Lab 7: Recursion I[1]

CSSSKL162: Programming Methodology Skills,

Summary

The purpose of this lab is to practice transforming recurrence relationships into working Java code. Start by reviewing the example code to gain an intuition and understanding for recursive behavior, and then build your own custom recursive methods.

1. Summation: An Example Recursive Function

Download the following classes for this lab: {RecursionLab.java}. In the main function, experiment with the two variations of summation functions (iterative versus recursive) by uncommenting the desired code paths. Read the comments throughout the code before moving on to modify this algorithm, and consider how you might rewrite the recursive function as an iterative function and vice versa.

2. From Summation to Factorial

The definition of the factorial function is defined below. Using the summation code as a guide, write a very similar recursive method that calculates the factorial of some input n. This method should be like all recursive functions in that it knows how to solve a trivial case (case1), and the function knows how to decompose larger cases into smaller cases (case2). Just as for and while loops use a boolean expression to govern loop termination, so too will we use a boolean condition in combination with an if to control repetition, as demonstrated here:

```
if (case1) {
   return base case solution;    //for summation, factorials, exponent, and Fibonacci, this is 1
} else (2) {
   result = recursive call         //typically the recursive call decrements by 1 or n/2
   return result;
}
```

Build this new recursive method and test your code with a short main driver.

[1] Original by Rob Nash. Edits by A.Retik, May 2016

3. The Exponent Function

In this section, **we will build two recursive functions and roughly compare their respective performance.** Write a method that produces the result of x^n by observing the following recurrence relationships:

(1) $x^0 = 1$
(2) $x^n = x * x^{(n-1)}$ when $n > 0$

Verify your method produces the correct value by writing a main that tests it. In the same class, write another function that will also recursively solve for exponential values, but will do so by cutting our problem in half at each step (rather than moving the solution along incrementally by subtracting one from n at each step). This new relationship is defined by the piecewise function:

(1) $x^0 = 1$
(2) $x^n = (x^{(n/2)})^2$ when n is even
(3) $x^n = x * (x^{((n-1)/2)})^2$ when n is odd

To obtain the desired speedup, make sure your new function recursively calls itself no more than once in the body of the method. Once you have built this new recursive method, you should be able to redirect your main to test its output. Do so, and compare the execution times for large n with the old implementation – which is faster, and why?

1. Answer the following questions in comments in your code
2. Build a polynomial f1(x) for the first exponent function
    a. Next, select a reference function g(x) that will dominate the f1(x) function in question
    b. Finally, for g(x) find witnesses c, k such that $|f1(x) \leq |c * g(x)|$ for all $x > k$?
3. Build a polynomial f2(x) for the second exponent function
    a. Select a g(x) reference function
    b. Find witnesses c, k to prove the class of Big O for the second

4. The Fibonacci Numbers

This series is described with by the following recursive relationship:

$f(0) = 0$ and $f(1) = 1$

$f(n) = f(n-1) + f(n-2)$ for $n > 1$

The above formulae produce the following output "1 1 2 3 5 8…" for some n (>4), and can be systematically calculated using a recursive function. Write a function Fib(n) that prints out the corresponding sequence of Fibonacci numbers, and write a main that invokes this function with some sample values of n.

5. Combinations, or “N Choose R”

Given a set of n items, how many ways can we pick r elements from n? This is known as the “choose function” (or binomial coefficient), and we can calculate the number of r-sized subsets of n (where order is unimportant) using the recurrence relationship defined below. Note that this definition builds on the notion of factorial, so make sure you understand the sample code first and have produced a working factorial method before moving on.

C(n,r) = n! / (r! * (n-r)!)

You can create a faster solution to this problem if you reuse the individual component calculations (rather than re-calculating these), which are r! (less than n!) and (n-r)! (also less than n!), but this is merely an observation and not required to be implemented as part of the lab.

6. Fractals & Depth-Limited Recursion

In this next section, we’ll create a new Shape Subclass that uses recursion to create a fractal snowflake pattern. We’ll manage the size of our flakes (as well as the number of recursive calls) using an integer called “`limit`”. This value will provide us with a way to abort the recursion after a specified number of recursive calls – this technique is known as depth-limited recursion, and is frequently used in graphics applications such as fractal design and reverse ray-tracing.

1. Download PolyDemo.java and Shape.java from the last homework.
2. Build a new class called FractalFlake.java that extends Shape
    a. Define a private final int to manage the snowflake’s `limit`.
        i. These should be values such as {1…50}
    b. Define a second private final int to manage the number of branches for the flake
        i. These should be between {5..12}
3. Build a suitable constructor for FractalFlake that invokes the superclass constructor as the first line of code.
    a. In our examples, these flakes will have a size variable, which you should pass to the constructor for your FractalFlake to store
    b. Our FractalFlake also has a branching factor, which should be handed to the constructor for our class to manage.
4. Override the draw function so that it invokes a second draw function that takes more parameters than just the Graphics object, as in:

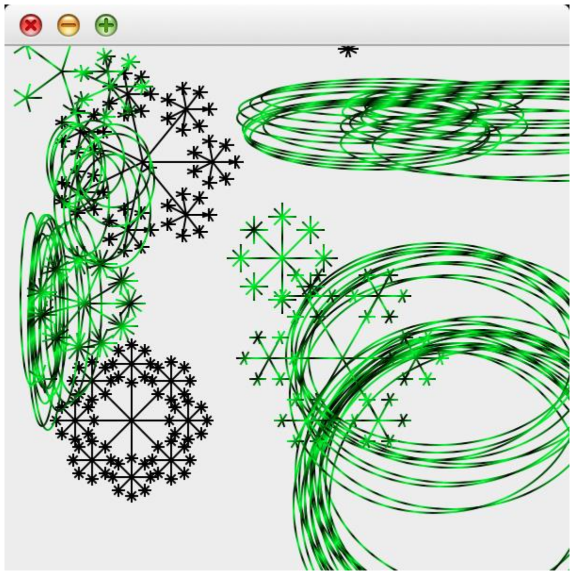
```
@Override
public void draw(Graphics g) {    //a redirect or facade
      draw(g,getX(), getY(),limit);
}
```

5. Overload the draw() function by defining a new draw function that takes a starting x,y coordinate pair, a `limit`, and a graphics object to render to, as in:

```
private void draw(Graphics g, int startX, int startY, int limit) {
      if(limit>= 3) {  //base case is depth <3
            for ( int i = 0; i < numBranches; i++ )
           {
            int x2 = startX + (int) (size *
                       Math.cos( (2 * Math.PI / numBranches) * i ));
            int y2 = startY - (int) (size *
                      Math.sin( (2 * Math.PI / numBranches) * i ));


                  g.drawLine( startX, startY, x2, y2 );  //do a branch
                  draw(g, x2, y2, limit/3);  //recursive call
               }
            }
      }
```

6. Finally, modify the PolyDemo driver so that in the getRandomShape() function, you can produce FractalFlakes as well as Sprays.
7. This type of recursion is limited by the value "`limit`" – when we've divided size enough times, eventually we will arrive at our base case (`limit`<3). In this way the `limit` variable provides a limit on the number of times the function will call itself; hence the name "depth-limited recursion"
8. Execute your driver and verify your recursive fractal patterns visually; this should look something like the snowflake patterns below.
9. Try altering the values for limit, numBranches, and the magic constant "3" above to obtain different snowflakes.



7. Recursive File Searching

In general, searching can take multiple forms depending on the structure and order of the set to search. If we can make promises about the data (this data is sorted, or deltas vary by no more than 10, etc.), then we can leverage those constraints to perform a more efficient search. Files in a file system are exposed to clients of the operating system and can be organized by filename, file creation date, size, and a number of other properties. We'll just be interested in the file names here, and we want perform a brute force (i.e, sequential) search of these files looking for a specific file. The way in which we'll get file information from the operating system, there will be no imposed ordering; as a result, a linear search is the best we can do. We'd like to search for a target file given a specified path and return the location of the file, if found. Let's sketch out this logic linearly before we attempt to tackle it recursively. Before attempting any of this section, be sure you have ready your recursive homework assignment in its entirety, as bits of that assignment will be referenced here. We'll start by imagining the steps required to find a file in a given directory or subdirectory on a very high level in pseudocode.

7.1 Pseudocode & Stepwise Refinement

Pseudocode is a useful tool for mapping out complex algorithms in a language-independent way before even writing one line of code. Google **Stepwise Refinement** and **Divide-and-Conquer.** We're going to practice this approach in detail here. We'll start with a very high level description of the program, and flesh out bits of detail in each stepwise refinement. Given the small amount of pseudocode below, can you take a small, incremental step to describe in more detail the steps required to accomplish this task?

Pseudocode Refinement Step 1 – The Problem Statement

This should be as short as possible, yet convey the full requirement of the program (however vague). Consider the following starting point for your stepwise refinement, which was pulled from the homework description.

"Given a target file to find and a starting directory, determine if and where the target file exists."

Pseudocode Refinement Step 2

Using the above sentence as an incomplete guide split the embedded concepts into 4 distinct steps, such as:

(1) Setup & Initialization (for the program, or a specific function, similar to preconditions)

____

(2) Input

____

(3) Processing

____

(4) Output

____

Pseudocode Refinement Step 3 to (N-1)

Using Step 2 above as a slightly more complete guide, split up the processing phase into more discrete steps required to search a directory. For example, a start on the next step might be…

```
//just for the (3) step above (Processing))
while(more directories to look at) {
    look at one file or directory and check for a match and return it if found
    if the current item is a directory, we must repeat these steps for this directory as well
}
```

target not found at this point

(Spoiler Alert!) Stepwise Refinement, Step N

Compare your final stepwise refinement to the refinement below of the processing step we examined above. Does the logic appear similar? **Submit your stepwise refinement in comments in your homework.**

```
Add the given directory to some structure to manage directories
while(more directories to examine) {
        Get a directory
        If a file, check for match
        If a directory,
                for(each file and directory in the directory) {
                        if a file, check for a match and return if found
                        if a directory, save this in a structure
                }
}
return not found
```

7.2 From Pseudocode to Java - File Search, Version 1.0

Build a linear search following the pseudocode approach above. You can put this code in simply a main, or you could design a static helper function in your HP static class that searches for files. Did you notice we used a stack to accomplish this directory searching? Using the main below and the outline above, complete the iterative file searching algorithm in the method ***searchFiles()***. Use fileObject.listFiles(), fileObject.isDirectory(), and fileObject.getName() to accomplish this below.

```
public static void main(String[] args) {

            System.out.println( searchFiles(new File("c:\\"), "hw3.zip") );

}

public static String searchFiles(File path, String target) {

//todo

}
```

7.3 File Search, Version 2.0

We made use of an explicit stack to track new directories as they appear in the above iterative code. Do you think we could use the method call stack in place of our Java stack to accomplish this recursively? If we remove the stack definition and code above and produce a recursive function, might that be shorter? Change the main function above to call recursiveSearch() instead, which is a new method that you will create. This method will take the same input as searchFiles, but instead of searching through files using explicit for loops and a Stack from java.util, we'll be using recursive function calls to facilitate some of the looping and use the method call stack to store our directory list instead. **Build a recursive linear search following the pseudocode you have refined to this point, or using the starter pseudocode approach below.** Note that multiple distinct correct solutions exist to this problem, so don't worry if your pseudocode isn't exactly like the code below.

```
public static String searchFiles(File path, String target) {

            check if path is actually a dir, abort if not

            loop over all files & dirs in the current directory

                        if a file, check for a match and return if found

                        if a directory, repeat these steps

                        if found in the directory, return found

            return "Not found";  //if we made it here, we didn't find it

}
```