2 \rceil$ where $x > 0$. For example:

| | |
|-----------------|----------------|
| <u>Input:</u> | <u>Output:</u> |
| ...##11111##... | ...##111##... |
| ^ | ^ |

Notice that both the input and output are unary numbers and the tape head must be pointing to the leftmost '1' of the output when M_1 halts.

Make use of M_1 to construct a Turing machine M to compute $\lceil \log_2 x \rceil$ where $x > 1$. For example:

| | |
|--------------------|---|
| <u>Input:</u> | <u>Output:</u> |
| ...##111##... | ...##11##... |
| ^ | ^ (since $\lceil \log_2 3 \rceil = 2$) |
| <u>Input:</u> | <u>Output:</u> |
| ...##11111111##... | ...##111##... |
| ^ | ^ (since $\lceil \log_2 8 \rceil = 3$) |

Similarly, both the input and output are unary numbers and the tape head must be pointing to the leftmost '1' of the output when M halts.

4. Consider the following language L :

$\{ (k, w) \mid T_k \text{ accepts } w \text{ in an even number of transitions.} \}$

- (a) Prove that L is recursively enumerable.
- (b) Is L recursive? Prove your answer.