Inner / Outer Product

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, y = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

1) Transpose

$x^T = [1 \quad 2 \quad 3]$

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

i) Inner Product. → scalar

$$x^T y = \underset{1\times3}{[1 \quad 2 \quad 3]} \underset{3\times1}{\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}} = \underset{1\times1}{4 + 2\cdot5 + 3\cdot6} = 32.$$

2) Outer Product.

$$xy^T = \underset{3\times1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} \underset{1\times3}{[4 \quad 5 \quad 6]} = \underset{3\times3}{\begin{bmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{bmatrix}}$$ //

Matrix Multiplication

$$A = \begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 4 \\ 19 & 5 \end{bmatrix}.$$

$$AB = \underbrace{\begin{bmatrix} 2 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}}_{2\times 2} + \underbrace{\begin{bmatrix} 4 \\ 5 \end{bmatrix} \begin{bmatrix} 3 & 1 \end{bmatrix}}_{2\times 2}$$

$$= \begin{bmatrix} 2 & 0 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 12 & 4 \\ 15 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 4 \\ 19 & 5 \end{bmatrix} //$$

LU

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 1 \\ -8 & -4 & -2 \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & -1 & 1 \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 8 & -5 \\ 0 & 0 & 5 \end{bmatrix}$$

$A = LU$

Lower Upper

$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 1 \\ -8 & -4 & -2 \end{bmatrix}, b = \begin{bmatrix} 6 \\ 15 \\ -22 \end{bmatrix}$ 일 때,

$Ax = b$를 만족하는 $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$를 구하시오.

$A = LU$?

$Ax = b$.

$\Rightarrow LUx = b$, $Ux = y$.

$\Rightarrow Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 15 \\ -22 \end{bmatrix}.$$

$y_1 = 6$

$2y_1 + y_2 = 15$, $y_2 = 3$

$-4y_1 - y_2 + y_3 = -22$, $y_3 = -22 + 24 + 3$

$= 5$

Determinants.

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 1 \\ -8 & -4 & -2 \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & -1 & 1 \end{bmatrix}, \mathbf{U} = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 8 & -5 \\ 0 & 0 & 5 \end{bmatrix}$$

$\det(A) = ?$

$$\det(AB) = \det(A)\cdot\det(B) = \det(BA)$$

$$\det(A) = \det(LU) = \det(L)\cdot\det(U)$$

$$= 1 \cdot 80 = 80$$

$$\det\left(A(A^TA)^{-1}A^T\right)$$

$$= \det\left((A^TA)^{-1}A^TA\right) = \det(I_n) = 1$$

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$ 가 positive semidefinite임을 보이시오.