Workshop 3

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* Vector arithmetic and properties.

Q1 Given the vectors $a=[2,-1,3]$, $b=[4,0,-3]$ and $c=[1,-2,+2]$

(i) $a+b = [(2+4),(-1+0),(3-3)]$.

$= [6,-1,0]$ //

(ii) $3a-4c = 3[2,-1,3] - 4[1,-2,+2]$.

$= [6,-3,9] - [4,-8,+8]$

$= [6-4, -3+8, 9-8]$

$= [2,5,1]$ //

(iii) $\|b\| = \sqrt{(4)^2+(0)^2+(-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$ //

(iv) $\hat{b} = \dfrac{b}{\|b\|} \Rightarrow \frac{1}{5}[4,0,-3]$

$\hat{b} = [4/5, 0, -3/5]$ //

(v) Same direction as b means $\hat{b}$. Same length as c means magnitude.

Assuming there is constant k for new vector.

New vector is $k[4/5, 0, -3/5]$

$$\underset{\sim}{x} = [4k/5, 0, -3k/5] \; //$$

$$\|c\| = \|x\|$$

$$\sqrt{(1)^2+(-2)^2+(2)^2} = \sqrt{(4k/5)^2+(-3k/5)^2}$$

$$1+4+4 = \frac{16k^2}{25} + \frac{9k^2}{25}$$

$$5 = \frac{\cancel{25}k^2}{\cancel{25}}$$

$k = \pm\sqrt{5}$ ~~Cannot be $-\sqrt{5}$~~ (Whether k is positive or negative is irrelavant as it is just the direction).

$\therefore$ New vector is $\pm\sqrt{5}[4/5, 0, -3/5]$ //

Date No

$\boxed{02}$ $A(2,-3)$ $B(4,1)$

$b = \overrightarrow{AB}$

$a = \overrightarrow{OA}$ $b = [(4-2), (1-(-3))]$

$a = [2, -3]$ // $b = [2, 4]$ //

Dot product and applications

$\boxed{03}$

(a) $a = [2, -4, \sqrt{5}]$ $b = [-2, 4, -\sqrt{5}]$

$$a.b = (2\times-2) + (-4\times4) + (\sqrt{5}\times-\sqrt{5})$$
$$= -4 - 16 - 5$$
$$= -25 \text{ //}$$

$$\text{Scalar projection} = \frac{a.b}{\|b\|} = \frac{-25}{\sqrt{(-2)^2+(4)^2+(-\sqrt{5})^2}}$$

$$= \frac{-25}{5}$$

$$= -5 \text{ //}$$

$$\text{Vector projection} = \text{scalar projection} \times \frac{b}{\|\underline{b}\|}$$

$$= \frac{-\cancel{5}^{1} \times [-2, 4, -\sqrt{5}]}{{}_{1}\cancel{\sqrt{(-2)^2+(4)^2+(-\sqrt{5})^2}}}$$

$$= -1[-2, 4, -\sqrt{5}]$$
$$= [+2, -4, +\sqrt{5}] \text{ //}$$

(b) $a = 2i + 10j - 11k$, $b = 2i + 2j + k$

$$a.b = (2\times2) + (10\times2) + (-11\times1)$$
$$= 4 + 20 - 11$$
$$= 13$$

$$\text{Scalar projection} = \frac{a.b}{\|b\|}$$
$$= \frac{13}{\sqrt{2^2+2^2+1}}$$
$$= 13/3$$

$$\text{Vector Projection} = \text{Scalar Projection} \times \frac{b}{\|b\|}$$
$$= \frac{13}{3} \times \frac{[2, 2, 1]}{3}$$
$$= \frac{13}{9} \times [2, 2, 1]$$

Date No

ProMate

Date No

(c) $a = i + k$, $b = i + j + k$.

$$a \cdot b = (1 \times 1 + 0 \times 1 + 1 \times 1)$$
$$= 2 \,//$$

$$\text{Scalar Projection} = \frac{a \cdot b}{\|\underline{b}\|}$$

$$= \frac{2}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$= 2/\sqrt{3} \,//$$

$$\text{Vector Projection} = \text{Scalar Projection} \times \frac{\underline{b}}{\|b\|}$$
$$= 2/\sqrt{3} \times \frac{[1, 1, 1]}{\sqrt{3}}$$
$$= 2/3 \, [1, 1, 1] \,//$$

Date No

$\boxed{04}$

$u = [2, -2, -1]$ and $v = [3, 5, -4]$.

$$\cos\theta = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{\|u\| \times \|v\|}$$

$$= \frac{(2\times 3 + (-2)\times 5 + (-1\times -4))}{\sqrt{2^2+(-2)^2+(-1)^2} \times \sqrt{3^2+5^2+(-4)^2}}$$

$$= \frac{0}{3 \times 5\sqrt{2}}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = 90^\circ$$

$\therefore$ Since these 2 vectors are at 90° to each other they are what is deemed as orthonogal.

Date No

$\boxed{05}$ $W = F \times s$ $F = 5\hat{j}$

$s =$ Origin to the point $[1,1,1]$

$= [1-0, 1-0, 1-0]$

$\because$ Origin is $[0,0,0]$

$W = [0,5,0].[1,1,1]$

$= (0\times1)+(5\times1)+(1\times0)$

$= 5$ (5 Newton Meters ?)

$\boxed{06}$

$W = F \times s$

$= 150 \times \cos 45^\circ \times 15$

$= 1590.99J$

$\boxed{07}$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$?