$\boxed{01}$

Workshop 4

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Cross product and applications.

$\boxed{01}$ $\quad \underline{a} = 2i - j$

$\underline{b} = i + 3j - 2k$

$$\begin{array}{ccc} a_1 & a_2 & a_3 \\ \left[\begin{array}{ccc} 2 & -1 & 0 \\ 1 & 3 & -2 \end{array}\right] & & \\ b_1 & b_2 & b_3 \end{array}$$

$$\underline{a} \times \underline{b} = [-1 \times -2 - 0 \times 3,\ 0 \times 1 - (-2) \times 2,\ 2 \times 3 - (-1) \times 1]$$

$$\underline{a} \times \underline{b} = [2, -4, 7].$$

$$\frac{a \times b . b}{||a \times b|| . ||b||} = \cos\theta$$

$$\cos\theta = \frac{[2, -4, 7] . [1, 3, -2]}{\sqrt{2^2 + (-4)^2 + 7^2} \times \sqrt{1^2 + 3^2 + (-2)^2}}$$

$$= \frac{2 \times 1 + (-4) \times 3 + 7 \times (-2)}{\sqrt{4 + 16 + 49} \times \sqrt{1 + 9 + 4}}$$

$$= \frac{-24}{\sqrt{29} \times \sqrt{14}}$$

Not orthogonal ?

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$\boxed{02}$ $P\ (1,-1,2)$

$Q\ (2,0,-1)$ $R(0,2,1)$.

$$\vec{PQ} = [2-1,\ 0-(-1),\ -1-2]$$
$$= [1,\ 1,\ -3]_{//}$$

$$\vec{PR} = [0-1,\ 2-(-1),\ 1-2]$$
$$= [-1,\ 3,\ -1]_{//}$$

Area Triangle $= \frac{1}{2} \times$ Area of Parallalogram

$$\vec{PQ} \times \vec{PR} = \begin{bmatrix} 1 & 1 & -3 \\ -1 & 3 & -1 \end{bmatrix}$$

$$= [1\times(-1) - (-3\times 3),\ (-3\times -1)-(1\times -1),\ (1\times 3)-(-1\times 1)]$$

$$= [8,\ 4,\ 4]$$

$$= \frac{1}{2} \times \|\vec{PQ} \times \vec{PR}\|$$

$$= \frac{1}{2} \times \sqrt{8^2 + 4^2 + 4^2}$$

$$= 2\sqrt{6}_{//}$$

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03 Area of parallalogram = $\frac{1}{2}\times$ base $\times$ height $\times$ angle

$= \|u\| \times \|v\| \times \sin\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

$= 32\sqrt{3}$ //

04 a b c are coplanar only if

$\underline{a}.(\underline{b}\times\underline{c}) = 0$ $a = [1, 2, -1]$

$$\underline{b}\times\underline{c} = \begin{bmatrix} -2 & 0 & 3 \\ 2 & -4 & -4 \end{bmatrix}$$

$= [0\times(-4) - 3\times(-4), 3\times 2 - (-4\times -2), (-2\times -4) - (0\times 2)]$

$\underline{b}\times\underline{c} = [12, -2, 8]$

$\underline{a}.(\underline{b}\times\underline{c}) = [1, 2, -1].[12, -2, 8]$

$= 1\times 12 - 4 - 8$

$= 0$ //

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05

(i) $A+B = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$

$= \begin{bmatrix} 9 & -5 & 0 \\ 5 & -9 & -1 \end{bmatrix}$ //

(ii) $-4B = -4 \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$

$= \begin{bmatrix} -28 & 20 & -4 \\ -4 & 16 & 12 \end{bmatrix}$ //

(iii) $AC = \overset{2\times3 \; \cancel{3\times2}}{\begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}} \times \overset{2\times2}{\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}}$

= Cannot multiply as it is 2x3 and 2x2.

(iv) $CB = \overset{2\times2}{\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}} \times \overset{2\times3}{\begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}}$

$= \begin{bmatrix} 1\times7+2\times1 & 1\times-5+2\times-4 & 1\times1+2\times-3 \\ -2\times7+1\times1 & -2\times-5+1\times-4 & -2\times1+1\times-3 \end{bmatrix}$

$= \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$ //

(v) $$AB^T = \overset{2\times 3}{\begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}} \times \overset{3\times 2}{\begin{bmatrix} 7 & 1 \\ -5 & -4 \\ 1 & -3 \end{bmatrix}} \quad 2\times 2$$

$$= \begin{bmatrix} 2\times 7 + 0\times -5 - 1\times 1 & 2\times 7 + 0\times -5 - 1\times 1 \\ 4\times 7 - 5\times -5 + 2\times 1 & 4\times 1 \ -5\times -4 \ 2\times -3 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 13 \\ 55 & 18 \end{bmatrix}$$ //

(vi) $C - 3I_2$

(vii) $$C^2 = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 1 + 2\times -2 & 1\times \cancel{1}2 + 2\times \cancel{-2}1 \\ -2\times \cancel{2}1 + 1\times -2 & -2\times 2 + 1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ -4 & -3 \end{bmatrix}$$ //

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06 If A is 6×4 and AB is 6×8 this means B is 4×8

$$[6\times4][4\times8] = [6\times8]$$

07 4 rows.

08 $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$

$$AB = \begin{bmatrix} 2\times4+5\times3 & 2\times-5+5\times k \\ -3\times4+1\times3 & -3\times-5+1\times k \end{bmatrix}$$

$$= \begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix}$$

$$BA = \begin{bmatrix} 4\times2 \;-5\times-3 & 4\times5 \;-5\times1 \\ 3\times2 \;+k\times-3 & 3\times5 \;+k\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 15 \\ 6-3k & 15+k \end{bmatrix}$$

$-10+5k = 15$
$5k = 25$
$k = 5$

$6-3k = -9$
$15 = 3k$
$5 = k$

09

$$A = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} \qquad B = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2\times -7 + 5\times 3 & 2\times -5 + 5\times 2 \\ -3\times -7 - 7\times 3 & -3\times -5 - 7\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\because$ P $AB = I_n$ $\therefore$ AB are the inverse of one another.

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