Topics in Control

Control of Multiagent Systems

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Overview

Multiagent Systems

- Agents are mechanical/software systems to imitate human/animals intelligence with
  - self sensing
  - self control
- We deal with dynamical agents modeled as difference/differential equations:
  - a mobile robot
  - a drone quadrotor
  - a power generator
  - a satellite
- Each agent has many limitations in the sensory system and actuation abilities.
- The idea of cooperative networked agents is considered.

Multiagent Systems

- A multiagent system (MAS) is a group of networked agents to accomplish a cooperative task:
  - multirobot systems
  - power micro grids
  - cooperative aerospace systems

Online at: texasmonthly.com, nocamels.com, and csail.mit.edu

Multiagent Systems

- The idea is using the advantages of a team:
  - Some missions may not be accomplished by a single agent.
  - Using simple and cheap agents instead of a sophisticated one.
  - They are more robust and flexible when some agents fail.

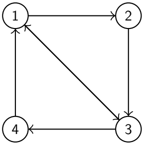
Online at: eu-robotics.net

Control of Multiagent Systems

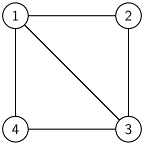
- In order to coordinate among the agents, interaction control laws are required.
- Interaction can be through a central computer or controller.
- It is similar to classic control of a MIMO system:
  - sensitivity to the central controller
  - communication problems in long maneuvers
  - need to high communication bandwidths
- Thus, decentralized/distributed control has been proposed $\rightsquigarrow$ based on local limited sensing and self decision making.
- The main idea in study of MASs is local control when only local information is available.

Graph Theory

- Interaction of agents is described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$
  - $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of N nodes/agents
  - $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of communication links
  - an edge (j, i) denotes that the ith agent receives information from the jth one
  - $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the adjacency matrix: $a_{ij} \in \mathbb{R}_{>0}$ if $(j, i) \in \mathcal{E}$
- Let $\mathcal{N}_i$ be the set of neighbors $j \in \mathcal{V}$ where $a_{ij} \neq 0$.
- $\mathcal{G}$ has a spanning tree if one node of the graph has a directed path to all the nodes.
- A graph is undirected if $a_{ij} = a_{ji}$.



(a) Directed.



(b) Undirected.

Graph Theory

- We define the Laplacian matrix of $\mathcal{G}$ as $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}^{N\times N}$:

$$\ell_{ij} = \begin{cases} \sum_{i=1,i\neq j}^{N} a_{ij} & i = j \\ -a_{ij} & i \neq j. \end{cases}$$

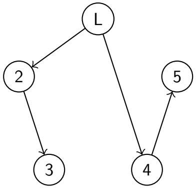
- If $\mathcal{G}$ has a spanning tree,
  - $\mathcal{L}$ has a zero eigenvalue
  - other eigenvalues of $\mathcal{L}$ lie in the open right half plane (RHP)
  - the right and left eigenvectors associated with the zero eigenvalue are $\mathbf{1}$ and $p \in \mathbb{R}^N$, respectively, where $p^\top \mathbf{1} = 1$.
- A graph is called balanced if all of its nodes are balanced as

$$\forall i \in \mathcal{V}, \sum_{j=1,j\neq i}^{N} a_{ij} = \sum_{j=1,j\neq i}^{N} a_{ji}$$

  and $p = \mathbf{1}/N$.
- An undirected graph is a special form of a balanced graph.

Leader-Following

- A main feature of a group of agents is common quantities of interests required for coordinated actions:
  - a common heading/orientation
  - a common velocity/position
- In some MASs, there is a leader determining those quantities.
- Other agents follow the trajectory of this leader hierarchically.



A leader-follower architecture.

- The main features of this structure:
  - The group trajectory are determined by the leader as the stable point of the MAS.
  - The interaction can be hierarchical.

Leader-Following

- We consider the model of the ith agent as follows:

$$\dot{x}_i(t) = u_i(t), i \in \{1, 2, \ldots, N\},$$

where $x_i(t)$ and $u_i(t)$ denote the state and input, respectively.

- If the leader is labeled as Agent 1, we consider

$$\begin{aligned} u_1(t) &= v(t, x_1), \\ u_i(t) &= \sum_{j \in \mathcal{N}_i} k(x_j(t) - x_i(t)), i \in \{2, 3, \ldots, N\}, \end{aligned}$$

- where $v(t, x_1)$ is a bounded control law and $k > 0$.
- Since all $u_i(t), i \in \{1, 2, \ldots, N\}$, are computed simultaneously, Agent i has no online access to $\dot{x}_j(t), j \in \mathcal{N}_i$.

Leader-Following

- By considering the tracking error $e_i(t) = \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t))$,

$$\dot{e}_i(t) = -\sum_{j \in \mathcal{N}_i} (k e_i(t) + \dot{x}_j(t)), i \in \{2, \dots, N\}.$$

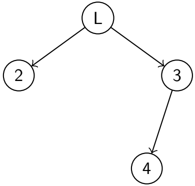
- Under the leader-follower control law:
  - no loops are in the interaction topology (hierarchical topology)
  - the local controllers are BIBO with respect to $\dot{x}_j(t)$:

$$G(s) = \frac{1}{s + \alpha}, \alpha > 0.$$

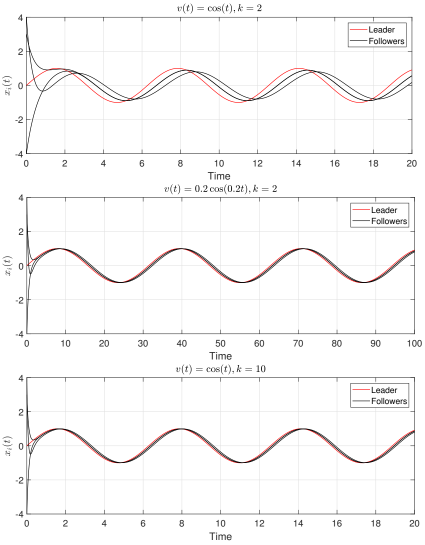
- Therefore, if $\dot{x}_1(t) = v(t, x_1)$ is bounded, all $e_i(t), i \in \{2, \dots, N\}$, remain bounded (sufficient condition).
- Note that $v(t, x_1)$ may be unbounded, while the errors remain bounded !
- Moreover,

$$\dot{x}_1(t) \to 0 \Longrightarrow e_i(t) \to 0,$$
$$k \to \infty \Longrightarrow e_i(t) \to 0 \text{ (not practical)}.$$

Leader-Following: Simulation



Network communication graph.



Consensus Problem

- The drawbacks of the leader-follower architecture are
  - The leader is the single point of failure.
  - No feedbacks are sent from the followers to the leader.
  - A leader decreases the degree of autonomy.

- If there is no a leader in the network, the agents require achieving consensus/agreement on the common quantities.

- Under a leaderless architecture, each agent follows other agents as its leaders:
  - This architecture does not have the above-mentioned drawbacks.
  - As no leader is available, convergence analysis is more complex. Hierarchical analysis is not possible.
  - It is not possible to track a desired state.

Consensus Problem

- We consider the following interaction protocol:

$$\dot{x}_i(t) \;=\; \sum_{j\in\mathcal{N}_i} a_{ij}(x_j(t)-x_i(t)), i\in\{1,2,\ldots,N\}.$$

- We should analyze the whole MAS as a single system.
- Let $x(t)=\begin{bmatrix}x_1(t) & x_2(t) & \ldots & x_N(t)\end{bmatrix}^\top$. Thus,

$$\dot{x}(t)=-\mathcal{L}x(t).$$

- For a matrix M, there exists a similarity transformation matrix:

$$M=TJT^{-1}$$

- where J has a Jordan canonical form.
- T and $T^{-1\top}$ are composed of the right and left eigenvectors of M, respectively.

Consensus Problem

- Accordingly,

$$\dot{x}(t) = -TJT^{-1}x(t). \tag{1}$$

- If the network has a spanning tree,

$$J = \mathrm{diag}(0, \acute{J})$$

- where $\acute{J}$ is a Jordan matrix with eigenvalues in the open RHP.
- The solution of (1) yields

$$x(t) = Te^{-Jt}T^{-1}x(0),$$

$$T = \begin{bmatrix} \mathbf{1} & \cdots & \cdots \end{bmatrix}, T^{-1} = \begin{bmatrix} p^\top \\ \vdots \\ \vdots \end{bmatrix}.$$

- As $-\acute{J}$ is Hurwitz,

$$\lim_{t\to\infty} e^{-Jt} = \mathrm{diag}(1, \mathbf{0}).$$

Consensus Problem

- Therefore,

$$\lim_{t\to\infty} x(t) = T\mathrm{diag}(1, \mathbf{0})T^{-1}x(0) = \mathbf{1}p^\top x(0),$$

- or

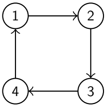
$$\lim_{t\to\infty} x_i(t) = p^\top x(0).$$

- If the network graph is balanced, $p = \mathbf{1}/N$ implying average consensus:

$$\lim_{t\to\infty} x_i(t) = \mathbf{1}^\top/Nx(0).$$

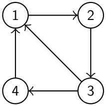
- In which case, we have $J = \mathrm{diag}(0, 0, \acute{J})$?
  How is the MAS steady state in that case?

Consensus Problem: Simulation



Network communication graph 1.

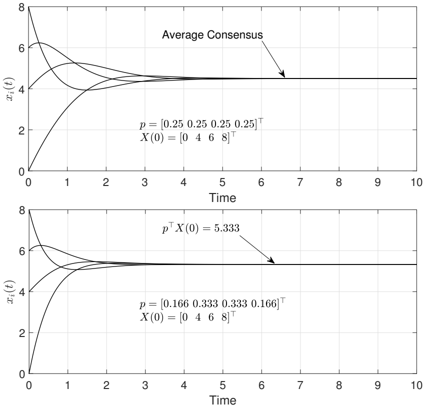
$a_{14} = 1,\ a_{21} = 1,\ a_{32} = 1,\ a_{43} = 1$



Network communication graph 2.

$a_{13} = 1,\ a_{14} = 1,\ a_{21} = 1,\ a_{32} = 1,$

$a_{43} = 1$



➢ How we can modify the weights to achieve average consensus?

Formation Control

- Inspired by biological behaviors such as flocking of geese, swarm control of MASs is a practical topic of research for
  - monitoring
  - surveillance
  - coverage
- It is defined as collective motion of mobile agents in groups:
  - ground vehicles
  - aerial vehicles
  - surface and underwater vehicles
- To optimize the agents swarm and simplify its mathematical analysis, the idea of swarming with formation is considered.
- It is defined as coordinated motion of mobile agents, while keeping desired geometric patterns.
- Based on the application can be: leader-follower and leaderless.

Leader-Follower Formation

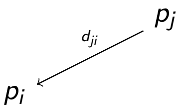
- Without loss of generality, we consider a MAS in 2D space:

$$\dot{p}_i(t) = u_i(t), i \in \{1, 2, \ldots, N\}.$$

- Inspired by the tracking leader-follower law, let

$$\begin{aligned} u_1(t) &= v(t, x_1), \\ u_i(t) &= \sum_{j \in \mathcal{N}_i} k(p_j(t) + d_{ji} - p_i(t)), i \in \{2, 3, \ldots, N\}, \end{aligned}$$

- where $d_{ji} \in \mathbb{R}^2$ describes a fixed desired relative position.



- They should be chosen to satisfy a feasible formation.

Leader-Follower Formation

- By defining $e_i(t) = \sum_{j \in \mathcal{N}_i}(p_i(t) - p_j(t) - d_{ji})$,

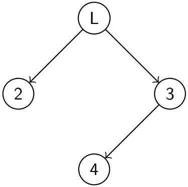
$$\dot{e}_i(t) = -\sum_{j \in \mathcal{N}_i}(ke_i(t) + \dot{p}_j(t)), i \in \{2, \dots, N\}.$$

- Therefore, if $\dot{p}_1(t)$ is bounded, $e_i(t)$ is bounded (sufficient condition).

- Moreover,

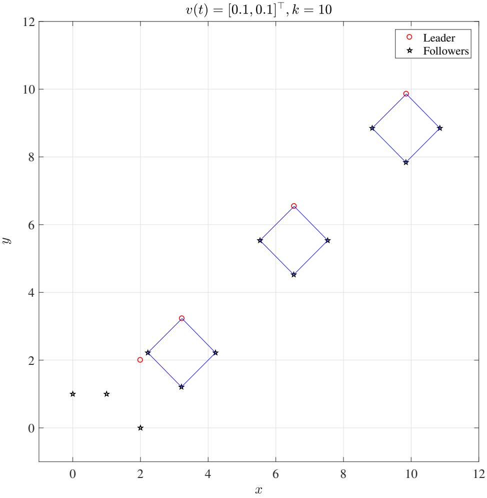
$$\dot{p}_1(t) \to \mathbf{0} \implies e_i(t) \to \mathbf{0},$$
$$k \to \infty \implies e_i(t) \to \mathbf{0} \text{ (not practical)}.$$

Leader-Follower Formation: Simulation Example



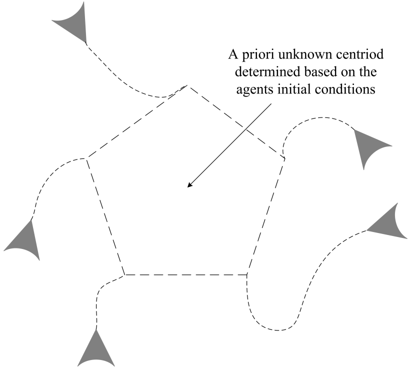
Network communication graph.

$$d_{12} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad d_{13} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad d_{34} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$



Consensus-Based Leaderless Formation

- As mentioned before, leader-following leads to some problems.
- In the case of leaderless networks, the agents should reach consensus on some quantities:
  - formation centroid
  - formation shape (size and orientation)



Achieving a formation in a leaderless group of mobile agents.

Consensus-Based Leaderless Formation

- We consider the following interaction law:

$$\dot{p}_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(p_j(t) + d_{ji} - p_i(t)), i \in \{1, 2, \ldots, N\}, \tag{2}$$

- which $d_{ji}, i, j \in \{1, 2, \ldots, N\}$, are fixed relative positions.
- They should be chosen to satisfy a feasible formation such that there exist $d_i, i \in \{1, 2, \ldots, N\}$,

$$d_i - d_j = d_{ji}, i, j \in \{1, 2, \ldots, N\}.$$

- Since there is no a leader in the network, integrated analysis is required.
- By defining $\bar{p}_i(t) = p_i(t) - d_i$, (2) implies

$$\dot{\bar{p}}_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{p}_j(t) - \bar{p}_i(t)), i \in \{1, 2, \ldots, N\}.$$

Consensus-Based Leaderless Formation

- Recalling the consensus problem, if the network communication graph has a spanning tree,

$$\lim_{t\to\infty} \bar{p}_i(t) = \bar{p}_j(t) = p_c, i, j \in \{1, 2, \dots, N\},$$

or

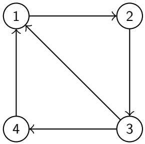
$$\lim_{t\to\infty} p_i(t) - d_i = p_j(t) - d_j = p_c.$$

- Therefore,

$$\lim_{t\to\infty} p_j(t) + d_{ji} - p_i(t) = \mathbf{0}.$$

- It implies achieving a desired formation about an *a priori* unknown centroid.

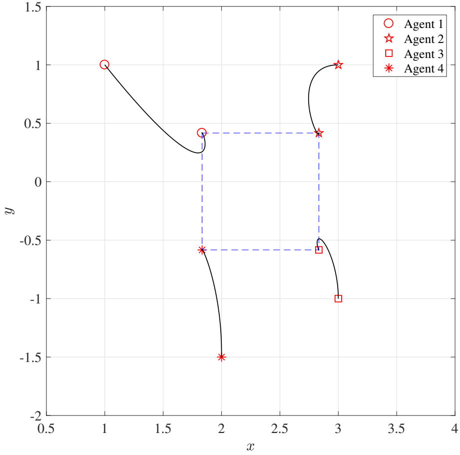
Consensus-Based Formation: Simulation



Network communication graph.

$$a_{13} = 1,\ a_{14} = 1,\ a_{21} = 1,$$

$$a_{32} = 1,\ a_{43} = 1$$



Consensus-Based Dynamic Formation

- The formation can be dynamic with a desired velocity $v_d(t)$.
- The interaction protocol is modified as follows:

$$\dot{p}_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(p_j(t) + d_{ji} - p_i(t)) + v_d(t), i \in \{1, 2, \ldots, N\}. \tag{3}$$

- Let $p_d(t)$ be an *a priori* unknown trajectory such that

$$\dot{p}_d(t) = v_d(t).$$

- By defining $\bar{p}_i(t) = p_i(t) - d_i - p_d(t)$, and since $d_i - d_j = d_{ji}$, (3) implies that

$$\dot{\bar{p}}_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(\bar{p}_j(t) - \bar{p}_i(t)), i \in \{1, 2, \ldots, N\}.$$

Consensus-Based Dynamic Formation

- Recalling the consensus problem, if the network communication graph has a spanning tree,

$$\lim_{t\to\infty} \bar{p}_i(t) = \bar{p}_j(t) = p_c, i, j \in \{1, 2, \ldots, N\},$$

or

$$\lim_{t\to\infty} p_i(t) - d_i - p_d(t) = p_j(t) - d_j - p_d(t) = p_c.$$

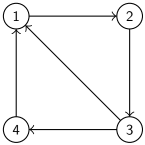
- Therefore,

$$\lim_{t\to\infty} p_j(t) + d_{ji} - p_i(t) = \mathbf{0},$$

$$\lim_{t\to\infty} \dot{\bar{p}}_i(t) = \dot{p}_i(t) - \dot{p}_d(t) = \mathbf{0}.$$

- They imply achieving a desired formation with velocity $v_d(t)$.

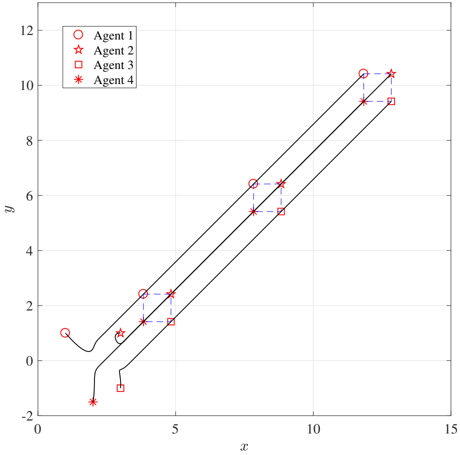
Consensus-Based Dynamic Formation: Simulation



Network communication graph.

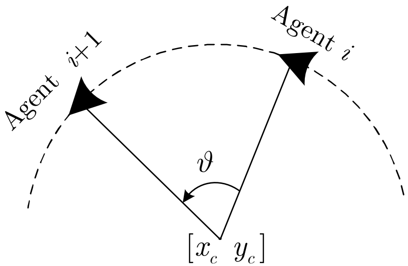
$$v_d(t) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$a_{13} = 1,\ a_{14} = 1,\ a_{21} = 1,$
$a_{32} = 1,\ a_{43} = 1$



Pursuit Formation

- Cyclic pursuit is another scenario for formation control of mobile autonomous agents.
- It is inspired by biological organisms such as ants and beetles for searching foods or avoiding predators.
- The rotational motion of a team of agents around a centroid increases their searching capability and coverage range.
- Full line of sight is feasible by a limited number of agents.



Cyclic pursuit configuration.

Pursuit Formation

- The desired position of the ith agent to achieve a regular polygon formation (with the angular rate ϖ) around the centroid $p_c = \begin{bmatrix} x_c & y_c \end{bmatrix}^\top$:

$$p_i^d(t) = p_c + \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix} \left(p_{i+1}(t) - p_c\right)$$

- where $\vartheta = \mathrm{sgn}(\varpi)\frac{2\pi}{N}$.
- To achieve a pursuit formation, we consider a control law as the combination of two terms:
  - a term to keep a desired formation
  - a term leading to rotational motion
- Therefore,

$$u_i(t) = \underbrace{-\lambda\left(p_i(t) - p_i^d(t)\right)}_{u_{i1}(t)} + \underbrace{\begin{bmatrix} 0 & -\varpi \\ \varpi & 0 \end{bmatrix} \left(p_i(t) - p_c\right)}_{u_{i2}(t)}.$$

Pursuit Formation

- The challenge is convergence analysis considering the coupled controllers $u_{i1}(t)$ and $u_{i2}(t)$.
- By defining $\mathbf{p}_i(t) = p_i(t) - p_c(t)$, for the whole MAS we have

$$\begin{bmatrix} \dot{\mathbf{p}}_1(t) \\ \vdots \\ \dot{\mathbf{p}}_N(t) \end{bmatrix} = C \begin{bmatrix} \mathbf{p}_1(t) \\ \vdots \\ \mathbf{p}_N(t) \end{bmatrix}$$

- where

$$C = \begin{bmatrix} C_1 & C_2 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & C_1 & C_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_2 & \mathbf{0} & \mathbf{0} & \dots & C_1 \end{bmatrix}$$

- which

$$C_1 = \begin{bmatrix} -\lambda & -\varpi \\ \varpi & -\lambda \end{bmatrix}, C_2 = \begin{bmatrix} \lambda \cos\vartheta & \lambda \sin\vartheta \\ -\lambda \sin\vartheta & \lambda\cos\vartheta \end{bmatrix}.$$

Pursuit Formation

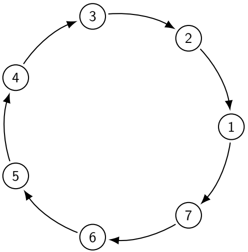
- Based on the theory of circulant matrices, C has
  - two imaginary eigenvalues $\pm j\varpi$
  - $2N-2$ eigenvalues in the open left half plane
- One can say the steady behavior will be determined by the imaginary eigenvalues:

$$\dot{\mathbf{p}}_i(t) \rightarrow \begin{bmatrix} 0 & -\varpi \\ \varpi & 0 \end{bmatrix} \mathbf{p}_i(t) \Longrightarrow \text{rotation}$$

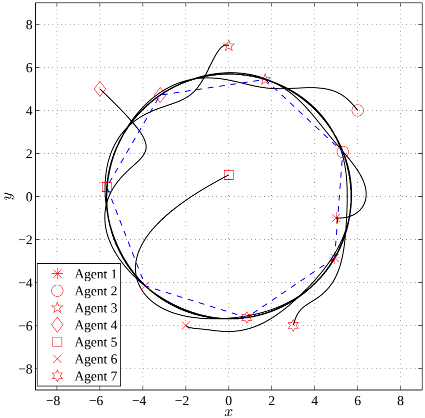
- Since $u_i(t) = \dot{\mathbf{p}}_i(t)$, it implies that $u_i(t) \rightarrow u_{2i}(t)$. Therefore,

$$u_{1i}(t) \rightarrow \mathbf{0} \Longrightarrow p_i(t) - p_i^d(t) \rightarrow \mathbf{0} \Longrightarrow \text{achieving a formation}$$

Pursuit Formation: Simulation



Network communication graph.



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