$\boxed{01}$

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Workshop - 2

Regions in the Argand plane

$\boxed{01}$

(i) $|z| < |z-i|$ Sub $z = x+iy$

$$|x+iy| < |x+iy-i|$$

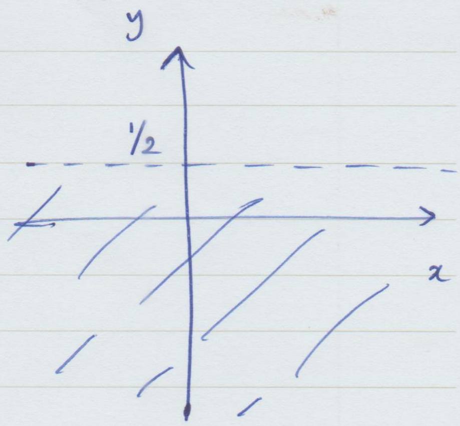
$$|x+iy| < |x+i(y-1)|$$

$$\left(\sqrt{x^2+y^2}\right)^2 < \left(\sqrt{x^2+(y-1)^2}\right)^2$$

$$\cancel{x^2}+\cancel{y^2} < \cancel{x^2}+\cancel{y^2}-2y+1$$

$$2y < 1$$

$$y < 1/2$$



(ii) $|z+i| < |z+1|$ Sub $z = x+iy$

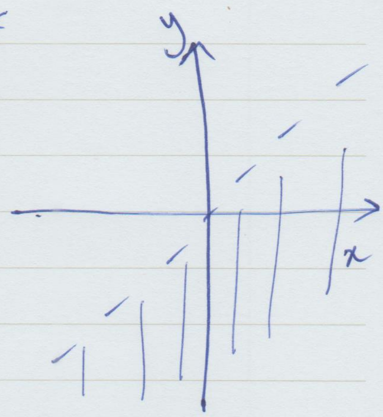
$$|x+iy+i| < |x+iy+1|$$

$$\left(\sqrt{(x)^2+(y+1)^2}\right)^2 < \left(\sqrt{(x+1)^2+y^2}\right)^2$$

$$\cancel{x^2}+\cancel{y^2}+2y+\cancel{1} < \cancel{x^2}+2x+\cancel{1}+\cancel{y^2}$$

$$\cancel{2}y < \cancel{2}x$$

$$y < x$$



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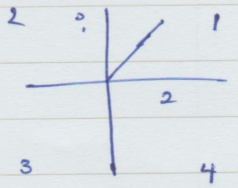
Polar and exponential representation

$\boxed{02}$

(i) $z = 2 + i$



$\phi = \tan^{-1}\left(\frac{1}{2}\right)$

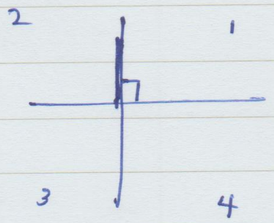
$r = \sqrt{(2)^2 + (1)^2}$

$\phi = 26.565^\circ // = 26.57^\circ //$

$r = \sqrt{5} //$

$z = \sqrt{5} \times e^{26.57i} //$

(ii) $z = -6i$



$\phi = \tan^{-1}\left(-\frac{6}{0}\right)$

$r = \sqrt{(-6)^2}$

$r = 6 //$

$\phi = 90^\circ //$

$z = 6 \times e^{90i} //$

(iii) $z = 2\,cis(-45^\circ)$

$r = 2$

$\theta = -45^\circ //$

Sub in equation

$z = re^{i\theta} //$

$\downarrow$

$z = 2 \times e^{-45i} //$

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$\boxed{03}$

(i) $z = 3e^{-30i}$

can be written as;

$z = 3\,cis(-30)$

$z = 3(\cos(-30) + i\sin(-30))$

$z = 3\left(\frac{\sqrt{3}}{2} + i \times -\frac{1}{2}\right)$

$z = \frac{3\sqrt{3}}{2} \cancel{+} - \frac{i}{2}$ //

(ii) $z = 5e^{120i}$

can be written as;

$z = 5\,cis(120)$

$z = 5(\cos(120) + i\sin(120))$

$z = 5\left(-\frac{1}{2} + i \times \frac{\sqrt{3}}{2}\right)$

$z = -\frac{5}{2} + \frac{\sqrt{3}}{2}i$ //

$\boxed{04}$ $z_1 = 2cis(15°)$, $z_2 = 15cis(55°)$, $z_3 = 3e^{70i}$

(i) $z_1 \times z_2 = 2cis(15°) \times 15\,cis(55°)$

$= 2 \times 15 \times cis(15+55)$

$= 30(\cos(70) + i\sin(70°))$ //

$= 30\cos 70 + \cancel{i\sin(70°)}) \; 30 i\sin(70)$ //

(ii) z_3 can be written as $\rightarrow$ $3\,cis(70°)$

$z_2/z_3 \rightarrow \frac{\cancel{15}^{5}\,cis(55)}{\cancel{3}^{1}\,cis(70)}$

$= 5\,cis(55-70)$

$= 5\,cis(-15)$

$= 5(\cos(-15) + i\sin(-15))$ //

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05

(i) De Moivre's states that

$$(r\,cis(\theta))^n = r^n \times cis(n\theta)$$

$\therefore$

$$(2cis(35°))^8$$

$$2^8 \times cis(35 \times 8)$$

$$256 \times cis(280)$$

(ii) Cartesian $\rightarrow$

$256((cos(280) + i sin(280))$

$\cancel{256(}$

$256 \times cos(280) + 256 \times i \times sin(280))$

$44.45 + (-252.11)i$

$44.45 - 252.11i$ //

Polar $\rightarrow$

$256((cos(280) + i sin(280))$

$256(cos(280°) + i sin(280°))$ //

(iii) $(6+5i)^3$ Turning z into polar

$z = 6+5i$

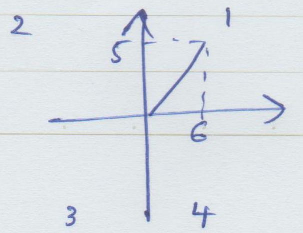
$r = \sqrt{6^2+5^2}$

$r = \sqrt{61}$ //

$\phi = tan^{-1}(5/6)$

$\phi = 39.81°$ //

$\theta = 39.81°$ //



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$\therefore$ this can be written as $\rightarrow$

~~$\sqrt{61}$ cis~~

$$\left(\sqrt{61}\, cis(39.81)\right)^3$$

$$\left(\sqrt{61}\right)^3 \times \left(\cos(39.81) + i\sin(39.81)\right)$$

Polar form $\longrightarrow$ $\left(\sqrt{61}\right)^3 \times \left(\cos(39.81) + i\sin(39.81)\right)$ //

Cartesian $\longrightarrow$ $\left(\sqrt{61}\right)^3 \times \left(\cos(39.81)\right) + i \times \sin(39.81) \times \left(\sqrt{61}\right)^3$

$$= 366 + 305i \;//$$

06

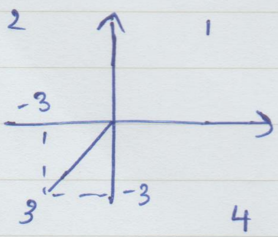
(i) Re-writing in polar form $\rightarrow$ $z = -3 - 3i$

$r = \sqrt{(-3)^2 + (-3)^2}$

$r = 3\sqrt{2}$ //

$\emptyset = \tan^{-1}\left(-3/-3\right)$

$\emptyset = 45°$ //



3rd quadrent so $\rightarrow$ $\emptyset - 180°$

$\theta = -135°$ //

$3\sqrt{2}\, cis(-135°)$

$\omega_k \rightarrow (3\sqrt{2})^{1/3} \times cis\left(\dfrac{\theta + k \times 360}{3}\right)$, $k = 0, 1, 2$

$\omega_0 = (3\sqrt{2})^{1/3} \times cis(-45°)$ //

$\omega_1 = (3\sqrt{2})^{1/3} \times cis(75°)$ //

$\omega_2 = (3\sqrt{2})^{1/3} \times cis(195°)$ //

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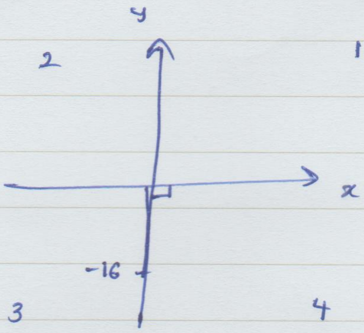
(ii) Re-writing $z = -16i$ in Polar form.

$r = \sqrt{(-16)^2}$ $\qquad\qquad$ $\phi = \tan^{-1}\left(-16/0\right)$

$r = 16$ $\qquad\qquad$ $\phi = -90°$



4^{th} so $\rightarrow$ $\phi = \theta$

$\theta = -90°$

can be written as $\rightarrow$ $16\,\text{cis}(-90°)$

$$\omega_k = (16)^{1/4} \times \text{cis}\left(\frac{\theta + k \times 360}{4}\right), \quad k = 0, 1, 2, 3$$

$\omega_0 = 2\,\text{cis}(-22.5°)$

$\omega_1 = 2\,\text{cis}(67.5°)$

$\omega_2 = 2\,\text{cis}(157.5°)$

$\omega_3 = 2\,\text{cis}(247.5°)$

07

(i) $x^2 + 9 = 0$

$x^2 = -9$ $\qquad\qquad$ $i^2 = -1$

$x = \sqrt{-9}$

$x = \sqrt{9 \times i^2}$

$x_1 = +3i$ or $x_2 = -3i$

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(ii) $\overset{a}{x^2} + \overset{b}{2x} + \overset{c}{5} = 0$

Using $\rightarrow \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

① $\rightarrow \quad = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-2+\sqrt{4-4\times 1\times 5}}{2\times 1}$

$= \dfrac{-2+\sqrt{-16}}{2}$

$= \dfrac{-2+\sqrt{4^2\times i^2}}{2}$

$= \dfrac{-2+4i}{2}$

root ① $= -1+2i$ // $\longrightarrow$ Because coefficients are real; the other root is;

$= -1-2i$ //

(iii) $x^4 + 2x^2 - 8 = 0$

sub $y = x^2$ Using $\rightarrow \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} \longrightarrow \overset{a}{y^2} + \overset{b}{2y} - \overset{c}{8} = 0$

① $\rightarrow \dfrac{-2+\sqrt{2^2-4\times 1\times(-8)}}{2\times 1}$

root ① $= 2$ //

② $\rightarrow \dfrac{-2-\sqrt{2^2-(4\times 1\times(-8)}}{2\times 1}$

root ② $= -4$ //

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For root ① sub in $y = x^2$

$$2 = x^2$$

$$+\sqrt{2} = x_1 \text{ or } x_2 = -\sqrt{2} //$$

For root ② sub in $y = x^2$

$$-4 = x^2$$

$$\pm\sqrt{-4} = x \qquad i^2 = -1 //$$

$$\pm\sqrt{4 \times i^2} = x$$

$$+2i = x_3 \text{ or } -2i = x_4 //$$