A prototype trade model with imports in two stages

# Purpose

To build a didactic trade model with a small number of goods and regions, focusing on the import behaviour. The initial model shall have two Armington style CES nests aggregating imports of various origins to a single import flow and then imports and domestic origins to total consumption. The model shall be useful for developing a revised model that allows the creation of new trade flows that none existed in the calibration point.

# The standard CES model

This chapter defines a standard economic trade model, focussing on details in the import behaviour and all but ignoring detail elsewhere. Therefore, it contains two levels of CES-nests for import demand, but rather simple supply functions. Consumer prices are linked to producer prices by fixed absolute margins, and trade costs are fixed per unit. We describe demand theory first, then turn to the rest of the model with supply functions, price linkages and market balances. Finally, we derive conditions for calibrating the various parameters to a given baseline situation.

## Theory of demand with two-stage CES for imports

We assume that for each good[[1]](#footnote-1), the single consumer in region acts as if minimizing the expenditure needed to reach a given level of utility . The good can be of either domestic origin, , or a bundle of imported commodities. Since we subsequently want to define another decision level defining the composition of the import bundle, we represent the import bundle by the utility level and the associated price index of imports . The expenditure minimization problem then becomes:

subject to

If we write down the first-order conditions for an optimal solution w.r.t. and , and divide the former by the latter, we obtain the familiar expression defining the ratio of the quantities as a constant elasticity function of the ratio of the associated prices , the share parameter and the parameter , which is the are constant elasticity of substitution:

(1)

The optimal solution w.r.t. the dual value of utility returns the primal constraint, which is the CES aggregator:

(2)

The import bundle is in turn a utility aggregate of the various import flows, and we assume that the consumer chooses a combination of import quantities from each of the origins , trading at prices , that minimize the expenditure needed to reach utility level , where the utility again is defined by a CES aggregator. The problem to solve is:

subject to

It can be shown that the first-order conditions for the solution to this problem can be arranged to obtain the quantity of imports from each origin as a function of the ratio of its price to a certain price index of imports, :

(3)

(4)

(5)

## Supply functions, price linkages and market balances

The quantity produced, , must equal the sum of domestic demand and exports:

(6)

The supply quantity is related to the producer price, , by a function exhibiting constant elasticity of supply:

(7)

The producer price is linked to domestic consumer prices and export prices with a fixed price mark-up . This implies that producers are indifferent to producing for the home market and the export market in this model:

(8)

(9)

The import price of each origin is linked to the export price in the region of origin via a fixed per-unit trade cost :

(10)

In order to check that the model contains the same number of variables as there are equations, it is useful to *pair* each variable with one equation that has the identical number of index positions.

Table 1: Pairing variables and equations

|  |  |  |  |
| --- | --- | --- | --- |
| Equation | Nr of instances | Variable | Nr of instances |
| (1) | *n* |  | *n* |
| (2) |  | (Not used) |  |
| (3) |  |  |  |
| (4) | *n* |  | *n* |
| (5) | *n* |  | *n* |
| (6) | *n* |  | *n* |
| (7) | *n* |  | *n* |
| (8) | *n* |  | *n* |
| (9) | *n* |  | *n* |
| (10) |  |  |  |

1. For simplicity of exposition, we assume that there is only one single good. The GAMS models allows for an additional index “comm” containing the set of commodities modelled. [↑](#footnote-ref-1)