Topics in Control: Model Order Reduction

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30 January 2020

Include your code in an appendix at the end of the report. Submit a single pdf file.

Standard plagiarism and late submission policies apply.

Solve points 1 to 6. If correctly done, these are enough for a First. Point 7 is optional and will give you bonus marks.

Mechanical models can be described by the second-order differential system

$$M\ddot{q} + C_d\dot{q} + Kq = B_d u$$

where $q \in \mathbb{R}^\kappa$, $M \in \mathbb{R}^{\kappa\times\kappa}$, $C_d \in \mathbb{R}^{\kappa\times\kappa}$ and $B_d \in \mathbb{R}^{\kappa\times 1}$, with M positive definite. This system can be written as

$$\dot{x} = Ax + Bu \qquad y = Cx, \tag{1}$$

with

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_d \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ -M^{-1}B_d \end{bmatrix},$$

and C taken according to the application.

Figure 1: Los Angeles Hospital building

In particular, we consider the model of a building with 8 floors, each having three degrees of freedom, *i.e.* displacements in $x-$ and $y-$directions, and rotation. The resulting system (1) has a state of dimension $n = 48$. The output of the system is the 25-th component of the state which corresponds to displacement in the $x-$direction of the first floor. The resulting matrices (A, B, C) of system (1) can be downloaded from Blackboard.

1. Select a matrix S and a vector L. As you complete your coursework, you will experience that the selection of S is fundamental to obtain a good reduced order model. You should update the chosen S and L often. In your report, explain how you evolved the selection of the matrices S and L. Why and how did you end up with your final selection?

2. Compute the matrix Π. Do not use the function "Sylvester" in MATLAB. Write your own routine exploiting more basic MATLAB commands (e.g. "kron"). Compare the solution of your routine with the solution given by the function "Sylvester" in MATLAB (it does not need to be better).

3. For your chosen S and L, take random initial conditions $x(0)$ and $\omega(0)$ and determine the output $y(t)$ when the input to (1) is given by $u(t) = L\omega(t)$. Plot the output $y(t)$ and the steady-state $y_{ss}(t) = C\Pi\omega(t)$. Explain the plot.

4. Select a set of eigenvalues Σ and determine a reduced order model (F, G, H) such that F has the eigenvalues Σ. Plot the output $y(t)$ and the output of the reduced order model $y_r(t)$. Explain the plot.

5. Show the Bode plots of (1) and of your reduced order model. Adjust S and Σ in order to get a better reduced order model. Explain your reasoning.

6. Using the same matrix S, use the output sequence $\{y(t)\}$ and the sequence $\{\omega(t)\}$ to compute an approximation of $C\Pi$ using the data-driven algorithm. Show the success of your algorithm by means of figures and/or computations.

7. BONUS: Read the paper [2]. With the previous result at point 6, estimate a transient sequence $y_{tr}(t)$ and use it to estimate an optimal $\widetilde{F}$ and $\widetilde{H}$ (you may use the function *ssest* of MATLAB). Use the result in point 6 to compute the optimal model $(\widetilde{F}, \widetilde{G}, \widetilde{H})$. Comment on all the numerical problems that you faced (if any). Show the Bode plots of (1) and of the new reduced order model and compare them with the previous results.

References

[1] G. Scarciotti, A. Astolfi. *Nonlinear Model Reduction by Moment Matching.* Foundations and Trends in Systems and Control: Vol. 4: No. 3-4, pp 224-409.

[2] G. Scarciotti, Z.-P. Jiang, A. Astolfi. *Constrained optimal reduced-order models from input/output data*, 55th IEEE Conference on Decision and Control, Pages: 7453-7458, 2016