Method

TA UPP JORDBRUKSMODELLER (TEX HEWITT) OCH JÄMFÖR VAD VÅR MODELL GÖR ANNORLUNDA

The economic model is a short term constrained optimization problem, which is based on the notion of a *fishery* (*f*)*,* which is a combination of using a particular type of vessel from a fleet *segment* (*seg*, type and size of vessel) with a particular *gear* in a particular geographical *area*. Each fishery has a fixed distribution of catch across the available *species s*, so that the observed average composition of the catch is replicated. Fishermen are assumed to choose fishing effort for each fishery as to maximize their profit including subsidies or taxes, using a given vessel fleet and a fixed fish stock (Equation 1).

Revenues stems from catch, which comes in two qualities: A and B, called sortA and sortB in the equations below. SortA is suitable for the market, whereas sortB commands a significantly lower price. If discards are allowed[[1]](#footnote-1), the agents are likely to discard all of sortB in order to fill their quotas with the more valuable sortA catch. The shares of sortA and sortB in total catch are fixed, see Equation 2 and Equation 3.

The marginal variable costs of fishing activities are assumed to increase in proportion to fishing effort, reflecting an assumption that higher efforts imply longer trips. The objective function also contains a calibration term per fishery that ensures that the observed fishing pattern is replicated by the model. Fixed costs per vessel are included in order to render the accounting complete, but have no impacts on behaviour since the number of vessels per segment is fixed.

Equation 1: Objective function

where

is fishing effort in days per fishery   
 are the prices of Sort A and Sort B respectively, for each fishery and species   
 are the (variable) quantities caught of sort A and sort B  
 Landing Obligation, parameter {0,1} defining whether catch has to be landed or not  
 Subsidy per day to fishery   
 Constant part of the marginal variable cost  
 Slope parameter of marginal variable cost function  
 The number of vessels operating in each segment *seg*  
 The fixed cost per vessel in each segment (impacting only on profits)  
 Calibration constant cost (or revenue if negative) term

Equation 2: Catch of sort A

Equation 3: Catch of sort B

Equation 4: Total catch as a function of effort

Similar to e.g. Frost et al 2009, we assume that total catch follows a Cobb-Doublas production function. Several authors, such as Kronbak (2005) and Hannesson (1983), have estimated catch-elasticities for such functions, and generally[[2]](#footnote-2) found them to imply decreasing marginal catch (. Equation 4 shows the catch function, where the (constant) stock factor is included in the factor . Thus, combines the distribution of catch across species with the scale of the total catch. Therefore, bears some relation to CPUE, Catch Per Unit of Effort, sometimes used in the literature, but is not fully equivalent since our catch is not constant.

Fishing efforts are constrained by quotas and effort restrictions. Only landed catch counts against the quotas, and hence there is a strong incentive to discard sortB if quotas are filled. The discards are currently modelled exogenously, assuming that all of sortB is discarded if there is no landing obligation () as defined in Equation 5 and Equation 6.

Equation 5: Landings

Equation 6: Discards

Quotas are defined for sets of fishing areas called *Quota Area* (*qa*) and for sets of species called *Quota Species* (*qs*). Quotas are modelled by Equation 7, where the indicator defines (= TRUE) if species *s* belongs to quota species *qs* AND fishery *f* is active in quota area *qa*. *TAC* is the quota (Total Allowable Catch). The construction with the indicator set appears in several places, hence we explain it thoroughly at this first place: The subscript reads as “*f* and *s* such that they belong to *qs* and *qa*”, i.e. for each quota species and quota area, the equation sums up the total landings of the relevant species caught by the relevant fisheries. For instance, there is a combined quota for “turbot and brill” (one quota species but two species in our model), in the area “North Sea”. Thus, the quota restriction sums up “Turbot” and “Brill” under this quota for all fisheries active in the “North Sea”. As another example, landing quotas may also differ by gear, such as for “Norwegian lobster”, where there are separate quotas for trawling and creeling. There, we create a quota species “Norwegian Lobster Creeling” caught by fishery activities using Creeling, and another quota species “Norwegian Lobster Trawling” caught by trawlers. Similarly, a landing quota may extend over several fishing areas, and as our fishing activities by definition is active in only one fishing area, the quota restriction sums up all the fisheries active in the larger quota area.

Equation 7: Catch quotas

Effort restrictions come in three types. For each *segment* there is a maximum number of annual fishing days available, based on the number of vessels (Equation 8). For each *fishery*, there is also a maximum number of days of fishing possible, based on e.g. season length and number of vessels (Equation 9). Similarly to the previous equation, the notation in the sum means “fisheries *f* that belong with segment *seg*”.

Equation 8: Effort restriction per segment

Equation 9: Effort restriction per fishery

In the effort regulation, there are various rules that apply to groups of fishing activities (sets of fisheries in our model) and sets of fishing areas. This is implemented in Equation 10, where is the installed engine class in kilowatts, *I* the indicator set defining (=1) if fishery *f* belongs to effort group *eg* and is active in effort area *ea*, and *maxEffortPerGroup* is the upper bound defined in regulation.

Equation 10: Effort regulation

# Seal damage compensation

Fishermen are eligible for economic compensation for seal damage. Compensation is paid in proportion to reported damage, but with an upper limit given by a total national budget. In the model, we assume a constant probability of seal damage for each fishing day within a given fishery, based on the reported frequency of damage. The frequency of reported damages ranges from 77% for the most strongly affected fishery for some coastal fisheries to zero for trawlers (see Figure 1).

Figure 1: Share of trips with reported seal damage (number of fisheries)

Seal damage affect fisheries in two ways: it reduces catch and increases marginal cost. As regards the catch reduction, the quantity actually lost is not observed. Following the Swedish regulation (HaV 2014), we assume that the quantity of catch that is lost due to seal damage equals the reported catch for fishing trips with reported seal damage, i.e. the reported landing is 50% of what it would have been if no seal damage had been reported. As regards the marginal cost of seal damage, we use estimated costs per day for seal damages, which is assumed constant and used to shift the marginal cost function of the fisheries with seal damage. We also need to ensure that the total budget available for compensation is not overshot. In order to do so, we sum up across fisheries, and compute the share of seal damage for each fishery in total national seal damage. This share is multiplied with the available national budget and divided by the annual fishing effort to derive the expected compensation per day for each fishery, denoted by in the objective function.

In order to ensure that the sum of the compensatory payments does not exceed the national budget, we define the subsidies in an iterative fashion. In the first iteration, we compute the subsidy per day based on historical fishing efforts. After solving the model with this subsidy per day and fishery, the total subsidy paid in the model may exceed the budget or not exhaust it. Therefore we re-compute it again using the simulated fishing efforts. This process is repeated until the change in simulated fishing efforts between two consecutive iteration is smaller than a tolerance for convergence. The iterative solution is used in order to avoid that the fishermen in the model collectively act as a monopolist towards the authorities, reducing the effort in order to have a higher subsidy per day.

# Estimation

The model is calibrated to the observed situation in 2012 (see section YY on data). Calibration implies that we assume that the observed situation is an equilibrium, which we want the model to reproduce. The calibration has to overcome that we have several parameters and variables that can be adjusted to reach the calibration point. One reason for this is that there may be errors in the observed variables such as costs and catches. Another reason might be that parameters from the literature are uncertain in themselves, or have been estimated in a different context, with e.g. different functional forms or for other vessel types, times horizons etc.

According to the terminology introduced by Buysse et al. (2007), our approach to calibration is that of Econometric Mathematical Programming, combining econometrics and mathematical programming. The calibration procedure is based on the necessary conditions for an optimal solution to the simulation model. This means that we form the Lagrangean function of (eq…) and derive the *first order conditions* (FOC) for a stationary point. The second order conditions are assumed to be satisfied due to the functional forms chosen and restrictions of certain signs.[[3]](#footnote-3) Exact calibration is obtained if the FOC are satisfied, but that is unlikely to be the case at the observed variable levels and the parameters used. Hence, we need to adjust selected variables and parameters in such a way that the FOC are satisfied, and the rest of this section aims at explaining how this is done.

We assumed that observations of variable costs and catches might contain errors, whereas the errors in observations of fishing effort and number of vessels can be neglected (due to regulatory reporting requirements and surveillance). The parameter limits the number of days a vessel can be active in a fishery (e.g. due to season length), but the existing data on this are vague.[[4]](#footnote-4) Furthermore, the parameter , is considered to be a residual error in the FOC tha cannot be observed but has to be estimated. We want to make changes that are “as small as possible”, and in order to define what is “small” for variables and parameters measured in different units, we cast the calibration problem in terms of a Bayesian estimation problem. This allows us to convert all adjustments into probability density values. We use as our estimates the variables and parameters that maximize the joint probability density, i.e. the combination of values that are “most likely” as measured against a-priori density functions while satisfying the FOC.

In general, we assumed all error terms and parameters to be normally distributed[[5]](#footnote-5). For the season-length parameter there is arguably a firm lower bound (zero) and an upper bound (365 days at most), whence a beta-density is more appropriate. Standard deviations for the a-priori distributions are not possible to obtain from available empirical sources, and we therefore had to use assumptions as reported in table T3. This means that the estimator may (for instance) reduce season length for a fishery so it becomes binding and a dual value appears in the first order condition, if this helps explain observed fishing effort better than adjustments in costs or the PMP term.

Table 1: Prior distributions for variables and parameters in the calibration

|  |  |  |  |
| --- | --- | --- | --- |
| Variable/Parameter | Modal value *m* | Standard deviation | Distribution family |
|  | Observed landings in log book | *m*/5 (high precision) | Normal |
|  | Observed discards from sample vessels | *m*/2 (low precision) | Normal |
|  | Average observed engine power per vessel in the segment | *m*/3 (medium precision) | Normal |
|  | Expert assumptions about average season length per fishery | *m*/5 (high precision) | Beta (with min = 0 and max given by expert assumption) |
|  | Observed average variable cost per day | *m*/4 (high precision) | Normal |
|  | Fixed, based on exogenous elasticity, according to appendix | (zero) | (spike) |
|  | Zero | Such that the variance is ten times that of the average cost. | Normal |

Since our model contains many inequality constraints, the necessary conditions for an optimal solution are the Karush-Kuhn-Tucker conditions, consisting of first order conditions (FOC) and complementary slackness conditions (CSC). The conditions are as follows:

Equation 11: FOC w.r.t. effort

This condition states (if the positive terms are taken first) that the marginal value of additional catch, plus subsidies, must equal the marginal cost plus PMP term plus the shadow costs of quotas, season length and effort restrictions. The various lambdas are the Lagrange multipliers associated with each restriction, whereas the is the Lagrange multiplier associated with the condition that for each fishery, effort has to be non-negative[[6]](#footnote-6).

Equation 12: FOC w.r.t. catch

means “marginal value of catch”, and thus the remaining terms mean the marginal cost of producing the catch in terms of sortA and sortB weighted with their respective shares.

Equation 13: FOC w.r.t. sortA

This condition states that the marginal revenue of selling fish as sortA equals the sum of the relevant quota rent, if any, and the dual of the sorting equation for sortA, which hence means “the marginal cost of catching sort A”.

Equation 14: FOC w.r.t. sortB

This condition implies that if there is *no landing obligation*, as in our simulations, the equation collapses to state that the dual value of the sort B constraint is zero (free disposal).

The CSC generally take the form , where the function g(z) is the relevant inequality constraint, the associated Lagrange multiplier, and the *complementarity gap* which is zero when the complementarity is satisfied and otherwise is something non-negative. The solution method described further below implies reducing the complementarity slacks until they become zero.

Equation 15: CSC for fleet capacity

Equation 16: CSC for season length

Equation 17: CSC for effort regulation

Equation 18: CSC for catch quotas

The model contains several inequality constraints, where we cannot always say a-priori if they should be binding – and hence a dual value present in the first order conditions – or if they are slack. Measurement errors in data and technicalities in accounting may cause e.g. a catch quota to appear not binding whereas it is binding in reality, and the same principle holds for other inequalities as well. The calibration method developed here includes the Karush-Kuhn-Tucker conditions (complementary slackness conditions, CSC, see Equation 15-Equation 18) so that the dual values and slacks are determined simultaneous with calibration. The presence of the CSC makes the calibration problem more difficult to solve, as it becomes a *mathematical program with equilibrium constraints* (MPEC) (Francisco Facchinei et al., 1999). The solution developed here is based on stepwise smooth approximations with a penalty function proposed by Dirkse et al. (2002), which was applied to a related estimation problem by Jansson and Heckelei (2009).

The *stepwise smooth approximations* mean that the estimation criterion function is augmented with a penalty function that penalizes the existence of complementarity gaps in the solution. The weight of the penalty function is initially very small, but iteratively increased until all complementarity gaps disappear. The estimation (calibration) criterion function including the penalty function is given by Equation 19.

The estimation criterion, to maximize, is derived from the logarithm of the product of all the prior density functions. The first five terms relate to normally distributed priors, where parameters given in Table 1, and the weights are computed as where is the standard deviation. The sixth term is the logarithm of the beta density function for after scaling it so that it falls into the interval (0,1).

Equation 19: Estimation criterion function

where the penalty is defined as the sum of complementarity gaps times a weight . In the solution of the estimation problem, the weight of the penalty is initially set to something small and stepwise increased until the complementarity gaps are zero and thus the CSC conditions satisfied and we have an optimal solution to the primal fisheries model.

# Data

Log book data for 2012

# References

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# Appendix XX. Derivation of the slope of the marginal cost function from assumed elasticities

The variable cost function has two parameters, but we only observe average variable costs for a single year. Therefore, not both parameters are simultaneously identified, unless we provide more information. We do this in the form of an assumed *elasticity* of effort to output prices, which we translate into the slope of the marginal cost function. In the literature of Positive Mathematical Programming, the use of exogenous elasticities has received some attention. In particular, Merel and Bucaram (2010) derive conditions under which exact calibration to exogenous elasticities, and show that the way we use exogenous elasticities here generally produces a model that reacts less elastic to changes than what the exogenous elasticity implies. The key reason is that the derivation only considers the slope of the marginal cost function, whereas the proper expression for the elasticity also would have to consider the interaction with the constraints (dual values influence the response of the model). Aware of this bias, we nevertheless use the approach since it produces results that are qualitatively correct, and the values of the exogenous elasticity is uncertain in itself.

The marginal cost function is assumed to be increasing in Effort. The variable cost (VC) function is

XX1

And the corresponding marginal cost function is the derivative,

XX2

Only data for average cost is known from the dataset, and thus this is used for estimating the marginal cost. Average cost is

XX3

This is our average cost per fishing day per métier from data input.

Solving (XX3) for and inserting into the expression for MC (XX2) gives us:

XX4

By using an assumption on the elasticity of Effort () to output prices combined with an assumption that MC = P (i.e. that MC is equal to the market price of fish in equilibrium) it is possible to calculate as a function of average cost and elasticity.

The elasticity η is defined as

η = XX5,

and with the assumption P = MC this becomes

η = XX6

solving for in equation XX2 and differentiating with respect to MC it is shown that

substituting this into XX6 it gives us the slope of the cost function as

XX7

Substituting XX4 for in XX7 and solving for we finally reach an expression for in terms of average cost and elasticity at the observed .

XX8

and thus the slope is a function of the known AC and , and an assumption of the elasticity η. In the estimation, η = 1.5 is used. The computed value of is held fixed in the estimation, so that in practice, the observed average cost is used to estimate the intercept so that the implied average cost (eq. XX3, included in the estimation) is “close” to the observed value as measured by the prior density function.

1. In the Common fisheries policy of the EU, discards are prohibited from the 2013 regulation (European Commission 2013), but were previously allowed. [↑](#footnote-ref-1)
2. Eide et al. (2003) found evidence of increasing returns to scale for Norwegian bottom trawlers targeting cod. Such behaviour is potentially problematic for modelling as it may render the objective function non-concave. However, it is also exceptional in the literature, which generally finds decreasing returns to scale. [↑](#footnote-ref-2)
3. The model has monotonously increasing marginal costs and decreasing catch, making the objective function concave. However, this is not sufficient, since the catch quota restriction is non-linear in fishery effort. Simulation tests indicate that our model is well behaved around the calibration point. [↑](#footnote-ref-3)
4. We also have the opportunity to calibrate the parameter in order for the model to be able to reflect the effort regulations. E.g. might the effort regulation be binding in practice, but due to administrative regulations some “effort days” are available in the system which makes the model restriction non-binding. [↑](#footnote-ref-4)
5. We have tried gamma distributed priors, with the support and modal values given by observations where available, but that does not seem to bring any clear benefits. [↑](#footnote-ref-5)
6. In the calibration, we only consider fisheries with non-zero effort, and hence is omitted from this point on. [↑](#footnote-ref-6)