Project 2: Periodical Cicadas

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Project Description: What are Cicada’s?

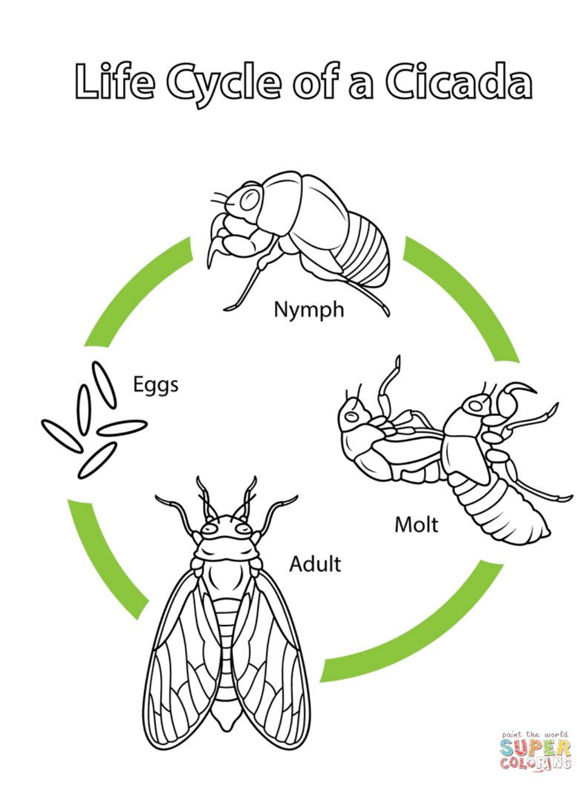
- 1300 different species worldwide
- Loud mating songs
- Not locusts

Project Description: Cicada Mortality

- Predators of Adults: birds, squirrels, domestic cats, dogs, turtles, snakes, spiders, and wasps
- Predators of eggs and nymphs: moles, ants, spiders, centipedes.
- Fungal infections
- Weather events can exterminate entire populations

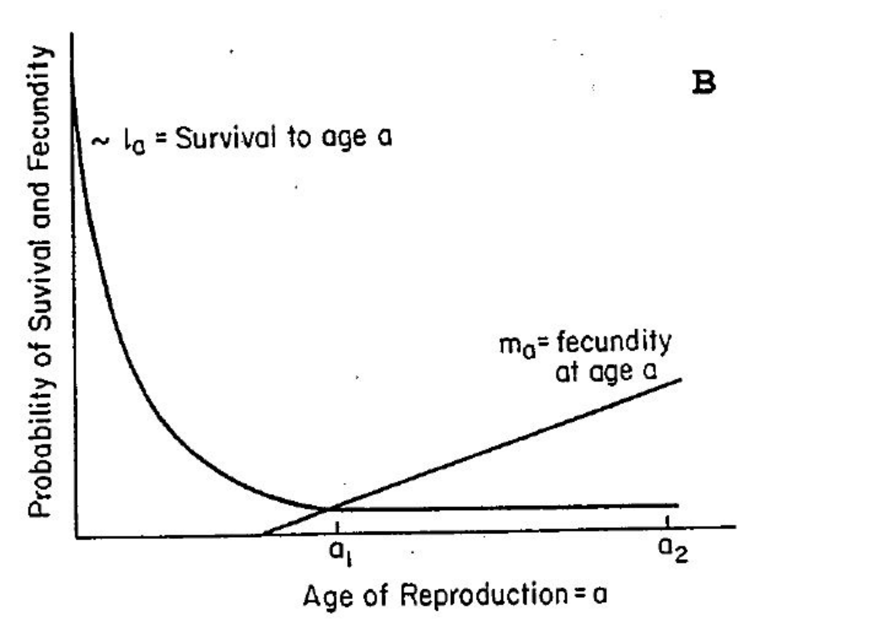
Project Description: Life Cycle

- Periodical Cicadas live for 15-17 years
- Entire population in an area is same age
- Only species that is synchronized
- Only in Eastern United States
- Adults emerge, mate, then die
- Lifecycle: Eggs → Nymphs → Molts → Adults



Project Description: Life Cycle

- Most organisms reproduce early, however Cicada's only reproduce during the adult stage
- Also as they age, the idea is that fecundity should go up because then they are given time to mature and develop to convert resources into making more eggs.
- As they age, their population decreases similar to a logarithmic function



Project Description: Eggs

- 600 eggs per female
- Hatch in 6-10 Weeks
- Eggs laid in tree branches



Project Description: Nymphs

- The nymphs then crawls from the grove to the ground and burrow 18 inches
- Feeds on roots
- Spends most of it's life in this stage (basically 17 years)
- Nymphs create small tunnels where they will emerge
- Cicada's use the tunnels to escape when it gets too wet
- For the last 7-8 years of their life, they stay about 4-5 inches below ground because of mole activity

Project Description: Molts

- Surface and molt

- Takes up to week


Project Description: Adults

- 2-4 weeks in this stage
- Only reproduce at this stage
- Females die immediately following laying up to 600 eggs.
- Males die shortly after mating.

Mathematical Modeling: Leslie Matrix

- Called the “Projection Matrix”, coined by Patrick H. Leslie in Oxford, England.
- Leslie Matrices have their applications in ecology to model population growth (or decline) over time.
- In order to construct a Leslie Matrix one only needs to know the survival rates, and fecundity rates of the species that they are modeling.
- Particularly useful because the eigenvalues of a Leslie Matrix show the rate of growth of an entire population over a specified amount of time.

Mathematical Modeling: Digraph

Mathematical Modeling: Ciada Leslie Matrix

	Egg	Nymph	Molts	Adult
Egg	0	0	0	300
Nymph	~0.196	0	0	0
Molts	0	0.02	0	0
Adult	0	0	0.85	0

Initial Pop.
0
0
0
1.5 Mil

Mathematical Solution: Population Rate

- By taking the matrix and raising it to a power '4n', we then see what the population of the cicadas look like after each 17-year cycle. So for the matrix P multiplied by an initial population vector v:

$$P^{(4n)}v^{(0)} = v$$

  We get a population of the cicadas after the given period.

- Why is it 4n?

Mathematical Solution: Increasing Population

- Starting with a population of 1.5 million, we see that after 1 life cycle the population is:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^{6} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.197 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^{4} = \begin{pmatrix} 0 \\ 0 \\ 1.50705\times10^{6} \end{pmatrix}$$

- After 4 life cycles the population is:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^{6} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.197 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^{16} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5284\times10^{6} \end{pmatrix}$$

- After 12 life cycles the population is:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^{6} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.197 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^{48} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.58682\times10^{6} \end{pmatrix}$$

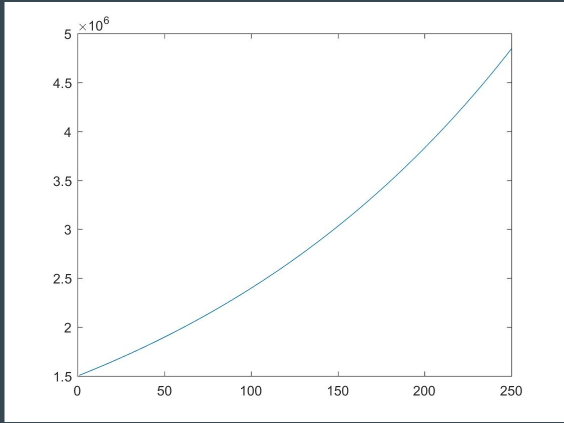
Mathematical Solution

- Then the population after a very long time amount of lifecycles shows:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^{6} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.197 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^{10^{17}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 6.38450262650035\times10^{50910056488238} \end{pmatrix}$$

- Since the max real e-value is greater than 1 the population is growing

$$\begin{pmatrix} -1.00117 + 0.\,i \\ \boxed{1.00117 + 0.\,i} \\ 2.77556\times10^{-16} + 1.00117\,i \\ 2.77556\times10^{-16} - 1.00117\,i \end{pmatrix}$$



Mathematical Solution: Decreasing Survival Rates

- Starting with a population of 1.5 million, we see that after 1 life cycle the population is:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^6 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.196 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.4994\times10^6 \end{pmatrix}$$

- After 4 life cycles:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^6 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.196 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^{16} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.4976\times10^6 \end{pmatrix}$$

- After 12 life cycles:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^6 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.196 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^{48} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.49282 \end{pmatrix}$$

Mathematical Solution: Changing egg survival rate

- In our original computation, we showed ~0.196 however we noticed using 0.196:

$$\begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.196 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}$$

- If we use mathematica to find the Eigenvalues, we get:

$$\begin{pmatrix} -0.9999 + 0.\,i \\ -1.66533 \times 10^{-16} + 0.9999\,i \\ -1.66533 \times 10^{-16} - 0.9999\,i \\ 0.9999 + 0.\,i \end{pmatrix}$$

- The population will decrease because the max real eigenvalue is less than 1
- Therefore the eggs must have a survival rate of higher than 0.196

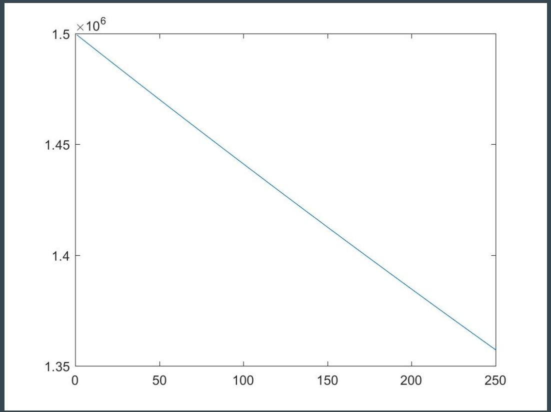
Mathematical Solution: Decreasing Survival Rate

- Then the population after a very long time amount of lifecycles shows:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1.5\times10^{6} \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.196 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix}^{10^{12}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- By finding the eigenvalues, we see what the total growth rate of the population is:

$$\begin{pmatrix} -0.9999 + 0.\, i \\ -1.66533\times10^{-16} + 0.9999\, i \\ -1.66533\times10^{-16} - 0.9999\, i \\ \boxed{0.9999 + 0.\, i} \end{pmatrix}$$



MatLab Bisection Search

```
P = [0, 0, 0, 300; 0.5, 0, 0, 0; 0, 0.02, 0, 0; 0, 0, 0.85, 0];
row = 2; col = 1;
maxpass = 50; desiredEigVal = 1;
minSearch = 0; maxSearch = 1;
eigVal = 0; pass = 0;
while(eigVal ~= desiredEigVal && pass<maxpass)
    pass = pass + 1;
    eigVal = max(abs(real(eig(P))));
    if(eigVal>desiredEigVal )
        maxSearch = P(row,col);
        P(2,1) = (minSearch+maxSearch)/2;
    end
    if(eigVal<desiredEigVal )
        minSearch = P(row,col);
        P(2,1) = (minSearch+maxSearch)/2;
    end
end
P(row, col)
```

Results for P(2,1):

0.196078431372549

Monomial Matrices

Matrix is a monomial matrix of order 4 = diag(300, a, .02, .85)*P(4,1,2,3)

P(4,1,2,3) has 1 disjoint cycle of length 4

Char Polynomial $\prod_{j=1}^{l}(t^k - \prod_{i=1}^{k}(a_{j_i})$ with L disjoint cycles of lengths k

The Char Polynomial is thus (t^4 – 300*.85*.02*a)

To find when 1 is an eigenvalue thus (1^4-5.099999*a)=0

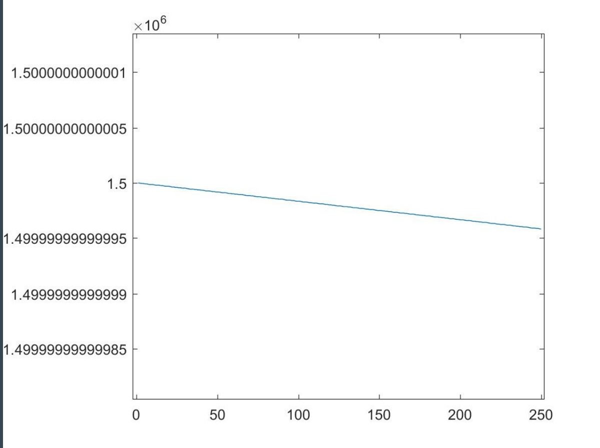
Thus a = 1/(5.099999) = 0.196078431372549

Mathematical Solution: Steady state population

- The egg survival rate has to be .196078431372549 for the population to be stable

$$\begin{pmatrix} 0 & 0 & 0 & 300 \\ 0.196078 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 \\ 0 & 0 & 0.85 & 0 \end{pmatrix} \qquad \begin{pmatrix} -1. + 0.\,i \\ 1. + 0.\,i \\ 1.249\times 10^{-16} + 1.\,i \\ 1.249\times 10^{-16} - 1.\,i \end{pmatrix}$$

- For every 300 eggs a stable population has
  - 58.82 become nymphs
  - 1.17 become molts
  - 1 becomes an adult and procreates.



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