Лабораторная работа №7

Группа: М8О-206Б-19

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Вариант: 19

In[1]:=
```
f = 11 * x ^ 2 - 2 * x * y - 2 * x * z + 2 * y * z + 9 * z ^ 2 - 4 * x + y + z;
```

In[2]:=
```
TraditionalForm [f]
```

Out[2]//TraditionalForm=

$$11 x^2 - 2 x y - 2 x z - 4 x + 2 y z + y + 9 z^2 + z$$

In[3]:=
```
A = {
{11, - 1, - 1},
{- 1, 0, 1},
{- 1, 1, 9}
};
```

In[4]:=
```
MatrixForm [A]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} 11 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & 9 \end{pmatrix}$$

Составим характеристическое уравнение

In[5]:=
```
l =.
AE = A - IdentityMatrix [3] * l;
MatrixForm [AE]
```

Out[7]//MatrixForm=

$$\begin{pmatrix} 11 - l & -1 & -1 \\ -1 & -l & 1 \\ -1 & 1 & 9 - l \end{pmatrix}$$

In[8]:=
```
myCharPoly = Det[AE]
```

Out[8]= $-18 - 96\,l + 20\,l^2 - l^3$

In[9]:=
```
wolframCharPoly = CharacteristicPolynomial [A, l]
```

Out[9]= $-18 - 96\,l + 20\,l^2 - l^3$

Проверка на равенство характеристических уравнений

In[10]:=
```
FullSimplify [myCharPoly == wolframCharPoly ]
```

Out[10]= True

Ищем собственные значения

In[11]:=
```
sol = Solve[myCharPoly == 0, l];
```

In[12]:=
```
myEigenVals = l /. sol;
```

In[13]:=
```
% // N
```

Out[13]=
```
{-0.18064 , 8.61753 , 11.5631}
```

Проверка на равенство собственных значений

In[14]:=
```
wolframEigenVals = Eigenvalues [A];
Sort[wolframEigenVals ] == Sort[myEigenVals ]
```

Out[15]=
```
True
```

In[16]:=
```
X = {x, y, z};
one = AE /. l → myEigenVals [[1]];
two = AE /. l → myEigenVals [[2]];
three = AE /. l → myEigenVals [[3]];
myOne = one.X
myTwo = two.X
myThree = three .X
```

Out[20]=
$$\left\{-y-z+x\left(11-\sqrt{-0.181\ldots}\right),\ -x+z-y\sqrt{-0.181\ldots},\ -x+y+z\left(9-\sqrt{-0.181\ldots}\right)\right\}$$

Out[21]=
$$\left\{-y-z+x\left(11-\sqrt{8.62\ldots}\right),\ -x+z-y\sqrt{8.62\ldots},\ -x+y+z\left(9-\sqrt{8.62\ldots}\right)\right\}$$

Out[22]=
$$\left\{-y-z+x\left(11-\sqrt{11.6\ldots}\right),\ -x+z-y\sqrt{11.6\ldots},\ -x+y+z\left(9-\sqrt{11.6\ldots}\right)\right\}$$

Собственные вектора

In[23]:=
```
myEigenVec1 = Solve[myOne == 0 /. z → 1];
myEigenVec2 = Solve[myTwo == 0 /. z → 1];
myEigenVec3 = Solve[myThree == 0 /. z → 1];
myEigenVec1 = {x, y, 1} /. myEigenVec1 [[1]]
myEigenVec2 = {x, y, 1} /. myEigenVec2 [[1]]
myEigenVec3 = {x, y, 1} /. myEigenVec3 [[1]]
```

Out[26]=
$$\left\{-\frac{-1-9\sqrt{-0.181\ldots}+\left(\sqrt{-0.181\ldots}\right)^2}{1+\sqrt{-0.181\ldots}},\ -\frac{8-\sqrt{-0.181\ldots}}{1+\sqrt{-0.181\ldots}},\ 1\right\}$$

Out[27]=
$$\left\{-\frac{-1-9\sqrt{8.62\ldots}+\left(\sqrt{8.62\ldots}\right)^2}{1+\sqrt{8.62\ldots}},\ -\frac{8-\sqrt{8.62\ldots}}{1+\sqrt{8.62\ldots}},\ 1\right\}$$

Out[28]=
$$\left\{-\frac{-1-9\sqrt{11.6\ldots}+\left(\sqrt{11.6\ldots}\right)^2}{1+\sqrt{11.6\ldots}},\ -\frac{8-\sqrt{11.6\ldots}}{1+\sqrt{11.6\ldots}},\ 1\right\}$$

In[29]:=
```
wolframEigenSys  = Eigensystem [A]
```

Out[29]=
```
{{11.6..., 8.62..., -0.181...},
 {{-2.28..., 0.284..., 1}, {0.447..., 0.0642..., 1}, {-0.804..., -9.98..., 1}}}
```

In[30]:=
```
myEigenVals [[1]]
myEigenVals [[2]]
myEigenVals [[3]]
```

Out[30]=
```
-0.181...
```

Out[31]=
```
8.62...
```

Out[32]=
```
11.6...
```

Проверка на равенство собственных векторов

In[33]:=
```
wolframEigenSys [[2, 3]] == N[myEigenVec1 ]
wolframEigenSys [[2, 2]] == N[myEigenVec2 ]
wolframEigenSys [[2, 1]] == N[myEigenVec3 ]
```

Out[33]=
```
True
```

Out[34]=
```
True
```

Out[35]=
```
True
```

Составляем матрицу из нормированных собственных векторов

In[36]:=
```
S = {
Normalize [myEigenVec1 ],
Normalize [myEigenVec2 ],
Normalize [myEigenVec3 ]
};
N[S] // MatrixForm
```

Out[37]//MatrixForm=

$$\begin{pmatrix} -0.0798259 & -0.991846 & 0.0993417 \\ 0.407145 & 0.0585251 & 0.911487 \\ -0.909869 & 0.113207 & 0.399153 \end{pmatrix}$$

Составляем каноническое уравнение

In[38]:=
```
(*fk1*)
a = {- 2, 0.5, 0.5};
a1 = S.a;
N[a1] // MatrixForm
fk1 = myEigenVals [[1]] * x1 ^ 2 + myEigenVals [[2]] * y1 ^ 2 +
myEigenVals [[3]] * z1 ^ 2 + 2 * a1[[1]] * x1 + 2 * a1[[2]] * y1 + 2 * a1[[3]] * z1;
fk1 = FullSimplify [fk1];
fk1 /. {x1 → x, y1 → y, z1 → z} // TraditionalForm
```

Out[40]//MatrixForm=

$$\begin{pmatrix} -0.286601 \\ -0.329284 \\ 2.07592 \end{pmatrix}$$

Out[43]//TraditionalForm=

$$(-0.18064\ x - 0.573201)\,x + y\,(8.61753\ y - 0.658567) + z\,(11.5631\ z + 4.15183)$$

In[44]:=
```
(* Дополняем члены до полного квадрата - fk2 *)
a0 = - myEigenVals [[1]] * (a1[[1]] / myEigenVals [[1]])^ 2 - myEigenVals [[2]] *
(a1[[2]] / myEigenVals [[2]])^ 2 - myEigenVals [[3]] * (a[[3]] / myEigenVals [[3]])^ 2;
fk2 = myEigenVals [[1]] * (x1 + a1[[1]] / myEigenVals [[1]])^ 2 +
myEigenVals [[2]] * (y1 + a1[[2]] / myEigenVals [[2]])^ 2 +
myEigenVals [[3]] * (z1 + a1[[3]] / myEigenVals [[3]])^ 2 + a0
TraditionalForm [
fk2]
```

Out[45]=

$$0.420512 + (1.58658 + x1)^2\ \sqrt{-0.181\ldots} + (-0.0382109 + y1)^2\ \sqrt{8.62\ldots} + (0.179529 + z1)^2\ \sqrt{11.6\ldots}$$

Out[46]//TraditionalForm=

$$(x1 + 1.58658)^2\ \sqrt{-0.181\ldots} + (z1 + 0.179529)^2\ \sqrt{11.6\ldots} + (y1 - 0.0382109)^2\ \sqrt{8.62\ldots} + 0.420512$$

Замена переменных:

x2 = x1 + 1.58658

y2 = y1 - 0.0382109

z2 = z1 + 0.179529

In[47]:=
```
(* fk3 *)
fk3 = myEigenVals [[1]] * x ^ 2 + myEigenVals [[2]] * y ^ 2 + myEigenVals [[3]] * z ^ 2 + a0;
TraditionalForm [
fk3 == 0]
```

Out[48]//TraditionalForm=

$$x^2\ \sqrt{-0.181\ldots} + z^2\ \sqrt{11.6\ldots} + y^2\ \sqrt{8.62\ldots} + 0.420512 = 0$$

In[49]:=
```
(* fk4 *)
fk4 = myEigenVals [[1]] / a0 * x ^ 2 +
myEigenVals [[2]] / a0 * y ^ 2 + myEigenVals [[3]] / a0 * z ^ 2 + 1;
TraditionalForm [
fk4 == 0]
```

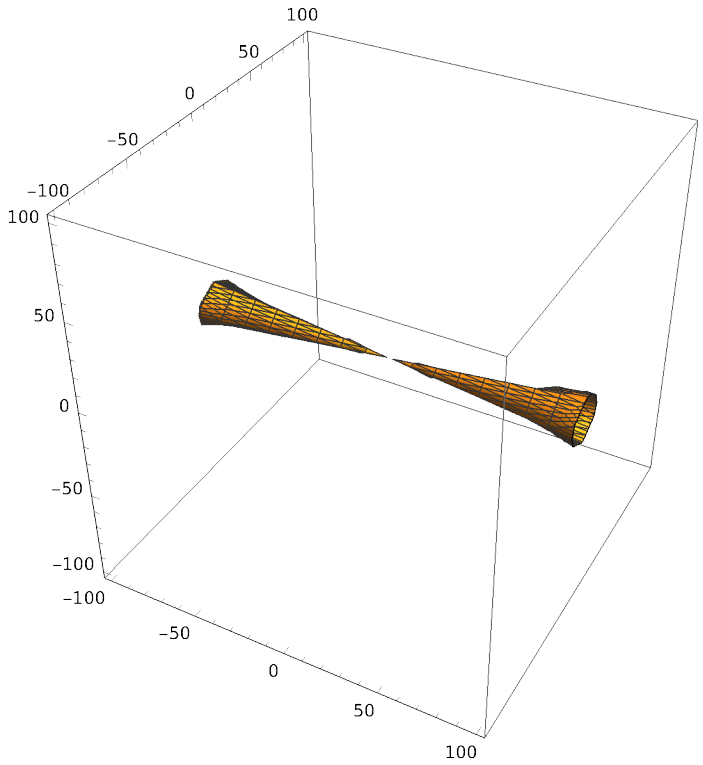
Out[50]//TraditionalForm=

$$-0.429573\ x^2 + 20.4929\ y^2 + 27.4977\ z^2 + 1 = 0$$

Полученная фигура

In[51]:=
```
x =.; y =.; z =.;
value = 100;
fk[a_, b_, c_] := fk4 /. {x2 → a, y2 → b, z2 → c}
ContourPlot3D [fk[x, y, z] == 0,
{x, - value , value}, {y, - value , value}, {z, - value , value}]
```

Out[54]=



Исходная фигура

In[55]:=
```
fnk[a_, b_, c_] := f
ContourPlot3D [fnk[x, y, z] == 0,
{x, - value , value}, {y, - value , value}, {z, - value , value}]
```

Out[56]=