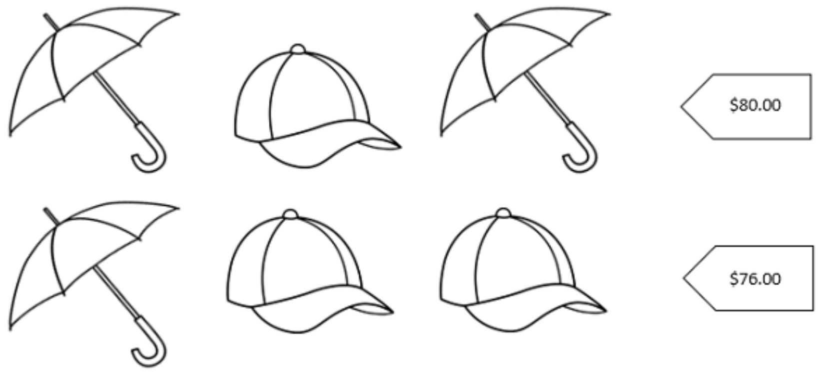
# Fostering Algebraic Reasoning


**Week 7 – Group 6**
**Samuel L. Peoples, CST, MA-C**
**August 11, 2016**


Umbrellas & Hats

The following picture shows the costs of two combinations of umbrellas and hats:




1. Without calculating the price of each, determine whether the cap or the umbrella is more expensive. What is the difference in price between the cap and the umbrella?

   Approaching the problem as if we were on a shopping trip, the members of group six were eager to solve this problem without a system of linear equations. We began by stating that, with our combined funds of $80.00, we could either get two umbrellas and a hat, or we could get two hats and an umbrella and still have money left to take the bus home. Thinking logically, we can derive that the umbrellas are marginally more expensive than hats. Because the difference in price is four dollars, and there is one less of each item between the two combinations, there is a four dollar difference between the hat and the umbrella.

2. Use the two pictures above to make a new combination of umbrellas and caps. Write down the cost of the combination.

   If two umbrellas and a hat is $80.00, and two hats and an umbrella is $76.00, we could remove an umbrella and a hat from each group to create two identical combinations of one hat and one umbrella. This combination costs $52.00; during this process, the price of each item was discovered, but will be revealed in part four.

3. Make a group of only caps or only umbrellas. Then find its price.

   Following the same trend from number two, group six was able to make a combination of three hats, already knowing the price of each item, we found the price of three hats to be $72.00, while the price of three umbrellas is $84.00

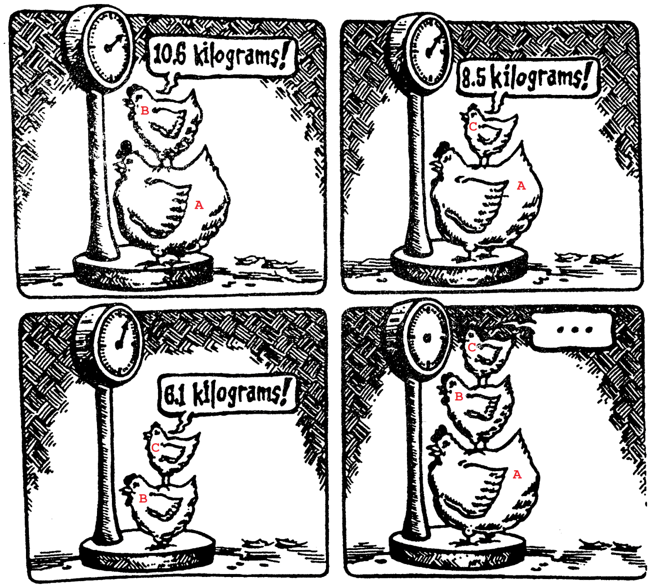
4. What is the price of one umbrella? One cap?

   The cost of an umbrella is $28.00 and a hat is $24.00.

Our group had trouble, at first, trying to think of an approach which would not include systems of equations. In reality, most students would end up doing what we did, where we established a pattern, and used skills of algorithmic thinking and doing-undoing to determine the prices of different combinations. While this trial and error style of problem solving is still a derivative of linear equations, where we replace and manipulate the problem until a solution is found. I believe students would have a much easier time with this, where they can be exposed to linear equations and how to solve for two unknowns, given a real-world scenario.

2. Chickens

   Three chickens were weighed in pairs; the first pair weighed in at $10.6kg$, the second pair weighed in at $8.5kg$, and the third pair weighed $6.1kg$.




   1. How much would the scale read if all three chickens were weighed at the same time?

| Comb. | Wt. | Diff. |
|---|---|---|
| AB | 10.6 | 2.1 |
| AC | 8.5 | |
| BC | 6.1 | 2.4 |
| ABC | 12.6 | |

      Our group began by making a table, where we can visibly see the association between each combination. We can then see that there exist two of the same entry if we add the combined weights. Where $10.6 + 8.5 + 6.1 = A + B + A + C + B + C = A + A + B + B + C + C$. Because we are able to have two of the same entry, the previous relationship is equivalent to $25.2 = 2A + 2B + 2C$; therefore, we can divide by two and get the combined weight of 12.6.

   2. How many kilograms does each chicken weigh?

      From the weight of all three, we can then subtract the weights of the combinations above it, which will reveal the weight of the missing chicken. For the first combination, $AB$, $C$ is revealed as 2. Following the pattern, $B$ is revealed as 4.1, and $A$ is revealed as 6.5.

   This exercise was quite a bit of fun for our group. By displaying the information neatly, we were able to see how we could manipulate the combinations to give us the information we needed. We believe that students would follow a similar path, developing their skills using symbols, and taking the steps towards understanding how it would work mathematically. We, again, had trouble making sure we weren't using systems of equations, and definitely think it must be difficult for teachers to think like thier students!

3. Around the Horn

Many families in the eastern United States migrated to California in the 1800s. Insdtead of going overland to reach California, some families migrated west by taking a ship that went around Cape Horn at the tip of South America. Suppose a ship leaves New York for San Francisco on the first of every month at noon, and at the same time, a ship leaves San Francisco for New York. Suppose also that each shup arrives exactly 6 months after it leaves. If you were on a ship leaving from New York, how many ships from Sand Francisco would you meet?

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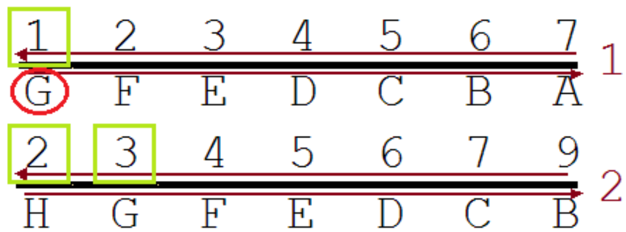


Our group began by making a drawing, which kept track of the ships in the San Francisco-bound ship's path. Similar to the image from the textbook, the ship, circled in red above, passes the inital ship, plus two additional ships along its path. Because the inital ship, for every month in its travels, has already been seen in the later months, we can create an equation, $S = 1 + 2n$, $0 \leq n \leq 6$, where $S$ is equal to the number of ships seen, and $n$ is equal to the number of months on the sea. Using this equation, or by following the image's pattern, for $n = 6$, the number of other boats passed is 13.

Group 6 had a lot of fun taking this problem in the two directions we did. We were able to quickly use an algorithmic equation and our skills of doing and undoing from the graphic that was generated. We were also able to use the symbols to follow the pattern until the sixth month, and were able to visually see the ships that were passed. We believe that students at all levels could have fun with this problem, as well as learn new tricks in a group.

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4. Mammals Brain Activity

In this activity, youll study the brain sizes of 62 mammals. Youll use Fathom to find the mammals with the largest and smallest brains. Youll see that your answers depend on how you define largest and smallest.

Q4 Brain weight of a human: 1320g.

Q5 Body weight of a rat: .28kg.

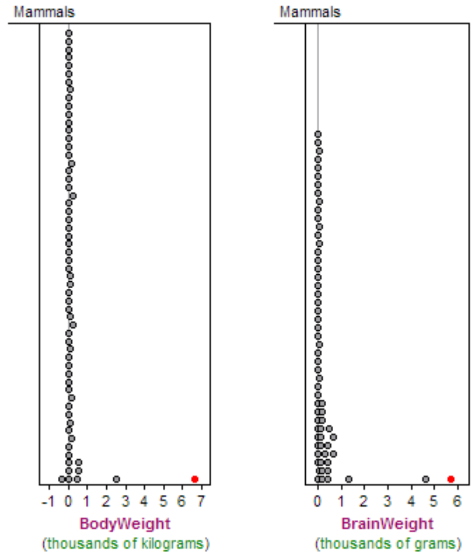
Q6 Highest brain weight: African Elephant: 5712g

Lowest brain weight: Lesser short-tailed shrew: .14g

Q7 List of African Elephant, mountain beaver, human, and rat, in order of largest to smallest brain weight:

| Species | BodyWe... | BrainWe... | Nondre... | Dreami... | TotalSle... | LifeSpan | Gestation | Predation | Exposure | Danger |
|---|---|---|---|---|---|---|---|---|---|---|
| African e... | 6654 kg | 5712 g | | | 3.3 h | 38.6 yr | 645 d | 3 | 5 | 3 |
| Human | 62 kg | 1320 g | 6.1 h | 1.9 h | 8 h | 100 yr | 267 d | 1 | 1 | 1 |
| Mountain... | 1.35 kg | 8.1 g | 8.4 h | 2.8 h | 11.2 h | | 45 d | 3 | 1 | 3 |
| Rat | 0.28 kg | 1.9 g | 10.6 h | 2.6 h | 13.2 h | 4.7 yr | 21 d | 3 | 1 | 3 |
| | | | | | | | | | | |

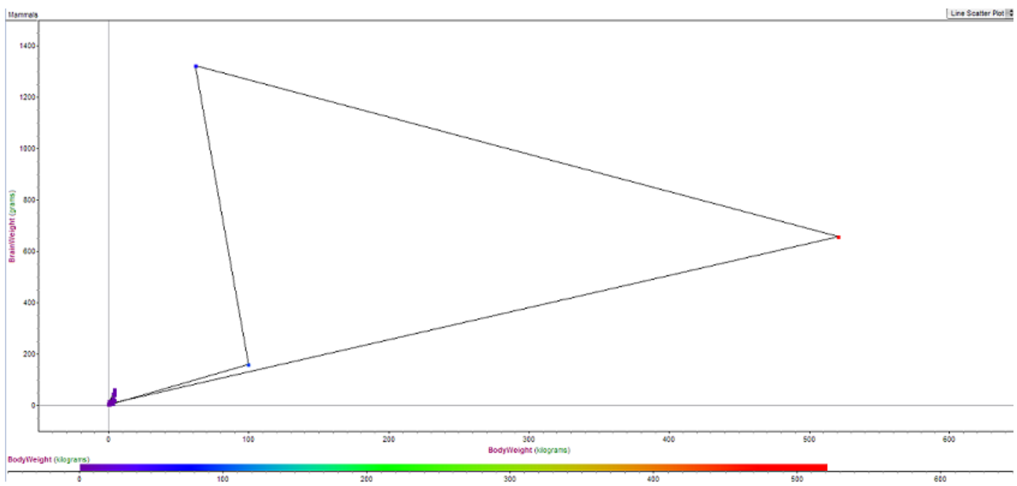
Q8 This is the same order for body and brain weights.




| Attribute | Value |
|---|---|
| Species | African elephant |
| BodyWe... | 6654 kg |
| BrainWe... | 5712 g |
| Nondre... | |
| Dreami... | |
| TotalSle... | 3.3 h |
| LifeSpan | 38.6 yr |

Q10 By using the CTRL button and clicking on the large stack of low-weight brain entries, I was able to soom into .14 grams and isolate the smallest value, being the Lesser short-tailed shrew.

Q11 Based on the following portion of the graph, we can conclude that there is not a direct, mutually exclusive formula for the relationship of all the mammals. The Human's brain to body ratio is not positive, like the others selected.



| Species | BodyWe... | BrainWe... | Nondre... | Dreami... | TotalSle... | LifeSpan | Gestation | Predation | Exposure | Danger |
|---|---|---|---|---|---|---|---|---|---|---|
| Human | 62 kg | 1320 g | 6.1 h | 1.9 h | 8 h | 100 yr | 267 d | 1 | 1 | 1 |
| Jaguar | 100 kg | 157 g | | | 10.8 h | 22.4 yr | 100 d | 1 | 1 | 1 |
| Horse | 521 kg | 655 g | 2.1 h | 0.8 h | 2.9 h | 46 yr | 336 d | 5 | 5 | 5 |

Q12 By removing the other columns, there are 57 cases between 500g and 1000g.

Q15 The ground squirrel has the highest brain to body ratio, while the african elephant has the lowest.

Q17 The proportion of brain to body is generally positively linear through the data set.

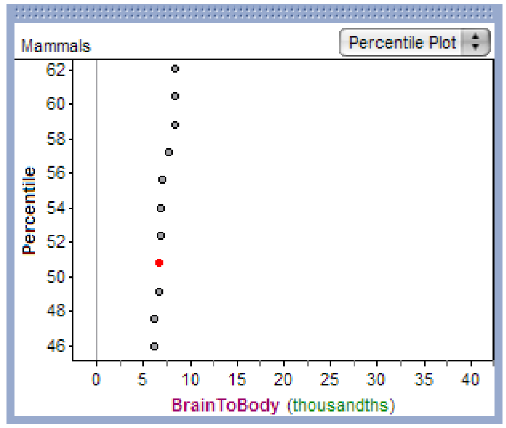
Q18 More mammals have a ratio below .01.

Q19 Using the program, the mean is .00962421.

Mammals

| | |
|---|---|
| BrainToBody | 0.00962421 |
| | 0.000858431 |
| | 0.00308571 |
| | 0.00661086 |
| | 0.0138425 |
| | 0.039604 |
| | 62 |
| | 0.00891504 |
| | 0.00113221 |
| | 0 |
| | 0.0107568 |

S1 = mean ( )
S2 = min ( )
S3 = Q1 ( )
S4 = median ( )
S5 = Q3 ( )
S6 = max ( )
S7 = count ( )
S8 = stdDev ( )
S9 = stdError ( )
S10 = count ( missing ( ) )
S11 = iqr ( )




Q20

Q21 By using the functions on the program, our group was able to find a percientile graph which displayed a value of .0066 to be in the 50th percentile, which makes it the most likely to be the 'typical' ratio.

This exercise was quite a lot of fun for our group. We were able to work through the program and use its features, while trying to find more exciting or 'better' solutions to the problems. This exercise uses large data to help the user understand how to interpret and create graphs, understand statistics and how they can be used to estimeate equations and relationships, as well as a representation of how these relationships can be used to understand more about the data than immediately present. Students would have a blast using this program, especially with the technical details displayed so easily for the user. Our group looks forward to more exercises like this.

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# Fostering Algebraic Reasoning

## Week 6 – Group 6
## Samuel L. Peoples, CST, MA-C
## August 4, 2016

4.2 Skyscraper Problem

> You manage the Milwaukee Skyscraper building. This building is 12 stories high and is covered entirely by windows on all four sides. Once a year, all the windows are washed. The cost for washing the windows is \$2.00 for each first-floor window, \$2.50 for each second-floor window, \$3.00 for each third-floor window, and so on. you have budgeted \$2,500.00 for window washing for the next year. Will this be enough to wash all of the windows? By how much are you off (+ or -)?
> (A picture of the building shows that it has 38 windows per floor.)

This exercise was immediately a favorite within the newly formed Group 6. We began by thinking algorithmically in attempt to find a function which would assist us in determining the cost of washing all the windows.

Knowing that there are thirty-eight windows per floor, and that there are twelve floors, we can begin the iteration at zero to form:

$$T = \sum_{F=0}^{11} [38 \cdot (2.00 + .50F)] = 2,166.00 \tag{1}$$

Where $T$ is the total cost, and $F$ is the current floor. By numbering from zero to eleven, the sum is easily found. Using this derivation, we can compute a general progression formula, which would be a possible solution for students studying algebra the first time to find.

By calculating the first five (0-4) iterations, we discover the set $\{95, 114, 133, 152, 171\}$, and notice a constant difference $d$ of 19.

Using a modified (variable names) definition of an arithmetic sequence:

$$T_F = T_1 + (F - 1) \cdot d \tag{2}$$

We could then calculate the cost of each floor, with the addition of the zeroth floor, calculated just as $38 \cdot 2$. The students would then be able to find the same value of 2,166.00.

Noticing that the original budget was \$2,500.00, the difference would be an excess of \$334.00.

This exercise was a fun activity for ourselves and for how we think the students would approach the problem. By thinking about how the problem could be applied to later work, the solutions were found with an open end, just in case the problem were to become much harder at a later time. By applying skills of algorithmic thinking and doing-undoing, the students would be able to successfully complete this task.

### 5.2 Bee Genealogy

Male bees hatch from unfertilized eggs and so have a mother but no father. Female bees hatch from fertilized eggs. How many ancestors does a male bee have in the twelfth generation back? How many of these are males? Generalize to any generation back.

Because male bees hatch from only one parent (the female), and female bees hatch from two parents (need two to fertilize). The pattern of the first twelve entries in the sequence are $\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144\}$. This means that there are 144 ancestors to the current male bee. Observing that there is a pattern after the first two generations, the same pattern is evident beginning with the third entry, thus, there would be 56 males.

Evidently, this pattern follows the Fibonacci Sequence, where:

$$F_n = F_{n-1} + F_{n-2} \tag{3}$$

For any generation, we can follow this sequence arithmetically for some time before needing a calculator to keep track of the numbers.

Truly interested students, or an overly eager teacher might introduce Binet's Formula:

$$F_n = \frac{\left(\frac{\sqrt{5}+1}{2}\right)^n - \frac{(-1)^n}{\left(\frac{\sqrt{5}+1}{2}\right)^n}}{\sqrt{5}} \tag{4}$$

The students would then be able to calculate an estimation on how many ancestors the bees had. For example, this equation would yield 143.99. . . 8 for $n = 12$.

Students would use their doing-undoing skills to work backwards to find the males in the sequence they discovered. Whether they recognized the pattern early, or brute forced the answer, our group observed that no matter which way the problem was approached, it was equally challenging and engaging.

Our group was interested in this activity because it was very open ended and did a great job of directing the students towards a larger goal with abstraction.

# Fostering Algebraic Reasoning

## Week 3 – Group 5

## Samuel L. Peoples, CST, MA-C

## July 10, 2016

1. Something Nu

   Consider the operation of counting the factors of a whole number. This function is usually called $v$ (the lowercase letter for *nu*). For example, the number 6 has the factors 1, 2, 3, and 6, so $v(6) = 4$. Here's some practice:

   a. If the input to $v$ is 5, what is the output? What if the input is 12?

   $v(5) = 2$, because the factors of 5 are 1, and 5.

   $v(12) = 6$, because the factors of 12 are 1, 2, 3, 4, 6, 12.

   b. What is $v(24)$? $v(288)$? $v(2^3 \cdot 3^2 \cdot 5^4)$?

   $v(24) = v(2^3 \cdot 3^1) = (3 + 1) \cdot (1 + 1) = 8$

   $v(288) = v(2^5 \cdot 3^2) = (5 + 1) \cdot (2 + 1) = 18$

   $v(2^3 \cdot 3^2 \cdot 5^4) = (3 + 1) \cdot (2 + 1) \cdot (4 + 1) = 60$

   c. Find some numbers that $v$ takes to 6.

   $v(x) = 6 = (5 + 1) = (2 + 1) \cdot (1 + 1)$

   This means that any prime number (Other than 1) raised to the fifth, or the product any two prime numbers, one of which has the exponent of 1 and the other with the exponent 2. A short set would include $\{12, 32, 45, \ldots\}$

   Generally: $\{n \mid m^5 = n, \text{ or } l^2 \cdot k = n; \text{ and } n, m, l, k \in \mathbb{Z}, \text{ and } m, l, k \text{ are prime}\}$

   d. Classify all numbers $n$ so that $v(n) = 3$. Classify all numbers $n$ so that $v(n) = 2$.

   Similarly as above. All $n$ such that $v(n) = 3$ is the set:
   $\{n \mid m^2 = n, \text{ and } n, m \in \mathbb{Z}, \text{ and } m \text{ is prime}\}$

   All squared primes will yield a *nu* of 3.

   e. What can you say about a number $m$ if $v(m) = 12$?

   Similarly as above. All $n$ such that $v(n) = 12$ is the set:
   $\{n \mid m^2 \cdot l \cdot k = n, \text{ or } m^2 \cdot l^2 = n; \text{ and } n, m, l, k \in \mathbb{Z}, \text{ and } m, k, \text{ and } l \text{ are prime}\}$

   The product of any squared prime by two other primes, or any squared prime with a cubed prime will yield a *nu* of 12.

   f. Find two numbers $n$ and $m$ so that $v(n \cdot m) = v(n) \cdot v(m)$. Find two more. Compare with what other people have found.

   A good way to approach this problem would be to begin by setting the two equations equal to a certain value.

   $v(n \cdot m) = v(n) \cdot v(m) = 1$

   This yields $n = m = 1$.

   $v(n \cdot m) = v(n) \cdot v(m) = 2$

This yields $n = \{n \mid n \text{ is prime, and} n \in \mathbb{Z}, n \neq 1\}$, $m = 1$

$v(n \cdot m) = v(n) \cdot v(m) = 5$

The larger the value becomes, the harder the problem becomes to solve. For the value of 5 and other larger values, a relationship can be determined between $n$ and $m$, where they contain no common factors except 1.

Once this pattern is recognized, students are capable of finding values for $n$ and $m$ which satisfy the equation $v(n \cdot m) = v(n) \cdot v(m)$

This problem was interesting and followed an iterative process to force students to analyze the different aspects of the problem. By applying the ideas of *doing and undoing*, the students can apply their knowledge of the *nu* function from the previous chapter to solve problems of the form $v(n) = ?$. Working with group 5, we were able to identify angles in which to tackle the problem, while working together to determine the best way to present the answers.

When dealing with *abstraction*, the students must think in terms of variables and algorithms to create rules in order to solve problems with larger values for their answer. For *nu* values of small numbers, the student is able to brute force the answer by finding all the prime factorizations by hand, but as the values grow, the students will realize that there must be a better way to approach the problem with more efficiency.

2. Difference of Sqaures

Which numbers can be expressed as the difference of two perfect squares?

As the book suggests, key skills in *Building Rules to Represent Functions*, *Abstraction*, and $Doing - Undoing$ are necessary to determine the answer to this question.

Observe that the difference between two consecutive squares is $2n + 1$, where $n \in \mathbb{Z}$, as displayed in previous chapters. This can be explored to determine odd numbers which are the difference of two squares.

Following the same logic, if we were to add two, rather than one, we would be able to determine the difference between two even squares. This can be reduced from $(n + 2)^2 - (n)^2 = 4n + 4$ to $4(n + 1)$. Thus, for a given integer $n$, there are two even integers which differ by two which are the difference between even squares that are also divisible by 4.

When conducting this exercise, students should be able to identify patterns and solutions from a few small values, then create rules around these patterns. By approaching the answers and rules in the separation between odds and evens, the students would be able to classify all solutions in the problem.

With the members of group 5, we were able to think back to previous exercises and apply the same logic from those problems to the one at-hand. Through the skills noted above, we were able to break the problem into pieces and attack it with algorithmic thinking, rather than brute-force.

3. Dog Days

In general, dogs do not live as long as humans. People often estimate that humans live about seven times as long as dogs. This means that you can estimate that for every "human year," a dog goes through 7 "dog years."

Liza got a dog just before last New Year's Day. On New Year's Day, Liza and her family celebrated the dog's first birthday, in dog years. How old, in dog yeras will the dog be next New Year's Day? Will the dog ever turn 51 in dog years on New Year's Day? Given an age, how can you tell whether Liza's dog will celebrate that birthday on New Year's Day?

> For this problem, students should be able to recognize that on New Year's Day of the first year of owning the Dog, the Dog is 1 Dog Year old.
>
> For the second New Year's Day that the family owns the dog, the dog will be 8.
>
> Following this pattern, for each New Year's Day, the dog will be $7n + 1$ dog years old, where $n$ is the integer value of the number of calendar years the family has owned the dog, on New Year's Day.
>
> To determine whether the dog will turn 51 on New Year's Day, the students would solve the equation $51 = 7n + 1$ for $n$ and find that there is no integer value for $n$ such that $50 = 7n$. Thus, the dog will never turn 51 on New Year's Day.
>
> For any integer age $m$, the students would be able to find the ages in which the dog will celebrate their birthday on New Year's Day. The students would apply the discovered equation to determine the solution.

By applying the skills of $Doing - Undoing$, *Rules* and *Abstraction*, students would be able to follow the same logic that the members of Group 5 did. We were able to determine the rule based on the word problem, apply the abstract ideas around it, and find soultions for the forward and reverse of the recursive function.

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In these exercises, the members of Group 5 frequently referenced work we had previously done and adjusted the values and rules to fit the problem at hand. Given a new problem that follows similar patterns, students should be able to reference their previous work and draw conclusions or extrapolate solutions which fit the new parameters.

The members of group 5 applied $Building\ Rules\ to\ Represent\ Functions$, *Abstraction*, and $Doing - Undoing$ to determine possible solutions to the problems, build rules, and test different situations in which they would not apply.

Asking questions such as "How can I describe the steps without using specific inputs?"; "What process reverses the one I used?"; and "How can I apply this $always$?"; help guide the students towards their answer and how to present it clearly.

As the course progresses, our group believes it will be crucial to be able to reference our own work and notes to determine simple and concise solutions, which can be related to students learning these subjects for the first time, which indicates the importance of building strong organization skills and the necessary abilities to represent abstract problems.

Talent Show Exercise

Given the problem, we were tasked with discovering an optimized profit based on a set of variables. With knowledge of excel, our group was able to expand the cells in columns H, I, J, and L to see how the different variables acted on the Cost and Revenue respectively.

| | |
|---|---|
| dj | IF(decision=”yes”,200,0) |
| times | IF(decision=”yes”,50,100) |
| maxprice | IF(decision=”yes”,16,8) |
| peo1 | INT(800-timesticket*ticketprice) |
| peo2 | IF(ticketprice,peo1,0) |
| peo3 | IF(peo2>800,800,peo2) |
| peo4 | IF(AND(ads>=2,poster+ads>=12),INT(1.05*peo3),peo3) |
| peo | IF(peo4>800,800,peo4) |
| revticket | INT(peo*ticketprice) |
| sodademand1 | IF(sodaprice<1.2,peo*(1.2-sodaprice),0) |
| sodademand | IF(sodademand1,sodaorder,INT(sodademand1)) |
| revsoda | INT(sodademand*sodaprice) |
| candydemand1 | IF(candyprice<=1.2,peo*(1.2-candyprice),0) |
| candydemand | IF(candydemand1,candyorder,INT(candydemand1)) |
| rev candy | INT(candyprice*candydemand) |
| postcost | poster*2 |
| adcost | ads*5 |
| sodacost | INT(peo*0.15) |
| candycost | INT(+candyorder*0.2) |

The table above displays the relationship between the different variables in the given problem:

| | |
|---|---|
| MC For the show? Cost: $200 | |
| Enter yes or no in box | |
| What price will you charge for tickets? | |
| How many ads @ $5 each? | |
| How many posters @ $2 each | |
| Number of sodas ordered @ 15c each | |
| How much will you charge for each soda? | |
| Number of candy bars ordered @ 20c each | |
| How much will you charge for each candy bar? | |

Noticing that the maximum price allowable with or without a DJ is 16 or 8, we needed to determine whether we would get more profit from the employment of the DJ or not. Observing that the value for peo1 is multiplied by 50 or 100, based on the decision to employ a DJ, an optimal profit can be achieved through committing the value for the DJ to “Yes”. This is because the profit from the tickets is greater with the DJ, and the $200 cost is easily offset by the profit from ticket sales.

From this point we were able to determine that the most optimal cost of the tickets would be $8.00.

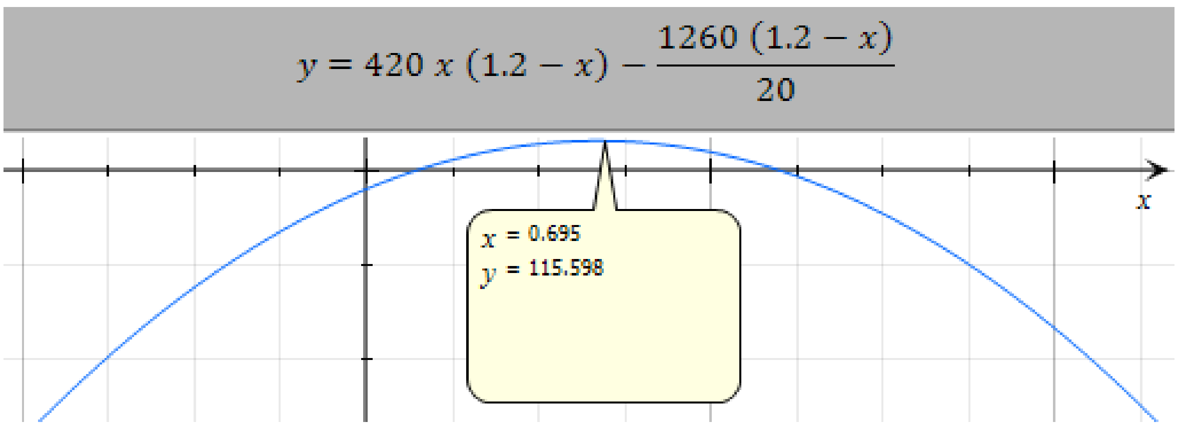
We then noticed that the function which acts on peo4 restricts the value for ads and posters to:

| | |
|---|---|
| peo4 | IF(AND(ads>=2,poster+ads>=12),INT(1.05*peo3),peo3) |

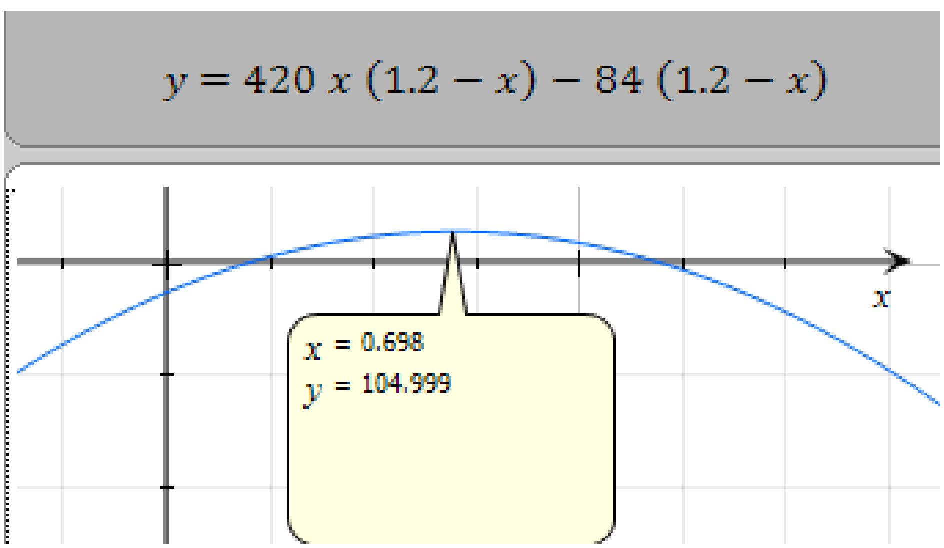
This allows us to set the value of ads to 2, and the value of posters to 10. Which is the value for the variables which yield the greatest profit.

The members of group 5 were also able to notice that the candy price and soda price were capped at $1.20, and that the value of peo·(1.2-price) determines the demand for the two variables.

In this exercise, the value for peo was 420, and we were able to create a function which displayed the optimal value for the number of sodas to order at $.15. The function is structured as: $y = 420(1.2 - x) \cdot x - .15 \cdot 420(1.2 - x)$. The graph below displays that the optimal cost will be .7 which yields a demand of 210 and a maximum profit of 116.




The same was done with the cost of candy at $.20 by changing the value in the equation.



Based on the findings we discovered, we were able to maximize the profit at $2,975.00. The figure below displays the values we chose.

**Now you make decisions:**

Do you want to hire a local radio personality as an MC for the show? It will cost $200 if you do.

| | | | | | |
|---|---|---|---|---|---|
| | Enter yes or no in box | | | | yes |
| What price will you charge for tickets? | | | | | $8.00 |
| How many ads @ $5 each? | | | | | 2 |
| How many posters @ $2 each | | | | | 10 |
| Number of sodas ordered @ 15¢ each | | | | | 210 |
| How much will you charge for each soda? | | | | | $0.70 |
| Number of candy bars ordered @ 20¢ each | | | | | 210 |
| How much will you charge for each candy bar? | | | | | $0.70 |
| | | | | | |
| | The show is over. Here is the treasurer's report: | | | | |
| | | | Attendanc | 420 | |
| | | Revenue | | | Costs |
| | Tickets | $3,360 | | Tickets | $16 |
| | | | | Programs | $200 |
| | | | | Posters | $20 |
| | | | | Ads | $10 |
| | | | | Chaperones | $160 |
| | Sodas | $147 | | Soda | $31 |
| | Candy | $147 | | Candy | $42 |
| | | | | DJ | $200 |
| | Total | $3,654 | | Total | $679 |
| | | | | | |
| | | Profit = Revenue - Cos | | $2,975 | |

Our group used the functions embedded in the excel document to find a reasonable optimization of the relationships between the variables listed. We were then able to make an argument for what values yield the highest profit.

Through trial and error, rule creation, and doing-undoing, we were able to build a model around this problem. Students tasked with the same problem would try to see how each variable changes the final profit and build intuitive rules around the behavior of the output.

Without knowledge of excel and how the functions in the cell actually act, the students would still be able to estimate the action each variable takes and then make minor adjustments to each variable down the column to find what scenarios are 'headed in the right direction'.

No matter which method the student chooses, this problem is a daunting and intimidating task which takes time and critical thinking. By introducing technology to the students' curriculum, it allows them to learn more by seeing the immediate outcome of large problems, while removing a lot of the unnecessary guesswork and errors that are associated with handwritten work. Our group looks forward to more problems such as these!

# Fostering Algebraic Reasoning


## Week 2 – Group 5
## Samuel L. Peoples, CST, MA-C


## July 7, 2016


1. A Crawling Snail

   A snail crawling up a pole 5 feet in length went up 4 feet each day and slid back 3 feet each night. How long would the snail take to go to the top? What if the pole were 7 feet in length? 28 feet? $m$ feet?

| Day | Start | Dist.↑ ($k$) - High pt. | Dist.↓ ($l$) | End pt. | Was ($m$) met? |
|---|---|---|---|---|---|
| 1 | 0 | 4 - 4 | 3 | 1 | m=5 - NO<br>m=7 - NO<br>m=28 - NO |
| 2 | 1 | 4 - 5 | 3 | 2 | m=5 - YES<br>m=7 - NO<br>m=28 - NO |
| 3 | 2 | 4 - 6 | 3 | 3 | m=5 - YES<br>m=7 - NO<br>m=28 - NO |
| 4 | 3 | 4 - 7 | 3 | 4 | m=5 - YES<br>m=7 - YES<br>m=28 - NO |
| ... | | | | | |
| 25 | 24 | 4 - 28 | 3 | 25 | m=5 - YES<br>m=7 - YES<br>m=28 - YES |

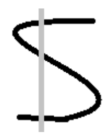
   In constructing the table above, it can be seen that for a snail that travels $k$=4 up and $l$=3 down each day, for any height $m$, it takes $m$-3 days to reach the top of the pole.

   For similar problems, rules can be made by following the iterative steps and determining where to create 'checks' to determine if the conditions have been met. By constructing algorithms in this manner, students can build the iterative and algebraic skills necessary to solve real world problems. By providing these real world problems to the students, a sense of accomplishment can be observed, while learning and understanding can be made simpler.

2. Cutting Through the Layers

   Imagine a single piece of string which can be bent back a forth. In the picture below, the string is bent so that it has three "layers."

   By cutting down the gray line, the result is four pieces of string.



   By making two cuts, as shown below, the result is seven pieces of string.

   Suppose that the number of layers is $l$, the number of cuts is $c$, and the number of pieces of string is $p$. Find an equation to determine the $p$, based on arbitrary values of $l$ and $c$.

| Inputs | | Output |
|---|---|---|
| Number of Layers($l$) | Number of Cuts ($c$) | Number of Pieces ($p$) |
| 3 | 0 | 1 |
| 3 | 1 | 4 |
| 3 | 2 | 7 |
| 4 | 3 | 13 |
| 4 | 4 | 17 |
| 5 | 4 | 21 |
| 5 | 5 | 26 |

   By populating the table above, a pattern can be observed, where $p = c \cdot l + 1$. Observe that for all values of $c$ and $l$ in the table above, one could draw or construct situations in which the value of $p$ matches the table's outputs.

   Because the pieces are all connected, except for the ends, the number of pieces is $c \cdot l + 1$. If the ends were connected, the number of pieces would equal $c \cdot l$.

   By using symbols, tables, and algebraic thinking, I was able to derive an equation which solves the problem. I believe that activities such as these are beneficial to a student's learning and could be altered to achieve different results based on the problem the teacher is trying to solve.

3. Age Problem

   a. Classify all numbers that leave a remainder of 3 when divided by 5 *and* a remainder of 1 when divided by 3.

      Observe that the situation described above is dependent upon a set defined as $\{n|n(\text{mod } 5) = 3 \text{ and } n(\text{mod } 3) = 1\}$

      The set defined by $\{n|n(\text{mod } 5) = 3\} \Leftrightarrow \{3, 8, 13, \ldots, \text{5n-2} \mid n \geq 1, n \in \mathbb{Z}\}$

      The set defined by $\{n|n(\text{mod } 3) = 1\} \Leftrightarrow \{1, 4, 7, \ldots, \text{3n-2} \mid n \geq 1, n \in \mathbb{Z}\}$

      Thus, the set $\{n|n(\text{mod } 5) = 3 \text{ and } n(\text{mod } 3) = 1\}$
      $\Leftrightarrow \{13, 28, 43, 58, 73, \ldots, \text{n}(5 \cdot 3) - 2 \mid n \geq 1, n \in \mathbb{Z}\}$

   b. If my age is divided by 3, the remainder is 2. If my age is divided by 5, the remainder is also 2. If my age is divided by 7, the remainder is 5. How old am I?

      Similaly as part $a$, the sets below list the values for each of the steps in this problem.

      $n(\text{mod } 3) = 2 \Rightarrow \{2, 5, 8, \ldots, \text{3n-1} \mid n \geq 1, n \in \mathbb{Z}\}$

      $n(\text{mod } 5) = 2 \Rightarrow \{2, 7, 12, \ldots, \text{5n-2} \mid n \geq 1, n \in \mathbb{Z}\}$

      $n(\text{mod } 7) = 5 \Rightarrow \{5, 12, 19, \ldots, \text{7n-2} \mid n \geq 1, n \in \mathbb{Z}\}$

      A pattern can be found between the first two sets, where the sets follow the pattern 15n+2.

      The pattern found when relating values which satisfy both the patterns 7n-2, and 15n+2 is a pattern of 105n+47.

      Thus, the only viable age is 47, where $n = 0$.

   In solving this problem, one must look at the different aspects individually, then find patterns which relate the solutions to each other. In order to avoid $brute-forcing$ the answer, patterns and variables must be implemented.

   The specific methods that were used included knowledge of the modulo theorem, where $a = b(\text{mod } n)$, properties of systems of equations, and algorithmic thinking. Students following the same pattern would create a list of values to find a pattern for each piece of the problem, then find how their answers relate to the problem at large. After finding a pattern, the solution presents itself.

4. Sneaking Up the Line

Eric the Sheep is at the end of a line of 50 sheep waiting to be shorn. But being an impatient sort of sheep, Eric sneaks up the line two places svery time the shearer takes a sheep from the front to be shorn. So, for example, while the first sheep is being shorn, eric moves ahead so that the are two sheep behind him in line. If at some point it is possible for Eric to move only one place, he does that instead of moving ahead two places.

a. How many sheep are shorn before Eric?

| Number of Sheep Shorn | Eric's Position |
|---|---|
| 0 | 50 |
| 1 | 47 |
| 2 | 44 |
| 3 | 41 |

Based on the pattern displayed in the table above, there are 17 sheep shorn before Eric is in the 1st position.

b. What pattern did you notice that could help you solve the original problem in which Eric is at the end of a line of 50 sheep?

The pattern followed can be attributed to the equation $s = \frac{n-p}{3}$, where $s$ is the number of sheep shorn, $n$ is the starting position of Eric, and $p$ is the number he jumps each time. After finding the number of sheep shorn until Eric reaches a position in which he cannot jump $p$ positions anymore, the student can then iterate as they see fit, being $p-1, p-2, \ldots, p-(p-1)$, depending on how the problem is presented.

A simpler approach would be to take the number of sheep ahead of Eric and divide it by $p+1$, then round to the next highest integer if the result of the division is a fraction or decimal. $s \approx \frac{n}{p+1}$

c. How would you determine how many were shorn before Eric based on $n$ number of sheep?

The answer to this question is described in part *b*.

d. Complete the table below.

| Number in front of Eric | 37 | 296 | 1000 | 37, 38, 39 | 61, 62, 63 | 7695 |
|---|---|---|---|---|---|---|
| Number shorn before Eric | 13 | 99 | 334 | 13 | 21 | 2565 |

e. Eric gets more (and more) impatient! Explore how your rule changes if Eric sneaks past 3, 4, or even 10 at a time.

As described above, the value of $p$ changes for each change to 3, 4, or 10. This would decrease $s$, as shown in the table below, where Eric begins at position 50.

| | | | | |
|---|---|---|---|---|
| n | 50 | 50 | 50 | 50 |
| p | 2 | 3 | 4 | 10 |
| s | 17 | 13 | 10 | 5 |

Similarly, a completely populated table is shown on the next page. In order to conserve space, redundant data has been omitted.

| Number of Sheep in Front of Eric | Number Shorn before Eric | | | |
|---|---|---|---|---|
| | p=2 | p=3 | p=4 | p=10 |
| 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 |
| 6 | 2 | 2 | 2 | 1 |
| 9 | 3 | 3 | 2 | 1 |
| 12 | 4 | 3 | 3 | 2 |
| 15 | 5 | 4 | 3 | 2 |
| 18 | 6 | 5 | 4 | 2 |
| 21 | 7 | 6 | 5 | 2 |
| 24 | 8 | 6 | 5 | 3 |
| 27 | 9 | 7 | 6 | 3 |
| 30 | 10 | 8 | 6 | 3 |
| 33 | 11 | 9 | 7 | 3 |
| 36 | 12 | 9 | 8 | 4 |
| 39 | 13 | 10 | 8 | 4 |
| 42 | 14 | 11 | 9 | 4 |
| 45 | 15 | 12 | 9 | 5 |
| 48 | 16 | 12 | 10 | 5 |
| 50 | 17 | 13 | 10 | 5 |

Solving this problem was not only engaging, but also difficult at times. Students must not only implement algebraic skills and pattern manipulation, but they must also test different possibile patterns and be comfortable with making mistakes. I believe that this exercise would not only help build the algebraic skills necessary to move forward with higher-level thinking, but also help them become accustomed to failure, and trial and error methods of problem solving.

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In working with other members of group 5, we were able to tackle the problems in small pieces and try to build solutions around said pieces. A frequent topic which we all thought about was $doing - undoing$ where we thought about the problem from the back-end and tried to find the question from an assumed solution. If it worked – Eureka! If not, we continued crunching numbers.

Each of the assigned problems in this week's assignment involved patterns and rules which defined functions or sets to solve a problem. In approaching the problems in different ways, we were able to build our own skills and also test the mentioned methods ourselves.

The assigned problems asked our group to think about a specific situation and to solve a problem based on those parameters. Following the solution, we were then asked to broaden our scope and view it abstractly. With $Abstraction\ from\ Computation$, we were able to assign variables to the problems of growing complexity, and approach the problem from values greater than our hands could manage with brute force.

I believe the skills we are building in our presentation of the problems, as well as seeing how each member is reasoning their solutions, is aiding in our growth and understanding of Algebra and how it is taught.

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