COURSEWORK

LECTURE An introduction to modelling and control of flexible and soft robots

QUESTION 1. Write the expression of the potential energy for a rigid-link model with $n = 4$ elastic joints. Marks 10%

Assume the following:

- the system is planar
- the stiffness k is the same for all joints, thus $K = \text{diag}\{k\}$
- the system is not subject to gravity (the potential energy V is only elastic)

QUESTION 2. Write the expression of the kinetic energy for a rigid-link model with $n = 4$ elastic joints. Use Matlab Mupad for symbolic computation. Marks 10%

Assume the following:

- all links have equal length l and equal mass m
- the mass of each link is concentrated in its midpoint

Hint: see Example 1 in the lecture notes.

*Bonus points: write the expression of the inertia matrix. Marks 10%

QUESTION 3. Write the equations of motion of the system in Port-Hamiltonian form. Provide an explicit expression of $\nabla_q H$ and $\nabla_p H$. Use Matlab Mupad. Marks 10%

Assume the following:

- the damping matrix is diagonal with equal elements $R = \text{diag}\{b\}$

Hint: see Equation (1) in the lecture notes. When computing $\nabla_p H$, recall that $p = M\dot{q}$

QUESTION 4. Write the expression of the matrix $G^{\perp}$ for $n = 4$ and the corresponding potential-energy PDE. Use Matlab Mupad. Marks 10%

Hint: see Example 4 in the lecture notes. Note that different choices of $G^{\perp}$ are possible. Verify that $G^{\perp}G = 0$ and that $\text{rank}\{G^{\perp}\} = n - 1$.

QUESTION 5. Compute the solution of the potential-energy PDE for $n = 4$. The solution should have a minimizer at $q = q^*$. Use Maple to solve the PDE and Matlab Mupad for verification. Marks 10%

Assume the following:

- the kinetic-energy PDE is solved with $M_d = k_m M$ and $J = 0$.

Hint: the Maple commands to solve the PDE for $n = 3$ are:

- $PDEs := [diff(V_d(q_1, q_2, q_3), q_1) - diff(V_d(q_1, q_2, q_3), q_2) = k(q_1 - q_2)/k_m,$
  $diff(V_d(q_1, q_2, q_3), q_2) - diff(V_d(q_1, q_2, q_3), q_3) = k(q_2 - q_3)/k_m]$
- $V_d := pdsolve(PDEs)$
- $pdetest(V_d, PDEs)$

Note that the result of $pdsolve$ includes a term $\_F1(q_1 + q_2 + q_3 + q_4)$, which can be freely chosen provided that it verifies the PDE. This should be chosen so that V_d has a minimizer at the position $q = q^*$. Verify that V_d solves the PDE using Matlab Mupad.

Hint: see Example 4 in the lecture notes and recall that $\theta = G^T q$ and $\theta^* = G^T q^*$.

QUESTION 6. Write the expression of the control law u for $n = 4$ (use Matlab Mupad).

Assume the following:

- the damping injection is $D_1 = G k_v G^T$, where $k_v > 0$ is a scalar

Hint: see Example 5 in the lecture notes. Marks 10%

QUESTION 7*. Write the expression of the payload compensation term u^* (use Matlab Mupad). Marks 5%

Assume the following:

- the payload δ is constant and known (i.e. torque affecting all joints in the same way)

Hint: see Example 6 in the lecture notes.

*this is a bonus question.

QUESTION 8. Program a simulation in Matlab using the ODE command and verify that the controller from Question 6 achieves the regulation goal $q = q^*$. Marks 20%

Further instructions:

- Use the following parameters for the simulation:

$l = 0.025$, $m = 4$, $k = 5$, $b = 0.01$, $q^* = \pi/6$, $k_p = 0.1$, $k_m = 5$, $k_v = 2$.

- Write a Matlab function called "Topics_in_control_ode_n4.m" and call the function using the Matlab script "Topics_in_control_simulations.m" provided.

Hints: copy the expression of $\nabla_q H$, $\nabla_p H$, M, M^{-1}, and $\dot{p}$ from Mupad into the Matlab function. Recall that $\dot{p} = M\ddot{q} + \dot{M}\dot{q}$ if the inertia matrix is not constant. Provide an explicit expression of M^{-1} in the Matlab function rather than using the command "$\mathrm{inv}(M)$".

QUESTION 9*. Verify with a simulation in Matlab using the ODE command that the controller from Question 7 achieves the regulation goal $\theta = \theta^*$ in the presence of a known constant payload $\delta = 0.1$. Marks 5%

*this is a bonus question.

TOTAL MARKS 100% (80% without bonus questions)

NOTE: marks are indicative and are provided as general guidance.