1. implement the following MATLAB programs clgs in Python.

[기존 그람-슈미츠 알고리즘 Classical Gram-Schmidt orthogonalization]

```
function [Q, R] = clgs(A)
[m, n] = size(A);
V=A; Q=eye(m,n);
R=zeros(n,n);
for j=1:n
    for i=1:j-1
        R(i,j)=Q(:,i)'*A(:,j);
        V(:,j)=V(:,j)-R(i,j)*Q(:,i);
end
R(j,j)=norm(V(:,j));
Q(:,j)=V(:,j)/R(j,j);
end
```

2. Implement the following MATLAB programs mgs in Python.

[수정 그람-슈미츠 알고리즘 Modified Gram-Schmidt orthogonalization]

```
function [Q, R] = mgs(A)
  [m, n] = size(A);
Q = A;
R=zeros(n,n);
for i = 1:n-1
    R(i,i)=norm(Q(:,i));
    Q(:,i)=Q(:,i)/R(i,i);
    R(i,i+1:n)=Q(:,i)'*Q(:,i+1:n);
    Q(:,i+1:n)=Q(:,i)'*R(i,i+1:n);
end
R(n,n)=norm(Q(:,n));
Q(:,n)=Q(:,n)/R(n,n);
```

3. Implement the following MATLAB code in Python and test it. Plot the graph(right).

```
[U,X]=qr(randn(80));
[V,X]=qr(randn(80));
J=1:80; S=diag(2.^(-J));
A=U*S*V;
[Qc,Rc]=clgs(A);
[Qm,Rm]=mgs(A);
```

Implementing the MATLAB program semilogy in Python, plot the diagonal elements r_{jj} produced by both computations with s_{jj} in one figure. Which is more numerically stable, classical or modified?