A MATLAB MEX-file environment of SLICOT [1]

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Abstract

Several MEX-files are developed based on SLICOT Fortran subroutines. The MEX-files provide new tools for the numerical solution of some classical control problems, such as the solution of linear or Riccati matrix equations computations in the MATLAB environment. Numerical tests show that the resulting MEX-files are equally accurate and much more efficient than the corresponding MATLAB functions in the control system toolbox and the robust control toolbox. In order to increase user-friendlyness the related m-files are also developed so that the MEX-file interface to the corresponding SLICOT routines can be implemented directly and easily.

1 Introduction

Because of its easy user interface, the MATLAB environment including various toolboxes [3, 4, 5] is currently widely used in the scientific community. Recently we can also observe an increasing acceptance of MATLAB even in solving industrial applications. This increasing popularity of MATLAB has led to the unfortunate side effect that standardized software libraries written in Fortran or C loose some of their importance, since they are relatively complicated to use for non-expert users. However the software in these low level libraries is often much more reliable and usually much more efficient than equivalent m-file implementations available in the MATLAB environment, since it can make use of data structures and system structures much more efficiently. Using the MATLAB MEX gateway it is possible to incorporate the efficiency of Fortran or C subroutines into the MATLAB environment.

The high demand for user-friendly interfaces and the desire to promote robust and efficient numerical software have led the partners of the NICONET Project to produce a collection of MEX-files representing MATLAB gateways for the most important subroutines of the SLICOT library.

In this report we will describe five MEX-files developed recently which belong to four different categories:

- Algebraic Riccati equation (ARE) solvers.
- Linear matrix equations (LME) solvers.
- System controllability, observability and minimal realization (COM) computational tools.
- System transformation tools.

For easy use of these MEX-files we have also developed another MATLAB interface consisting of m-files, that represents a single function for each single problem which then calls the appropriate MEX-file.

In Section 2 we discuss the functions and structures of the MEX gateway and m-files and their relations to SLICOT subroutines. In section 3 we present some numerical tests to show the superiority of the gateways compared to the corresponding MATLAB control system toolbox and robust control toolbox functions. In Appendix A we present the category tables for the presented mex-files.

2 MEX subroutines and m-files

SLICOT subroutines are based on the Fortran libraries LAPACK and BLAS. Moreover, SLICOT usually has several routines to deal with a class of problems of the same type. On the other hand MATLAB provides a flexible environment for inputs and outputs as well as basic matrix operations. We combine several SLICOT subroutines from the same class into a single MEX interface. Such a MEX file is obviously complicated so it is suitable only for experts to use. For general users we generated another simple MATLAB interface, consisting of a sequence of m-files calling these MEX files.

The basic strategy in the development of these interfaces is the following:

- The MEX interfaces are written for experts only.

  Based on these MEX files it should be easy for experts to create the related m-files. Moreover, since the SLICOT library is an ongoing project, the MEX interfaces have simple logical structures so that it is easy for experts to modify or expand these MEX routines.

- The m-files are prepared for users.

  They should be easy to use for all kinds of users without much knowledge about SLICOT routines and computational details.

Based on these requirements the structures of the MEX routines and m-files are designed as shown in the following figures. Here to simplify the subroutine structures we developed two MEX subroutines for standard linear matrix equation solvers, one for linear matrix equations and one for generalized linear matrix equations.

All MEX files are written in Fortran to comply with the SLICOT Fortran library.

In Figure 1, the MEX routine ARESOL.f includes the SLICOT routines **SB02MD.f, SB02ND.f, SB02MT.f, SB02OD.f** for the solution of the algebraic Riccati equations

$$
\begin{aligned}
0 &= Q + A^T X + XA - (L + XB)R^{-1}(L + XB)^T, & (1)\\
0 &= Q + A^T X + XA - XGX, & (2)\\
X &= Q + A^T XA - (L + A^T XB)(R + B^T XB)^{-1}(L + A^T XB)^T, & (3)\\
X &= Q + A^T X(I + GX)^{-1}A. & (4)
\end{aligned}
$$

The new m-files that call the MEX-file are the following:

- `slcaregs`: solves the CARE (1) and computes the associated feedback gains of the linear quadratic control problem by using the generalized Schur method on extended matrix pencils.

- `slcares`: solves the CAREs (1) and (2) and the related control problems by using the Schur method on the related double sized matrices.

- `sldaregs`: solves the DARE (3) and the associated feedback gain of the linear quadratic control problem by using the generalized Schur method on extended matrix pencils.

- `sldares`: solves the DAREs (3) and (4) and the related control problems by using the Schur method on the related double sized matrices.

- `sldaregsv`: solves the DAREs (3) and (4) and the related control problems by using the generalized Schur method on the related double sized pencils. (In `sldaregs` the size of the pencil is larger than twice the size of the state A.)

In Figure 2, the MEX gateway GENLEQ.f combines the SLICOT subroutines **SB04OD.f, SG03AD.f. SG03BD.f** for solving the generalized linear matrix equations

$$AX - YB = C, \qquad DX - YE = F, \tag{5}$$

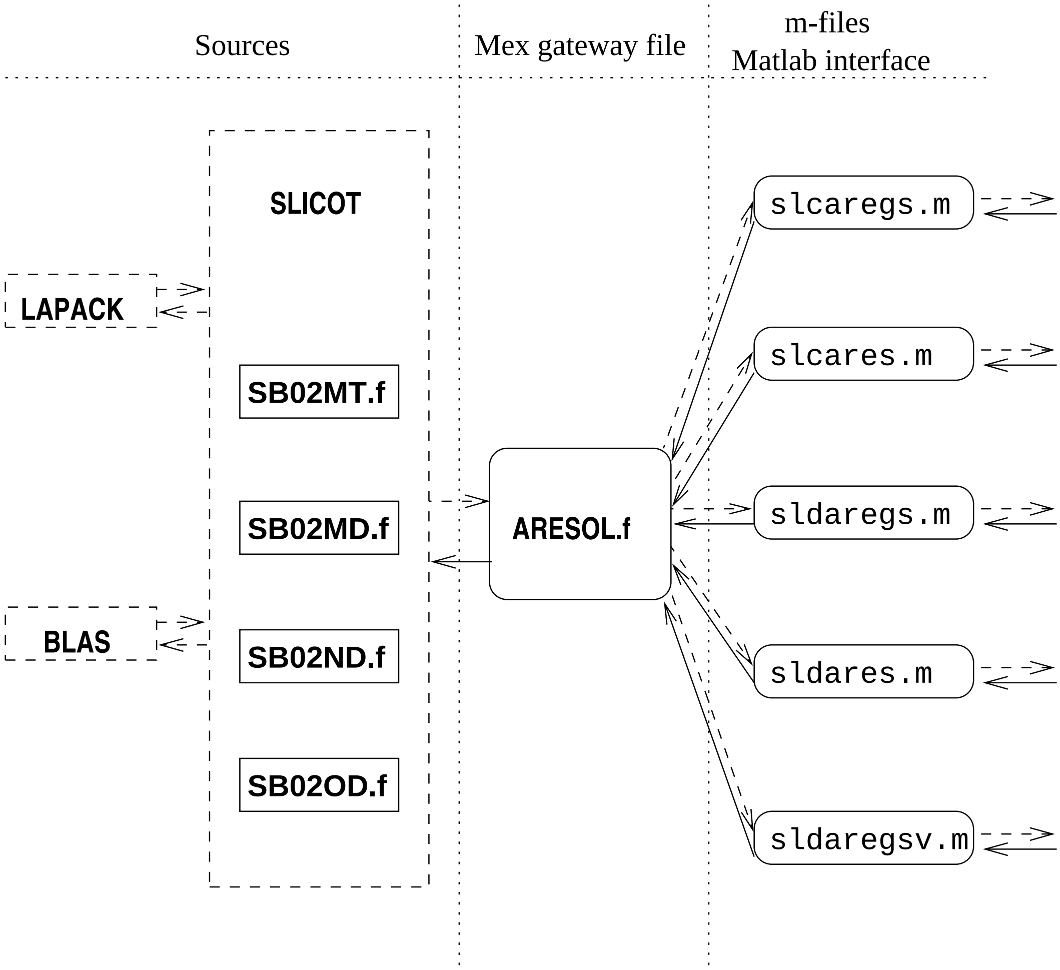

Figure 1: Structure of the ARE solver

$$A^T X + D^T Y = C, \qquad X B^T + Y E^T = -F, \tag{6}$$

$$AXB - DXE = C. \tag{7}$$

$$\mathtt{op}(A)^T X \mathtt{op}(E) + \mathtt{op}(E)^T X \mathtt{op}(A) = C, \tag{8}$$

$$\mathtt{op}(A)^T X \mathtt{op}(A) - \mathtt{op}(E)^T X \mathtt{op}(E) = C, \tag{9}$$

$$\begin{aligned}\mathtt{op}(A)^T \left(\mathtt{op}(X)^T \mathtt{op}(X)\right) \mathtt{op}(E) + \mathtt{op}(E)^T \left(\mathtt{op}(X)^T \mathtt{op}(X)\right) \mathtt{op}(A)& \\ = -\mathtt{op}(C)^T \mathtt{op}(C),&\end{aligned} \tag{10}$$

$$\begin{aligned}\mathtt{op}(A)^T \left(\mathtt{op}(X)^T \mathtt{op}(X)\right) \mathtt{op}(A) - \mathtt{op}(E)^T \left(\mathtt{op}(X)^T \mathtt{op}(X)\right) \mathtt{op}(E)& \\ = -\mathtt{op}(C)^T \mathtt{op}(C).&\end{aligned} \tag{11}$$

Here and hereafter $\mathtt{op}(A)$ is either matrix A or A^T.

The m-files have the following functions:

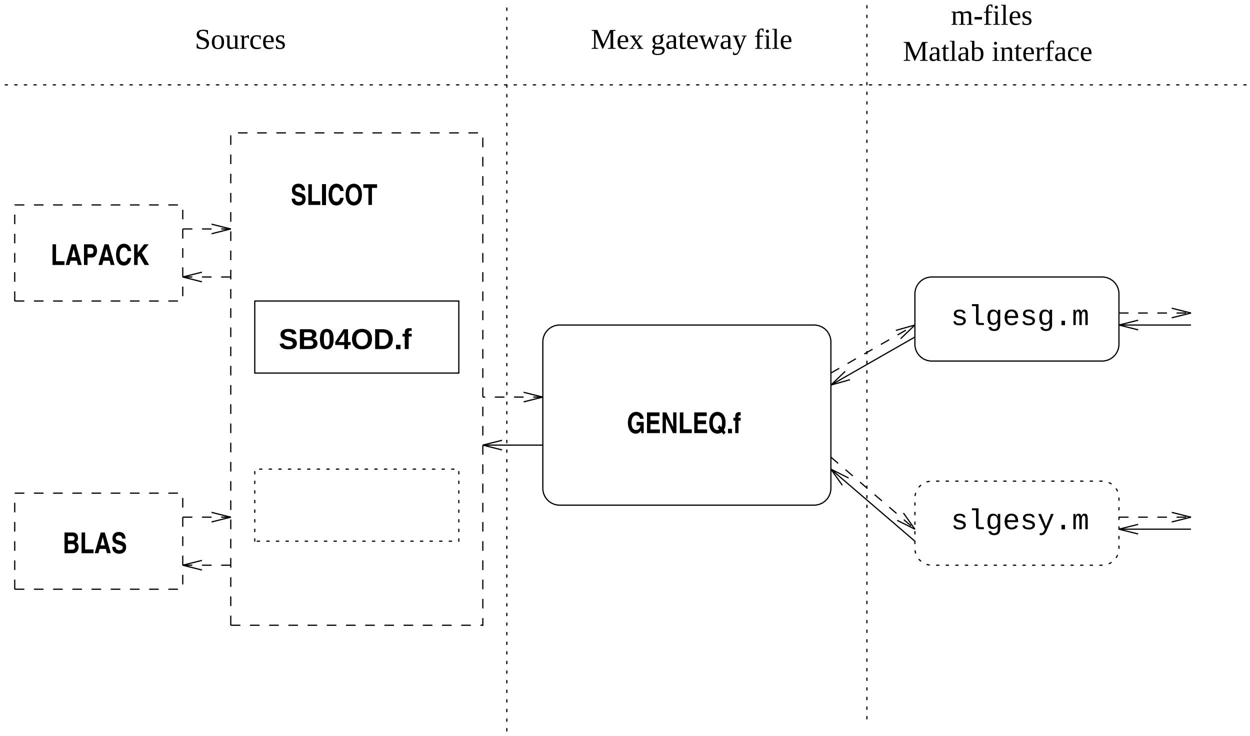

Figure 2: Structure of the GLME solver

- `slgesg`: solves the pair of generalized equations (5).
- `slgesy`*: solves the generalized equations (7) [1].
- `slgely`: solves the generalized continuous-time Lyapunov equations (8).
- `slgest`: solves the generalized discrete-time Lyapunov equations (9).
- `slgsly`: solves the generalized stable continuous-time Lyapunov equations (10).
- `slgsst`: solves the generalized stable discrete-time Lyapunov equations (11).

In Figure 3, the MEX routine LINMEQ.f combines all SLICOT subroutines **SB03MD.f, SB03OD.f, SB04MD.f, SB04ND.f** for solving the ordinary linear matrix equations

$$\mathtt{op}(A)X + X\mathtt{op}(B) = C, \tag{12}$$
$$\mathtt{op}(A)X\mathtt{op}(B) + X = C, \tag{13}$$
$$\mathtt{op}(A)^T X + X\mathtt{op}(A) = C, \tag{14}$$
$$\mathtt{op}(A)^T X\mathtt{op}(A) - X = C, \tag{15}$$
$$\mathtt{op}(A)^T \left(\mathtt{op}(X)^T \mathtt{op}(X)\right) + \left(\mathtt{op}(X)^T \mathtt{op}(X)\right) \mathtt{op}(A)$$

[1]All functions with superscript * are still unavailable at current stage.

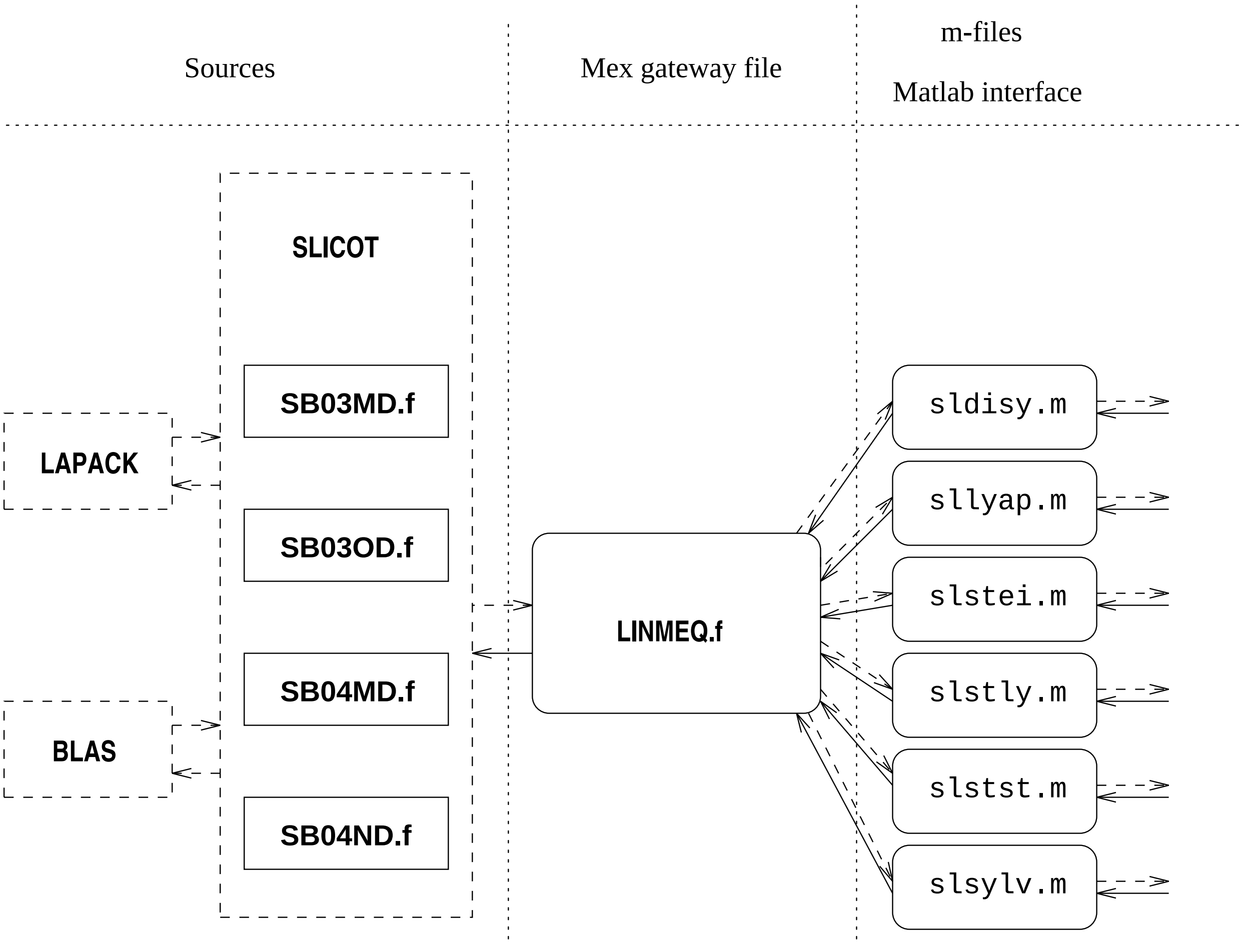

Figure 3: Structure of the LME solver

$$= -\mathtt{op}(C)^T \mathtt{op}(C), \tag{16}$$

$$\mathtt{op}(A)^T \left(\mathtt{op}(X)^T \mathtt{op}(X)\right) \mathtt{op}(A) - \mathtt{op}(X)^T \mathtt{op}(X) = -\mathtt{op}(C)^T \mathtt{op}(C). \tag{17}$$

The m-files are designed as follows:

- `sldisy`*: solves the discrete type Sylvester equations (13).
- `sllyap`: solves the Lyapunov equation (14).
- `slstei`: solves the Stein equation (15).
- `slstly`: solves the stable Lyapunov equation (16).
- `slstst`: solves the stable Stein equation (17).

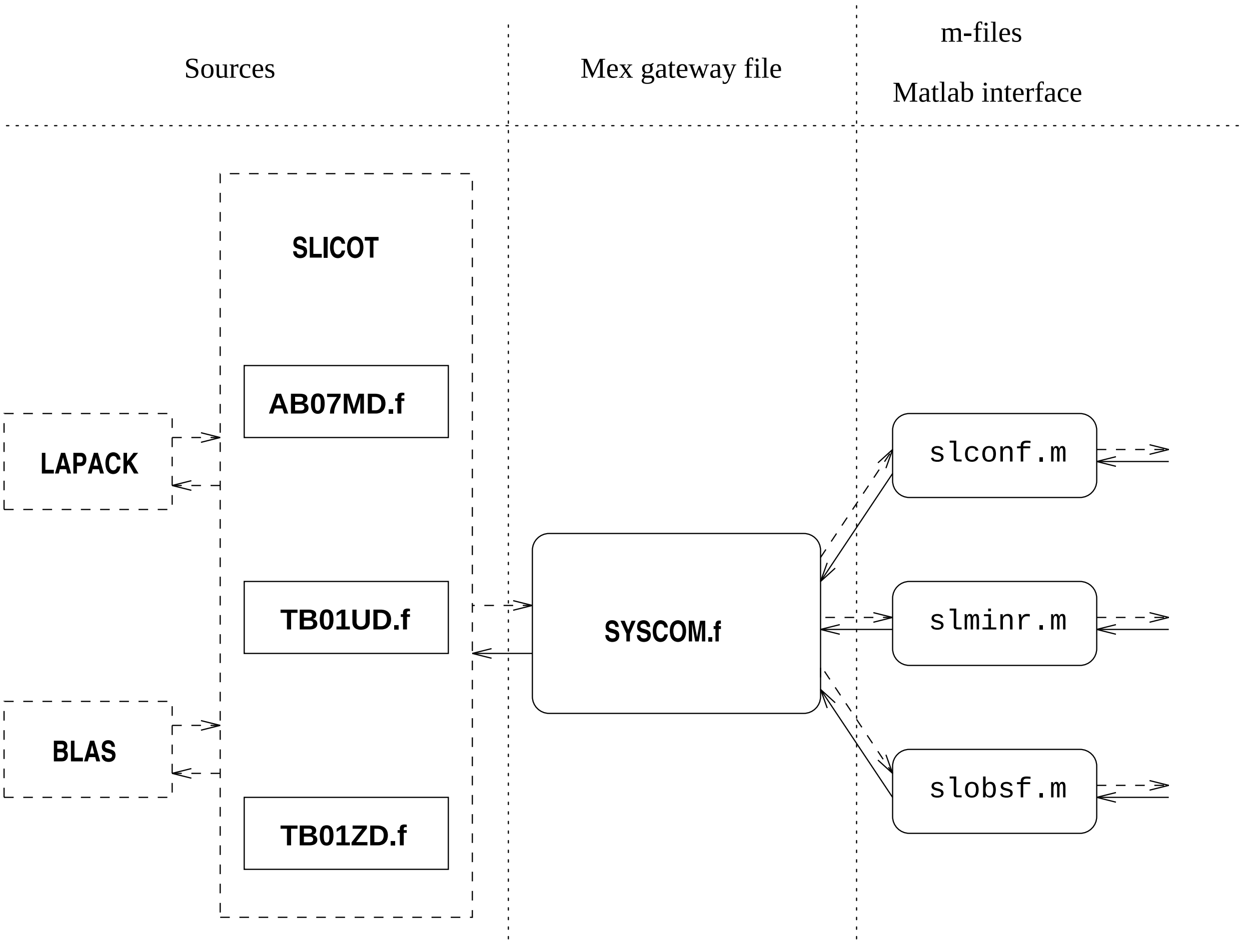

Figure 4: Structure of the System COM solver

- `slsylv`: solves the Sylvester equation (12).

In Figure 4 the MEX routine SYSCOM.f combines the SLICOT subroutines **AB07MD.f, TB01PD.f, TB01UD.f, TB01ZD.f** to compute the controllability, observability forms and the minimal realization sub-systems. Corresponding m-files are given for the following tasks:

- `slconf`: computes a transformed controllable system and checks controllability.
- `slminr`: computes a minimal realization sub-system.
- `slobsf`: computes a transformed observable system and checks observability.

In Figure 5 the MEX routine SYSTRA.f combines the SLICOT subroutines **TB01ID.f, TB01KD.f, TB01LD.f, TB01WD.f** to balance systems or to transform state matrices to triangular forms. The associated m-files perform the following tasks:

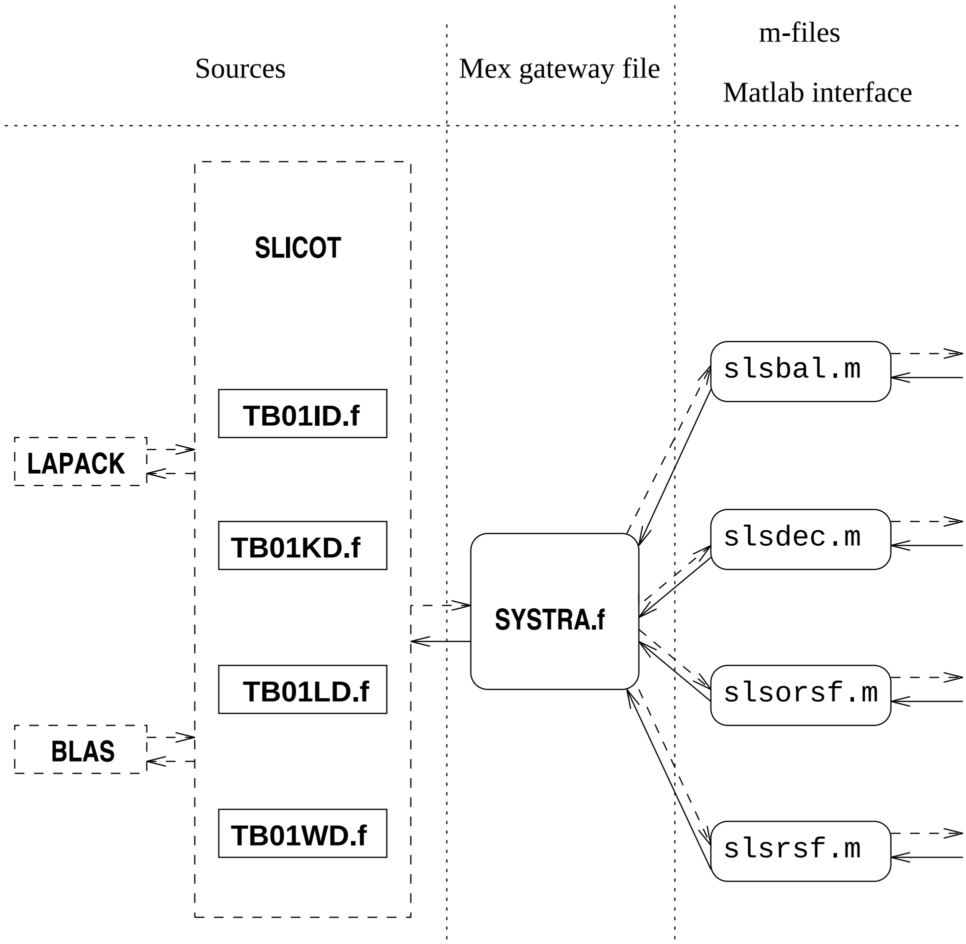

Figure 5: Structure of the System Transformation solver

- `slsbal`: balances a given system with matrices (A, B, C).
- `slsdec`: transforms the state matrix to block diagonal form with the eigenvalues in the required order.
- `slsorsf`: transforms the state matrix to Schur form with the eigenvalues in the required order.
- `slsrsf`: transforms the state matrix to Schur form.

Brief explanations of the functionality of the MEX routines and m-files are given in the category tables in Appendix A. The MEX routines are designed as bridges to connect the SLICOT library and the MATLAB m-files. The MEX subroutines have the following characteristics:

- Each subroutine has been written in a compact, but structured form, and the same functional blocks of code appear in all subroutines. An option argument in the first input, to indicate the method to be used or the task to be performed.

- All subroutines use MATLAB pointer functions for data inputs and outputs. This makes the MEX routines portable for different types of computers. We do not use the MATLAB `%val` construction, since it is not available for all machines.

- To make the routines efficient for all types of arrays we use Fortran 90 allocatable types to reduce the storage requirements during the computation. Considering that Fortran 90 compilers are not available everywhere, we give a comment `assign Fortran 90 allocatable arrays` before the block of `ALLOCATE` statements, and a comment `free Fortran 90 allocatable arrays` before the block of `DEALLOCATE` statements. This may make it easy for users to modify the MEX routines if only a Fortran 77 compiler is available by simply deleting all `ALLOCATE` and `DEALLOCATE` statements and assigning the arrays with fixed dimensions in the beginning. (In this case the user should refer to the related SLICOT subroutines for the proper sizes and lengths of the arrays.)

- For all subroutines the numbers of input arguments and output arguments are variable. This reduces unnecessary computations.

- Error messages are provided when the routines break down. The error messages will report incorrect input-output arguments or SLICOT routine error messages created during the computations.

The m-files are built as the interface for the MEX routines. Since they are user functions, we made the input-output arguments flexible and in such a way that they are easy to use. This leads to the following properties.

- All m-files work for minimal necessary input data. The default option values will be automatically set by the functions.

- The outputs are arranged in order based on the importance of the arguments.

- In some m-files there are several important outputs but the user may only be interested in some of them. For this we put an additional option in the input to let the user choose the wanted outputs.

3 Numerical results

We have performed many numerical calculations using the developed mexfiles. The results reported below have been obtained using the Fortran 77 versions of the mexfiles, run on a SUN Ultra 2 Creator 2200 workstation under SunOS 5.5, and Matlab 5.3. Also available are numerical results obtained using the Fortran 90 versions of the mexfiles, run on a PC at 500 MHz, with 128 Mb memory, Digital Visual Fortran V5.0, and Matlab 5.3.0.10183 (R11).

We have tested the m-files associated to the MEX file **genleq**. The test results for **slgesg** are listed in the Tables 1 – 4. For Tables 1, 2 all matrices are randomly generated with various dimensions. In Table 1 the estimator *Dif* is computed while in Table 2 it is not computed.

Size		Time	Rel. residual		Rel. error		Dif
n	m	slgesg	in X	in Y	in X	in Y	
10	2	0.01	1.3×10^{-15}	3.2×10^{-15}	3.6×10^{-15}	7.0×10^{-15}	0.122
	4	0.01	5.1×10^{-15}	4.7×10^{-15}	4.0×10^{-14}	2.7×10^{-14}	0.022
	6	0.00	2.5×10^{-15}	2.7×10^{-15}	1.1×10^{-13}	1.9×10^{-13}	0.003
	8	0.00	3.4×10^{-15}	3.8×10^{-15}	1.7×10^{-13}	3.0×10^{-13}	0.004
	10	0.01	1.0×10^{-14}	8.9×10^{-15}	2.6×10^{-13}	4.3×10^{-13}	0.003
20	4	0.01	1.2×10^{-14}	1.2×10^{-14}	3.4×10^{-14}	6.8×10^{-14}	0.023
	8	0.02	1.0×10^{-14}	1.1×10^{-14}	4.1×10^{-14}	7.3×10^{-14}	0.044
	12	0.02	1.2×10^{-14}	1.1×10^{-14}	4.4×10^{-13}	4.3×10^{-13}	0.010
	16	0.03	1.4×10^{-14}	1.6×10^{-14}	4.5×10^{-14}	4.5×10^{-14}	0.021
	20	0.04	1.8×10^{-14}	2.1×10^{-14}	1.3×10^{-13}	1.5×10^{-13}	0.015
30	6	0.04	1.6×10^{-14}	1.5×10^{-14}	5.9×10^{-14}	7.6×10^{-14}	0.029
	12	0.05	1.7×10^{-14}	1.4×10^{-14}	4.6×10^{-13}	8.3×10^{-13}	0.002
	18	0.07	2.0×10^{-14}	2.2×10^{-14}	3.5×10^{-13}	3.8×10^{-13}	0.006
	24	0.08	3.0×10^{-14}	2.8×10^{-14}	7.7×10^{-13}	7.6×10^{-13}	0.002
	30	0.12	3.3×10^{-14}	3.2×10^{-14}	1.0×10^{-12}	1.2×10^{-12}	0.003
40	8	0.09	2.7×10^{-14}	2.7×10^{-14}	3.9×10^{-13}	7.3×10^{-13}	0.007
	16	0.11	2.8×10^{-14}	3.2×10^{-14}	1.1×10^{-12}	2.2×10^{-12}	0.002
	24	0.14	3.6×10^{-14}	3.2×10^{-14}	5.5×10^{-13}	8.8×10^{-13}	0.005
	32	0.17	4.7×10^{-14}	4.7×10^{-14}	1.4×10^{-12}	1.8×10^{-12}	0.002
	40	0.24	5.7×10^{-14}	6.5×10^{-14}	2.4×10^{-13}	2.3×10^{-13}	0.012
50	10	0.16	3.5×10^{-14}	3.0×10^{-14}	1.9×10^{-13}	2.9×10^{-13}	0.011
	20	0.19	5.2×10^{-14}	4.4×10^{-14}	4.3×10^{-13}	6.4×10^{-13}	0.011
	30	0.25	6.0×10^{-14}	5.9×10^{-14}	3.2×10^{-13}	3.2×10^{-13}	0.008
	40	0.33	7.2×10^{-14}	6.7×10^{-14}	5.1×10^{-13}	6.2×10^{-13}	0.007
	50	0.44	8.6×10^{-14}	8.6×10^{-14}	1.1×10^{-12}	5.0×10^{-13}	0.002

Table 1: Test **slgesg**. Random matrices I

Size		Time	Rel. residual		Rel. error	
n	m	slgesg	in X	in Y	in X	in Y
16	4	0.00	6.6×10^{-15}	7.1×10^{-15}	1.4×10^{-14}	2.0×10^{-14}
	8	0.01	6.2×10^{-15}	7.8×10^{-15}	5.0×10^{-14}	4.2×10^{-14}
	12	0.02	1.1×10^{-14}	1.3×10^{-14}	3.4×10^{-14}	4.5×10^{-14}
	16	0.02	1.3×10^{-14}	1.3×10^{-14}	1.4×10^{-13}	1.1×10^{-13}
32	8	0.04	2.2×10^{-14}	2.2×10^{-14}	1.5×10^{-13}	3.3×10^{-13}
	16	0.05	2.1×10^{-14}	2.3×10^{-14}	1.4×10^{-13}	1.5×10^{-13}
	24	0.08	3.4×10^{-14}	2.9×10^{-14}	5.9×10^{-13}	5.2×10^{-13}
	32	0.11	3.7×10^{-14}	3.4×10^{-14}	8.5×10^{-14}	1.3×10^{-13}
64	16	0.30	5.9×10^{-14}	6.6×10^{-14}	4.6×10^{-13}	8.3×10^{-13}
	32	0.39	8.8×10^{-14}	8.2×10^{-14}	2.4×10^{-11}	3.6×10^{-11}
	48	0.55	9.5×10^{-14}	9.7×10^{-14}	1.2×10^{-8}	1.4×10^{-8}
	64	0.77	1.3×10^{-13}	1.4×10^{-13}	6.7×10^{-13}	6.6×10^{-13}
128	32	2.27	1.6×10^{-13}	1.5×10^{-13}	1.0×10^{-12}	1.9×10^{-12}
	64	2.77	2.2×10^{-13}	2.2×10^{-13}	1.1×10^{-12}	1.6×10^{-12}
	96	4.02	2.7×10^{-13}	2.8×10^{-13}	2.2×10^{-11}	2.4×10^{-11}
	128	6.02	3.7×10^{-13}	3.8×10^{-13}	6.8×10^{-12}	6.6×10^{-12}

Table 2: Test **slgesg**. Random matrices II

In Table 3 the results were computed with the pencil (A, D) in randomly generated Schur form and (B, E) still in general form. In Table 4 both pencils (A, D) and (B, E) are in Schur forms.

The tables 5 – 7 list the test results for the m-file **slgely** which solves the generalized continuous-time Lyapunov equations.

Tables 8 – 10 show the test results for the m-file **slgest** for solving the generalized discrete-time Lyapunov equations.

Size		Time	Rel. residual		Rel. error	
n	m	slgesg	in X	in Y	in X	in Y
16	4	0.00	1.3×10^{-15}	1.1×10^{-14}	6.8×10^{-16}	7.6×10^{-15}
	8	0.00	2.5×10^{-15}	1.3×10^{-14}	8.2×10^{-16}	5.1×10^{-15}
	12	0.01	8.8×10^{-15}	2.3×10^{-14}	1.9×10^{-14}	2.8×10^{-13}
	16	0.01	6.6×10^{-15}	2.6×10^{-14}	1.5×10^{-14}	2.9×10^{-13}
32	8	0.01	2.7×10^{-15}	4.0×10^{-14}	7.5×10^{-15}	1.9×10^{-13}
	16	0.02	5.6×10^{-15}	3.1×10^{-14}	1.3×10^{-15}	3.1×10^{-14}
	24	0.04	1.0×10^{-14}	5.6×10^{-14}	2.5×10^{-15}	5.0×10^{-14}
	32	0.07	2.0×10^{-14}	7.0×10^{-14}	3.5×10^{-15}	6.3×10^{-14}
64	16	0.03	7.6×10^{-15}	8.6×10^{-14}	1.3×10^{-15}	2.6×10^{-14}
	32	0.10	1.9×10^{-14}	1.5×10^{-13}	2.8×10^{-15}	7.7×10^{-14}
	48	0.23	3.0×10^{-14}	1.6×10^{-13}	3.4×10^{-15}	8.2×10^{-14}
	64	0.45	4.4×10^{-14}	1.8×10^{-13}	3.6×10^{-15}	7.6×10^{-14}
128	32	0.19	1.9×10^{-14}	2.7×10^{-13}	2.0×10^{-15}	7.6×10^{-14}
	64	0.66	4.4×10^{-14}	3.9×10^{-13}	1.3×10^{-13}	7.1×10^{-12}
	96	1.63	8.0×10^{-14}	4.3×10^{-13}	6.8×10^{-15}	3.2×10^{-13}
	128	3.45	1.1×10^{-13}	5.2×10^{-13}	6.2×10^{-14}	2.6×10^{-12}

Table 3: Test **slgesg**. Random Schur form I

Size		Time	Rel.residual		Rel. error	
n	m	slgesg	in X	in Y	in X	in Y
16	4	0.00	4.8×10^{-16}	3.1×10^{-15}	8.7×10^{-16}	2.9×10^{-15}
	8	0.00	6.1×10^{-16}	3.4×10^{-15}	9.5×10^{-16}	2.1×10^{-15}
	12	0.00	7.0×10^{-16}	3.4×10^{-15}	7.8×10^{-15}	1.1×10^{-14}
	16	0.01	8.6×10^{-16}	3.7×10^{-15}	1.3×10^{-14}	1.2×10^{-14}
32	8	0.01	5.8×10^{-16}	7.4×10^{-15}	1.2×10^{-15}	4.4×10^{-15}
	16	0.02	1.4×10^{-15}	8.0×10^{-15}	6.8×10^{-15}	1.3×10^{-14}
	24	0.02	1.5×10^{-15}	8.6×10^{-15}	5.0×10^{-15}	6.7×10^{-15}
	32	0.02	1.2×10^{-15}	1.0×10^{-14}	1.5×10^{-14}	1.5×10^{-14}
64	16	0.02	1.4×10^{-15}	1.8×10^{-14}	9.1×10^{-15}	3.6×10^{-14}
	32	0.05	1.5×10^{-15}	2.1×10^{-14}	8.6×10^{-15}	1.7×10^{-14}
	48	0.08	2.0×10^{-15}	2.4×10^{-14}	6.6×10^{-14}	8.8×10^{-14}
	64	0.11	2.0×10^{-15}	2.6×10^{-14}	4.1×10^{-14}	4.2×10^{-14}
128	32	0.12	2.5×10^{-15}	5.2×10^{-14}	1.1×10^{-14}	4.2×10^{-14}
	64	0.27	2.7×10^{-15}	6.1×10^{-14}	3.7×10^{-14}	7.5×10^{-14}
	96	0.45	3.6×10^{-15}	6.5×10^{-14}	7.7×10^{-14}	1.0×10^{-13}
	128	0.71	3.8×10^{-15}	7.0×10^{-14}	8.3×10^{-14}	8.4×10^{-14}

Table 4: Test **slgesg**. Random Schur form II

n	Time	Rel. residual	Rel. error	sep
10	0.01	3.5×10^{-14}	2.7×10^{-13}	0.001
20	0.03	2.6×10^{-13}	1.6×10^{-12}	0.182
30	0.09	5.9×10^{-13}	3.8×10^{-12}	0.126
40	0.19	1.4×10^{-12}	4.0×10^{-12}	6×10^{-4}
50	0.35	2.2×10^{-12}	2.2×10^{-11}	1×10^{-5}

Table 5: Test **slgely**. Random matrices I

n	Time	Rel. residual	Rel. error
16	0.01	1.5×10^{-13}	1.1×10^{-12}
32	0.06	6.4×10^{-13}	1.5×10^{-11}
64	0.37	4.3×10^{-12}	5.8×10^{-11}
128	2.83	2.0×10^{-11}	6.1×10^{-10}

Table 6: Test **slgely**. Random matrices II

n	Time	Rel. residual	Rel. error
16	0.01	1.1×10^{-14}	1.2×10^{-15}
32	0.02	3.6×10^{-14}	1.7×10^{-14}
64	0.08	1.3×10^{-13}	5.6×10^{-14}
128	0.64	4.1×10^{-13}	1.3×10^{-13}

Table 7: Test **slgely**. Random Schur form

n	Time	Rel. residual	Rel. error	sep
10	0.00	2.0×10^{-14}	7.8×10^{-14}	0.048
20	0.03	1.3×10^{-13}	2.9×10^{-12}	8×10^{-4}
30	0.11	2.5×10^{-13}	1.9×10^{-12}	7×10^{-4}
40	0.19	5.0×10^{-13}	2.7×10^{-11}	7×10^{-5}
50	0.32	1.3×10^{-12}	5.3×10^{-12}	0.064

Table 8: Test **slgest**. Random matrices I

n	Time	Rel. residual	Rel. error
16	0.01	1.1×10^{-13}	8.4×10^{-13}
32	0.06	2.8×10^{-13}	1.5×10^{-12}
64	0.41	1.8×10^{-12}	1.9×10^{-10}
128	2.90	7.2×10^{-12}	2.7×10^{-11}

Table 9: Test **slgest**. Random matrices II

n	Time	Rel. residual	Rel. error
16	0.01	8.9×10^{-14}	3.4×10^{-16}
32	0.02	4.4×10^{-13}	4.0×10^{-16}
64	0.09	2.2×10^{-12}	5.2×10^{-16}
128	0.63	1.2×10^{-11}	7.1×10^{-16}

Table 10: Test **slgest**. Random Schur form

The test results for **slgsly**, which solves the stable generalized continuous-time Lyapunov equations, are given in Table 11 – 13.

Finally the results for **slgsst** or the stable generalized discrete-time Lyapunov equation are reported in Table 14 – 16.

We also tested the m-files **slgely, slgest, slgsly, slgsst** with the benchmack collection matrices in *ctlex, dtlex.* The results are reported in the tables 17 – 24.

n	m	Time	Rel. residual
10	2	0.02	7.5×10^{-15}
	4	0.00	1.6×10^{-14}
	6	0.01	2.3×10^{-14}
	8	0.00	3.6×10^{-14}
	10	0.00	5.4×10^{-14}
20	4	0.02	2.4×10^{-14}
	8	0.01	7.8×10^{-14}
	12	0.01	8.7×10^{-14}
	16	0.02	1.2×10^{-13}
	20	0.02	1.6×10^{-13}
30	6	0.04	1.1×10^{-13}
	12	0.05	1.8×10^{-13}
	18	0.04	2.7×10^{-13}
	24	0.05	4.5×10^{-13}
	30	0.05	4.2×10^{-13}
40	8	0.08	2.2×10^{-13}
	16	0.10	4.7×10^{-13}
	24	0.09	4.8×10^{-13}
	32	0.11	9.1×10^{-13}
	40	0.09	9.5×10^{-13}
50	10	0.17	3.4×10^{-13}
	20	0.17	7.2×10^{-13}
	30	0.17	1.1×10^{-12}
	40	0.18	1.5×10^{-12}
	50	0.19	1.9×10^{-12}

Table 11: Test **slgsly**. Random matrices I

n	m	Time	Rel. residual
16	4	0.01	3.6×10^{-14}
	8	0.01	3.4×10^{-14}
	12	0.01	8.1×10^{-14}
	16	0.01	1.8×10^{-13}
32	8	0.05	1.4×10^{-13}
	16	0.06	2.7×10^{-13}
	24	0.05	3.3×10^{-13}
	32	0.06	3.8×10^{-13}
64	16	0.32	9.6×10^{-13}
	32	0.35	1.3×10^{-12}
	48	0.37	2.4×10^{-12}
	64	0.36	2.6×10^{-12}
128	32	2.58	3.4×10^{-12}
	64	2.72	7.3×10^{-12}
	96	2.74	1.1×10^{-11}
	128	2.81	1.4×10^{-11}

Table 12: Test **slgsly**. Random matrices II

n	m	Time	Rel. residual
16	4	0.01	1.3×10^{-14}
	8	0.00	1.3×10^{-14}
	12	0.00	1.6×10^{-14}
	16	0.01	2.6×10^{-13}
32	8	0.02	3.4×10^{-14}
	16	0.01	9.0×10^{-14}
	24	0.01	9.0×10^{-14}
	32	0.01	1.2×10^{-13}
64	16	0.06	1.7×10^{-13}
	32	0.08	2.8×10^{-13}
	48	0.08	4.1×10^{-13}
	64	0.08	6.9×10^{-13}
128	32	0.39	7.3×10^{-13}
	64	0.44	2.0×10^{-12}
	96	0.46	3.4×10^{-12}
	128	0.49	3.2×10^{-12}

Table 13: Test **slgsly**. Random Schur form

n	m	Time	Rel. residual
10	2	0.00	6.0×10^{-15}
	4	0.01	8.4×10^{-15}
	6	0.00	3.4×10^{-14}
	8	0.00	2.4×10^{-14}
	10	0.01	1.9×10^{-14}
20	4	0.02	3.2×10^{-14}
	8	0.02	5.7×10^{-14}
	12	0.02	1.0×10^{-13}
	16	0.02	9.6×10^{-14}
	20	0.02	1.8×10^{-13}
30	6	0.05	6.4×10^{-14}
	12	0.05	1.9×10^{-13}
	18	0.05	2.6×10^{-13}
	24	0.04	3.3×10^{-13}
	30	0.05	3.0×10^{-13}
40	8	0.10	1.9×10^{-13}
	16	0.10	3.6×10^{-13}
	24	0.10	3.9×10^{-13}
	32	0.10	5.6×10^{-13}
	40	0.10	9.7×10^{-13}
50	10	0.17	2.6×10^{-13}
	20	0.18	5.9×10^{-13}
	30	0.18	8.8×10^{-13}
	40	0.18	1.1×10^{-12}
	50	0.19	1.3×10^{-12}

Table 14: Test **slgsst**. Random matrices I

n	m	Time	Rel. residual
16	4	0.01	2.6×10^{-14}
	8	0.01	5.0×10^{-14}
	12	0.01	9.7×10^{-14}
	16	0.01	1.5×10^{-13}
32	8	0.05	1.5×10^{-13}
	16	0.05	2.1×10^{-13}
	24	0.05	2.9×10^{-13}
	32	0.05	5.1×10^{-13}
64	16	0.35	6.2×10^{-13}
	32	0.38	1.3×10^{-12}
	48	0.38	1.9×10^{-12}
	64	0.36	3.0×10^{-12}
128	32	2.74	4.2×10^{-12}
	64	2.73	7.6×10^{-12}
	96	2.85	1.0×10^{-11}
	128	2.86	1.2×10^{-11}

Table 15: Test **slgsly**. Random matrices II

n	m	Time	Rel. residual
16	4	0.01	4.8×10^{-15}
	8	0.01	1.3×10^{-14}
	12	0.01	1.6×10^{-14}
	16	0.01	4.3×10^{-14}
32	8	0.02	3.7×10^{-14}
	16	0.02	6.6×10^{-14}
	24	0.02	9.1×10^{-14}
	32	0.02	1.1×10^{-13}
64	16	0.08	1.5×10^{-13}
	32	0.09	3.0×10^{-13}
	48	0.09	4.5×10^{-13}
	64	0.09	5.9×10^{-13}
128	32	0.45	6.8×10^{-13}
	64	0.47	1.6×10^{-12}
	96	0.50	2.1×10^{-12}
	128	0.53	4.6×10^{-12}

Table 16: Test **slgsst**. Random Schur form

n	Time	Rel. residual	Rel. error
10	0.00	7.9×10^{-14}	5.3×10^{-15}
20	0.02	2.0×10^{-13}	6.4×10^{-14}
30	0.05	5.7×10^{-13}	4.6×10^{-13}
40	0.12	1.2×10^{-12}	2.6×10^{-13}

Table 17: Test **slgely**. Benchmark *ctlex*. Ex 4.3. I

n	Time	Rel. residual	Rel. error
16	0.01	1.6×10^{-13}	3.1×10^{-14}
32	0.06	8.5×10^{-13}	4.4×10^{-13}

Table 18: Test **slgely**. Benchmark *ctlex*. Ex 4.3. II

n	Time	Rel. residual	Rel. error
10	0.01	2.5×10^{-13}	6.7×10^{-15}
20	0.02	8.6×10^{-13}	2.2×10^{-14}
30	0.06	3.4×10^{-12}	9.8×10^{-14}
40	0.11	6.9×10^{-12}	9.3×10^{-14}

Table 19: Test **slgest**. Benchmark *dtlex*. Ex 4.3. I

n	Time	Rel. residual	Rel. error
16	0.01	4.3×10^{-13}	2.2×10^{-14}
32	0.06	3.6×10^{-12}	5.2×10^{-14}

Table 20: Test **slgest**. Benchmark *dtlex*. Ex 4.3. II

n	Time	Rel. residual
12	0.01	3.5×10^{-13}
24	0.02	9.7×10^{-12}
36	0.06	7.3×10^{-11}

Table 21: Test **slgsly**. Benchmark *ctlex*. Ex 4.4. I

n	Time	Rel. residual
18	0.01	1.8×10^{-12}
36	0.06	7.3×10^{-11}

Table 22: Test **slgsly**. Benchmark *ctlex*. Ex 4.4. II

n	Time	Rel. residual
12	0.01	1.0×10^{-14}
24	0.02	6.5×10^{-14}
36	0.07	2.1×10^{-13}

Table 23: Test **slgsst**. Benchmark *dtlex*. Ex 4.4. I

We tested the m-files associated to the MEX file **linmeq**. The first group of data was used to test the Sylvester equation solver **slsylv** and to compare it with the Matlab Toolbox code **lyap**. The results are listed in Tables 25 – 28.

Tables 29 – 31 list the test results for the Lyapunov equation solver **sllyap** and its comparison with Matlab code **lyap**.

The numerical results for the discrete-time Lyapunov equation solver **slstei** and its comparison with Matlab code **dlyap** are shown in Tables 32 – 34.

n	Time	Rel. residual
18	0.01	5.7×10^{-14}
36	0.07	2.1×10^{-13}

Table 24: Test **slgsst**. Benchmark *dtlex*. Ex 4.4 II

Size		Time		Rel. residual		Rel. error	
n	m	slsylv	lyap	slsylv	lyap	slsylv	lyap
10	2	0.00	0.01	1.3×10^{-15}	6.8×10^{-15}	6.0×10^{-15}	4.5×10^{-14}
	4	0.00	0.01	5.4×10^{-15}	6.9×10^{-15}	2.1×10^{-15}	4.6×10^{-15}
	6	0.00	0.01	2.7×10^{-15}	5.6×10^{-15}	6.5×10^{-15}	8.3×10^{-15}
20	4	0.00	0.02	9.1×10^{-15}	2.4×10^{-14}	1.0×10^{-14}	9.1×10^{-15}
	8	0.01	0.03	1.5×10^{-14}	2.9×10^{-14}	3.8×10^{-14}	1.8×10^{-14}
	12	0.01	0.04	1.4×10^{-14}	1.8×10^{-14}	4.4×10^{-14}	1.7×10^{-14}
30	6	0.01	0.05	2.7×10^{-14}	2.4×10^{-14}	7.9×10^{-15}	1.3×10^{-14}
	12	0.01	0.10	2.1×10^{-14}	2.2×10^{-14}	4.6×10^{-14}	3.4×10^{-14}
	18	0.01	0.10	2.5×10^{-14}	2.8×10^{-14}	8.2×10^{-14}	9.4×10^{-14}
40	8	0.02	0.09	3.1×10^{-14}	3.7×10^{-14}	2.7×10^{-13}	7.9×10^{-14}
	16	0.02	0.12	2.7×10^{-14}	4.4×10^{-14}	5.2×10^{-13}	3.5×10^{-13}
	24	0.04	0.17	4.0×10^{-14}	4.6×10^{-14}	8.4×10^{-14}	3.5×10^{-14}
50	10	0.02	0.15	3.5×10^{-14}	3.8×10^{-14}	1.9×10^{-13}	1.8×10^{-13}
	20	0.04	0.21	4.7×10^{-14}	4.5×10^{-14}	3.4×10^{-13}	3.5×10^{-13}
	30	0.06	0.27	7.0×10^{-14}	6.1×10^{-14}	6.8×10^{-13}	5.4×10^{-13}

Table 25: **slsylv** vs **lyap**. Random matrices I

Size		Time		Rel. residual		Rel. error	
n	m	slsylv	lyap	slsylv	lyap	slsylv	lyap
16	4	0.00	0.02	9.1×10^{-15}	1.3×10^{-14}	4.2×10^{-15}	3.4×10^{-14}
	8	0.00	0.02	7.7×10^{-15}	1.2×10^{-14}	1.0×10^{-14}	3.1×10^{-14}
32	8	0.00	0.06	3.4×10^{-14}	2.5×10^{-14}	6.5×10^{-14}	9.1×10^{-14}
	16	0.02	0.08	2.2×10^{-14}	3.5×10^{-14}	9.5×10^{-14}	9.9×10^{-14}
64	16	0.05	0.32	5.1×10^{-14}	7.0×10^{-14}	6.3×10^{-12}	2.6×10^{-12}
	32	0.10	0.49	5.5×10^{-14}	1.0×10^{-13}	1.6×10^{-13}	1.2×10^{-13}
128	32	0.43	2.21	1.0×10^{-13}	1.7×10^{-13}	2.3×10^{-12}	5.3×10^{-13}
	64	0.79	3.29	1.5×10^{-13}	2.4×10^{-13}	1.3×10^{-12}	1.5×10^{-12}

Table 26: **slsylv** vs **lyap**. Random matrices II

Size		Time		Rel. residual		Rel. error	
n	m	slsylv	lyap	slsylv	lyap	slsylv	lyap
16	4	0.01	0.01	4.8×10^{-16}	1.8×10^{-15}	7.5×10^{-15}	1.1×10^{-14}
32	8	0.01	0.05	1.1×10^{-15}	3.4×10^{-15}	4.4×10^{-14}	1.2×10^{-13}
64	16	0.03	0.18	2.0×10^{-15}	1.1×10^{-14}	1.1×10^{-11}	2.7×10^{-10}
128	32	0.14	1.18	4.6×10^{-15}	2.1×10^{-14}	3.8×10^{-11}	2.4×10^{-10}

Table 27: **slsylv** vs **lyap**. A Random Schur & B Random Hessenberg

Size		Time		Rel. residual		Rel. error	
n	m	slsylv	lyap	slsylv	lyap	slsylv	lyap
16	4	0.00	0.02	4.6×10^{-16}	1.3×10^{-15}	8.1×10^{-16}	4.5×10^{-16}
	8	0.00	0.02	4.3×10^{-16}	6.3×10^{-16}	6.8×10^{-16}	1.1×10^{-15}
32	8	0.00	0.04	5.6×10^{-16}	8.0×10^{-16}	1.2×10^{-15}	1.9×10^{-15}
	16	0.00	0.06	1.2×10^{-15}	1.8×10^{-15}	1.1×10^{-14}	1.7×10^{-14}
64	16	0.01	0.18	1.3×10^{-15}	2.0×10^{-15}	3.4×10^{-15}	4.4×10^{-15}
	32	0.01	0.31	1.8×10^{-15}	3.3×10^{-15}	5.3×10^{-14}	1.3×10^{-13}
128	32	0.04	1.18	3.2×10^{-15}	4.0×10^{-15}	1.2×10^{-14}	3.5×10^{-14}
	64	0.10	2.24	3.2×10^{-15}	5.0×10^{-15}	2.0×10^{-14}	1.9×10^{-14}

Table 28: **slsylv** vs **lyap**. A, B Random Schur

Size	Time		Rel. residual		Rel. error	
n	sllyap	lyap	sllyap	lyap	sllyap	lyap
10	0.00	0.01	2.9×10^{-14}	2.1×10^{-14}	5.6×10^{-14}	7.9×10^{-14}
20	0.01	0.04	2.7×10^{-14}	3.1×10^{-14}	3.4×10^{-14}	9.5×10^{-14}
30	0.03	0.10	4.8×10^{-14}	5.1×10^{-14}	5.6×10^{-14}	4.6×10^{-14}
40	0.05	0.21	7.0×10^{-14}	7.5×10^{-14}	7.5×10^{-13}	6.3×10^{-13}
50	0.10	0.36	9.9×10^{-14}	1.1×10^{-13}	3.0×10^{-13}	1.9×10^{-13}

Table 29: **sllyap** vs **lyap**. Random matrices I

Size	Time		Rel. residual		Rel. error	
n	sllyap	lyap	sllyap	lyap	sllyap	lyap
16	0.00	0.03	2.3×10^{-14}	3.3×10^{-14}	1.7×10^{-14}	9.7×10^{-15}
32	0.03	0.12	7.4×10^{-14}	3.9×10^{-14}	5.1×10^{-13}	7.9×10^{-13}
64	0.20	0.75	1.6×10^{-13}	1.3×10^{-13}	6.6×10^{-13}	7.9×10^{-13}
128	1.42	5.52	3.9×10^{-13}	3.5×10^{-13}	1.5×10^{-12}	1.6×10^{-12}

Table 30: **sllyap** vs **lyap**. Random matrices II

Size	Time		Rel. residual		Rel. error	
n	sllyap	lyap	sllyap	lyap	sllyap	lyap
16	0.00	0.03	2.9×10^{-16}	4.6×10^{-15}	1.1×10^{-14}	2.3×10^{-14}
32	0.01	0.12	6.4×10^{-16}	1.6×10^{-14}	8.0×10^{-15}	2.4×10^{-14}
64	0.03	0.76	8.7×10^{-16}	2.5×10^{-14}	2.8×10^{-14}	1.1×10^{-13}
128	0.27	5.53	1.2×10^{-15}	4.7×10^{-14}	2.6×10^{-13}	2.5×10^{-12}

Table 31: **sllyap** vs **lyap**. A Random Schur

Size	Time		Rel. residual		Rel. error	
n	slstei	dlyap	slstei	dlyap	slstei	dlyap
10	0.01	0.02	2.9×10^{-14}	5.7×10^{-14}	8.2×10^{-14}	5.1×10^{-14}
20	0.01	0.05	1.7×10^{-13}	2.4×10^{-13}	2.2×10^{-12}	1.3×10^{-12}
30	0.02	0.10	3.2×10^{-13}	7.1×10^{-12}	3.6×10^{-12}	1.1×10^{-10}
40	0.05	0.21	8.1×10^{-13}	1.2×10^{-10}	1.8×10^{-12}	2.1×10^{-10}
50	0.10	0.37	1.3×10^{-12}	3.6×10^{-12}	4.8×10^{-11}	3.2×10^{-11}

Table 32: **slstei** vs **dlyap**. Random matrices I

Size	Time		Rel. residual		Rel. error	
n	slstei	dlyap	slstei	dlyap	slstei	dlyap
16	0.00	0.03	7.7×10^{-14}	5.0×10^{-14}	4.9×10^{-14}	7.9×10^{-14}
32	0.02	0.12	5.4×10^{-13}	9.4×10^{-13}	9.9×10^{-13}	1.2×10^{-12}
64	0.20	0.75	2.0×10^{-12}	9.1×10^{-12}	1.6×10^{-11}	3.3×10^{-11}
128	1.48	5.77	9.9×10^{-12}	1.1×10^{-10}	3.9×10^{-11}	5.3×10^{-8}

Table 33: **slstei** vs **dlyap**. Random matrices II

We also tested the stable Lyapunov equation solvers **slstly, slstst**. There is no corresponding code in the Matlab toolbox, but we still compare them with **lyap, dlyap** respectively. The results are reported in Tables 35 – 37, and Tables 38 – 40, respectively. Note in both cases the errors for **slstly, slstst** are evaluated for $X = U^T U$.

Size	Time		Rel. residual		Rel. error	
n	slstei	dlyap	slstei	dlyap	slstei	dlyap
16	0.01	0.03	9.8×10^{-16}	8.7×10^{-14}	1.1×10^{-15}	5.7×10^{-14}
32	0.00	0.12	9.0×10^{-16}	5.9×10^{-14}	1.6×10^{-14}	1.4×10^{-13}
64	0.04	0.76	3.9×10^{-15}	2.6×10^{-13}	7.0×10^{-14}	2.2×10^{-13}
128	0.32	5.71	1.1×10^{-14}	6.0×10^{-12}	2.2×10^{-12}	3.5×10^{-11}

Table 34: **slstei** vs **dlyap**. A Random Schur

Size	Time		Rel. residual		Rel. error	
n	slstly	lyap	slstly	lyap	slstly	lyap
10	0.01	0.01	4.8×10^{-14}	4.8×10^{-14}	2.4×10^{-15}	2.4×10^{-15}
20	0.01	0.04	1.2×10^{-13}	1.3×10^{-13}	2.9×10^{-15}	3.1×10^{-15}
30	0.03	0.10	2.4×10^{-13}	2.1×10^{-13}	4.1×10^{-15}	3.5×10^{-15}
40	0.05	0.20	3.7×10^{-13}	3.7×10^{-13}	4.6×10^{-15}	4.6×10^{-15}
50	0.10	0.35	5.1×10^{-13}	4.5×10^{-13}	5.1×10^{-15}	4.5×10^{-15}

Table 35: **slstly** vs **lyap**. Random matrices I

Size	Time		Rel. residual		Rel. error	
n	slstly	lyap	slstly	lyap	slstly	lyap
16	0.00	0.03	1.0×10^{-13}	1.2×10^{-13}	3.3×10^{-15}	3.6×10^{-15}
32	0.04	0.12	2.6×10^{-13}	2.5×10^{-13}	4.2×10^{-15}	4.0×10^{-15}
64	0.19	0.73	6.7×10^{-13}	7.2×10^{-13}	5.3×10^{-15}	6.7×10^{-15}
128	1.36	5.58	1.7×10^{-12}	1.8×10^{-12}	5.8×10^{-15}	7.2×10^{-15}

Table 36: **slstly** vs **lyap**. Random matrices II

Size	Time		Rel. residual		Rel. error	
n	slstly	lyap	slstly	lyap	slstly	lyap
16	0.01	0.03	1.1×10^{-13}	8.1×10^{-14}	3.4×10^{-15}	2.5×10^{-15}
32	0.03	0.12	2.6×10^{-13}	2.4×10^{-13}	4.1×10^{-15}	3.8×10^{-15}
64	0.17	0.70	7.1×10^{-13}	6.5×10^{-13}	5.6×10^{-15}	5.1×10^{-15}
128	1.38	5.49	2.2×10^{-12}	1.9×10^{-12}	8.4×10^{-15}	7.5×10^{-15}

Table 37: **slstly** vs **lyap**. A Random Schur

Size	Time		Rel. residual		Rel. error	
n	slstst	dlyap	slstst	dlyap	slstst	dlyap
10	0.00	0.01	3.0×10^{-15}	2.1×10^{-15}	3.2×10^{-15}	2.1×10^{-15}
20	0.01	0.05	4.5×10^{-15}	3.3×10^{-15}	4.5×10^{-15}	3.3×10^{-15}
30	0.02	0.13	4.3×10^{-15}	3.6×10^{-15}	4.3×10^{-15}	3.6×10^{-15}
40	0.06	0.22	6.9×10^{-15}	4.1×10^{-15}	6.9×10^{-15}	4.1×10^{-15}
50	0.10	0.38	6.6×10^{-15}	4.7×10^{-15}	6.6×10^{-15}	4.7×10^{-15}

Table 38: **slstst** vs **dlyap**. Random matrices I

Size	Time		Rel. residual		Rel. error	
n	slstst	dlyap	slstst	dlyap	slstst	dlyap
16	0.00	0.03	3.3×10^{-15}	2.8×10^{-15}	3.4×10^{-15}	2.8×10^{-15}
32	0.03	0.12	5.4×10^{-15}	3.9×10^{-15}	5.4×10^{-15}	3.9×10^{-15}
64	0.22	0.75	7.0×10^{-15}	5.5×10^{-15}	7.0×10^{-15}	5.5×10^{-15}
128	1.47	6.08	8.4×10^{-15}	7.7×10^{-15}	8.4×10^{-15}	7.7×10^{-15}

Table 39: **slstst** vs **dlyap**. Random matrices II

Size	Time		Rel. residual		Rel. error	
n	slstst	dlyap	slstst	dlyap	slstst	dlyap
16	0.01	0.03	3.8×10^{-15}	2.6×10^{-15}	3.9×10^{-15}	2.7×10^{-15}
32	0.03	0.12	5.3×10^{-15}	4.2×10^{-15}	5.4×10^{-15}	4.2×10^{-15}
64	0.20	0.76	6.6×10^{-15}	5.5×10^{-15}	6.6×10^{-15}	5.5×10^{-15}
128	1.42	6.10	8.2×10^{-15}	7.2×10^{-15}	8.2×10^{-15}	7.2×10^{-15}

Table 40: **slstst** vs **dlyap**. A Random Schur

Finally we also tested the m-files with the numerical examples in the benchmack collections *ctlex, dtlex*, [1, 2]. The related results and the comparisons are listed in the Tables 41–56. Note that in Table 47 for $n = 30, 40$, Table 48 for $n = 32$ and Table 55 for $n = 30$ there were warnings for both compared Matlab functions to indicate that the reciprocal of the condition number is less than 10^{-17}.

Size	Time		Rel. residual		Rel. error	
n	sllyap	lyap	sllyap	lyap	sllyap	lyap
10	0.01	0.00	1.5×10^{-13}	3.0×10^{-14}	6.1×10^{-15}	2.4×10^{-15}
20	0.00	0.03	1.4×10^{-10}	9.5×10^{-11}	3.4×10^{-11}	9.4×10^{-12}
30	0.02	0.05	2.9×10^{-7}	1.2×10^{-7}	1.4×10^{-5}	2.6×10^{-6}
40	0.03	0.17	3.4×10^{-3}	3.0×10^{-4}	8.2	1.0

Table 41: **sllyap** vs **lyap**. Benchmark *ctlex*. Ex 4.1. I

Size	Time		Rel. residual		Rel. error	
n	sllyap	lyap	sllyap	lyap	sllyap	lyap
16	0.00	0.02	4.1×10^{-12}	1.0×10^{-12}	2.4×10^{-13}	1.7×10^{-13}
32	0.02	0.06	7.0×10^{-7}	1.1×10^{-7}	2.2×10^{-4}	4.9×10^{-5}

Table 42: **sllyap** vs **lyap**. Benchmark *ctlex*. Ex 4.1. II

Size	Time		Rel. residual	
n	sllyap	lyap	sllyap	lyap
10	0.01	0.01	1.9×10^{-11}	8.4×10^{-11}
20	0.01	0.04	1.5×10^{-4}	4.3×10^{-4}
30	0.02	0.10	6.7×10^{3}	1.3×10^{3}
40	0.05	0.20	5.7×10^{-1}	3.5×10^{2}

Table 43: **sllyap** vs **lyap**. Benchmark *ctlex*. Ex 4.2. I

4 Conclusion

We have described the functions of some MEX files and the related MATLAB m-files which are based on SLICOT Fortran routines for the solution of several important control problems. The m-files have extended functionality compared to the corresponding codes in the control system toolbox and robust control toolbox, but they can be implemented as easy as the latter ones. The numerical tests show that both the new m-files and the MATLAB codes essentially get the same numerical accuracy, but the former ones are usually several times faster than the others.

The SLICOT routines and the related MEX files and m-files are freeware. We sincerely hope the users will provide their suggestions and comments to help us improve the present codes.

Size	Time		Rel. residual	
n	sllyap	lyap	sllyap	lyap
16	0.00	0.04	2.3×10^{-7}	7.2×10^{-7}
32	0.03	0.11	6.9×10^{3}	4.2×10^{3}

Table 44: **sllyap** vs **lyap**. Benchmark *ctlex*. Ex 4.2. II

Size	Time		Rel. residual		Rel. error	
n	slstei	dlyap	slstei	dlyap	slstei	dlyap
10	0.01	0.01	9.0×10^{-16}	1.8×10^{-15}	1.9×10^{-15}	2.5×10^{-15}
20	0.00	0.02	2.8×10^{-14}	1.3×10^{-13}	5.7×10^{-11}	4.1×10^{-10}
30	0.02	0.07	5.3×10^{-11}	1.0×10^{-8}	3.1×10^{-6}	4.3×10^{-4}
40	0.04	0.19	1.1×10^{-7}	1.5×10^{-3}	4.1×10^{-2}	3.0×10^{3}

Table 45: **slstei** vs **dlyap**. Benchmark *dtlex.* Ex 4.1. I

Size	Time		Rel. residual		Rel. error	
n	slstei	dlyap	slstei	dlyap	slstei	dlyap
16	0.00	0.02	2.6×10^{-15}	3.6×10^{-15}	2.1×10^{-13}	1.7×10^{-13}
32	0.02	0.07	3.0×10^{-10}	7.3×10^{-8}	1.3×10^{-5}	2.4×10^{-2}

Table 46: **slstei** vs **dlyap**. Benchmark *dtlex.* Ex 4.1. II

Size	Time		Rel. residual	
n	slstei	dlyap	slstei	dlyap
10	0.00	0.02	3.1×10^{-12}	1.8×10^{-11}
20	0.01	0.05	3.8×10^{-5}	7.1×10^{-2}
30	0.02	0.10	1.2×10^{4}	2.3×10^{10}
40	0.06	0.22	1.3×10^{4}	1.2×10^{13}

Table 47: **slstei** vs **dlyap**. Benchmark *dtlex.* Ex 4.2. I

Size	Time		Rel. residual	
n	slstei	dlyap	slstei	dlyap
16	0.01	0.03	4.1×10^{-8}	6.8×10^{-6}
32	0.03	0.14	9.2×10^{5}	5.8×10^{11}

Table 48: **slstei** vs **dlyap**. Benchmark *dtlex.* Ex 4.2. II

Size	Time		Rel. residual		Rel. error	
n	slstly	lyap	slstly	lyap	slstly	lyap
10	0.01	0.01	1.1×10^{-13}	3.0×10^{-14}	6.6×10^{-15}	2.4×10^{-15}
20	0.00	0.02	1.0×10^{-10}	9.5×10^{-11}	3.4×10^{-11}	9.4×10^{-12}
30	0.02	0.05	2.9×10^{-7}	1.2×10^{-7}	1.4×10^{-5}	2.6×10^{-6}

Table 49: **slstly** vs **lyap**. Benchmark *ctlex.* Ex 4.1. I

Size	Time		Rel. residual		Rel. error	
n	slstly	lyap	slstly	lyap	slstly	lyap
16	0.01	0.01	4.4×10^{-12}	1.0×10^{-12}	2.4×10^{-13}	1.7×10^{-13}
32	0.02	0.06	5.8×10^{-7}	1.1×10^{-7}	2.2×10^{-4}	4.9×10^{-5}

Table 50: **slstly** vs **lyap**. Benchmark *ctlex*. Ex 4.1. II

Size	Time		Rel. residual	
n	slstly	lyap	slstly	lyap
10	0.00	0.02	1.7×10^{-11}	8.4×10^{-11}
20	0.01	0.06	1.1×10^{-4}	4.3×10^{-4}
30	0.02	0.11	6.9×10^{3}	1.3×10^{3}

Table 51: **slstly** vs **lyap**. Benchmark *ctlex*. Ex 4.2. I

Size	Time		Rel. residual	
n	slstly	lyap	slstly	lyap
8	0.01	0.01	4.1×10^{-12}	1.3×10^{-12}
16	0.00	0.03	3.3×10^{-7}	7.2×10^{-7}

Table 52: **slstly** vs **lyap**. Benchmark *ctlex*. Ex 4.2. II

Size	Time		Rel. residual		Rel. error	
n	slstst	dlyap	slstst	dlyap	slstst	dlyap
10	0.00	0.01	8.5×10^{-16}	1.8×10^{-15}	8.7×10^{-16}	2.5×10^{-15}
20	0.01	0.03	1.7×10^{-14}	1.3×10^{-13}	1.8×10^{-15}	4.1×10^{-10}
30	0.02	0.05	3.3×10^{-11}	1.0×10^{-8}	2.3×10^{-15}	4.3×10^{-4}

Table 53: **slstst** vs **dlyap**. Benchmark *dtlex*. Ex 4.1. I

Size	Time		Rel. residual		Rel. error	
n	slstst	dlyap	slstst	dlyap	slstst	dlyap
16	0.01	0.02	2.5×10^{-15}	3.6×10^{-15}	1.4×10^{-15}	1.7×10^{-13}
32	0.02	0.07	2.1×10^{-10}	7.3×10^{-8}	7.3×10^{-13}	2.4×10^{-2}

Table 54: **slstst** vs **dlyap**. Benchmark *dtlex*. Ex 4.1. II

Size	Time		Rel. residual	
n	slstst	dlyap	slstst	dlyap
10	0.01	0.01	5.3×10^{-12}	1.8×10^{-11}
20	0.01	0.04	6.5×10^{-5}	7.1×10^{-2}
30	0.02	0.10	1.7×10^{4}	2.3×10^{10}

Table 55: **slstst** vs **dlyap**. Benchmark *dtlex*. Ex 4.2. I

Size	Time		Rel. residual	
n	slstst	dlyap	slstst	dlyap
8	0.00	0.01	1.6×10^{-12}	9.4×10^{-13}
16	0.01	0.03	1.3×10^{-7}	6.8×10^{-6}

Table 56: **slstst** vs **dlyap**. Benchmark *dtlex*. Ex 4.2. II

References

[1] D. Kressner, V. Mehrmann und T. Penzl, 'CTLEX– a Collection of Benchmark Examples for Continuous-Time Lyapunov Equations', *SLICOT Working Note SLWN1999-6*, siehe auch *http:/www.esat.kuleuven.ac.be/pub/WGS/REPORTS/*

[2] D. Kressner, V. Mehrmann und T. Penzl, 'DTLEX– a Collection of Benchmark Examples for Discrete-Time Lyapunov Equations, *SLICOT Working Note SLWN1999-7*, siehe auch *http:/www.esat.kuleuven.ac.be/pub/WGS/REPORTS/*

[3] The MathWorks, Inc., Cochituate Place, 24 Prime Park Way, Natick, MA 01760. *Using* MATLAB, 1996.

[4] The MathWorks, Inc., Cochituate Place, 24 Prime Park Way, Natick, Mass, 01760. *The MATLAB Control Toolbox, Version 3.0b*, 1993.

[5] The MathWorks, Inc., Cochituate Place, 24 Prime Park Way, Natick, Mass, 01760. *The MATLAB Robust Control Toolbox, Version 2.0b*, 1994.

Appendix A: Category Tables

Algebraic Riccati Equation (ARE) Solver		
`aresol`	Collection of ARE routines in SLICOT	MEX-file.
`slcaregs`	Solves CARE with generalized Schur method.	m-files
`slcares`	Solves CARE with Schur method.	
`sldaregs`	Solves DARE with generalized Schur method.	
`sldares`	Solves DARE with Schur method.	
`sldaregsv`	Solves DARE with generalized Schur method on double sized pencils.	

Generalized Linear Matrix Equation (GLME) Solver		
`genleq`	Collection of GLME routines in SLICOT	MEX-file.
`slgely`	Solves generalized Lyapunov equations.	m-files
`slgesg`	Solves generalized pairs of matrix equations.	
`slgest`	Solves generalized Stein equations.	
`slgsly`	Solves generalized stable Lyapunov equations.	
`slgstst`	Solves generalized stable Stein equations.	
`slgesy`*	Solves generalized Sylvester equations.	

Linear Matrix Equation (LME) Solver		
`linmeq`	Collection of LME routines in SLICOT	MEX-file.
`sldisy`*	Solves discrete-time Sylvester equations.	m-files
`sllyap`	Solves Lyapunov equations.	
`slstei`	Solves Stein equations.	
`slstly`	Solves stable Lyapunov equations with right hand side in factorized form.	
`slstst`	Solves stable Stein equations with right hand side in factorized form.	
`slsylv`	Solves Sylvester equations.	

System Controllability, Orbservability, Minimal Realization (COM) Forms Computational Tool		
`syscom`	Collection of system COM computational routines in SLICOT	MEX-file.
`slconf`	Computes controllability staircase form.	m-files
`slminr`	Computes minimal realization sub-system.	
`slobsf`	Computes observability staircase form.	

System Transformations		
`systra`	Collection of system transformation routines in SLICOT	MEX-file.
`slsbal`	Balances the system.	m-files
`slsdec`	Transforms state matrix to block diagonal form with ordered eigenvalues.	
`slsorsf`	Transforms state matrix to Schur form with ordered eigenvalues.	
`slsrsf`	Transforms state matrix to Schur form.	