

MATRICES

- Matrices are shown in the following way:

The diagram illustrates two matrices, Matrix A and Matrix B, on a light blue background. Matrix A is a 4x2 matrix with elements [7, 3; 2, 5; 6, 8; 9, 0]. Matrix B is a 2x3 matrix with elements [7, 4, 9; 8, 1, 5]. Red arrows indicate the dimensions: a vertical arrow on the left of Matrix A points down from the top row to the bottom row, labeled "4 rows". A horizontal arrow at the bottom of Matrix A points right from the first column to the second column, labeled "2 columns". A vertical arrow on the left of Matrix B points down from the top row to the bottom row, labeled "2 Rows". A horizontal arrow at the bottom of Matrix B points right from the first column to the third column, labeled "3 Columns".

Matrix A

$$\begin{bmatrix} 7 & 3 \\ 2 & 5 \\ 6 & 8 \\ 9 & 0 \end{bmatrix}$$

2 columns
4 rows

Matrix B

$$\begin{bmatrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{bmatrix}$$

2 Rows
3 Columns

The digits shown horizontally are called ROWS

The digits shown vertically are in COLUMNS

The ORDER of the matrix is given by [ROWS x COLUMNS]

- For addition and subtraction of matrices, the order of the matrices has to be same and the corresponding values are added or subtracted under the given circumstances.

$$\begin{array}{c} \mathbf{a} \\ \left[\begin{array}{c} 1 \\ 0.6 \\ -2 \end{array} \right] + \left[\begin{array}{c} -1 \\ 0 \\ 0.2 \end{array} \right] + \left[\begin{array}{c} 3 \\ 1 \\ 0 \end{array} \right] = \left[\begin{array}{c} 3 \\ 1.6 \\ -1.8 \end{array} \right] \end{array}$$

$$\begin{array}{c} \mathbf{b} \\ \left[\begin{array}{cc} 1 & 0 \\ -1 & 0 \end{array} \right] + \left[\begin{array}{cc} 0 & -1 \\ -1 & 0 \end{array} \right] = \left[\begin{array}{cc} 1 & -1 \\ -2 & 0 \end{array} \right] \end{array}$$

- There are 2 types of multiplication. Firstly, scalar, which is demonstrated in the following:

$$3 \begin{bmatrix} 5 & 2 & 11 \\ 9 & 4 & 14 \end{bmatrix} = \begin{bmatrix} 3*5 & 3*2 & 3*11 \\ 3*9 & 3*4 & 3*14 \end{bmatrix} = \begin{bmatrix} 15 & 6 & 33 \\ 27 & 12 & 42 \end{bmatrix}$$

The number outside the matrix is multiplied to all the numbers inside.

The second type is matrix multiplication, the following:

Matrix

$$\begin{matrix} A \\ \left[\begin{matrix} 7 & 3 \\ 2 & 5 \\ 6 & 8 \\ 9 & 0 \end{matrix} \right] \end{matrix}$$

Matrix

$$\begin{matrix} B \\ \left[\begin{matrix} 7 & 4 & 9 \\ 8 & 1 & 5 \end{matrix} \right] \end{matrix}$$

2 columns = 2 Rows

4 rows

3 Columns

Dimension of Product Matrix

4 x 3

It is necessary for the number of columns of first matrix to be equal to number of rows of second. Otherwise multiplication cant be done.

The product of 2 matrices

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix} \times \begin{bmatrix} 1 & 8 & 9 \\ 2 & 7 & 10 \\ 3 & 6 & 11 \\ 4 & 5 & 12 \end{bmatrix} = \begin{bmatrix} 50 & 94 & 178 \\ 60 & 120 & 220 \end{bmatrix}$$

The product is found by multiplying the first row of matrix A with first column of matrix B with corresponding values to give answer 50. $[(1*1)+(3*2)+(5*3)+(7*4)]=50$. Likewise the first column is multiplied with second row and then third row.

Then the second column is multiplied with first, second, and third rows respectively.

- These are identity matrices:

$$I_1 = [1], \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \dots, \quad I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Multiplication of matrices with identity matrix gives the same matrix as an answer.

- A zero matrix is one with all numbers 0 and addition or subtraction gives the same matrix as answer.
- Finally the inverse:

$$\mathbf{B} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \text{If } AD - BC \neq 0, \text{ then } \mathbf{B} \text{ has an inverse, denoted } \mathbf{B}^{-1}$$

$$\mathbf{B}^{-1} = \frac{1}{AD-BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

$AD-BC$ is known as determinant and it is multiplied with the matrix with digits of main diagonal displaced and signs of the other changed.