Sahas Gunasekara
20462075
IT21100666

Date No

Arithmetic

Workshop 1 *

01 $i^2 = -1$

(i) $i^3 = (i)^2 \times i$
$= -1 \times i$
$= i$ //.

(ii) $i^4 = ((i)^2)^2$
$= (-1)^2$
$= 1$ //

(iii) $i^{17} = i^{16} \times i$
$= 1 \times i$
$= i$ //

∴ All even powers of i^2 is $+1$ //.

02 $z_1 = 5+2i$, $z_2 = 4-3i$, $z_3 = 1-2i$

(i) $z_1 + z_2 = 5+2i+4-3i \rightarrow (5+4)+(2i-3i)$
$= 9-i$ //

(ii) $3z_1 + \overline{z}_2 \Rightarrow \overline{z}_2 = 4+3i$
$= 3(5+2i) + 4+3i$
$= 15+6i+4+3i$
$= (15+4)+(6+3)i$
$= 19+9i$ //

(iii) $z_2 z_3 = (4-3i)(1-2i)$
$= 4-8i-3i+6i^2$
$= 4-11i+6(-1)$
$= 4-11i-6$
$= -2+(-11)i$ //

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(iv) $\frac{z_2}{z_1} = \frac{4-3i}{5+2i}$

$= \frac{(4-3i)}{(5+2i)} \times \frac{(5-2i)}{(5-2i)}$

$= \frac{20-8i-15i+6i^2}{25-4(i)^2}$

$= \frac{20-6-23i}{25+4}$

$= \frac{14-23i}{29}$

$= 14/29 - 23/29\,i$ //.

(v) $(z_3)^2 = (1-2i)^2$

$= 1-4i+4i^2$

$= 1-4i-4$

$= -3-4i$ //

03 $(a-2bi)-(b-ai) = 2+i$

$a-b-2bi+ai = 2+i$

Non-imaginary part → $a-b = 2$ — (1)

Imaginary part → $-2b+a = 1$ — (2)

(1) − (2)

$a-b+2b-a = 2-1$

$b = +1$ //

Sub b in equation (1)

$a-1 = 2$

$a = 3$ //

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04 $\frac{-6}{i^7} = \frac{-6}{i^6 \times i}$

$= \frac{-6}{(i^2)^3 \times i}$

$= \frac{-6}{-1 \times i}$

$= \frac{6 \times i}{i \quad i}$

$= \frac{6i}{i^2}$

$= -6i$ //

Cartesian and Polar Representation.

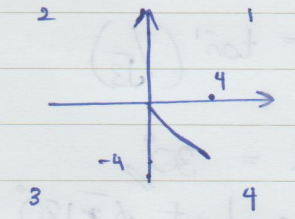
05

(i) $z = 4 - 4i$

$\tan\alpha = \frac{-4}{4}$

$\alpha = \tan^{-1}(-4/4)$

$\alpha = -45°$ //



$r^2 = 4^2 + (-4)^2$

$r = 4\sqrt{2}$ //

$z = 4\sqrt{2}\,cis(-45°)$ //

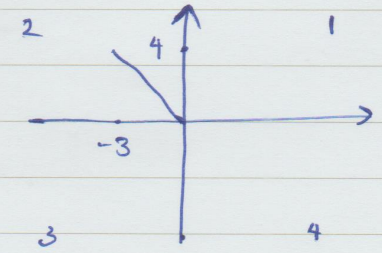
(ii) $z = -3 + 4i$

$\tan\alpha = -4/3$

$\alpha = \tan^{-1}(-4/3)$

$\alpha = -53.13°$ //

∴ 2nd quadrant → $\alpha + 180° = 126.87°$ //



$r^2 = (-3)^2 + (4)^2$

$r = 5$ //,

$z = 5\,cis(126.87°)$ //

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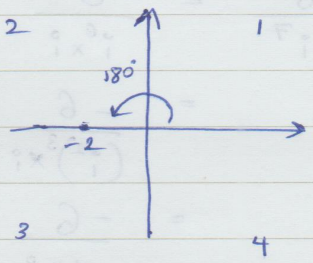
(iii) $z = -2 + 0i$

$\tan\alpha = 0/-2$

$\alpha = \tan^{-1}(0)$

$\alpha = 0^\circ$ //

∴ Second quadrent, $\alpha + 180^\circ = 180^\circ$ //



$r^2 = (-2)^2 + 0$

$r = 2$ //

$z = 2\,cis(180^\circ)$ //

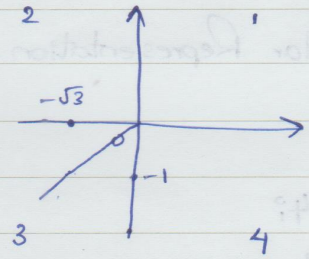
(iv) $z = -\sqrt{3} - i$

$\tan\alpha = +1/+\sqrt{3}$

$\alpha = \tan^{-1}(1/\sqrt{3})$

$\alpha = 30^\circ$ //

∴ Third quadrent $\alpha - 180^\circ = -150^\circ$ //



$r^2 = (-\sqrt{3})^2 + (-1)^2$

$r^2 = 3 + 1$

$r = 2$ //

$z = 2\,cis(-150^\circ)$ //

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(i) $z = 3\text{cis}(-135^\circ)$

$= 3(\cos(-135) + i\sin(-135))$

$= 3\left(-\frac{\sqrt{2}}{2} + i \times -\frac{\sqrt{2}}{2}\right)$

$= -\frac{3\sqrt{2}}{2} + \left(-\frac{3\sqrt{2}}{2}\right)i$ //.

(ii) $z = 4\text{cis}(300^\circ)$

$= 4(\cos(300) + i\sin(300))$

$= 4\left(\frac{1}{2} + i \times \left(-\frac{\sqrt{3}}{2}\right)\right)$

$= 2 - 2\sqrt{3}i$ //

(iii) $z = \pi\text{cis}(17^\circ)$

$= \pi(\cos(17^\circ) + i\sin(17^\circ))$

$= \pi(0.96 + 0.29i)$

$= 0.96\pi + 0.29\pi i$ //

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Points and regions in the Argand plane

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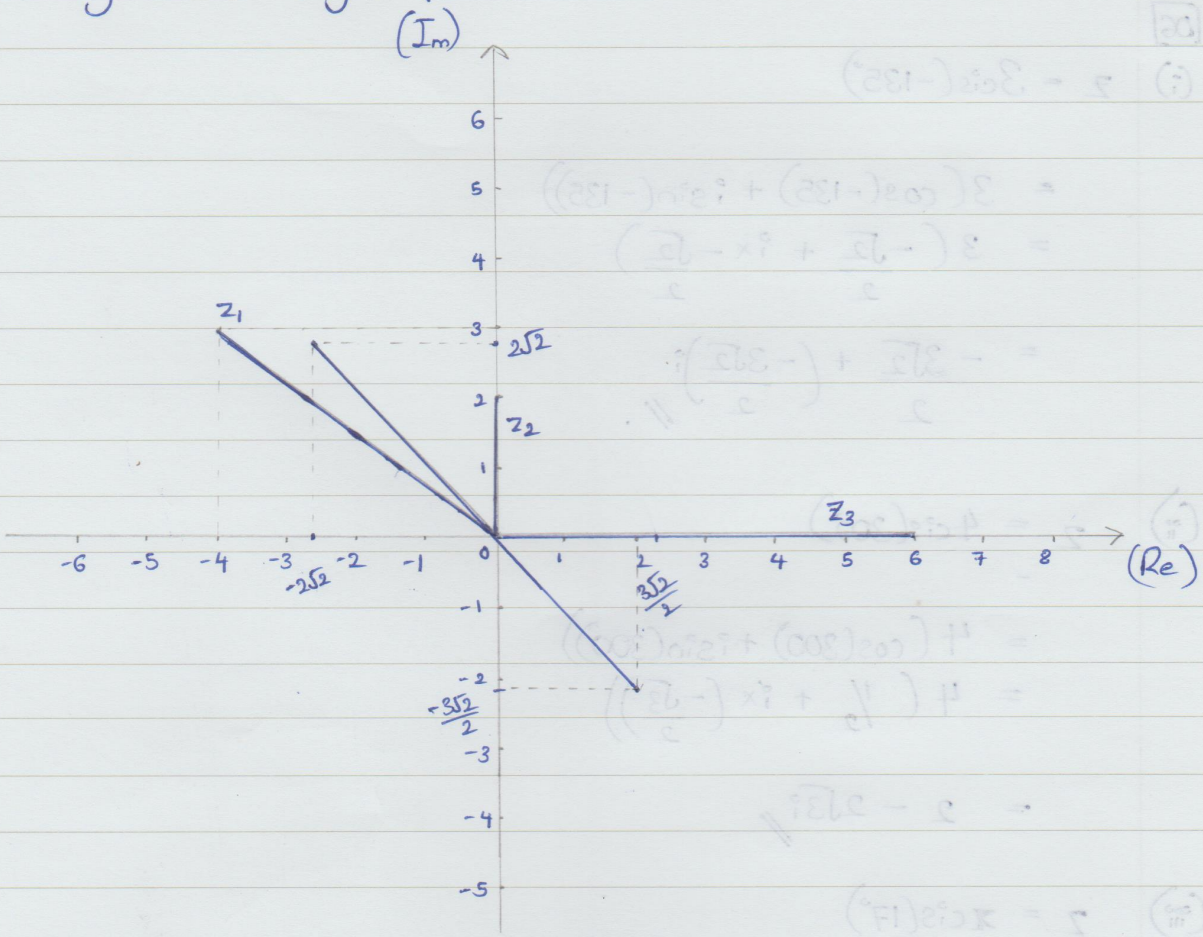

$z_4 = 4\,\mathrm{cis}(135^\circ)$

$= 4(\cos(135^\circ) + i\sin(135^\circ))$

$= 4\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$

$z_4 = -2\sqrt{2} + 2\sqrt{2}i$ //

$z_5 = 3\,\mathrm{cis}(-45^\circ)$

$= 3(\cos(-45^\circ) + i\sin(-45^\circ))$

$z_5 = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$ //

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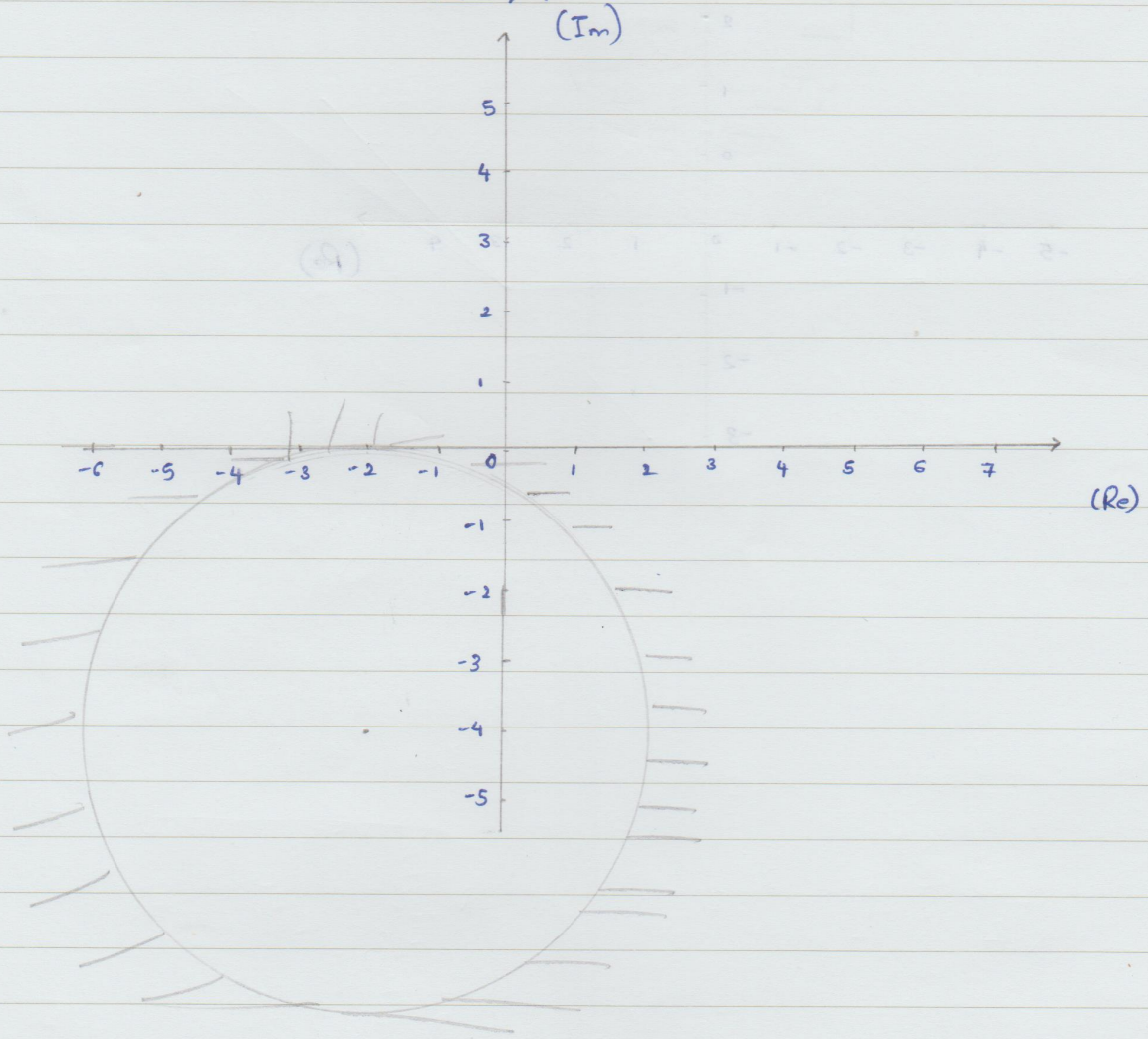
(i) $|z - z_0|$ ← form

$|z + 2 + 4i| \geq 4$

$|z - (-2 - 4i)| \geq 4$



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(ii) $|z-2| < 2$

$|z-(2)| < 2$