Analysis of Students Geometric Thinking with Technology

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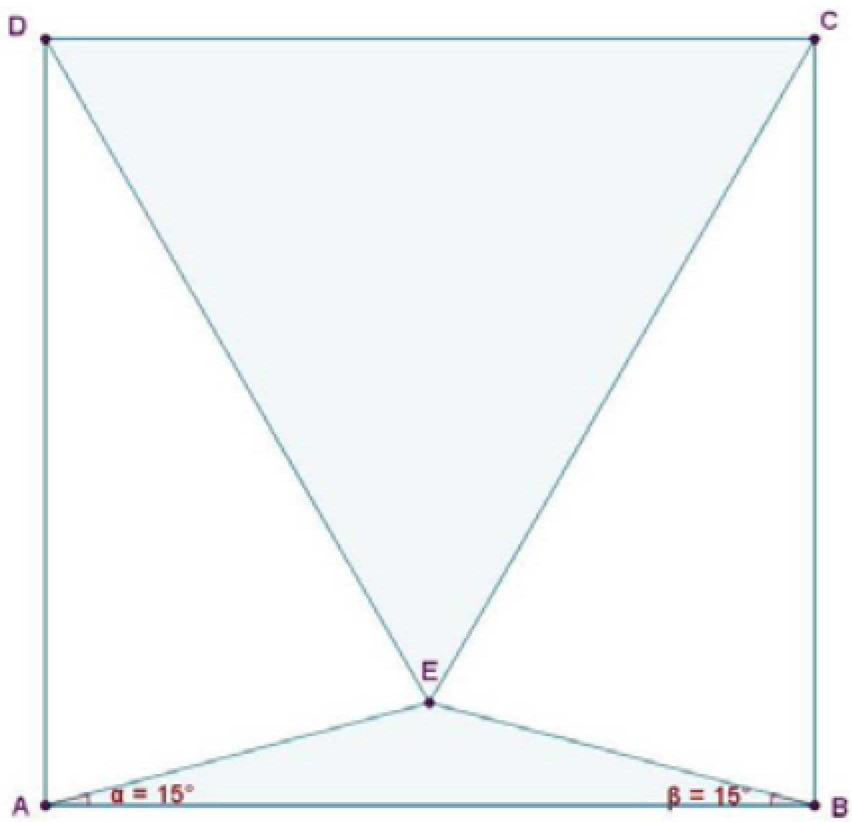
1 Introduction

In this activity, each of the four group members evaluated students from different majors at the University of Washington, Bothell; these students were from the Chemistry, Education, Mathematics, and Business departments. Each student was provided with a *Van Hiele* test, and then guided through the written solutions and then were provided the opportunity to solve the problem using *Geogebra*. Our group then compared the results and solutions that each student preferred, and connected our observations to the practice of Geometry education and the concepts covered by this course.

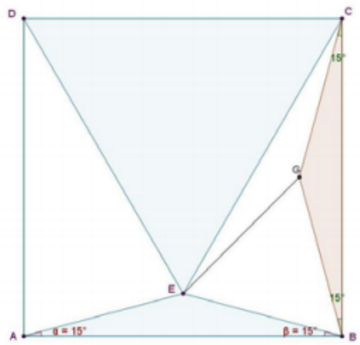
1.1 Task

Let E be inside square ABCD such that EAB = EBA=15. Show that CDE is to equilateral.



1.2 Solutions

Solution1 (classical):

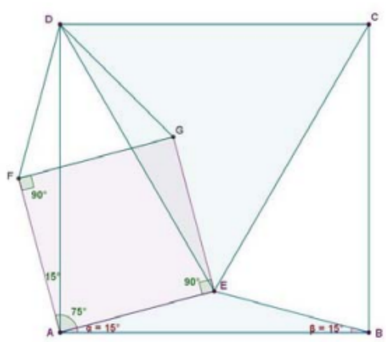


A good example for the heuristic*: draw an auxiliary object*

- Construct ΔBCG congruent with ΔABE.
- It is easy to notice that ΔBEG is equilateral.
- That implies $\Delta BCG \cong \Delta CGE$ (SAS congruence criterion)
- $CE \cong CB$ implies ΔCDE is equilateral

□

Solution2 (classical): This is one student's solution:



Draw an auxiliary object, again.

- Construct the square AEGF (G inside the square)
- $\Delta ADF \cong \Delta ABE$ (SAS c. criterion)
- $FG \cong FD$, $\angle DFG = 60°$, $\angle DGE = 150°$
- $\Delta DEG \cong \Delta ABE$ (SAS)
- $DE \cong AB$ implies ΔCDE is equilateral

□

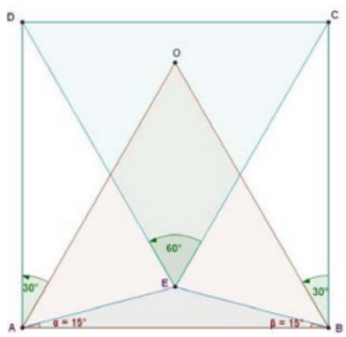
Solution3 (classical): (*Mathematical Excalibur, vol.13, number3, p.2, example 2.)*

Method: *using geometric transformation*

We introduced one very useful fact:

***Fact**: If $\alpha + \beta$ is not multiple of $360°$, then $R(O_2, \beta)\ R(O_1, \alpha) = R(O, \alpha + \beta)$, where $\angle OO_1O_2 = \alpha/2$, $\angle O_1O_2O = \beta/2$*

The students were asked to consider the proof of the fact as their homework.

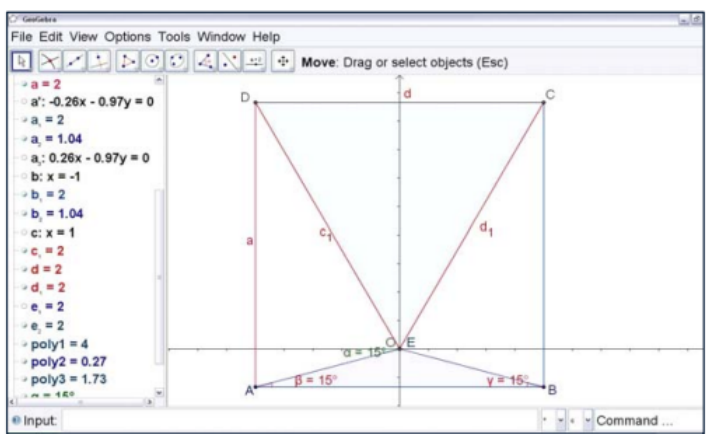


- Let O be inside the square such that ΔABO is equilateral
- $R(B, 30°)$ sends C to O, $R(A, 30°)$ sends O to D
- Since $\angle EAB = 15° = \angle EBA$ according to the above fact

  $R(A, 30°)\ R(B, 30°) = R(E, 60°)$
- So $R(E, 60°)$ sends C to D
- Therefore ΔCDE is equilateral

□

Solution4 (GeoGebra)



2 Chemistry Student

-Wenxi Wang

I interviewed a UW Bothell senior female student in chemistry major, who only had one year basic geometry class in her grade 10th back to high school. The key for this interview was to help the student developing some geometric thinking and introducing her the mathematics tool Geogebra. We spent one and half hours to accomplish the interview. There were three main parts in this one-on-one interview, the Van Hiele level test, introduction of Geogebra and task solving by using both pen and Geogebra tool. I let the student started with the Van Hiele geometry assessment. It took her twenty-five minutes total to finish these 25 questions. The students spent only 8 minutes on first 15 questions and 20 mins on the rest 10 questions. I asked her if she think it is easy or not. Her answer was first 10 question she learned when she was in junior high so they were super easy and 5 questions after starting to get hard but she knew how to do, the rest she needs to think hard and doing with guess for some question. She said she realized the problems got harder and harder. I compared her answer with the answer key and got which Van Hiele level she is in without telling her the results.

As I knew, the student has never heard about Geogebra or used other geometry tool before. Since Geogebra was new to the student, I showed her 3 minutes long youtube video about introduction of Geogebra. She was amazed by how much Geogebra can do and how easy visualization tool it is. I ran Geogebra and showed how to use those function keys in Geogebra. She was very excited to learn. Then I let her play around some function keys and get to know Geogebra for 15 minutes. At the beginning of the investigation, in order to making the student feel stress-free I told her couple times that I was not looking for a right answer from her, instead I was investigating the way she thinking in geometry and how she pursuing this question. I let her use pen and paper to solve the problem first and switched to the Geogebra tool. I observed that she started the problem by labelling the useful information that the problem provided. Then she self-reviewed some related concepts such as the conditions to be an isosceles triangle, equiangular triangle, square and so on. She told me that she knew the side length AE and BE were equal because the angle is equal, and in order to proving triangle CDE is an equilateral is to show side length CD equals to both CE and DE. After 5 minutes thinking, she told me that she had hard time to find the piece of information that can help her to get there. I provide some hints that she could draw extra side length or shape to help her find the solution. She was confused and did not know where to start with. I asked her to review again those concepts that she just reviewed and think backwards.

At the end, I showed her three different solutions to solve this problem and let her use one of the solution method she learned to try it on Geogebra to see if this way helped her understanding or not. She found Geogebra is a great tool to use because it has a coordinate system and you can draw lines and angle precisely to solve the problem. She said she also can use the slopes and distances to measure the lines and make sure they are equal. I told her she did a great job and her score for the previous Van Hiele assessment which was level 3. She was not surprised by the result and she wish she could learn more about Geogebra in future to develop her geometric thinking.

There are lots of concepts embodied in this task, which are equilateral triangle, isosceles triangle, equiangular triangle, square and so on. The student should know the definitions and the conditions to be those shapes to help them start the problem. After I gave a short tutorial of Geogebra to the student, student learned some functions keys of this tool and helped them understand and solve the problem in a better visual way. By using the Geogebra, student has to think about which step to use first to make the next move without creating a mess non logical thought. To anticipate technology whether or not would enhance students work on the task by asking them some feedback about it. We mainly focused on collecting the geometry thinking from different major student that has different math background. We provided pen, paper and computer for student to use during the interview. I can tell the students understandings of the concepts that were embodied in the task by the way she explained to me how to solve the problem and what did she know based on the information from the task. Student used pen and paper to do the task first and then use Geogebra to help her solve the problem logically and visually. The student was very interested to use Geogebra and ask me questions whenever she had a trouble to access. She learned most function keys at the same time she also finished solving the problem. There are lots of representations the student used in solving this task such as numbers, drawing and symbolic notations. Those representations hindered them keep on track what information they had and what else needs to pursue in order to proving the definition. The convincing argument they used to justify their answer to the problem was by using the backward proof and the definition of the isosceles triangles.

Overall, I was surprised that she focused on the definitions, which normally people will focus on the side length and angles. Also, the student was a fast learner and she learned how to use Geogebra in a short amount of time. Especially, she knew how to connect the different key functions and use them easily. I found the Van Hiele geometric assessment was a good way for me roughly know which geometry level the student is. The 25 questions and times the student spend can be a good data to analyze how much they knew about geometry and their mathematical knowledge. Despite some natural development of geometric thinking, instruction is necessary to move student through several levels of geometric understanding and reasoning skill. The van Hiele theory puts forward a hierarchy of levels of thinking for different grade students and others. Visual plays a key role in the level one. Then, descriptive, which focus on identify and describe the properties of shapes such as an equilateral triangle can be distinguished because of its three equal sides with equal angles and symmetries. This is very basic knowledge the interviewee needs to know before we start the interview. Level three is informal deduction which is student needs to be able to know the properties of shape are logically ordered to reach this level.For instance, the basis of formal deduction, definitions, and the role of axioms.

3 Education Student

-Dhanya Regith

Proving that the triangle CDE is equilateral, multiple concepts can be employed depending on the method adopted. Properties of square and triangles, congruence of triangles, trigonometric angles are some of the concepts that could be used to solve this problem. For the analysis paper, we had chosen GeoGebra as the technology, through which students are able to construct the shapes and arrive at the proofs through construction. GeoGebra provides opportunities for the students to construct shapes and angles and then measure the lengths of the sides.

This task would involves analyzing the properties of shapes such as square and triangles and thus the GHOM of reasoning with relationships would be elicited in the students. The congruence properties of the triangles are also concepts that could attribute to developing reasoning with relationships. Another GHOM which is utilized here is balancing exploration and reflection, since the student works out the task using paper and pencil and then advances on to using GeoGebra. This enables the student to explore deeply about the strategy to be employed.

An Education major with prior education in Math was interviewed for this task. The student initially used paper and pencil method to work on the task and then progressed on to use GeoGebra.

The Education major had taken basic Math courses in High School, Linear Algebra and a basic course in Geometry. The student was also familiar with technology tools in Math like AutoCAD, MATLAB and GeoGebra. As a result the student had a good understanding of the concepts embodied in this task. While doing the task by paper and pencil method, the student used the concepts of similar triangles, trigonometric angles, and quadratic equations to attempt to solve the task. The student had demonstrated a Van Hiele Level 3 in the test provided and it shows that the student has good analytical skills, abstraction and deduction. The student is comfortable with deductive reasoning and the paper-pencil work indicates that the student is able to generalize geometric ideas and reason with relationships.

The student used GeoGebra to investigate the different properties, like stretching a side of a shape to analyze the change in properties. By using GeoGebra, the student demonstrated the ability to balance exploration and reflection in conjecture with the paper-pencil method. The student was not able to solve the task by paper-pencil method but using GeoGebra the student was able to measure the lengths of the triangle and show that the triangle is equilateral. But the student did not believe that work through GeoGebra could be accepted as formal proofs or even setting the side of the square could make the proof more robust.

The student used the following symbols while solving the task:

- Short lines to depict the congruence of sides of the square and triangle
- Marking the right angles
- Assigned variables to the unknown sides and used these variables to develop equations
- Question marks to depict the unknown factors
- Quadratic equations and trigonometric angles
- x, y variables to represent the unknown sides and alpha, beta to represent angles

Use of these representations helped the student very much in navigating through the strategy to solve the equations. The use of x, y variables to represent the unknown sides and alpha, beta to represent angles clearly indicates that the student has exposure to geometric representations and proofs.

Though the student was not able to completely arrive at the solution by paper-pencil method, the student had clearly written down each of the steps that could potentially lead to the solution.

The student was thinking aloud and explained the reason for taking each of the step. The reasoning that the student had provided clearly encompassed the concepts in geometry and algebra.

The fact that the student was able to solve the problem using GeoGebra made me think that technology can be intricately integrated with learning Geometry. But the student clarified that paper-pencil method was preferred since it helps in arranging the students thoughts. But had the student been more proficient in GeoGebra it would have been easier to construct geometrically accurate representations of the task. But even then in this task, constructing through GeoGebra could only add validity to the proof, but cannot be proofs themselves, since they are not based on logical reasoning.

The task for this paper is a complex task, which requires an in-depth knowledge of geometry and proofs. GeoGebra helped in constructing the geometric representation and show that the triangle is equilateral, but the GHOM of generalizing geometric ideas cannot be applied in this case, since the equality is shown for a specific case.

The Education Major was able to clearly understand the steps to be followed to arrive at the solution for this task. The students educational and background knowledge in Math and geometry clearly played a significant role in developing the solution. But the task being a complex one, the student was not able to arrive at a formal proof. The student used quadratic equations, trigonometric angles, congruence theorems, and properties of shapes like square and triangle to lead to a solution. Thus it can be inferred that this problem can be introduced to 11th or 12th grade students and help them to develop the GHOMs of reasoning with relationships and generalizing geometric ideas. At this stage, the Van Hiele level of the students could be at Level 2. To advance them to foster GHOM of balancing exploration and reflection, use of technology can be employed in class. GeoGebra, which is a powerful tool in learning geometry, provides students with the opportunity to explore different possibilities. For example, students could construct a geometric figure and then alter the dimensions to analyze the properties of the modified figures and generalize ideas. This exploration process will also help them to reflect upon their already existing understanding of the concepts and build off from there. Use of technology like GeoGebra will most certainly help in visualizing and concreting the abstract thinking of the students. But as an educator, it is important to note that, we would have to give some leads in between to help the students to arrive at a solution in this task. Also, students should have explored GeoGebra prior to getting introduced to the task. Only then will they be able to effectively use the tools of GeoGebra to explore, investigate and arrive at conclusions for the task.

4 Mathematics Student

-Samuel Peoples

One of the students we evaluated was a college Mathematics major; this particular student was tested in a public space, with minimal distraction. When approaching this task, the student was provided a Van Hiele test and then asked to prove that a triangle formed from the 150 vertex of an isosceles triangle inside of a square is equilateral by hand and with technology. By providing the student with the Van Hiele test, we were able to ease the student into geometric thought, as well as reminding them of key concepts and skills that they may have forgotten since they last took a geometry course. As we are well-aware, the Van Hiele exam steps through each of the four Geometric Habits of Mind, being Reasoning with Relationships, Generalizing Geometric Ideas, Investigating Invariants, and Balancing Exploration with Reflection; while the proof exercise focused mainly on generalization, and reflection. We expected that the Mathematics student would have a much easier time operating at a higher level of thinking, while students from other majors would have a harder time, mainly because geometric proofs are viewed negatively by most students.

We have read that students with higher Van Hiele scores have much greater success with geometric proofs than those with lower scores, so by providing this test to students from different backgrounds, we are able to use this idea to see how our peers approach these topics. We utilized pencil and paper to approach the proof by hand, and Geogebra to complete the activity with technology. We noted that the student had not taken a formal geometry course since their sophomore year of high school, and that the student was out of practice; while they were initially apprehensive to the written proof, the student was seemingly eager to find a solution. Unfortunately, they were unable to complete the proof using only pencil and paper; their geometric skills were evident in their success when taking the Van Hiele test, displaying a strong intuition on how they would solve the problem. They drew a series of pictures and diagrams, making connections between different geometric ideas, but continuously returned to their algebraic skills. When asked about this particular aspect of their thinking, the student made a note that they were most familiar with the algebraic approach, and that they were more uncomfortable with geometry.

The student initially noted that there were theorems and axioms that they knew were available, and that they could not remember some of outright. By using supplementary and complementary angles, the student attempted to solve the problem; ultimately requiring much assistance although their Van Hiele score was four. The student noted that the distance formula could be used if they were working with a coordinate plane, which allowed us to easily transition to the technological approach. When asked if these lengths from the distance formula could serve as proof, the student noted that the problem would need to be manipulated to determine whether the conclusions found were true for all squares.

When the student was given the opportunity to use Geogebra, they noted that they had only used the program briefly in another class, so they were given a short tutorial of the abilities of the program by drawing the required figures. By locking a polygon to maintain equal sides, the student then created a point inside the square which formed a triangle with two locked 15 angles and one 150 angle. Next, the student connected the opposite vertices of the square to the 150 vertex, and labeled the sides A, B, C. Following this, the student manipulated the corners of the square, through rotation, reflection, and scaling. They then showed that the lengths each side of triangle ABC were equal to the length of each side of the square, which proved that the interior triangle was equilateral.

The Mathematics student was evidently capable of completing the exercise with minimal assistance, but did have a hard time returning to a mindset of geometric thought. The student was able to show the relationships between angles algebraically, and clearly understood when some aspects of the problem were able to be generalized; this allowed the student to make hypotheses on how to approach it. The student was primarily visually oriented, which may have been the reason why Geogebra was their preferred method of solving this problem. When shown the four solutions, the student described the written approach as nontrivial and that it was difficult to reach the same conclusions without assistance. This contradiction from their expectation served as a useful topic to explore following the exercise.

We found that the student preferred Geogebra because they were able to manipulate the figures and directly connect their ideas about the problem to a precise figure in the program. Because the student was visually oriented, they were able to recall a lot of skills that they had forgotten since the last time they had seen a geometric proof. The student found themselves frequently experimenting with different transformations, figures, and algebraic interpretations when using Geogebra, and quickly began to draw conclusions through their discovery. They said that they felt quite comfortable with the program, and that they wished it was used in their earlier learning; the student admitted that a formal written proof would still be difficult without axioms, and that the proof would be moderately rigorous. Geogebra and its functions came easily to the Mathematics student, and they frequently noted the intuitive nature of the program. They said that it would be interesting to see how this problem could be extended to higher dimensions, and explained how their conclusions could be applied to a cube and pyramid, wondering if there were ways to do so in the program.

It was surprising to see that a student that hadnt used geometry formally for multiple years was able to recall their skills when using Geogebra, but remained reserved when asked to write the proof formally, even after seeing the solution. The student has had multiple proofs courses in their Mathematics curriculum, but still preferred the technological approach. This suggests that Geogebra is a useful stepping stone between geometric reasoning and proofs, where the program allows students to directly apply their expectations, or verify that something doesnt work. The student argued that Geogebra could be used to help students develop the axioms required to find a written solution, and that the two methods could be combined to help guide younger students towards a formal conclusion.

5 Business Student

-Puiyam Wong

We would like to use GeoGebra as our technological tools and we interviewed people in different majors, such as Mathematics, Education, Chemistry and Business. I did interview someone, who is majoring in business. We would like to investigate that GeoGebra can help them understand more about geometric reasoning and proof, and feel easier to work on geometric problem by using GeoGebra. The interviewee has high school mathematical background, such as geometry proof, linear algebra, statistic, and so on. They also took at least one college-level class during college year. I took an hour and half to finish the interview. This interview involved testing the level of geometry by Van Hiele test, doing a problem with geometric proof with what they know, having a tutorial of using GeoGebra, and using this technological tools to do this problem. For our problem task, we require our interviewees to prove that a triangle in a square with some information. This problem involves properties of different triangles and square.

First, he spent 20 minutes to finish 25 questions on the Van Hiele test. He told me that it was so easy at first half of the test but he felt confused about second half since he said there was some parts that he was not sure about that and just did it by guess. I checked the answers afterward and he got level 3 but I did not let him know immediately. Next, I showed him that how to use GeoGebra and how GeoGebra can help him for solving the problem with geometric reasoning or proof. At the beginning of using GeoGebra, he thought about which step he should do first for doing this task. He required to get some papers and pencils for solving my hands first since he said he was used to do everything about geometric proof with papers and pencils while he was in high school. I observed that he started to write all information what is given on the paper first, and try to find some connection in this information. He told me that triangle ABE is an isosceles triangle since angle EAB and EBA are the same. Also, he told me that he just needed to prove side CE and DE are equals to CD that in order to prove triangle CDE is an equilateral triangle. He thought side BC and CE are equals and side AD and DE are equals as well but he was in trouble of proving these are fact. He tried to do it conversely that assume triangle CDE is an equilateral triangle and angle CDE, DCE and CED are 60 degrees. Then he can prove that triangle ADE and CBE are isosceles triangles by using calculation of angles. I told him he had the good reasoning but not exactly solution. He tried different ways for 5 10 minutes but he still cannot get the solution. Last, I showed him that there are 4 different solutions for this problem and he said he never thought about those.

Then, I ask him try to use GeoGebra to prove this problem. He spent a few minutes and found the answer immediately since he saw the measurement of each side and angle. It showed that length of side CE, DE and CD are the same length. He thought this tool is very convenient since it contains all data with the shape, such as length, angle, slope, etc. It is easy to do the problem solving in geometry by using measurement unit. He never uses any technological tool for geometric proof while he was studying in high school. By checking the result of Van Hiele assessment, I told him he was in level 3 that he expected after finishing test. He knew he would get at most level 3 4 since he felt confused about some properties of shapes. After this interview, he would like to use this software in future to enrich his reasoning in geometry for teaching his brother, who is learning geometry in secondary school.

There are several concepts embodied in this task, like properties of equilateral triangle, isosceles triangle, square, etc. Students have to know a lot of definitions to inspire their reasoning and proof, for example, they should know sum of all angles in a triangle is 180 degrees, sum of all angles in a square is 360 degrees and right angle is 90 degrees that can help them think about what will have a connection with given information. As I provide some instructions of GeoGebra to student, who can learn how to do problem solving by using GeoGebra, and let students solve it visually. Student can see the measurement of each side and angle via GeoGebra, especially it is not given to them. For instance, student can see each angle of triangle CDE is 60 degrees, which represents triangle CDE is an equilateral triangle. I can say about the students understandings of the concepts that were embodied in the task that is how he thought about this problem, what he should mention for getting correct solution. For example, he knew the properties of an equilateral triangle that contains three equal sides and three equal angles. He understood what he needed to find but he had no idea how to start and how to find this information correctly. He preferred to add some lines for reasoning that required to do it by paper and pencil. He did the same way via GeoGebra and he could do it more accurate than paper and pencil since he could do it with exact data. He said he thought the problem solving in a converse way because he could assume what he hoped to get first. Then he did the problem solving backwards with step-by-step. Since GeoGebra provided more accurate data than paper and pencils, it inspired him how he got a solution with step-by-step. He used GeoGebra, paper and pencil in solving this task. These could let him have a great geometric thinking. He understood every proof could not be satisfied without words which was not a formal proof that was part of convincing arguments to justify his answer to the problem.

I was surprised that he thought about GeoGebra is more convenient to them for problem solving and he knew to think this problem with backward proof that I did not expect on him. He understood what he needed for solving this problem and how he could find all connection from the given information. He had a clear concept of this problem solving task that made him solve the problem easily. He could find the advantage of GeoGebra and used it as part of proof that helped him save a lot of time. He also mentioned that he would be interested in geometric proof more if he could use GeoGebra as his assistance while he was learning geometry. This software can inspire students to learn geometry since they can try different ways via technological tools conveniently. He thought he would get inspired more geometric thinking as well when he could use it during class time. Although GeoGebra is a good technological tool to students, it should allow students to use this kind of tool after they get understand a lot of basic concept and practice with many problems. Otherwise, students would rely on it when they do the geometric problems.

6 Comparisons

	Chemistry	Education	Math	Business
Van Hiele Score	3	3	4	3
Preferred Method	Geogebra	Paper-Pencil	Geogebra	Paper-Pencil
Style of Thought	Visual, Definition	Symbolic representation , Algebraic, Geometric	Visual, Algebraic	Visual, geometric concept
Assistance Required	Draft thought, review definitions, help with tools	Concepts, Critical Questions, review of GeoGebra	Written, needed axioms	Measurement, definitions

Table 1. Summary and comparison from different major students.

When comparing the student's preferences and abilities, we were able to connect our learning in the classroom to our findings from the activity. The Van Hiele scores of our students were a clear representation of the students mathematics skills, wherein those that had recently taken courses in mathematics were able to draw conclusions and make appropriate hypotheses for their expectations. The four students had each completed a collegiate-level calculus course, which helped place each student at level three, with the assistance of their recollection of their geometric courses in the past. The Mathematics student was more familiar with proofs and proof methods with a Van Hiele score of four, which gave them an advantage when finding a solution.

Students which had been exposed to more technology-based problem solving (Chemistry, Mathematics) and algebraic problems were more inclined to use Geogebra, while students which deal more with humanities (Education, Business) preferred to use pencil and paper. This could have been because of our presentation and abilities with Geogebra, but could also be tied to the individual students exposure to technology-based problem solving. The students each became more comfortable with Geogebra as they experimented with its functions, and displayed more independence as the exercise was completed.

Each of the students had a firm understanding of algebra and were able to make the appropriate connections while being guided through the written solutions; each also made note of the seemingly nontrivial nature of the written solutions when compared to the Geogebra approach. Each of the written solutions had a trick or hint which made it more apparent, and the students had little trouble with understanding the conclusions each of the solutions made, even if they were not immediately noticeable. The STEM students were more skilled in Reasoning with Relationships and Making Generalizations, while the two other students had trouble understanding abstraction and identification of whether proofs were sufficiently completed.

Each of the students were surprised by the versatile nature of Geogebra and expressed their desire for the program when they had initially taken geometry in High School. Our group noticed that the individual students skills in geometry were diminished with time, and that a college-level geometry course based in application could help students recall the skills taught in high school. While each of the students were familiar with Euclidian geometry, the level of abstraction required for this exercise made the students initially hesitant to attempt a solution. Our group would be interested in expanding this exercise to a larger sample so as to facilitate a statistical conclusion, which may show a necessity for more geometry courses in college. Our group found that students with a more robust toolkit of mathematics skills, especially with the aide of technology, results in a higher level of confidence in their ability to find a solution, while a majority of the intuition necessary for geometry can be recalled with practice.

7 Conclusion

After interviewing four candidates from different majors in UW Bothell, namely Chemistry, Education, Math and Business, it can be observed that STEM students displayed more confidence in their ability to find a solution. STEM students exhibited the GHOMs of reasoning with relationships and balancing exploration with reflections. With the strong background knowledge in Geometry, the Math major was able to generalize geometric ideas even while using paper-pencil methodology to solve the task. Variation was also observed in their preferences to paper-pencil methodology or technology to solve the task. While the STEM majors preferred the technology over the traditional method, they felt that introducing the technology prior to the tasks would be an effective strategy. As educators, it would be best to facilitate the students to explore the technology first and be accustomed with the various options provided by the tool, so that later on when the tasks are given, they can make effective use of the technology and focus on the task, rather than being overwhelmed by having to learn a new technology and having to do the task as well.

One interesting observation was that our group's intuition was that the STEM students would have been more inclined to use pencil and paper, as they would have been more skilled in written proofs based on their $VanHiele$ score. The opposite was true, where the humanities-oriented students fell back on their high school instruction. All four students quickly displayed geometric skills that surprised them, as they had expressed that they had not practiced geometric proofs for at least two years each.

The Van Hiele score played a key role in students ability to find the solution and also help us as interviewer to know which geometry level they were at. There are lots of advantages for using Geogebra as some students has comments such as Geogebra can help them see more initial task from the task, it also helps student visualize related concepts and how they affect each other. Students conceptual understanding also can get enhanced at the same time; We found students were exchanging methods between geometry and algebra during the interview. We think GeoGebra is a new suitable working environment for problem solving. It not only helps student visualizing but also easier to use for some advanced mathematical problems. By connecting the refined skills the students had in algebra with their geometric intuition, Geogebra was a useful tool for visualizing tasks and solving more abstract problems. In the future, we would like to interview more people in different majors and different math background to help analyze whether the Geogebra should be introduced to all student in high school or not. By exploring the application of technology in the classroom, a better understanding of geometry can possibly be achieved, where students would be able to retain their geometric abilities with proofs over time. By bridging the gap between algebra and geometry, students can apply their skills to a more diverse range of problems.

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